# CHAPTER 14

14.1-1 Let player I be the labor union with strategy i being to decrease its wage demand by 10(i-1)¢.
Let player II be the management with strategy i being to increase its offer by 10(i-1)¢.

The payoff matrix is:

| I \ II | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| 1 | 1.35 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 |
| 2 | 1.5 | 1.35 | 1.3 | 1.4 | 1.5 | 1.6 |
| 3 | 1.4 | 1.4 | 1.35 | 1.4 | 1.5 | 1.6 |
| 4 | 1.3 | 1.3 | 1.3 | 1.35 | 1.5 | 1.6 |
| 5 | 1.2 | 1.2 | 1.2 | 1.2 | 1.35 | 1.6 |
| 6 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.35 |

14.1-2 Label the products respectively A and B. Then the strategies for each manufacturer are:

1- Normal development of both products
2- Crash development of product A
3- Crash development of product B

Let $p_{ij} = \dfrac{(\%\text{ increase to I from A}) + (\%\text{ increase to I from B})}{2}$

The payoff matrix then becomes

| I \ II | 1 | 2 | 3 | Row min |
|---|---|---|---|---|
| 1 | 8 | 10 | 10 | 8 ← max |
| 2 | 4 | -4 | 13 | -4 |
| 3 | 4 | 13 | -4 | -4 |
| Column Maximum: | 8 ↑ min | 13 | 13 | |

Hence, both manufacturers should use normal development and I will increase his share of the market by 8%.

14.1-3 Each player has the same strategy set. A strategy must specify the first chip chosen and, for each possible first choice by the opponent, choices of second and third chips. For example, a typical strategy is "Pick $i$ first. If opponent chooses W, pick $j_1$ and $k_1$. If opponent chooses R, pick $j_2$ and $k_2$. If opponent chooses B, pick $j_3$ and $k_3$" where, for $\ell = 1,2,3$ $\{i, j_\ell, k_\ell\} = \{W, R, B\}$ There are 3 choices for $i$ and, for each $i$, 8 choices of "conditional" strategies, forming 24 distinct strategies. Payoffs are determined from the table (all net payoffs are either \$120, 0 or -120, depending on whether player I wins 3 times, wins 1 time and ties 1 time or loses 3 times).

14.2-1

a) Strategies 4, 5 and 6 of player II are dominated by strategy 3.
Strategies 4, 5 and 6 of player I are dominated by strategy 3.
Strategy 1 of player II is dominated by strategy 3.
Strategy 1 of player I is dominated by strategy 3.
Strategy 2 of player II is dominated by strategy 3.
Strategy 2 of player I is dominated by strategy 3.
Therefore, the optimal strategy is for the labor union to decrease its demand by 20¢ and for management to increase its offer by 20¢.
A wage of \$1.35 will be decided.

b)

| | | II: 1 | 2 | 3 | 4 | 5 | 6 | Row Minimum |
|---|---|---|---|---|---|---|---|---|
| I | 1 | 1.35 | 1.2 | 1.3 | 1.4 | 1.5 | 1.6 | 1.2 |
| | 2 | 1.5 | 1.35 | 1.3 | 1.4 | 1.5 | 1.6 | 1.3 |
| | 3 | 1.4 | 1.4 | 1.35 | 1.4 | 1.5 | 1.6 | 1.35 ← max |
| | 4 | 1.3 | 1.3 | 1.3 | 1.35 | 1.5 | 1.6 | 1.3 |
| | 5 | 1.2 | 1.2 | 1.2 | 1.2 | 1.35 | 1.6 | 1.2 |
| | 6 | 1.1 | 1.1 | 1.1 | 1.1 | 1.1 | 1.35 | 1.1 |
| Column Maximum | | 1.5 | 1.4 | 1.35 ↑ min | 1.4 | 1.5 | 1.6 | |

14.2-2

Strategy 3 of player I is dominated by strategy 2.
Strategy 3 of player II is dominated by strategy 1.
Strategy 1 of player I is dominated by strategy 2.
Strategy 2 of player II is dominated by strategy 1.
Therefore, the optimal strategy is for player I to choose strategy 2 and player II to choose strategy 1 resulting in a payoff of 1 to player I.

