# 12
# Superconductivity

## *Chapter Outline*



Most of the material covered in this chapter has to do with the phenomenon of superconductivity. As we shall see, magnetic fields play an important role in the field of superconductivity, so it is important to understand the magnetic properties of materials before discussing the properties of superconductors.

All magnetic effects in matter can be explained on the basis of the current loops associated with atomic magnetic dipole moments. These atomic magnetic moments arise both from the orbital motion of the electrons and from an intrinsic property of the electrons known as *spin.* Our description of magnetism in matter is based in part on the experimental fact that the presence of bulk matter generally modifies the magnetic field produced by currents. For example, when a material is placed inside a current-carrying solenoid, the material sets up its own magnetic field, which adds vectorially to the field that was already present.

The phenomenon of superconductivity has always been very exciting, both for its fundamental scientific interest and because of its many applications.[1] The discovery in the 1980s of high-temperature superconductivity in certain metallic oxides sparked even greater excitement in the scientific and business communities. Many scientists consider this major breakthrough to be as important as the invention of the transistor. For this reason, it is important that all students of science and engineering understand the basic electromagnetic properties of superconductors and become aware of the scope of their current applications.

Superconductors have many unusual electromagnetic properties, and most applications take advantage of such properties. For example, once a current is produced in a superconducting ring maintained at a sufficiently low temperature, that current persists with no measurable decay. The superconducting ring exhibits no electrical resistance to direct currents, no heating, and no losses. In addition to the property of zero resistance, certain superconductors expel applied magnetic fields so that the field is always zero everywhere inside the superconductor.

As we shall see, classical physics cannot explain the behavior and properties of superconductors. In fact, the superconducting state is now known to be a special quantum condensation of electrons. This quantum behavior has been verified through such observations as the quantization of magnetic flux produced by a superconducting ring.

In this chapter we also give a brief historical review of superconductivity, beginning with its discovery in 1911 and ending with recent developments in high-temperature superconductivity. In describing some of the electromagnetic properties displayed by superconductors, we use simple physical arguments whenever possible. We explain the essential features of the theory of superconductivity with the realization that a detailed study is beyond the scope of this text. Finally, we discuss many of the important applications of superconductivity and speculate on potential applications.

## 12.1 MAGNETISM IN MATTER

The magnetic field produced by a current in a coil of wire gives a hint as to what might cause certain materials to exhibit strong magnetic properties. In general, any current loop has a magnetic field and a corresponding magnetic moment. Similarly, the magnetic moments in a magnetized substance are associated with internal currents on the atomic level. One can view such currents as arising from electrons orbiting around the nucleus and protons orbiting about each other inside the nucleus. However, as we shall see, the intrinsic

[1] M. Brian Maple, a research physicist at the University of California at San Diego, was asked what he found so fascinating about superconductivity. He responded as follows. “For me the fascination of superconductivity is associated with the words perfect, infinite, and zero. A superconductor has the property of being a perfect conductor, or having infinite conductivity, or zero resistance. Whenever you see something that’s perfect, infinite, or zero, truly zero, that’s obviously a special state of affairs.”

magnetic moment associated with the electron is the main source of magnetism in matter.

We begin with a brief discussion of the magnetic moments due to electrons. The mutual forces between these magnetic dipole moments and their interaction with an external magnetic field are of fundamental importance to an understanding of the behavior of magnetic materials. We shall describe three categories of materials—paramagnetic, ferromagnetic, and diamagnetic. **Paramagnetic** and **ferromagnetic** materials are those that have atoms with permanent magnetic dipole moments. **Diamagnetic** materials are those whose atoms have no permanent magnetic dipole moments.

## Magnetic Moments of Atoms

As we learned in Section 8.2, the total magnetic moment of an atom has orbital and spin contributions. For atoms or ions containing many electrons, the electrons in closed shells pair up with their spins and orbital angular momenta opposite each other, a situation that results in a net magnetic moment of zero. However, atoms with an odd number of electrons must have at least one "unpaired" electron and a spin magnetic moment of at least one Bohr magneton, $\mu_B$, where

$$\mu_B = \frac{e\hbar}{2m_e} = 9.274 \times 10^{-24}\ \text{J/T} \qquad (12.1)$$

The total magnetic moment of an atom is the vector sum of the orbital and spin magnetic moments, and an unpaired outer electron can contribute both an orbital moment and a spin moment. For example, if the unpaired electron is in an $s$ state, $L = 0$ and consequently the orbital moment is zero. However, if the unpaired electron is in a $p$ or $d$ state, $L \neq 0$ and the electron contributes both an orbital moment and a spin moment. The orbital moment is about the same order of magnitude as the Bohr magneton. Table 12.1 gives a few examples of total magnetic moments for different elements. Note that helium and neon have zero moments because their closed shells cause individual moments to cancel.

**Table 12.1 Magnetic Moments of Some Atoms and Ions**

| Atom (or Ion) | Magnetic Moment ($10^{-24}$ J/T) |
|---|---|
| H | 9.27 |
| He | 0 |
| Ne | 0 |
| $Ce^{3+}$ | 19.8 |
| $Yb^{3+}$ | 37.1 |

The nucleus of an atom also has a magnetic moment associated with its constituent protons and neutrons. However, the magnetic moment of a proton or neutron is small compared with the magnetic moment of an electron and can usually be neglected. Because the masses of the proton and neutron are much greater than that of the electron, their magnetic moments are smaller than that of the electron by a factor of approximately $10^3$.

## Magnetization and Magnetic Field Strength

The magnetization of a substance is described by a quantity called the **magnetization vector, M.** *The magnitude of the magnetization vector is equal to the magnetic moment per unit volume of the substance.* As you might expect, the total magnetic field in a substance depends on both the applied (external) field and the magnetization of the substance.

Consider a region where there exists a magnetic field $\mathbf{B}_0$ produced by a current-carrying conductor. If we now fill that region with a magnetic substance, the total magnetic field $\mathbf{B}$ in that region is $\mathbf{B} = \mathbf{B}_0 + \mathbf{B}_m$, where $\mathbf{B}_m$ is the field produced by the magnetic substance. This contribution can be expressed in terms of the magnetization vector as $\mathbf{B}_m = \mu_0\mathbf{M}$; hence the total magnetic field in the region becomes

$$\mathbf{B} = \mathbf{B}_0 + \mu_0\mathbf{M} \tag{12.2}$$

It is convenient to introduce a field quantity **H**, called the **magnetic field strength.** This vector quantity is defined by the relationship $\mathbf{H} = (\mathbf{B}/\mu_0) - \mathbf{M}$, or

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) \tag{12.3}$$

In SI units, the dimensions of both **H** and **M** are amperes per meter.

To better understand these expressions, consider the space enclosed by a solenoid that carries a current $I$. (We call this space the *core* of the solenoid.) If this space is a vacuum, then $\mathbf{M} = 0$ and $\mathbf{B} = \mathbf{B}_0 = \mu_0\mathbf{H}$. Since $B_0 = \mu_0 nI$ in the core, where $n$ is the number of turns per unit length of the solenoid, then $H = B_0/\mu_0 = \mu_0 nI/\mu_0$, or

$$H = nI \tag{12.4}$$

That is, the magnetic field strength in the core of the solenoid is due to the current in its windings.

