Respostas

1. a. $3x+3 < x+6 \iff 2x < 3 \iff x < \frac{3}{2}$

   $\{x \in \mathbb{R} | x < \frac{3}{2}\}$

   b. $x-3 > 3x+1 \iff -2x > 4 \iff 2z < -4 \iff x < -2$

   $\{x \in \mathbb{R} | x < -2\}$

   c. $2x-1 \geq 5x+3 \iff -3x \geq 4 \iff 3x \leq -4 \iff x \leq -\frac{4}{3}$

   $\{x \in \mathbb{R} | x \leq -\frac{4}{3}\}$

   d. $x+3 \leq 6x-2 \iff -5x \leq -5 \iff 5x \geq 5 \iff x \geq 1$

   $\{x \in \mathbb{R} | x \geq 1\}$

   e. $1-3x > 0 \iff -3x > -1 \iff 3x < 1 \iff x < \frac{1}{3}$

   $\{x \in \mathbb{R} | x < \frac{1}{3}\}$

   f. $2x+1 \geq 3x \iff -x \geq -1 \iff x \leq 1$

   $\{x \in \mathbb{R} | x \leq 1\}$

2. a. $4x-3 < 6x+2 \iff -2x < 5 \iff 2x > -5 \iff x > -\frac{5}{2}$

   $\{x \in \mathbb{R} | 4x-3 < 6x+2\} =]-\frac{5}{2}, +\infty[$

   b. $|x| < 1 \iff -1 < x < 1$

   $\{x \in \mathbb{R} || x| < 1\} =]-1, 1[$

   c. $|2x-3| \leq 1 \iff -1 \leq 2x-3 \leq 1 \iff -1+3 \leq 2x \leq 1+3 \iff 2 \leq 2x \leq 4 \iff 1 \leq x \leq 2$

   $\{x \in \mathbb{R} || 2x-3| \leq 1\} = [1, 2]$

   d. $3x+1 < \frac{x}{3} \iff 9x+3 < x \iff 8x < -3 \iff x < -\frac{3}{8}$

   $\{x \in \mathbb{R} | 3x+1 < \frac{x}{3}\} =]-\infty, -\frac{3}{8}[$

3. a. $(x-a)(x+a) = x^2 + ax - ax - a^2 = x^2 - a^2$

   b. $(x-a)(x^2+ax+a^2) = x^3 + ax^2 + a^2x - ax^2 - a^2x - a^3 = x^3 - a^3$

   c. $(x-a)(x^3+ax^2+a^2x+a^3) = x^4 + ax^3 + a^2x^2 + a^3x - ax^3 - a^2x2 - a^3x - a^4 = x^4 - a^4$

   d. $(x-a)(x^4+ax^3+a^2x^2+a^3x+a^4) = x^5 + ax^4 + a^2x^3 + a^3x^2 + a^4x_ax^4 - a^2x^3 - a^3x^2 - a^4x - a^5 = x^5 - a^5$

4. a. $\frac{(-x^2+2x)-(-1+2)}{x-1} = \frac{(-x^2+2x)-1}{x-1} = \frac{-(x-1)^2}{x-1}$

   b. Vamos primeiro calcular $f(x+h)$: $f(x+h) = -(x+h)^2 + 2(x+h) = -x^2 - 2xh - h^2 + 2x + 2h$

   Então: $\frac{f(x+h)-f(x)}{h} = \frac{-x^2-2xh-h^2+2x+2h-(-x^2+2x)}{h} = \frac{-2xh-h^2+2h}{h} = -2x - h + 2, h \neq 0.$

5. a. $f(x+h) = 2(x+h)+1 = 2x+2h+1$

$\frac{f(x+h)-f(x)}{h} = \frac{2x+2h+1-(2x+1)}{h} = \frac{2h}{h} = 2$

b. $f(x+h) = 3(x+h)-8 = 3x+3h-8$

$\frac{f(x+h)-f(x)}{h} = \frac{3x+3h-8-(3x-8)}{h} = \frac{3h}{h} = 3$

c. $f(x+h) = -2(x+h)+4 = -2x-2h+4$

$\frac{f(x+h)-f(x)}{h} = \frac{-2x-2h+4-(-2x+4)}{h} = -\frac{2h}{h} = -2$

d. $f(x+h) = (x+h)^2 = x^2+2hx+h^2$

$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2hx+h^2-x^2}{h} = \frac{2hx+h^2}{h} = 2x+h$