14.2-3

Strategy 1 of player II is dominated by strategy 3.
Strategy 4 of player II is dominated by strategy 2.
Strategies 1 and 2 of player I are dominated by strategy 3.
Strategy 2 of player II is dominated by strategy 3.
Therefore, the optimal strategy is for player I to choose strategy 3 and player II to choose strategy 3 resulting in a payoff of 1 to player II.

14.2-4

| | | II: 1 | 2 | 3 | Row Minimum |
|---|---|---|---|---|---|
| I | 1 | 1 | -1 | 1 | -1 |
| | 2 | -2 | 0 | 3 | -2 |
| | 3 | 3 | 1 | 2 | 1 ← max |
| Column Maximum | | 3 | 1 ↑ min | 3 | |

v=1, player I uses strategy 3 and player II uses strategy 2.

The game is stable with saddlepoint (3,2)

14.2-5

| | | II: 1 | 2 | 3 | 4 | Row Minimum |
|---|---|---|---|---|---|---|
| I | 1 | 3 | -3 | -2 | -4 | -4 |
| | 2 | -4 | -2 | -1 | 1 | -4 |
| | 3 | 1 | -1 | 2 | 0 | -1 ← max |
| Column Maximum | | 3 | -1 ↑ min | 2 | 1 | |

v=-1, player I uses strategy 3 and player II uses strategy 2.

The game is stable with saddlepoint (3,2)

14.2-6 (a)

| | | II: 1 | 2 | 3 | Row Minimum |
|---|---|---|---|---|---|
| I | 1 | 2 | 3 | 1 | 1 ← max |
| | 2 | 1 | 4 | 0 | 0 |
| | 3 | 3 | -2 | -1 | -2 |
| Column maximum | | 3 | 4 | 1 ↑ min | |

v=1, player I uses strategy 1 and player II uses strategy 3.

14.2-6 (b) Strategy 1 of player II is dominated by strategy 3.
Strategy 3 of player I is dominated by strategies 1 and 2.
Strategy 2 of player II is dominated by strategy 3.
Strategy 2 of player I is dominated by strategy 1.
Therefore, the optimal strategy is for player I to choose strategy 1 and player II to choose strategy 3 resulting in a payoff of 1 to player I.

14.2-7 (a)

| I \ II | 1 | 2 | 3 | Row Minimum |
|---|---|---|---|---|
| 1 | 7 | -1 | 3 | -1 |
| 2 | 1 | 0 | 2 | 0 ← max |
| 3 | -5 | -3 | 1 | -5 |
| Column Maximum: | 7 | 0 | 3 | |
| | | ↑ min | | |

Hence, $v=0$ with Politician I using Issue 2 and Politician II using Issue 2.

(b) Let $p_{ij} = P\{\text{winning or tying election for Politician I}\}$
Then the payoff matrix becomes

| I \ II | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 0 | 3/5 |
| 2 | 1/5 | 0 | 2/5 |
| 3 | 0 | 0 | 1/5 |

Strategy 3 of Politician I dominated by strategy 2.
Strategy 2 of Politician I dominated by strategy 1.
Strategies 1 and 3 of Politician II dominated by strategy 2.
Hence, eliminating dominated strategies gives $v=0$ with Politician I using Issue 1 and Politician II using Issue 2. Therefore, Politician II can prevent Politician I from winning or tying.

(c) Let $p_{ij} = \begin{cases} 1 & \text{if Politician I will win or tie} \\ 0 & \text{if Politician II will win} \end{cases}$

Then the payoff matrix becomes

| I \ II | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 0 | 0 |
| 2 | 0 | 0 | 0 |
| 3 | 0 | 0 | 0 |

(CONT)

14.2-7 (c) cont. Min[Column maxima] = 0 = Max[Row minima].
Hence the minimax criterion yields $v=0$ (Politician I cannot win). Politician I can use any issue and Politician II can use Issue 2 or 3. However, since Issue 1 offers Politician I his only chance of winning, he should use that and hope Politician II makes an error and also uses Issue 1.