If the solenoid core is now filled with some substance and the current $I$ is kept constant, **H** inside the substance remains unchanged and has magnitude $nI$. This is because the magnetic field strength **H** is due solely to the current in the solenoid. However, the total field **B** changes when the substance is introduced. From Equation 12.3, we see that part of **B** arises from the term $\mu_0\mathbf{H}$ associated with the current in the solenoid; the second contribution to **B** is the term $\mu_0\mathbf{M}$, due to the magnetization of the substance filling the core.

## Classification of Magnetic Substances

In a large class of substances, specifically paramagnetic and diamagnetic substances, the magnetization vector **M** is proportional to the magnetic field strength **H**. For these substances we can write

$$\mathbf{M} = \chi\mathbf{H} \tag{12.5}$$

where $\chi$ (Greek letter chi) is a dimensionless factor called the **magnetic susceptibility.** If the sample is paramagnetic, $\chi$ is positive, in which case **M** is in the same direction as **H**. If the substance is diamagnetic, $\chi$ is negative and **M** is opposite **H**. It is important to note that this linear relationship between **M** and **H** does not apply to ferromagnetic substances. Table 12.2 gives the susceptibilities of some substances. Substituting Equation 12.5 for **M** into Equation 12.3 gives

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(\mathbf{H} + \chi\mathbf{H}) = \mu_0(1 + \chi)\mathbf{H}$$

or

$$\mathbf{B} = \mu_m\mathbf{H} \tag{12.6}$$

where the constant $\mu_m$ is called the **magnetic permeability** of the substance and has the value

$$\mu_m = \mu_0(1 + \chi) \tag{12.7}$$

**Table 12.2 Magnetic Susceptibilities of Some Paramagnetic and Diamagnetic Substances at 300 K**

| Paramagnetic Substance | $\chi$ | Diamagnetic Substance | $\chi$ |
|---|---|---|---|
| Aluminum | $2.3 \times 10^{-5}$ | Bismuth | $-1.66 \times 10^{-5}$ |
| Calcium | $1.9 \times 10^{-5}$ | Copper | $-9.8 \times 10^{-6}$ |
| Chromium | $2.7 \times 10^{-4}$ | Diamond | $-2.2 \times 10^{-5}$ |
| Lithium | $2.1 \times 10^{-5}$ | Gold | $-3.6 \times 10^{-5}$ |
| Magnesium | $1.2 \times 10^{-5}$ | Lead | $-1.7 \times 10^{-5}$ |
| Niobium | $2.6 \times 10^{-4}$ | Mercury | $-2.9 \times 10^{-5}$ |
| Oxygen (STP) | $2.1 \times 10^{-6}$ | Nitrogen (STP) | $-5.0 \times 10^{-9}$ |
| Platinum | $2.9 \times 10^{-4}$ | Silver | $-2.6 \times 10^{-5}$ |
| Tungsten | $6.8 \times 10^{-5}$ | Silicon | $-4.2 \times 10^{-6}$ |

Substances may also be classified in terms of how their magnetic permeabilities $\mu_m$ compare to $\mu_0$ (the permeability of free space), as follows:

Paramagnetic $\mu_m > \mu_0$

Diamagnetic $\mu_m < \mu_0$

Ferromagnetic $\mu_m \gg \mu_0$

Since $\chi$ is very small for paramagnetic and diamagnetic substances (Table 12.2), $\mu_m$ is nearly equal to $\mu_0$ in these cases. For ferromagnetic substances, however, $\mu_m$ is typically several thousand times larger than $\mu_0$ but is not a constant. Although Equation 12.6 provides a simple relationship between **B** and **H**, it must be interpreted with care in the case of ferromagnetic substances. As mentioned earlier, **M** is not a linear function of **H** for ferromagnetic substances. This is because the value of $\mu_m$ *is not a characteristic of the substance* but rather depends on the previous state and treatment of the ferromagnetic material.

**EXAMPLE 12.1 An Iron-Filled Toroid**

A toroid carrying a current of 5.00 A is wound with 60 turns/m of wire. The core is made of iron, which has a magnetic permeability of $5000\mu_0$ under the given conditions. Find $H$ and $B$ inside the iron.

**Solution** Using Equations 12.4 and 12.6, we get

$$H = nI = \left(60.0\ \frac{\text{turns}}{\text{m}}\right)(5.00\ \text{A}) = 300\ \frac{\text{A}\cdot\text{turns}}{\text{m}}$$

$$B = \mu_m H = 5000\mu_0 H$$

$$= 5000\left(4\pi \times 10^{-7}\ \frac{\text{Wb}}{\text{A}\cdot\text{m}}\right)\left(300\ \frac{\text{A}\cdot\text{turns}}{\text{m}}\right) = 1.88\ \text{T}$$

This value of $B$ is 5000 times larger than the value in the absence of iron!

***Exercise*** Determine the magnitude and direction of the magnetization inside the iron core.

***Answer*** $M = 1.5 \times 10^6$ A/m; **M** is in the direction of **H**.

## Ferromagnetism

Spontaneous magnetization occurs in some substances whose atomic constituents have permanent magnetic dipole moments. The magnetic moments exhibit long-range order, which can take on various forms, as shown in Figure

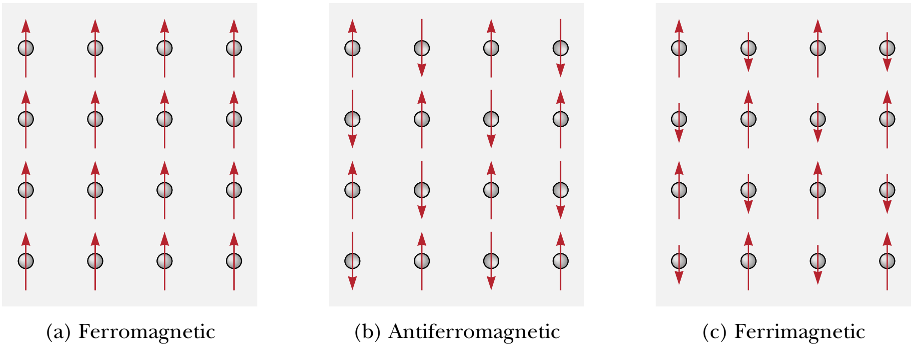



**Figure 12.1** Magnetic ordering in three types of solids. (a) In a ferromagnetic substance, all spins are aligned. (b) In an antiferromagnetic substance, spins in two sublattices have the same magnitude but are opposite in direction. (c) In a ferrimagnetic substance, spins in the two sublattices have different magnitudes *and* opposite directions.

12.1. The magnetic moments of a ferromagnet tend to be aligned as in Figure 12.1a, and hence a ferromagnetic substance has a net magnetization. This permanent alignment is due to a strong coupling between neighboring moments, which can be understood only in quantum mechanical terms. In an antiferromagnetic substance (Fig. 12.1b), the magnetic moments all have the same magnitude. However, because the magnetic moments in the sublattices are oppositely directed, the net magnetization of an antiferromagnet is zero. In a ferrimagnetic substance (Fig. 12.1c), the magnetic moments of the atoms in the two sublattices are oppositely directed, but their magnitudes are not the same. Hence a ferrimagnetic substance has a net magnetization.

Iron, cobalt, and nickel are ferromagnetic at sufficiently low temperatures. Rare earths such as gadolinium and terbium are ferromagnetic below room temperature, while other rare earths are ferromagnetic at very low temperatures. At extremely high temperatures, all transition and rare-earth metals become paramagnetic.

