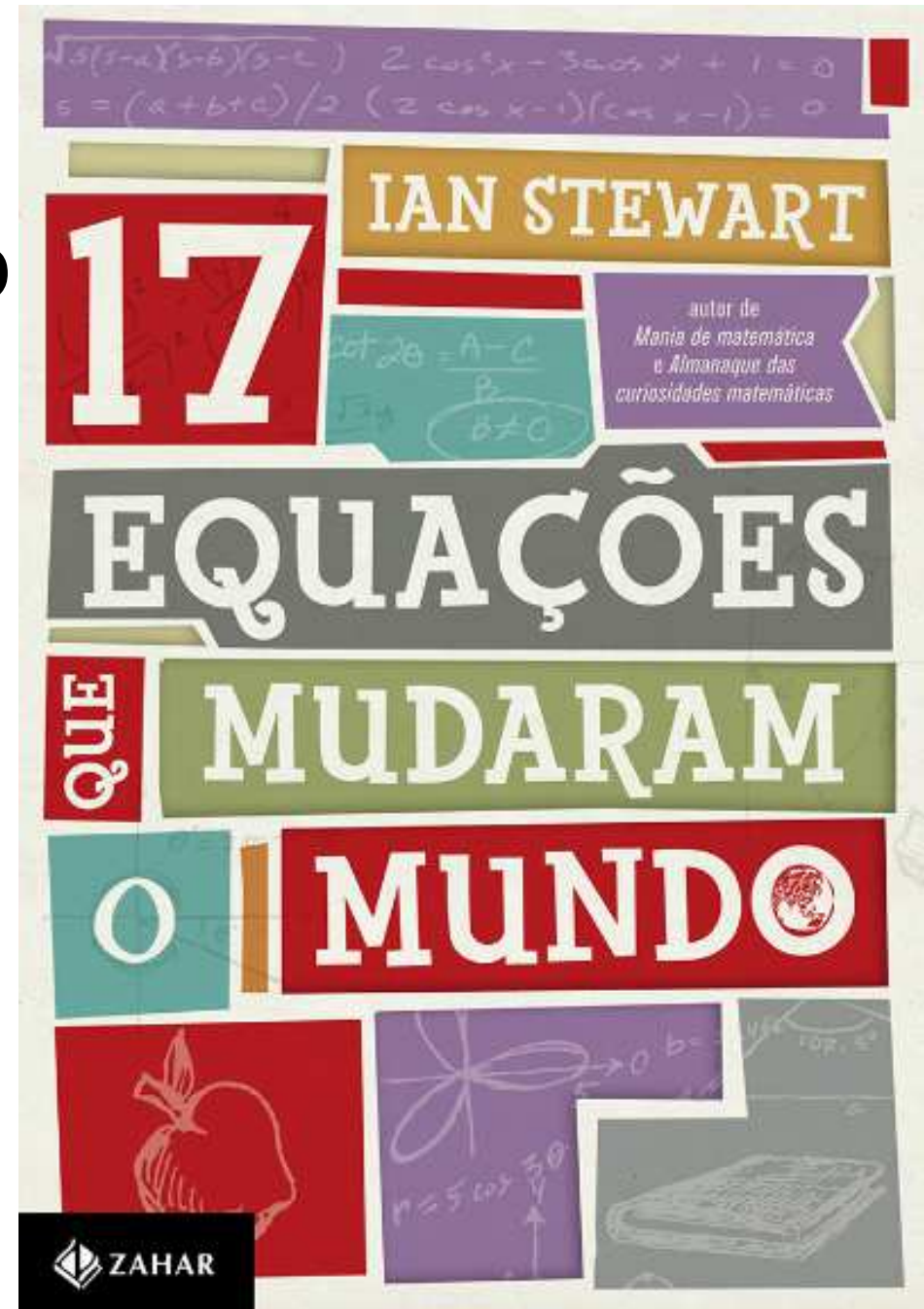


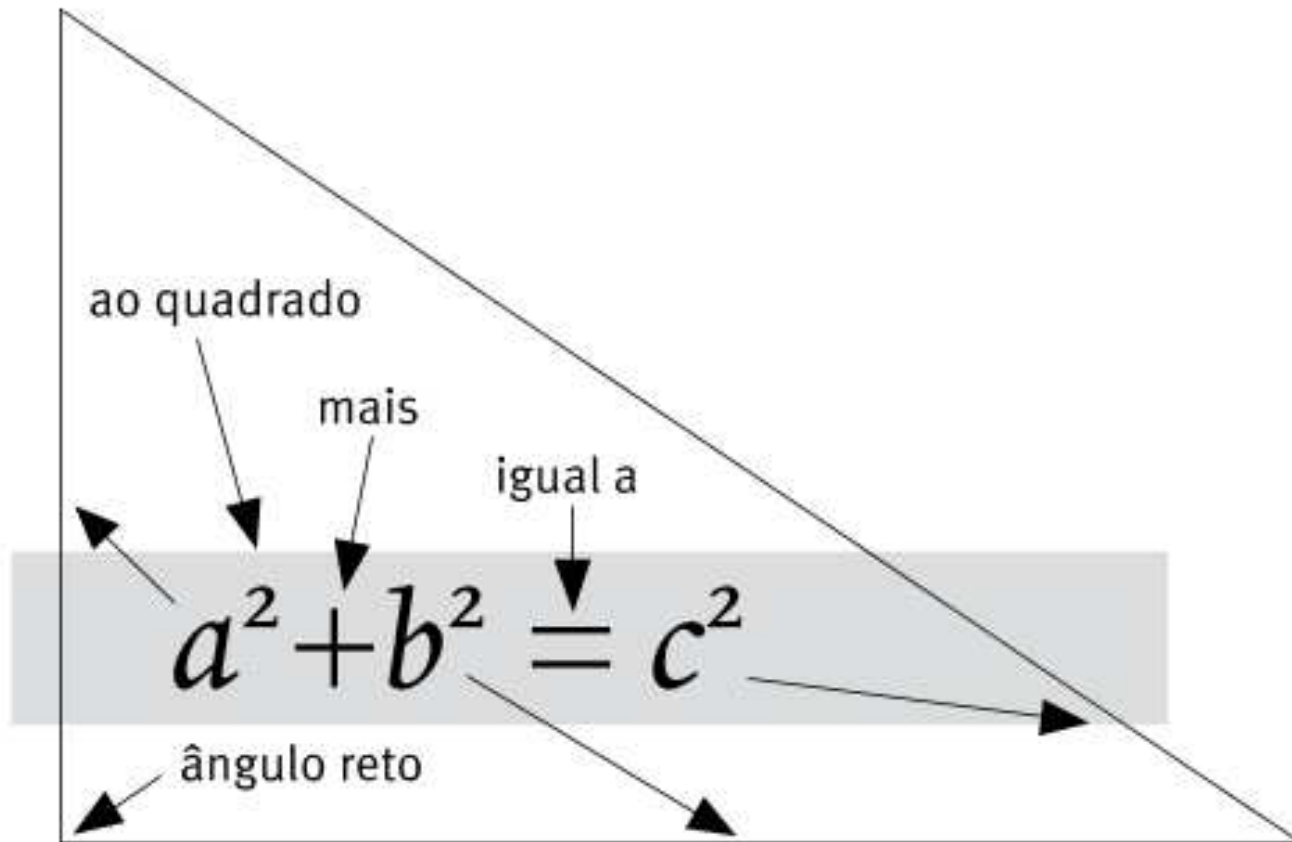
17 Equações que Mudaram o Mundo

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1 – O Teorema de Pitágoras



2 – 0 Logaritmo

multiplica

soma

$$\log xy = \log x + \log y$$

logaritmo

3 – A Definição de Derivada

The diagram illustrates the definition of a derivative with the following annotations:

- taxa de variação** (rate of change) points to $\frac{df}{dt}$.
- grandeza** (quantity) points to f .
- limite** (limit) points to \lim .
- variação no valor da grandeza** (variation in the value of the quantity) points to the entire fraction $\frac{f(t+h) - f(t)}{h}$.
- valor novo** (new value) points to $f(t+h)$.
- valor antigo** (old value) points to $f(t)$.
- menos** (minus) points to the subtraction sign $-$.
- em relação a tempo** (with respect to time) points to dt .
- intervalo de tempo** (time interval) points to h .
- dividido por** (divided by) points to h .
- tende a zero** (tends to zero) points to $h \rightarrow 0$.
- (torna-se muito pequeno)** (becomes very small) is a note under **tende a zero**.