14.2-8 Advantages: It provides the best possible guarantee on what the worst outcome can be, regardless of how skillfully the opponent plays the game. Therefore it reduces the risk of very undesirable outcomes to a minimum.

Disadvantages: It is a very conservative approach, and, therefore, it may yield far from the best attainable results if the opponent is not skillful.

14.3-1 (a)

| Strategies for player I | Strategies for player II |
|---|---|
| 1- Pass on heads or tails | 1- If player I bets, call. |
| 2- Bet on heads or tails | 2- If player I bets, pass |
| 3- Pass on heads, bet on tails | |
| 4- Pass on tails, bet on heads | |

b)

| I \ II | 1 | 2 |
|---|---|---|
| 1 | -5 | -5 |
| 2 | 0 | 5 |
| 3 | -15/2 | 0 |
| 4 | 5/2 | 0 |

Strategies 1 and 3 of player I are dominated by strategy 2. Eliminating the dominated strategies we obtain the payoff table:

| I \ II | 1 | 2 |
|---|---|---|
| 2 | 0 | 5 |
| 4 | 5/2 | 0 |

14.3-1 (c) The payoff matrix is:

| I \ II | 1 | 2 | Row minimum |
|---|---|---|---|
| 1 | -5 | -5 | -5 |
| 2 | 0 | 5 | 0 |
| 3 | -15/2 | 0 | -15/2 |
| 4 | 5/2 | 0 | 0 |
| Column maximum | 5/2 | 5 | |

max

min

Min[Column maxima] $\neq$ Max[Row minima], therefore there is no saddle point.

If either player chose a pure strategy, the other player could adjust his strategy in such a way as to cause the first player to want to change his strategy, too. Mixed strategies are needed.

d) expected payoff $= p_{21}x_2y_1 + p_{22}x_2y_2 + p_{41}x_4y_1 + p_{42}x_4y_2$,

where $x_2+x_4=1$, $y_1+y_2=1$ are the probabilities of each player using each non-dominated strategy (from (b)).

case (i): $y_1=1,\ y_2=0$ $\qquad \frac{5}{2}x_4 = \frac{5}{2}(1-x_2)$

(ii): $y_1=0,\ y_2=1$ $\qquad 5x_2 = 5(1-x_4)$

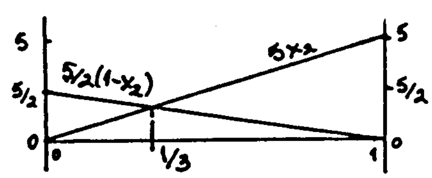
(iii): $y_1=\frac{1}{2}=y_2$ $\qquad 5x_2(\frac{1}{2}) + 5/2\, x_4(\frac{1}{2})$

$= \frac{5}{2}x_2 + \frac{5}{4}(1-x_2)$

$= \frac{5}{4}x_2 + \frac{5}{4}$

14.4-1

| $(y_1, y_2)$ | Expected Payoff |
|---|---|
| (1, 0) | $5/2(1-x_2)$ |
| (0, 1) | $5x_2$ |




$5/2(1-x_2) = 5x_2 \Rightarrow (x_1^*, x_2^*, x_3^*, x_4^*) = (0, 1/3, 0, 2/3)$ and $v = 5/3$

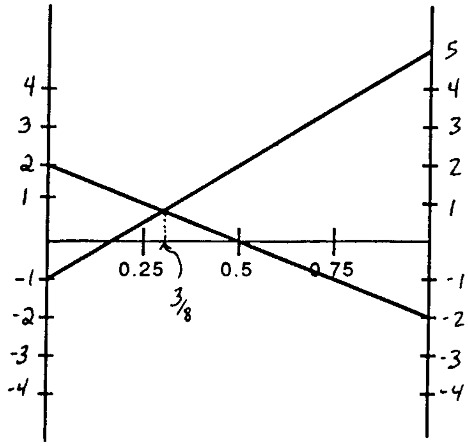
$y_1^*(5/2)(1-x_2) + y_2^*(5x_2) = 5/3$ for $0 \le x_2 \le 1 \Rightarrow \frac{5}{2}\cdot y_1^* = 5/3$