All ferromagnetic materials contain microscopic regions called **domains,** within which all magnetic moments are aligned. Each of these domains has a volume of about $10^{-12}$ to $10^{-8}$ $m^3$ and contains $10^{17}$ to $10^{21}$ atoms. The boundaries between domains having different orientations are called **domain walls.** In an unmagnetized sample, the domains are randomly oriented such that the net magnetic moment is zero, as shown in Figure 12.2a. When the sample is placed in an external magnetic field, the domains tend to align with the field, which results in a magnetized sample, as in Figure 12.2b. Observations show that domains initially oriented along the external field grow in size at the expense of the less favorably oriented domains. When the external field is removed, the sample may retain a net magnetization in the direction of the original field.[2] At ordinary temperatures, thermal agitation is not sufficiently high to disrupt this preferred orientation of magnetic moments.

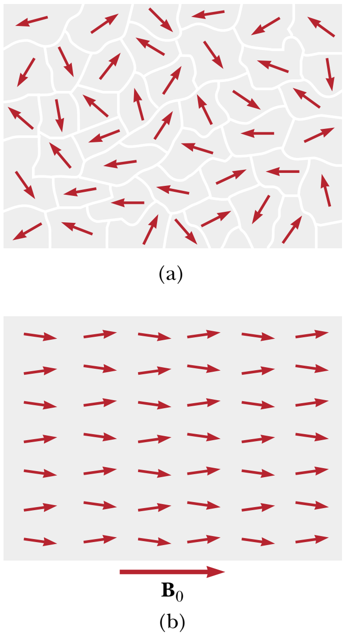



**Figure 12.2** (a) Random orientation of atomic magnetic dipoles in an unmagnetized substance. (b) When an external magnetic field $\mathbf{B}_0$ is applied, the magnetic dipoles tend to align with the field, giving the sample a net magnetization **M**.

[2] It is possible to observe the domain walls directly and follow their motion under a microscope. In this technique, a liquid suspension of powdered ferromagnetic substance is applied to the sample. The fine particles tend to accumulate at the domain walls and shift with them.

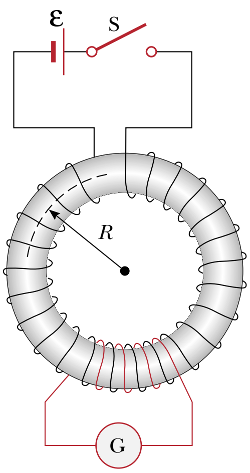



**Figure 12.3** A toroidal winding arrangement used to measure the magnetic properties of a substance. The material under study fills the core of the toroid, and the circuit containing the galvanometer measures the magnetic flux.

A typical experimental arrangement used to measure the magnetic properties of a ferromagnetic material consists of a toroid-shaped sample wound with $N$ turns of wire, as in Figure 12.3. This configuration is sometimes referred to as a **Rowland ring.** A secondary coil connected to a galvanometer is used to measure the magnetic flux. The magnetic field **B** within the core of the toroid is measured by increasing the current in the toroid coil from zero to $I$. As the current changes, the magnetic flux through the secondary coil changes by $BA$, where $A$ is the cross-sectional area of the toroid. Because of this changing flux, an emf is induced in the secondary coil that is proportional to the rate of change in magnetic flux. If the galvanometer in the secondary circuit is properly calibrated, one can obtain a value for **B** corresponding to any value of the current in the toroidal coil. The magnetic field **B** is measured first in the empty coil and then with the same coil filled with the magnetic substance. The magnetic properties of the substance are then obtained from a comparison of the two measurements.

Now consider a toroid whose core consists of unmagnetized iron. If the current in the windings is increased from zero to some value $I$, the field intensity $H$ increases linearly with $I$ according to the expression $H = nI$. Furthermore, the total field $B$ also increases with increasing current, as shown in Figure 12.4. At point $O$, the domains are randomly oriented, corresponding to $B_m = 0$. As the external field increases, the domains become more aligned until all are nearly aligned at point $a$. At this point, the iron core is approaching saturation. (The condition of saturation corresponds to the case where all domains are aligned in the same direction.) Next, suppose the current is reduced to zero, thereby eliminating the external field. The $B$-versus-$H$ curve, called a **magnetization curve,** now follows the path $ab$ shown in Figure 12.4. Note that at point $b$, the field **B** is not zero, although the external field is equal to zero. This is explained by the fact that the iron core is now magnetized due to the alignment of a large number of domains (that is, $\mathbf{B} = \mathbf{B}_m$). At this point, the iron is said to have a remanent magnetization and could be considered to be a "permanent" magnet. If the external field is reversed in direction and increased in strength by reversal of the current, the domains reorient until the sample is again unmagnetized at point $c$, where $\mathbf{B} = 0$. A further increase in the reverse current causes the iron to be magnetized in the opposite direction, approaching saturation at point $d$. A similar sequence of events occurs as the current is reduced to zero and then increased in the original (positive) direction. In this case, the magnetization curve follows the path $def$. If the current is increased sufficiently, the magnetization curve returns to point $a$, where the sample again has its maximum magnetization.

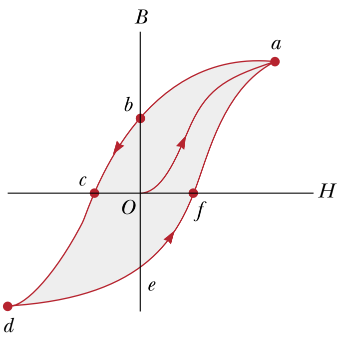



**Figure 12.4** A hysteresis curve for a ferromagnetic material.

The effect just described, called **magnetic hysteresis,** shows that the magnetization of a ferromagnetic substance depends on the history of the substance as well as the strength of the applied field. (The word *hysteresis* literally means "to lag behind.") One often says that a ferromagnetic substance has a "memory" since it remains magnetized after the external field is removed. The closed loop in Figure 12.4 is referred to as a hysteresis loop. Its shape and size depend on the properties of the ferromagnetic substance and on the strength of the maximum applied field. The hysteresis loop for "hard" ferromagnetic materials (those used in permanent magnets) is characteristically wide as in Figure 12.5a, corresponding to a large remanent magnetization. Such materials cannot be easily demagnetized by an external field. This is in contrast with "soft" ferro-

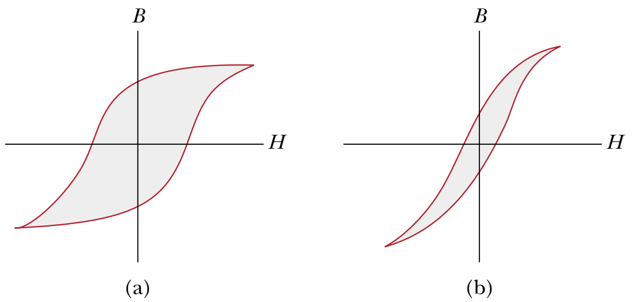



**Figure 12.5** Hysteresis curves for (a) a hard ferromagnetic material and (b) a soft ferromagnetic material.

magnetic materials, such as iron, which have very narrow hysteresis loops and small remanent magnetizations (Fig. 12.5b). Such materials are easily magnetized and demagnetized. An ideal soft ferromagnet would exhibit no hysteresis and hence would have no remanent magnetization. One can demagnetize a ferromagnetic substance by carrying the substance through successive hysteresis loops and gradually decreasing the applied field, as in Figure 12.6.

