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Game Theory
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Exercises
Static games of incomplete information

Exercise 1.

(a) What is a static Bayesian game?

(b) What is a (pure) strategy in a static Bayesian game?

(c) What is a (pure-strategy) Bayesian Nash equilibrium in such a static Bayesian game?

Exercise 2. Consider a Cournot duopoly operating in a market with inverse demand $P(Q) = a - Q$, where $Q = q_1 + q_2$ is the aggregate quantity on the market. Both firms have total costs $c_i(q_i) = cq_i$, but demand is uncertain: it is high ($a = a_H$) with probability θ and low ($a = a_L$) with probability $1 - \theta$. Furthermore, information is asymmetric: firm 1 knows whether demand is high or low, but firm 2 does not. All of this is common knowledge. The two firms simultaneously choose quantities.

(a) What are the strategy spaces for the two firms?

(b) Make assumptions concerning a_H, a_L, θ, and c such that all equilibrium quantities are positive. What is the Bayesian Nash equilibrium of this game?

Exercise 3. Find all the pure-strategy Bayesian Nash equilibria in the following static Bayesian game:

	L	R
T	$1,1$	$0,0$
B	$0,0$	$0,0$

Game 1

	L	R
T	$0,0$	$0,0$
B	$0,0$	$2,2$

Game 2

(i) Nature determines whether the payoffs are as in Game 1 or as in Game 2, each game being equally likely.

(ii) Player 1 learns whether nature has drawn Game 1 or Game 2, but player 2 does not.

(iii) Player 1 chooses either T or B; player 2 simultaneously chooses either L or R.

(iv) Payoffs are given by the game drawn by nature.

Exercise 4. Consider the following asymmetric-information model of Bertrand duopoly with differentiated products. Demand for firm i is $q_i(p_i, p_j) = a - p_i - b_i p_j$. Costs are zero for both firms. The sensitivity of firm i's demand to firm j's price is either high or low. That is, b_i is either b_H or b_L, where $b_H > b_L > 0$. For each firm, $b_i = b_H$ with probability θ and $b_i = b_L$ with probability $1 - \theta$, independent of the realization of b_j. Each firm knows its own b_i, but not its competitor's. All of this is common knowledge.

(a) What are the action spaces, type spaces, beliefs, and utility functions in this game?

(b) What are the strategy spaces?

(c) What conditions define a symmetric pure-strategy Bayesian Nash equilibrium of this game? Solve for such an equilibrium.