$5y_2^* = 5/3$

$\Rightarrow (y_1^*, y_2^*) = (2/3, 1/3)$

14.4-2

| $(y_1, y_2)$ | Expected Payoff |
|---|---|
| $(1,0)$ | $3x_1 - 1(1-x_1) = 4x_1 - 1$ |
| $(0,1)$ | $-2x_1 + 2(1-x_1) = -4x_1 + 2$ |



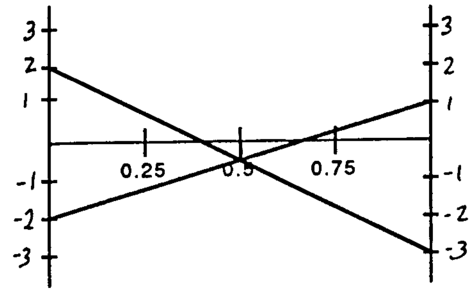

$4x_1 - 1 = -4x_1 + 2 \Rightarrow x_1^* = 3/8,\ x_2^* = 5/8$

$v = 4(3/8) - 1 = 1/2$

$$\left.\begin{aligned} 3y_1^* - 2y_2^* &= 1/2 \\ -y_1^* + 2y_2^* &= 1/2 \end{aligned}\right\} \Rightarrow y_1^* = y_2^* = 1/2$$

The payoff matrix for player II is:

$$\begin{array}{cc|cc} & & \multicolumn{2}{c}{II} \\ & & 1 & 2 \\ \hline \multirow{2}{*}{I} & 1 & -3 & 2 \\ & 2 & 1 & -2 \end{array}$$

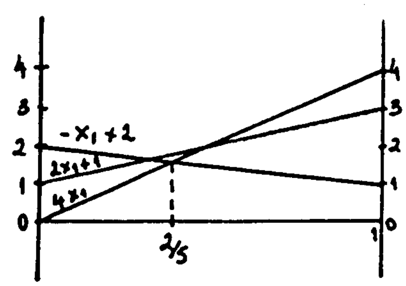



| $(x_1, x_2)$ | Expected Payoff |
|---|---|
| $(1,0)$ | $-3y_1 + 2(1-y_1) = -5y_1 + 2$ |
| $(0,1)$ | $y_1 - 2(1-y_1) = 3y_1 - 2$ |

$-5y_1 + 2 = 3y_1 - 2 \Rightarrow y_1^* = \frac{1}{2},\ y_2^* = 1/2$

14.4-3

| $(y_1, y_2, y_3)$ | Expected payoff |
|---|---|
| $(1, 0, 0)$ | $4x_1$ |
| $(0, 1, 0)$ | $3x_1 + (1-x_1) = 2x_1 + 1$ |
| $(0, 0, 1)$ | $x_1 + 2(1-x_1) = -x_1 + 2$ |




$4x_1 = -x_1 + 2 \Rightarrow (x_1^*, x_2^*) = (2/5, 3/5)$

and $v = 8/5$

$y_1^*(4x_1) + y_3^*(-x_1 + 2) = 8/5$ for $0 \le x_1 \le 1 \Rightarrow \begin{aligned} 2y_3^* &= 8/5 \\ 4y_1^* + y_3^* &= 8/5 \end{aligned}$

$\Rightarrow (y_1^*, y_2^*, y_3^*) = (1/5, 0, 4/5)$

14.4-4 (a)

| Strategies for A.J. Team | Strategies for G.N. Team |
|---|---|
| 1. John does not swim butterfly | 1. Mark does not swim butterfly |
| 2. John does not swim backstroke | 2. Mark does not swim backstroke |
| 3. John does not swim breaststroke | 3. Mark does not swim breaststroke |

Let the payoff entries be the total points won in all three events when a given pair of strategies are used by the teams. Then the payoff matrix becomes:

| AJ \ GN | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 14 | 13 | 12 |
| 2 | 13 | 12 | 12 |
| 3 | 12 | 12 | 13 |