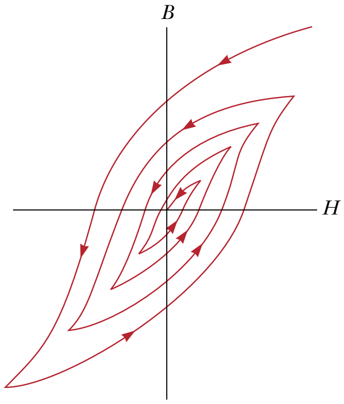



**Figure 12.6** Demagnetizing a ferromagnetic material by carrying it through successive hysteresis loops.

The magnetization curve is useful for another reason. *The area enclosed by the magnetization curve represents the work required to take the material through the hysteresis cycle.* The source of the external field—that is, the emf in the circuit of the toroidal coil—supplies the energy acquired by the sample in the magnetization process. When the magnetization cycle is repeated, dissipative processes within the material due to realignment of the domains result in a transformation of magnetic energy into internal thermal energy, which raises the temperature of the substance. For this reason, devices subjected to alternating fields (such as transformers) use cores made of soft ferromagnetic substances, which have narrow hysteresis loops and correspondingly small energy losses per cycle.

## Paramagnetism

A paramagnetic substance has a positive but small susceptibility ($0 < \chi \ll 1$), which is due to the presence of atoms (or ions) with permanent magnetic dipole moments. These dipoles interact only weakly with each other and are randomly oriented in the absence of an external magnetic field. When the substance is placed in an external magnetic field, its atomic dipoles tend to line up with the field. However, this alignment process must compete with thermal motion, which tends to randomize the dipole orientations.

Experimentally, one finds that the magnetization of a paramagnetic substance is proportional to the applied field and inversely proportional to the absolute temperature under a wide range of conditions. That is,

$$M = C\frac{B}{T} \tag{12.8}$$

**Curie's law**

This is known as **Curie's law** after its discoverer, Pierre Curie (1859–1906), and the constant $C$ is called **Curie's constant** (Problem 13). This shows that the magnetization increases with increasing applied field and with decreasing temperature. When $B = 0$, the magnetization is zero, corresponding to a random orientation of dipoles. At very high fields or very low temperatures, the

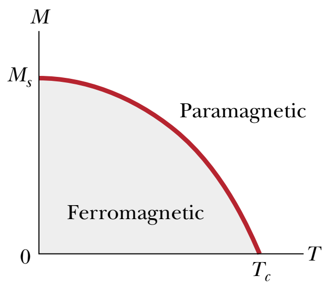



**Figure 12.7** A plot of the magnetization versus absolute temperature for a ferromagnetic substance. The magnetic moments are aligned (ordered) below the Curie temperature $T_c$, where the substance is ferromagnetic. The substance becomes paramagnetic, that is, disordered above $T_c$.

magnetization approaches its maximum (saturation) value, corresponding to a complete alignment of its dipoles, and Equation 12.8 is no longer valid.

Interestingly, when the temperature of a ferromagnetic substance reaches or exceeds a critical temperature, called the **Curie temperature,** the substance loses its spontaneous magnetization and becomes paramagnetic (Fig. 12.7). Below the Curie temperature, the magnetic moments are aligned and the substance is ferromagnetic. Above the Curie temperature, the thermal energy is large enough to cause a random orientation of dipoles; hence the substance becomes paramagnetic. For example, the Curie temperature for iron is 1043 K. Table 12.3 lists Curie temperatures and saturation magnetization values for several ferromagnetic substances.

## Diamagnetism

A diamagnetic substance is one whose atoms have no permanent magnetic dipole moment. When an external magnetic field is applied to a diamagnetic substance such as bismuth or silver, a weak magnetic dipole moment is induced in the direction opposite the applied field (Lenz's law). Although the effect of diamagnetism is present in all matter, it is weak compared to paramagnetism or ferromagnetism.

We can obtain some understanding of diamagnetism by considering two electrons of an atom orbiting the nucleus in opposite directions but with the same speed. The electrons remain in these circular orbits because of the attractive electrostatic force (the centripetal force) of the positively charged nucleus. Because the magnetic moments of the two electrons are equal in magnitude and opposite in direction, they cancel each other, and the dipole moment of the atom is zero. When an external magnetic field is applied, the electrons experience an additional force $q\mathbf{v} \times \mathbf{B}$. This added force modifies the central force and thereby increases the orbital speed of the electron whose magnetic moment is antiparallel to the field and decreases the speed of the electron whose magnetic moment is parallel to the field. As a result, the magnetic moments of the electrons no longer cancel, and the substance acquires a net dipole moment that opposes the applied field. It is important to note that

**Table 12.3 Curie Temperatures and Saturation Magnetizations for Several Ferromagnetic Substances**

| Substance | $T_c$ (K) | $M_s$ ($10^6$ A/m) |
|---|---|---|
| Iron | 1043 | 1.75 |
| Cobalt | 1404 | 1.45 |
| Nickel | 631 | 0.512 |
| Gadolinium | 289 | 2.00 |
| Terbium | 230 | 1.44 |
| Dysprosium | 85 | 2.01 |
| Holmium | 20 | 2.55 |

Source of data: D. W. Gray, ed., *American Institute of Physics Handbook*, New York, McGraw-Hill, 1963.

this is a classical explanation. Quantum mechanics is needed for a complete explanation of diamagnetism.

**EXAMPLE 12.2 Saturation Magnetization of Iron**

Estimate the maximum magnetization in a long cylinder of iron, assuming there is one unpaired electron spin per atom.

**Solution** The maximum magnetization, called the saturation magnetization, is attained when all the magnetic moments in the sample are aligned. If the sample contains $n$ atoms per unit volume, then the saturation magnetization $M_s$ has the value

$$M_s = n\mu$$

where $\mu$ is the magnetic moment per atom. Since the molecular weight of iron is 56 g/mol and its density is 7.9 g/cm$^3$, the value of $n$ is $8.5 \times 10^{28}$ atoms/m$^3$. Assuming each atom contributes one Bohr magneton (due to one unpaired spin) to the magnetic moment, we get

$$M_s = \left(8.5 \times 10^{28} \frac{\text{atoms}}{\text{m}^3}\right)\left(9.27 \times 10^{-24} \frac{\text{A} \cdot \text{m}^2}{\text{atom}}\right)$$
$$= 7.9 \times 10^5 \text{ A/m}$$

This is about one-half the experimentally determined saturation magnetization for annealed iron, which indicates that there are actually two unpaired electron spins per atom.

## 12.2 A BRIEF HISTORY OF SUPERCONDUCTIVITY

The era of low-temperature physics began in 1908 when the Dutch physicist Heike Kamerlingh Onnes first liquefied helium, which boils at 4.2 K at standard pressure. Three years later, in 1911, Kamerlingh Onnes and one of his assistants discovered the phenomenon of superconductivity while studying the resistivity of metals at low temperatures.[3] They first studied platinum and found that its resistivity, when extrapolated to 0 K, depended on purity. They then decided to study mercury because very pure samples could easily be prepared by distillation. Much to their surprise, the resistance of the mercury sample dropped sharply at 4.15 K to an unmeasurably small value. It was quite natural that Kamerlingh Onnes would choose the name **superconductivity** for this new phenomenon of perfect conductivity. Figure 12.8 shows the experimental results for mercury and platinum. Note that platinum does not exhibit superconducting behavior, as indicated by its finite resistivity as $T$ approaches 0 K. In 1913 Kamerlingh Onnes was awarded the Nobel prize in physics for the study of matter at low temperatures and the liquefaction of helium.