$$\frac{df}{dt} = \lim_{h \rightarrow 0} \frac{f(t+h) - f(t)}{h}$$

4 – A Lei da Gravitação de Newton

The diagram shows the equation $F = G \frac{m_1 m_2}{d^2}$ with labels pointing to each component:

- força de atração** (force of attraction) points to F .
- constante gravitacional** (gravitational constant) points to G .
- massa do corpo 1** (mass of body 1) points to m_1 .
- massa do corpo 2** (mass of body 2) points to m_2 .
- dividido por** (divided by) points to the fraction bar.
- distância entre os corpos** (distance between the bodies) points to d .
- ao quadrado** (squared) points to the exponent 2 .

$$F = G \frac{m_1 m_2}{d^2}$$

5 – 0 Número Imaginário

número
imaginário

ao quadrado

menos um

um

$$i^2 = -1$$
The diagram illustrates the equation $i^2 = -1$. It features a light gray rectangular background. Above the equation, there are four labels in black text: 'número imaginário' (imaginary number) pointing to the 'i', 'ao quadrado' (squared) pointing to the '2', 'menos um' (minus one) pointing to the '-' sign, and 'um' (one) pointing to the '1'. The equation itself is centered on the gray background.

6 – A Fórmula de Euler para Poliedros

número de faces número de arestas número de vértices

$$F - A + V = 2$$

7 – A Distribuição Normal

The diagram illustrates the normal distribution function $\Phi(x)$ with the following annotations:

- probabilidade...** points to $\Phi(x)$.
- ...de obter este número** points to the equals sign.
- é igual a** points to the equals sign.
- um** points to the numerator 1 .
- dividido por** points to the denominator $\sqrt{2\pi\sigma}$.
- menos** points to the minus sign in $(x - \mu)$.
- média** points to μ .
- ao quadrado** points to the exponent 2 in $(x - \mu)^2$.
- dois** points to the coefficient 2 in $2\sigma^2$.
- desvio padrão** points to σ .
- elevado à potência** points to the exponent 2 in σ^2 .
- raiz quadrada** points to the square root symbol $\sqrt{\quad}$.
- dois** points to the coefficient 2 in 2π .
- 3,14159** points to π .
- desvio padrão** points to σ .
- 2,71828** points to the base e .

The formula is:
$$\Phi(x) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(x-\mu)^2}{2\sigma^2}}$$

8 – A Equação da Onda

The diagram shows the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$ with several labels pointing to its parts:

- deslocamento**: points to the variable u in both partial derivatives.
- derivada parcial de segunda ordem**: points to the second-order partial derivatives ∂^2 on both sides.
- em relação ao tempo**: points to the denominator ∂t^2 .
- em relação ao espaço**: points to the denominator ∂x^2 .
- velocidade**: points to the constant c .
- ao quadrado**: points to the constant c^2 .

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$$

9 – A Transformada de Fourier

The diagram shows the Fourier transform equation $\hat{f}(\xi) = \int_{-\infty}^{\infty} f(x) e^{-2\pi i x \xi} dx$ with various parts annotated in Portuguese. The annotations are as follows:

- transformação de**: points to the hat symbol on $\hat{f}(\xi)$.
- função**: points to $f(x)$.
- infinito**: points to the upper limit ∞ .
- menos infinito**: points to the lower limit $-\infty$.
- integral**: points to the integral symbol \int .
- espaço**: points to the variable x in the exponent.
- raiz quadrada de -1**: points to the imaginary unit i .
- 2,718...**: points to the constant e .
- 3,141...**: points to the constant π .
- função**: points to the variable ξ in the exponent.
- frequência**: points to the variable ξ in the exponent.