Strategy 2 of A.J. Team dominated by strategy 1.
Strategy 1 of G.N. Team dominated by strategy 2

Final payoff table

| A.J. \ G.N. | 2 | 3 |
|---|---|---|
| 1 | 13 | 12 |
| 3 | 12 | 13 |

| $(y_2, y_3)$ | Expected Pay off |
|---|---|
| $(1, 0)$ | $13x_1 + 12(1-x_1) = x_1 + 12$ |
| $(0, 1)$ | $12x_1 + 13(1-x_1) = -x_1 + 13$ |

$\Rightarrow x_1 + 12 = -x_1 + 13$

$\therefore x_1^* = 1/2,\ x_2^* = 0,\ x_3^* = 1/2$

and $V = 12\ 1/2$

14.4-4 (a) (cont.)

$$y_2^*(x_1+12)+y_3^*(-x_1+13) = 12\tfrac{1}{2} \quad \text{for } 0 \le x_1 \le 1 \Rightarrow \begin{array}{l} 12y_2^* + 13y_3^* = 12\tfrac{1}{2} \\ 13y_2^* + 12y_3^* = 12\tfrac{1}{2} \end{array}$$

$\therefore\ y_1^* = 0,\ y_2^* = 1/2,\ y_3^* = 1/2$

That is, John should always swim the backstroke and should swim the butterfly or breaststroke each with probability 1/2. Also, Mark should always swim the butterfly and should swim the backstroke or breaststroke each with probability 1/2. And the A.J. Team can expect to get $12\tfrac{1}{2}$ points in the three events.

(b) The strategies for the two teams are as in part (a). Let $p_{ij} = \begin{cases} 1/2 & \text{if } p_{ij} \ge 13 \text{ for part (a); that is, A.J. Team wins} \\ -1/2 & \text{if } p_{ij} < 13 \text{ for part (a); that is, A.J. Team loses} \end{cases}$

The payoff matrix becomes

| A.J. \ G.N. | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1/2 | 1/2 | -1/2 |
| 2 | 1/2 | -1/2 | -1/2 |
| 3 | -1/2 | -1/2 | 1/2 |

Strategy 2 of A.J. Team dominated by strategy 1.
Strategy 1 of GN Team dominated by strategy 2.
After eliminating the dominated strategies the matrix is

| AJ \ GN | 2 | 3 |
|---|---|---|
| 1 | 1/2 | -1/2 |
| 3 | -1/2 | 1/2 |

If $12\tfrac{1}{2}$ is added to each entry, the optimal strategies are unchanged. Furthermore, the payoff matrix of part (a) is obtained. Hence the strategies given in part (a) are still optimal and $v = 12\tfrac{1}{2} - 12\tfrac{1}{2} = 0$

14.4-4(c) Since John and Mark are the best swimmers on their respective teams, they will always swim in two events since the team can do no better if they swim in only one or no events. Hence, if either does not swim in the first event, the butterfly, he will surely swim the last two events.

Thus the strategies for the A.J. team are:

1. John enters the butterfly and then enters the backstroke regardless of whether Mark enters the butterfly.
2. John enters the butterfly and then swims the backstroke if Mark enters the butterfly, but swims breaststroke if Mark does not.
3. John enters the butterfly and then swims breaststroke if Mark enters the butterfly, but swims the backstroke if Mark does not.
4. John enters the butterfly and then swims the breaststroke regardless of whether Mark enters the butterfly.
5. John does not swim the butterfly and then enters both the breaststroke and the backstroke.

The strategies for the G.N. Team are as above but with the roles of John and Mark reversed.

The payoff matrix is

| A.J. \ G.N. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 1 | $1/2$ | $1/2$ | $-1/2$ | $-1/2$ | $-1/2$ |
| 2 | $1/2$ | $1/2$ | $-1/2$ | $-1/2$ | $1/2$ |
| 3 | $-1/2$ | $-1/2$ | $-1/2$ | $-1/2$ | $-1/2$ |
| 4 | $-1/2$ | $-1/2$ | $-1/2$ | $-1/2$ | $1/2$ |
| 5 | $-1/2$ | $1/2$ | $-1/2$ | $1/2$ | $1/2$ |

Strategy 3 of G.N. Team dominates all others.