We now know that the resistivity of a superconductor is truly zero. Soon after the discovery by Kamerlingh Onnes, many other elemental metals were found to exhibit zero resistance when their temperatures were lowered below a certain characteristic temperature of the material, called the **critical temperature,** $T_c$.

The magnetic properties of superconductors are as dramatic and as difficult to understand as their complete lack of resistance. In 1933 W. Hans Meissner and Robert Ochsenfeld studied the magnetic behavior of superconductors and found that when certain ones are cooled below their critical temperatures in the presence of a magnetic field, *the magnetic flux is expelled from the interior of the*

[3] H. Kamerlingh Onnes, *Leiden Comm.*, 120b, 122b, 124c, 1911.

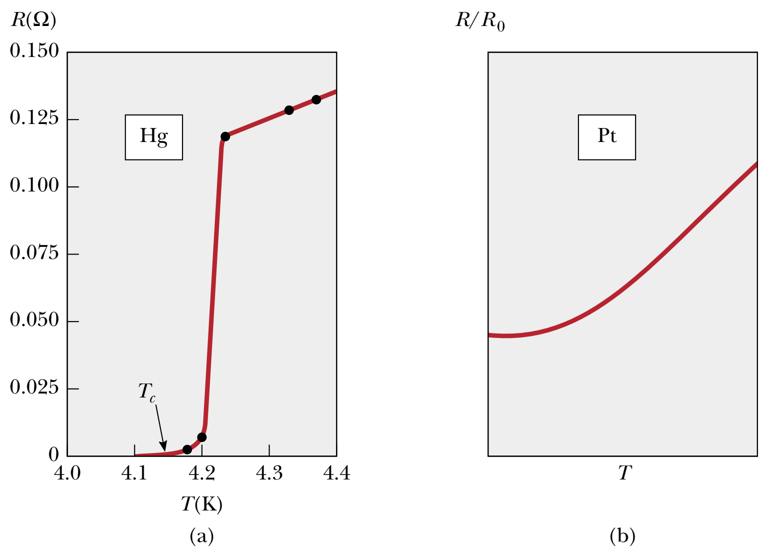



**Figure 12.8** Plots of resistance versus temperature for (a) mercury (the original data published by Kamerlingh Onnes) and (b) platinum. Note that the resistance of mercury follows the path of a normal metal above the critical temperature, $T_c$, and then suddenly drops to zero at the critical temperature, which is 4.15 K for mercury. In contrast, the data for platinum show a finite resistance $R_0$ even at very low temperatures.

*superconductor.*[4] Furthermore, these materials lost their superconducting behavior above a certain temperature-dependent **critical magnetic field,** $B_c(T)$. In 1935 Fritz and Heinz London developed a phenomenological theory of superconductivity,[5] but the actual nature and origin of the superconducting state were first explained by John Bardeen, Leon N. Cooper, and J. Robert Schrieffer in 1957.[6] A central feature of this theory, commonly referred to as the BCS theory, is the formation of bound two-electron states called **Cooper pairs.** In 1962 Brian D. Josephson predicted a tunneling current between two superconductors separated by a thin ($<2$ mm) insulating barrier, where the current is carried by these paired electrons.[7] Shortly thereafter, Josephson's predictions were verified, and today there exists a whole field of device physics based on the Josephson effect. Early in 1986 J. Georg Bednorz and Karl Alex Müller reported evidence for superconductivity in an oxide of lanthanum, barium, and copper at a temperature of about 30 K.[8] This was a major breakthrough in superconductivity because the highest known value of $T_c$ at that time was about 23 K in a compound of niobium and germanium. This remarkable discovery, which marked the beginning of a new era of high-temperature superconductivity, received worldwide attention in both the scientific community and the business world. Recently, researchers have reported critical tempera-

[4] W. H. Meissner and R. Ochsenfeld, *Naturwisschaften* 21:787, 1933.

[5] F. London and H. London, *Proc. Roy. Soc. (London)* A149:71, 1935.

[6] J. Bardeen, L. N. Cooper, and J. R. Schrieffer, *Phys. Rev.* 108:1175, 1957.

[7] B. D. Josephson, *Phys. Letters* 1:251, 1962.

[8] J. G. Bednorz and K. A. Müller, *Z. Phys.* B64:189, 1986.

tures as high as 150 K in more complex metallic oxides, but the mechanisms responsible for superconductivity in these materials remain unclear.

Until the discovery of high-temperature superconductors, the use of superconductors required coolant baths of liquefied helium (rare and expensive) or liquid hydrogen (very explosive). On the other hand, superconductors with $T_c > 77$ K require only liquid nitrogen, which boils at 77 K and is comparatively inexpensive, abundant, and relatively inert. If superconductors with $T_c$'s above room temperature are ever found, technology will be drastically altered.

## 12.3 SOME PROPERTIES OF TYPE I SUPERCONDUCTORS

### Critical Temperature and Critical Magnetic Field

Table 12.4 lists the critical temperatures of some superconducting elements, classified as **type I superconductors.** Note the absence of copper, silver, and gold, which are excellent electrical conductors at ordinary temperatures but do not exhibit superconductivity.

In the presence of an applied magnetic field $\mathbf{B}$, the value of $T_c$ decreases with increasing magnetic field, as indicated in Figure 12.9 for several type I superconductors. When the magnetic field exceeds the critical field, $\mathbf{B}_c$, the superconducting state is destroyed and the material behaves as a normal conductor with finite resistance.

The magnitude of the critical magnetic field varies with temperature according to the approximate expression

$$B_c(T) = B_c(0)\left[1 - \left(\frac{T}{T_c}\right)^2\right] \tag{12.9}$$

**Table 12.4 Critical Temperatures and Critical Magnetic Fields (at $T = 0$ K) of Some Elemental Superconductors**

| Superconductor | $T_c$ (K) | $B_c(0)$ (T) |
|---|---|---|
| Al | 1.196 | 0.0105 |
| Ga | 1.083 | 0.0058 |
| Hg | 4.153 | 0.0411 |
| In | 3.408 | 0.0281 |
| Nb | 9.26 | 0.1991 |
| Pb | 7.193 | 0.0803 |
| Sn | 3.722 | 0.0305 |
| Ta | 4.47 | 0.0829 |
| Ti | 0.39 | 0.010 |
| V | 5.30 | 0.1023 |
| W | 0.015 | 0.000115 |
| Zn | 0.85 | 0.0054 |

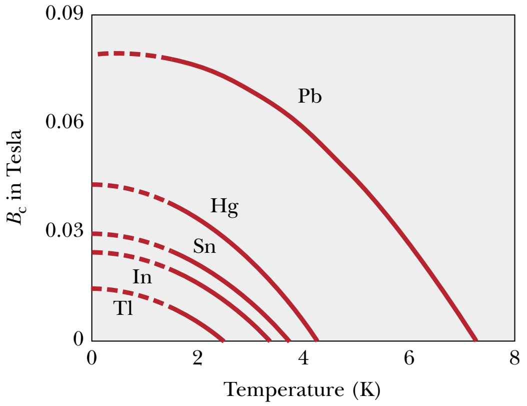



**Figure 12.9** Critical magnetic field versus critical temperature for several type I superconductors. Extrapolations of these fields to 0 K give the critical fields listed in Table 12.4. For a given metal, the material is superconducting at fields and temperatures below its critical temperature and behaves as a normal conductor above that curve.