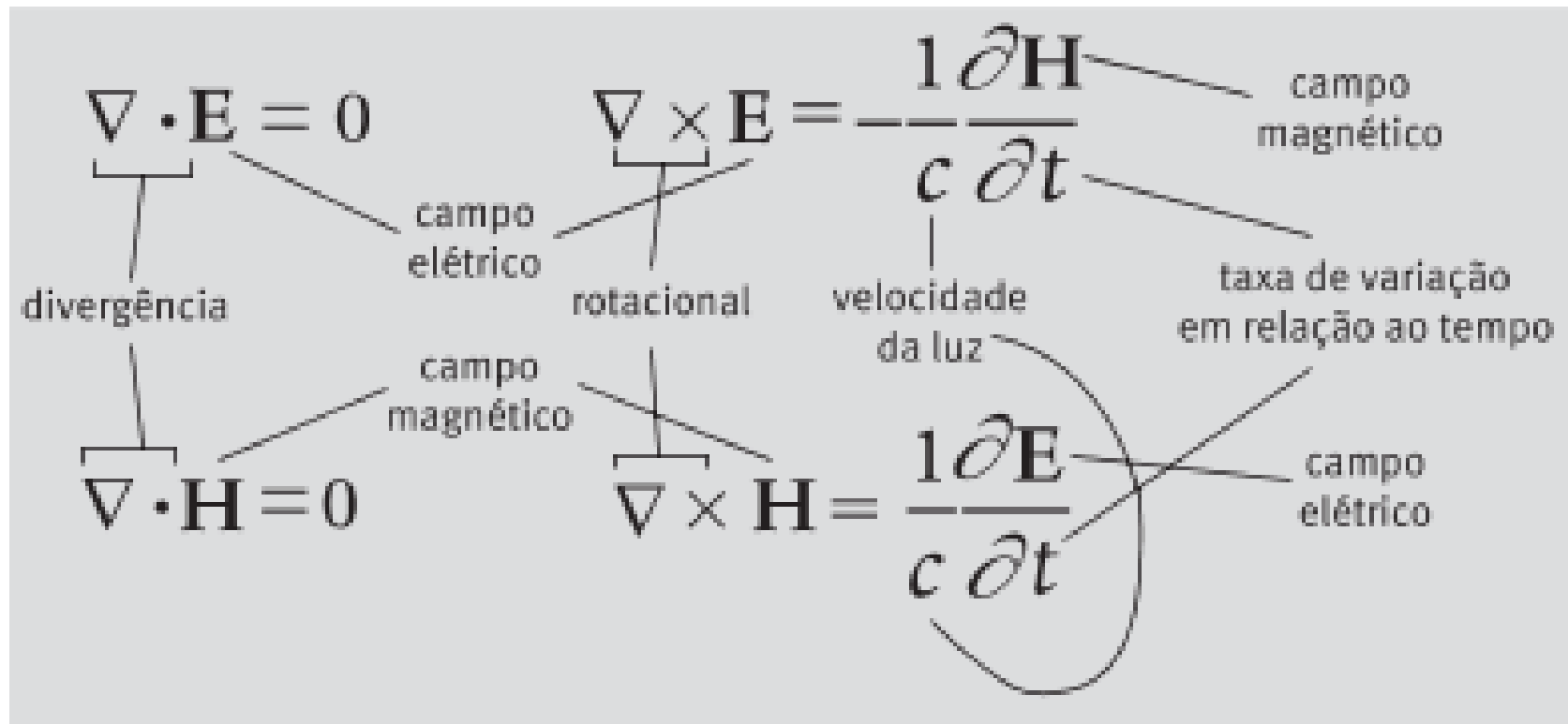
10 - A Equação de Navier-Stokes

The diagram shows the Navier-Stokes equation with labels for its components:

- densidade** (density) points to ρ .
- velocidade** (velocity) points to \mathbf{v} in both $\frac{\partial \mathbf{v}}{\partial t}$ and $\mathbf{v} \cdot \nabla \mathbf{v}$.
- derivada em relação ao tempo** (derivative with respect to time) points to $\frac{\partial}{\partial t}$.
- produto escalar** (scalar product) points to the dot product \cdot in $\mathbf{v} \cdot \nabla$.
- gradiente** (gradient) points to ∇ in $\nabla \mathbf{v}$.
- pressão** (pressure) points to p .
- tensão** (stress) points to \mathbf{T} .
- forças do corpo** (body forces) points to \mathbf{f} .
- divergência** (divergence) points to $\nabla \cdot$.