Since the resulting payoff matrix is:

| A.J. \ G.N. | 3 |
|---|---|
| 1 | $-1/2$ |
| 2 | $-1/2$ |
| 3 | $-1/2$ |
| 4 | $-1/2$ |
| 5 | $-1/2$ |

if G.N. Team uses strategy 3, it will win, regardless of the strategy chosen by the A.J. Team.

(d) Strategy 2 of AJ Team dominates strategies 1, 3, 4.

Thus, if the coach for the GN Team may choose any of his strategies at random, the coach for the A.J. Team should choose either strategy 2 or 5.

The payoff matrix becomes (after eliminating the dominated strategies of A.J Team):

| A.J. \ G.N. | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| 2 | $1/2$ | $1/2$ | $-1/2$ | $-1/2$ | $1/2$ |
| 5 | $-1/2$ | $1/2$ | $-1/2$ | $1/2$ | $1/2$ |

The two rows are identical except for columns 1 and 4.

Thus, if the coach for the A.J. Team knows that the other coach has a tendency to enter Mark in butterfly and backstroke more often than breast-stroke, that means column 1 is more likely to be chosen than column 4. Therefore, the coach for the A.J. Team should choose strategy 2.

14.5-1 Adding 3 to the entries of table 12.6 we obtain the payoff table

| I \ II | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 3 | 1 | 5 |
| 2 | 8 | 7 | 0 |

The new linear programming model for player I is:

Maximize $x_3$

subject to
$$3x_1 + 8x_2 - x_3 \geq 0$$
$$x_1 + 7x_2 - x_3 \geq 0$$
$$5x_1 - x_3 \geq 0$$
$$x_1 + x_2 = 1$$
$$x_1, x_2, x_3 \geq 0$$

The new linear programming model for player II is:

Minimize $y_4$

subject to
$$3y_1 + y_2 + 5y_3 - y_4 \leq 0$$
$$8y_1 + 7y_2 - y_4 \leq 0$$
$$y_1 + y_2 + y_3 = 1$$
$$y_1, y_2, y_3, y_4 \geq 0$$

Based on the information given in Section 12.5, the optimal solutions for these new models are:

$(x_1^*, x_2^*, x_3^*) = (7/11, 4/11, 35/11)$ and $(y_1^*, y_2^*, y_3^*, y_4^*) = (0, 5/11, 6/11, 35/11)$

Note that $x_3^* = y_4^*$ and also $x_3^* = y_4^* = v + 3$ where $v$ is the original game value.

14.5-2

a) Maximize $x_4$

subject to
$$5x_1 + 2x_2 + 3x_3 - x_4 \geq 0$$
$$4x_2 + 2x_3 - x_4 \geq 0$$
$$3x_1 + 3x_2 - x_4 \geq 0$$
$$x_1 + 2x_2 + 4x_3 - x_4 \geq 0$$
$$x_1 + x_2 + x_3 = 1$$
$$x_1, x_2, x_3, x_4 \geq 0$$

b)

Solve Automatically by the Simplex Method:

**Optimal Solution**

Value of the
Objective Function: Z = 2.36842105

| Variable | Value |
|---|---|
| $X_1$ | 0.05263 |
| $X_2$ | 0.73684 |
| $X_3$ | 0.21053 |
| $X_4$ | 2.36842 |

14.5-3 To insure $x_4 \geq 0$ add 3 to each entry of the payoff table.

a)

$$\begin{aligned} \text{Maximize} \quad & x_4 \\ \text{subject to:} \quad & 7x_1 + 2x_2 + 5x_3 - x_4 \geq 0 \\ & 5x_1 + 3x_2 + 6x_3 - x_4 \geq 0 \\ & 6x_2 + x_3 - x_4 \geq 0 \\ & x_1 + x_2 + x_3 = 1 \\ & x_i \geq 0 \text{ for } i = 1,2,3,4. \end{aligned}$$