As you can see from this equation and Figure 12.9, the value of $B_c$ is a maximum at 0 K. The value of $B_c(0)$ is found by determining $B_c$ at some finite temperature and extrapolating back to 0 K, a temperature that cannot be achieved in the laboratory. The value of the critical field limits the maximum current that can be sustained in a type I superconductor.

Note that $B_c(0)$ is the maximum magnetic field that is required to destroy superconductivity in a given material. If the applied field exceeds $B_c(0)$, the metal never becomes superconducting at any temperature. Values for the critical field for type I superconductors are quite low, as Table 12.4 shows. For this reason, type I superconductors are not used to construct high-field magnets, called superconducting magnets, because the magnetic fields generated by modest currents destroy, or "quench," the superconducting state.

## Magnetic Properties of Type I Superconductors

One can use simple arguments based on the laws of electricity and magnetism to show that the magnetic field inside a superconductor cannot change with time. According to Ohm's law, the electric field inside a conductor is proportional to the resistance of the conductor. Thus, since $R = 0$ for a superconductor, *the electric field in its interior must be zero.* Now recall that Faraday's law of induction can be expressed as

$$\oint \mathbf{E} \cdot d\mathbf{s} = -\frac{d\Phi_B}{dt} \qquad (12.10)$$

That is, the line integral of the electric field around any closed loop is equal to the negative rate of change in the magnetic flux $\Phi_B$ through the loop. Since $\mathbf{E}$ is zero everywhere inside the superconductor, the integral over any closed path inside the superconductor is zero. Hence $d\Phi_B/dt = 0$, which tells us that *the magnetic flux in the superconductor cannot change.* From this we can conclude that $B\ (= \Phi_B/A)$ *must remain constant inside the superconductor.*

Prior to 1933 it was assumed that superconductivity was a manifestation of perfect conductivity. If a perfect conductor is cooled below its critical temperature in the presence of an applied magnetic field, the field should be trapped in the interior of the conductor even after the field is removed. The final state of a perfect conductor in an applied magnetic field should depend upon which occurs first, the application of the field or the cooling below the critical temperature. If the field is applied after cooling below $T_c$, the field should be expelled from the superconductor. On the other hand, if the field is applied before cooling, the field should not be expelled from the superconductor after cooling below $T_c$.

When experiments were conducted in the 1930s to examine the magnetic behavior of superconductors, the results were quite different. In 1933 Meissner and Ochsenfeld[4] discovered that, when a metal became superconducting in the presence of a weak magnetic field, the field was expelled so that **B** equaled 0 everywhere in the interior of the superconductor. Thus the same final state, $\mathbf{B} = 0$, was achieved whether the field was applied before or after the material was cooled below its critical temperature. Figure 12.10 illustrates this effect for a material in the shape of a long cylinder. Note that the field penetrates the cylinder when its temperature is greater than $T_c$ (Fig. 12.10a). As the temperature is lowered below $T_c$, however, the field lines are spontaneously expelled from the interior of the superconductor (Fig. 12.10b). Thus a type I superconductor is more than a perfect conductor (resistivity $\rho = 0$); it is also a perfect diamagnet ($\mathbf{B} = 0$). The phenomenon of the expulsion of magnetic fields from the interior of a superconductor is known as the Meissner effect. The property that $\mathbf{B} = 0$ in the interior of a type I superconductor is as fundamental as the property of zero resistance and shows the important role that magnetism plays in superconductivity. If the applied field is sufficiently large ($B > B_c$), the superconducting state is destroyed and the field penetrates the sample.

Because a superconductor is a perfect diamagnet, it repels a permanent magnet. In fact, one can perform a dazzling demonstration of the Meissner effect by floating a small permanent magnet above a superconductor and

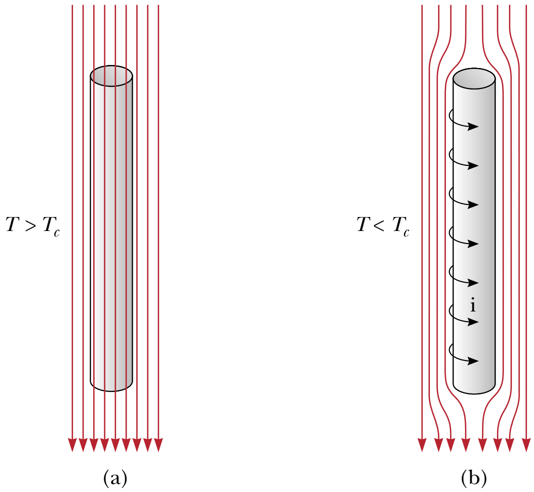



**Figure 12.10** A type I superconductor in the form of a long cylinder in the presence of an external magnetic field. (a) At temperatures above $T_c$, the field lines penetrate the sample because it is in its normal state. (b) When the rod is cooled to $T < T_c$ and becomes superconducting, magnetic flux is excluded from its interior by the induction of surface currents.

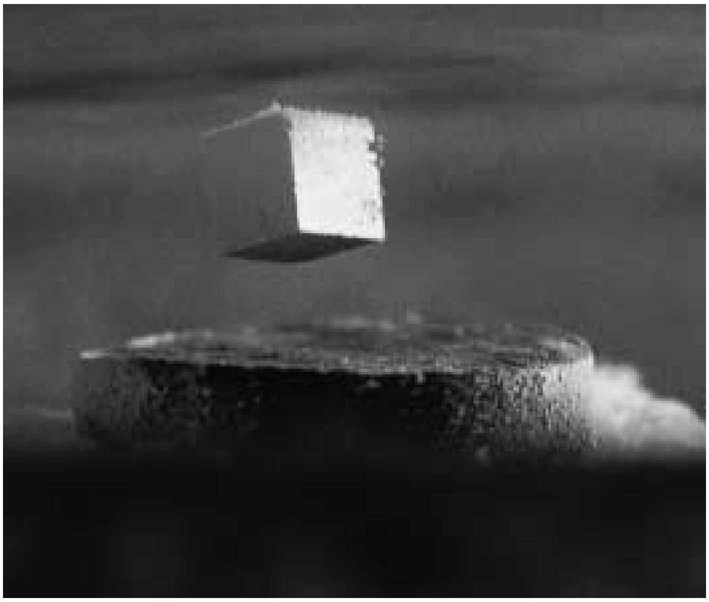


**Figure 12.11** A small permanent magnet levitated above a pellet of the $YBa_2Cu_3O_{7-\delta}$ superconductor cooled to the temperature of liquid nitrogen, 77 K. *(Courtesy of IBM Research)*

achieving magnetic levitation. Figure 12.11 is a dramatic photograph of magnetic levitation. The details of this demonstration are provided in Questions 16 through 19.

You should recall from your study of electricity that a good conductor expels static electric fields by moving charges to its surface. In effect, the surface charges produce an electric field that exactly cancels the externally applied field inside the conductor. In a similar manner, a superconductor expels magnetic fields by forming surface currents. To illustrate this point, consider again the superconductor in Figure 12.10. Let us assume that the sample is initially at a temperature $T > T_c$, as in Figure 12.10a, so that the field penetrates the cylinder. As the cylinder is cooled to a temperature $T < T_c$, the field is expelled as in Figure 12.10b. In this case, surface currents are induced on the superconductor, producing a magnetic field that exactly cancels the externally applied field inside the superconductor. As you might expect, the surface currents disappear when the external magnetic field is removed.

### EXAMPLE 12.3 Critical Current in a Pb Wire

A lead wire has a radius of 3.00 mm and is at a temperature of 4.20 K. Find (a) the critical magnetic field in lead at this temperature and (b) the maximum current the wire can carry at this temperature and still remain superconducting.