e. $f(x+h) = (x+h)^2-2(x+h) = x^2+2hx+h^2-2x-2h$

$\frac{f(x+h)-f(x)}{h} = \frac{x^2+2hx+h^2-2x-2h-(x^2-2x)}{h} = \frac{2hx+h^2-2h}{h} = 2x+h-2$

f. $f(x+h) = \frac{1}{x+h}$

$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h}-\frac{1}{x}}{h} = \frac{\frac{x-(x+h)}{x(x+h)}}{h} = \frac{\frac{-h}{x^2+xh}}{h} = \frac{-h}{h(x^2+xh)} = \frac{-h}{hx^2+h^2x} = \frac{-1}{x^2+hx} = \frac{-1}{x(x+h)}$

g. $f(x+h) = \frac{1}{(x+h)+2}$

$\frac{f(x+h)-f(x)}{h} = \frac{\frac{1}{x+h+2}-\frac{1}{x+2}}{h} = \frac{\frac{(x+2)-(x+h+2)}{(x+h+2)(x+2)}}{h} = \frac{\frac{-h}{(x+h+2)(x+2)}}{h} = \frac{-h}{h(x+h+2)(x+2)} = \frac{-1}{(x+h+2)(x+2)}$

6. a. $\{x \in \mathbb{R} | x \neq 1\}$

b. $\{x \in \mathbb{R} | x \neq 1 \quad e \quad x \neq -1\}$

c. $\mathbb{R}$

d. $\{x \in \mathbb{R} | x \neq -2\}$

e. $\{x \in \mathbb{R} | x \geq -2\}$

f. $\{x \in \mathbb{R} | x \neq 0 \quad e \quad x \neq -1\}$

g. $\{x \in \mathbb{R} | x < -1 \quad e \quad x \geq 1\}$

7. a. $h(x) = 3(x+2)+1 = 3x+7$

b. $h(x) = \sqrt{2+x^2}$

c. $h(x) = \frac{(x^2+3)+1}{(x^2+3)-2} = \frac{x^2+4}{x^2+1}$

d. $h(x) = -(2x-3)^2+3(2x-3)+1 = -(4x^2-12x+9)+(6x-9)+1 = -4x^2+18x-17$

e. $h(x) = \frac{2}{(x+1)-2} = \frac{2}{x-1}$

f. $h(x) = \frac{(\frac{x}{x+1})+1}{(\frac{x}{x+1})-1} = \frac{\frac{x+(x+1)}{x+1}}{\frac{x-(x+1)}{x+1}} = \frac{\frac{2x+1}{x+1}}{\frac{-1}{x+1}} = \frac{(x+1)(2x+1)}{(x+1)(-1)} = -(2x+1)$, para $x \neq -1$

g. $h(x) = \frac{(\frac{2x+1}{x-1})+1}{(\frac{2x+1}{x-1})-2} = \frac{\frac{2x+1+(x-1)}{x-1}}{\frac{2x+1-2(x-1)}{x-1}} = \frac{\frac{3x}{x-1}}{\frac{3}{x-1}} = x$, para $x \neq 1$

8. a. $\sum_{i=1}^{n}(x_i-\bar{x})^2 = \sum_{i=1}^{n}(x_i^2-2x_i\bar{x}+\bar{x}^2) = \sum_{i=1}^{n}x_i^2 - \sum_{i=1}^{n}(2x_i\bar{x}) + \sum_{i=1}^{n}\bar{x}^2 = \sum_{i=1}^{n}x_i^2 - 2\bar{x}\sum_{i=1}^{n}x_i + n\bar{x}^2 = \sum_{i=1}^{n}x_i^2 - (2\bar{x}n\bar{x}) + n\bar{x}^2 = \sum_{i=1}^{n}x_i^2 - 2n\bar{x}^2 + n\bar{x}^2 = \sum_{i=1}^{n}x_i^2 - n\bar{x}^2$

b. $\frac{1}{n-1}\sum_{i=1}^{n} = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - 2x_i\bar{x} + \bar{x}^2] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - 2\bar{x}\sum_{i=1}^{n} x_i + \sum_{i=1}^{n} \bar{x}^2] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - 2\bar{x}n\bar{x} + n\bar{x}^2] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - 2n\bar{x}^2 + n\bar{x}^2] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - n\bar{x}^2] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - n\frac{(\sum_{i=1}^{n} x_i)^2}{n^2}] = \frac{1}{n-1}[\sum_{i=1}^{n} x_i^2 - \frac{(\sum_{i=1}^{n} x_i)^2}{n}]$