Solve Automatically by the Simplex Method:

b) **Optimal Solution**

Value of the Objective Function: Z = 3.79166667

| Variable | Value |
|---|---|
| $X_1$ | 0.33333 |
| $X_2$ | 0.625 |
| $X_3$ | 0.04167 |
| $X_4$ | 3.79167 |

14.5-4

a) To insure $x_5 \geq 0$ add 4 to each entry of the payoff table.

$$\begin{aligned} \text{maximize} \quad & \\ \text{subject to:} \quad & 5x_1 + 6x_2 + 4x_3 - x_5 \geq 0 \\ & x_1 + 7x_2 + 8x_3 + 4x_4 - x_5 \geq 0 \\ & 6x_1 + 4x_2 + 3x_3 + 2x_4 - x_5 \geq 0 \\ & 2x_1 + 7x_2 + x_3 + 6x_4 - x_5 \geq 0 \\ & 5x_1 + 2x_2 + 6x_3 + 3x_4 - x_5 \geq 0 \\ & x_1 + x_2 + x_3 + x_4 = 1 \end{aligned}$$

$x_i \geq 0$ for $i = 1,2,3,4,5$

Solve Automatically by the Simplex Method:

b) **Optimal Solution**

Value of the Objective Function: Z = 3.98101266

| Variable | Value |
|---|---|
| $X_1$ | 0.31013 |
| $X_2$ | 0.26582 |
| $X_3$ | 0.20886 |
| $X_4$ | 0.21519 |
| $X_5$ | 3.98101 |

14.5-5 Following Table 6.14, the dual of player I's problem is:

$$\begin{aligned} \text{min.} \quad & y_{m+1} \\ \text{s.t.} \quad & p_{11} y_1' + p_{12} y_2' + \cdots + p_{1m} y_m' + y_{m+1} \geq 0 \\ & p_{21} y_1' + p_{22} y_2' + \cdots + p_{2m} y_m' + y_{m+1} \geq 0 \\ & \vdots \\ & p_{m1} y_1' + p_{m2} y_2' + \cdots + p_{mm} y_m' + y_{m+1} \geq 0 \\ & -y_1' - y_2' - \cdots - y_m' = 1 \\ & y_i' \leq 0, \quad i = 1,2,\ldots,m \quad (y_{m+1} \text{ free}) \end{aligned}$$

Now let $y_i = -y_i'$, $i = 1,2,\ldots,m$

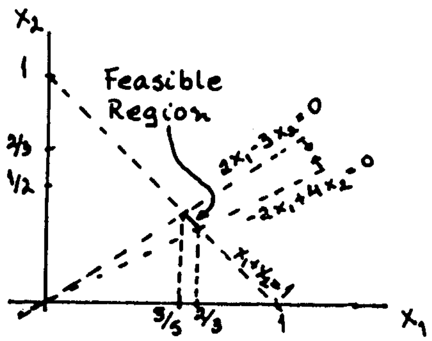
and we get the linear program given for player II.

14.5-6 Taking the dual of the player I problem gives:

$$\begin{array}{ll} \min & y_4 \\ \text{s.t.} & -2y_2' + 2y_3' + y_4 \geq 0 \\ & 5y_1' + 4y_2' - 3y_3' + y_4 \geq 0 \\ & -y_1' - y_2' - y_3' = 1 \\ & y_1', y_2', y_3' \leq 0 \quad y_4 \text{ free} \end{array}$$

let $y_i = -y_i'$, $i = 1,2,3$
→ substitution produces the given player II problem.