**Solution** (a) We can use Equation 12.9 to find the critical field at any temperature if $B_c(0)$ and $T_c$ are known. From Table 12.4 we see that the critical magnetic field of lead at 0 K is 0.0803 T and its critical temperature is 7.193 K. Hence Equation 12.9 gives

$$B_c(4.2\text{ K}) = (0.0803\text{ T})\left[1 - \left(\frac{4.20}{7.193}\right)^2\right] = 0.0529\text{ T}$$

(b) According to Ampère's law, if a wire carries a steady current $I$, the magnetic field generated at an exterior point a distance $r$ from the wire is

$$B = \frac{\mu_0 I}{2\pi r}$$

When the current in the wire equals a certain critical current $I_c$, the magnetic field at the wire surface equals the critical magnetic field $B_c$. (Note that $B = 0$ inside, because all the current is on the wire surface.) Using the preceding expression and taking $r$ equal to the radius of the wire, we find

$$I = \frac{2\pi r B}{\mu_0} = 2\pi\frac{(3.00\times 10^{-3}\text{ m})(0.0529\text{ T})}{4\pi\times 10^{-7}\text{ N/A}^2} = 794\text{ A}$$

## Penetration Depth

As we have seen, magnetic fields are expelled from the interior of a type I superconductor by the formation of surface currents. In reality, these currents are not formed in an infinitesimally thin layer on the surface. Instead, they penetrate the surface to a small extent. Within this thin layer, which is about 100 nm thick, the magnetic field **B** decreases exponentially from its external value to zero, according to the expression

$$B(x) = B_0 e^{-x/\lambda} \qquad (12.11)$$

where it is assumed that the external magnetic field is parallel to the surface of the sample. In this equation, $B_0$ is the value of the magnetic field at the surface, $x$ is the distance from the surface to some interior point, and $\lambda$ is a parameter called the **penetration depth.** The variation of magnetic field with distance inside a type I superconductor is plotted in Figure 12.12. The superconductor occupies the region on the positive side of the $x$ axis. As you can see, the magnetic field becomes very small at depths a few times $\lambda$ below the surface. Values for $\lambda$ are typically in the range 10 to 100 nm.

Penetration depth varies with temperature according to the empirical expression

$$\lambda(T) = \lambda_0\left[1 - \left(\frac{T}{T_c}\right)^2\right]^{-1/2} \qquad (12.12)$$

where $\lambda_0$ is the penetration depth at 0 K. From this expression we see that $\lambda$ becomes infinite as $T$ approaches $T_c$. Furthermore, as $T$ approaches $T_c$, while the sample is in the superconducting state, an applied magnetic field penetrates more and more deeply into the sample. Ultimately, the field penetrates the entire sample ($\lambda$ becomes infinite), and the sample becomes normal.

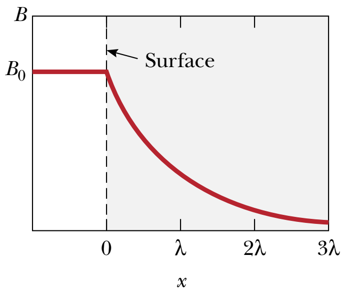



**Figure 12.12** The magnetic field **B** inside a superconductor versus distance $x$ from the surface of the superconductor. The field outside the superconductor (for $x < 0$) is $B_0$, and the superconductor is to the right of the dashed line.

## Magnetization

When a bulk sample is placed in an external magnetic field $\mathbf{B}_0$, the sample acquires a magnetization **M** (see Section 12.1). The magnetic field **B** inside the sample is related to $\mathbf{B}_0$ and **M** through the relationship $\mathbf{B} = \mathbf{B}_0 + \mu_0\mathbf{M}$. When the sample is in the superconducting state, $\mathbf{B} = 0$; therefore it follows that the magnetization is

$$\mathbf{M} = -\frac{\mathbf{B}_0}{\mu_0} = \chi\mathbf{H} \tag{12.13}$$

where $\chi$ (= − 1) is the magnetic susceptibility. That is, whenever a material is in a superconducting state, its magnetization opposes the external magnetic field and the magnetic susceptibility has its maximum negative value. Again we see that a type I superconductor is a perfect diamagnetic substance.

Figure 12.13a is a plot of magnetic field inside a type I superconductor versus external field (parallel to a long cylinder) at $T < T_c$. (Long cylinders are used to minimize end effects.) The magnetization versus external field at some constant temperature is plotted in Figure 12.13b. Note that when $B_0 > B_c$, the magnetization is approximately zero.

With the discovery of the Meissner effect, Fritz and Heinz London were able to develop phenomenological equations for type I superconductors based on equilibrium thermodynamics. They could explain the critical magnetic field in terms of the energy increase of the superconducting state, an increase resulting from the exclusion of flux from the interior of the superconductor. According to equilibrium thermodynamics, a system prefers to be in the state having the lowest free energy. Hence, the superconducting state must have a lower free energy than the normal state. If $E_s$ represents the energy of the superconducting state per unit volume and $E_n$ the energy of the normal state per unit volume, then $E_s < E_n$ below $T_c$ and the material becomes superconducting. The exclusion of a field $B$ causes the total energy of the superconducting state to increase by an amount equal to $B^2/2\mu_0$ per unit volume. The critical field value is defined by the equation

$$E_s + \frac{B_c^2}{2\mu_0} = E_n \tag{12.14}$$

Because the London theory also gives the temperature dependence of $E_s$, an exact expression for $B_c(T)$ could be obtained. Note that the field exclusion energy $B_c^2/2\mu_0$ is just the area under the curve in Figure 12.13b.

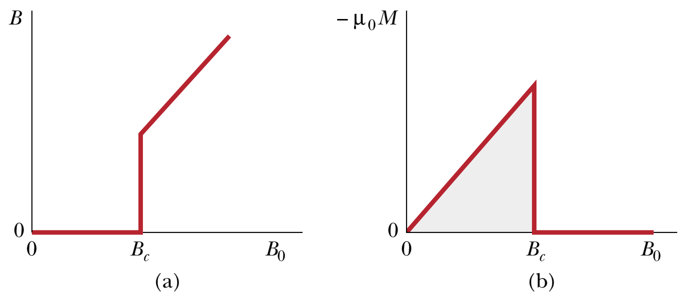



**Figure 12.13** The magnetic field–dependent properties of a type I superconductor. (a) A plot of internal field versus applied field, where $\mathbf{B} = 0$ for $B_0 < B_c$. (b) A plot of magnetization versus applied field. Note that $\mathbf{M} \approx 0$ for $B_0 > B_c$.

## 12.4 TYPE II SUPERCONDUCTORS

By the 1950s researchers knew there was another class of superconductors, which they called type II superconductors. These materials are characterized by two critical magnetic fields, designated $B_{c1}$ and $B_{c2}$ in Figure 12.14. When the external magnetic field is less than the lower critical field $B_{c1}$, the material is entirely superconducting and there is no flux penetration, just as with type I superconductors. When the external field exceeds the upper critical field $B_{c2}$, the flux penetrates completely and the superconducting state is destroyed, just as for type I materials. For fields lying between $B_{c1}$ and $B_{c2}$, however, the material is in a mixed state, referred to as the **vortex state.** (This name is given because of swirls of currents that are associated with this state.) While in the vortex state, the material can have zero resistance and has partial flux penetration. Vortex regions are essentially filaments of normal material that run through the sample when the external field exceeds the lower critical field, as illustrated in Figure 12.15. As the strength of the external field increases, the number of filaments increases until the field reaches the upper critical value, and the sample becomes normal.