$$\rho \left(\frac{\partial \mathbf{v}}{\partial t} + \mathbf{v} \cdot \nabla \mathbf{v} \right) = -\nabla p + \nabla \cdot \mathbf{T} + \mathbf{f}$$

11 – As Equações de Maxwell



12 – A Segunda Lei da Termodinâmica

A diagram illustrating the second law of thermodynamics. The equation $dS \geq 0$ is centered on a light gray rectangular background. Four labels with thin lines pointing to the equation are positioned above it: 'variação em' points to the 'd' in dS ; 'entropia' points to the 'S' in dS ; 'maior ou igual a' points to the \geq symbol; and 'zero' points to the '0' on the right side of the equation.

variação em entropia maior ou igual a zero

$$dS \geq 0$$

13 – Teoria da Relatividade

energia de
repouso da matéria

massa

velocidade
da luz

ao quadrado

$$E = mc^2$$

The diagram shows the equation $E = mc^2$ centered on a light gray rectangular background. Four labels are positioned above the equation, each with a thin black line pointing to a specific part of the equation: 'energia de repouso da matéria' points to the letter 'E', 'massa' points to the letter 'm', 'velocidade da luz' points to the letter 'c', and 'ao quadrado' points to the superscript '2'.

14 – A Equação de Schroedinger

The diagram shows the Schrödinger equation $i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$ with several labels and arrows pointing to its parts:

- raiz quadrada de menos um** (square root of minus one) points to the imaginary unit i .
- constante de Planck dividida por 2π** (Planck constant divided by 2π) points to the reduced Planck constant \hbar .
- em relação ao tempo** (with respect to time) points to the partial derivative $\frac{\partial}{\partial t}$.
- taxa de variação** (rate of variation) points to the partial derivative $\frac{\partial}{\partial t}$.
- função de onda quântica** (quantum wave function) points to the wave function Ψ on the left side of the equation.
- operador hamiltoniano** (Hamiltonian operator) points to the operator \hat{H} .

The equation is displayed as:

$$i\hbar \frac{\partial}{\partial t} \Psi = \hat{H} \Psi$$

15 – Teoria da Informação

The diagram shows the Shannon entropy formula $H = - \sum_x p(x) \log p(x)$ with several labels and arrows pointing to its components:

- informação**: points to the variable H .
- soma**: points to the summation symbol \sum .
- probabilidade de símbolo**: points to the probability term $p(x)$.
- logaritmo de base 2**: points to the logarithm function \log .
- símbolos**: points to the variable x in the summation index.

$$H = - \sum_x p(x) \log p(x)$$

16 – Teoria do Caos

The diagram shows the logistic equation $x_{t+1} = kx_t(1-x_t)$ centered in a gray rectangular box. Five labels with arrows point to specific parts of the equation: 'tamanho da população da próxima geração' points to x_{t+1} ; 'taxa irrestrita de crescimento' points to k ; 'tamanho da população agora' points to x_t ; 'agora' points to the second x_t in the term $(1-x_t)$; and 'tamanho da população' points to the first x_t in the term $(1-x_t)$.

tamanho da população da próxima geração

taxa irrestrita de crescimento

$$x_{t+1} = kx_t(1-x_t)$$

tamanho da população agora

17 – A Equação de Black-Scholes

taxa de variação da taxa de variação

volatilidade

preço do derivativo financeiro

taxa de variação

$$\frac{1}{2} \sigma^2 S^2 \frac{\partial^2 V}{\partial S^2} + rS \frac{\partial V}{\partial S} + \frac{\partial V}{\partial t} - rV = 0$$

preço da mercadoria

em relação à taxa de juros livre de risco

tempo

Detailed description: The diagram shows the Black-Scholes equation with several labels and arrows pointing to specific parts. 'volatilidade' points to the sigma symbol. 'preço da mercadoria' points to the S symbol. 'em relação à taxa de juros livre de risco' points to the r symbol. 'tempo' points to the t symbol in the denominator of the third term. 'preço do derivativo financeiro' points to the V symbol. 'taxa de variação' points to the partial derivative with respect to V in the third term. 'taxa de variação da taxa de variação' points to the partial derivative with respect to S in the second term.