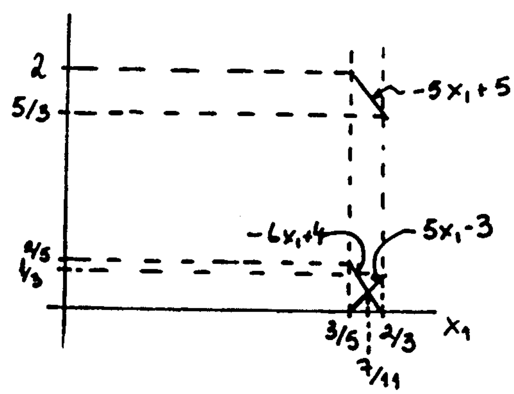
14.5-7




There fore the feasible region may be algebraically described by : $x_2 = 1 - x_1 \quad 3/5 \leq x_1 \leq 2/3$

The restrictions may be rewritten as:

$$\begin{array}{ll} x_3 \leq -5x_1 + 5 & 3/5 \leq x_1 \leq 2/3 \\ x_3 \leq -6x_1 + 4 & 3/5 \leq x_1 \leq 2/3 \\ x_3 \leq 5x_1 - 3 & 3/5 \leq x_1 \leq 2/3 \end{array}$$




$$-6x_1 + 4 = 5x_1 - 3$$
$$\Rightarrow x_1 = 7/11$$

There fore the algebraic expression for the maximizing value of $x_3$ for any point in the feasible region is:

$$x_3 = \begin{cases} 5x_1 - 3 & \text{for } 3/5 \leq x_1 \leq 7/11 \\ -6x_1 + 4 & \text{for } 7/11 \leq x_1 \leq 2/3 \end{cases}$$

Hence, the optimal solution is:

$$x_1^* = 7/11$$
$$x_2^* = 1 - 7/11 = 4/11$$
$$x_3^* = 5\,(7/11) - 3 = 2/11$$

14.5-8 AUTOMATIC SIMPLEX METHOD: FINAL TABLEAU

| Bas Var | Eq No | Z | X1 | X2 | X3 | X4 | X5 | X6 | X7 | X8 | X9 | X10 | Right side |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | 1M | 1M | 1M | 1M | |
| Z | 0 | 1 | 0 | 0 | 0 | 0 | 0.455 | 0.545 | 0 | -0.45 | -0.55 | 0.182 | 0.182 |
| X2 | 1 | 0 | 0 | 1 | 0 | 0 | -0.09 | 0.091 | 0 | 0.091 | -0.09 | 0.364 | 0.364 |
| X4 | 2 | 0 | 0 | 0 | 0 | 1 | -0.91 | -0.09 | -1 | 0.909 | 0.091 | 1.636 | 1.636 |
| X1 | 3 | 0 | 1 | 0 | 0 | 0 | 0.091 | -0.09 | 0 | -0.09 | 0.091 | 0.636 | 0.636 |
| X3 | 4 | 0 | 0 | 0 | 1 | 0 | 0.455 | 0.545 | 0 | -0.45 | -0.55 | 0.182 | 0.182 |

The optimal primal solution is $(x_1, x_2) = (0.636, 0.364)$ with a payoff = 0.182

The optimal dual solution is $(y_1, y_2, y_3) = (0, 0.455, 0.545)$

14.5-9 (a) Since saddlepoints can be found from the linear programming formulation of the game, part (a) follows from part (b).

(b) Consider the linear programming formulation of the problem for Player II. The $i^{th}$ and $k^{th}$ constraints are

$$p_{i1}y_1 + p_{i2}y_2 + \cdots + p_{in}y_n \le y_{n+1}$$

$$p_{k1}y_1 + p_{k2}y_2 + \cdots + p_{kn}y_n \le y_{n+1}$$

If row k weakly dominates row i, then

$$p_{i1}y_1 + \cdots + p_{in}y_n \le p_{k1}y_1 + \cdots + p_{kn}y_n \text{ for all } y_1, \ldots, y_n$$

That is, the $i^{th}$ constraint is redundant since it is implied by the $k^{th}$ constraint. Hence, eliminating weakly dominated pure strategies for Player I corresponds to eliminating redundant constrainst in the linear program for Player II. Similarly, eliminating weakly dominated pure strategies for Player II corresponds to eliminating redundant constraints in the linear program for Player I.

Since this process cannot eliminate feasible solutions or create new ones, all optimal strategies cannot be eliminated and new ones cannot be created.