One can view the vortex state as a cylindrical swirl of supercurrents surrounding a cylindrical normal-metal core that allows some flux to penetrate the interior of the type II superconductor. Associated with each vortex filament is a magnetic field that is greatest at the core center and falls off exponentially outside the core with the characteristic penetration depth $\lambda$. The supercurrents are the "source" of **B** for each vortex. In type II superconductors, the radius of the normal-metal core is smaller than the penetration depth.

Table 12.5 gives critical temperatures and $B_{c2}$ values for several type II superconductors. The values of $B_{c2}$ are very large in comparison with those of $B_c$ for type I superconductors. For this reason, type II superconductors are well suited for the construction of high-field superconducting magnets. For example, using the alloy NbTi, superconducting solenoids may be wound to produce magnetic fields in the range 5 to 10 T. Furthermore, they require no power to maintain the field. Iron-core electromagnets rarely exceed 2 T and consume

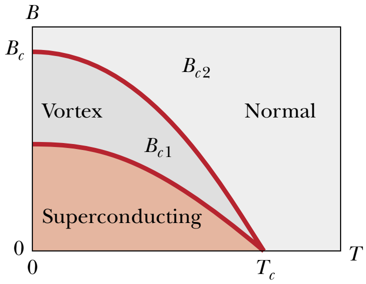



**Figure 12.14** Critical magnetic fields as a function of temperature for a type II superconductor. Below $B_{c1}$, the material behaves as a type I superconductor. Above $B_{c2}$, the material behaves as a normal conductor. Between these two fields, the superconductor is in the vortex (mixed) state.

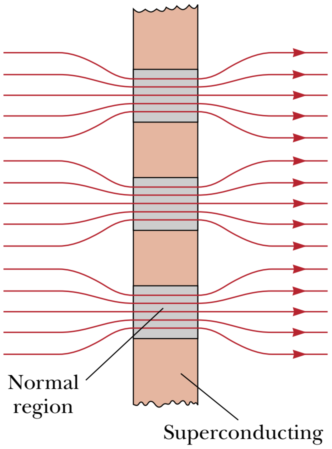



**Figure 12.15** A schematic diagram of a type II superconductor in the vortex state. The sample contains filaments of normal (unshaded) regions through which magnetic field lines can pass. The field lines are excluded from the superconducting (shaded) regions.

**Table 12.5 Critical Temperatures and Upper Critical Magnetic Fields (at $T$ = 0 K) of Some Type II Superconductors**

| Superconductor | $T_c$ (K) | $B_{c2}(0)$ (T) |
|---|---|---|
| $Nb_3Al$ | 18.7 | 32.4 |
| $Nb_3Sn$ | 18.0 | 24.5 |
| $Nb_3Ge$ | 23 | 38 |
| NbN | 15.7 | 15.3 |
| NbTi | 9.3 | 15 |
| $Nb_3(AlGe)$ | 21 | 44 |
| $V_3Si$ | 16.9 | 23.5 |
| $V_3Ga$ | 14.8 | 20.8 |
| PbMoS | 14.4 | 60 |

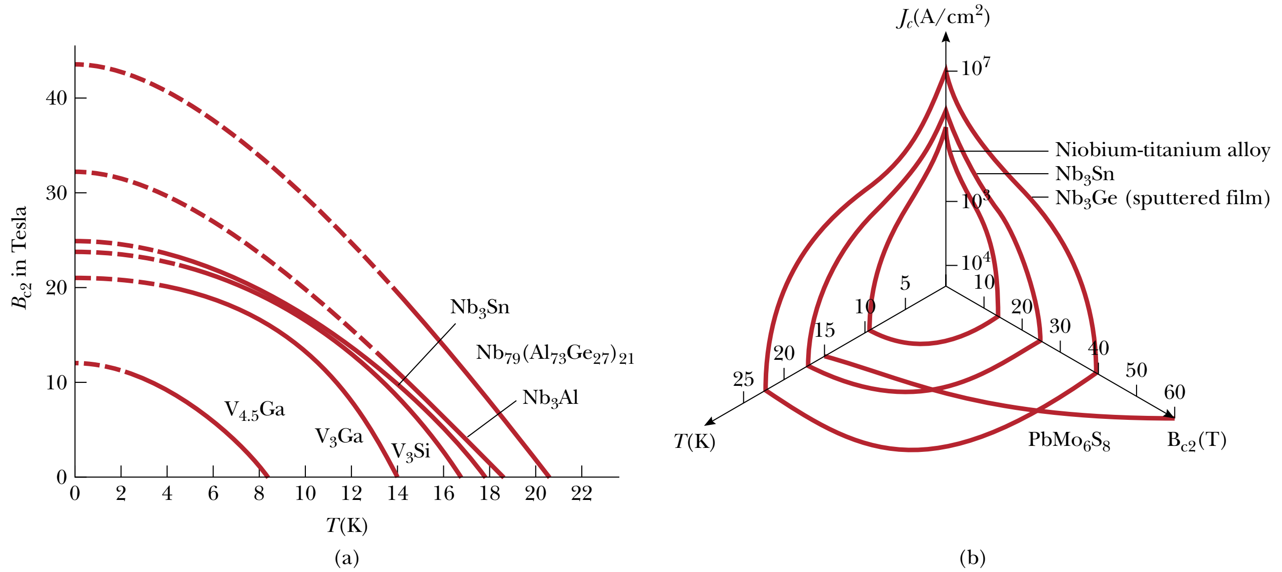



**Figure 12.16** (a) Upper critical field, $B_{c2}$, as a function of temperature for several type II superconductors. *(From S. Foner, et al.,* Physics Letters *31A:349, 1970)* (b) A three-dimensional plot showing the variation of critical current density, $J_c$, with temperature, and the variation of the upper critical field with temperature for several type II superconductors.

power to maintain the field. Notice also that type II superconductors are compounds formed from elements of the transition and actinide series. Plots of $B_{c2}$ variations with temperature appear in Figure 12.16a. The three-dimensional plot in Figure 12.16b shows the variation of critical temperature with both $B_{c2}$ and critical current density, $J_c$.

Figure 12.17a shows internal magnetic field versus external field for a type II superconductor, while Figure 12.17b shows the corresponding magnetization versus external field.

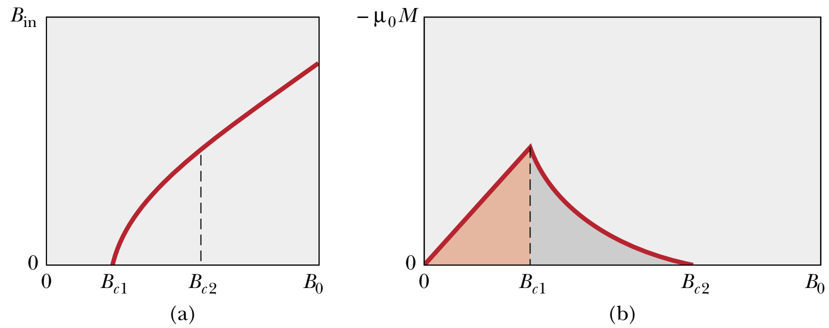



**Figure 12.17** The magnetic behavior of a type II superconductor. (a) A plot of internal field versus external field. (b) A plot of magnetization versus external field.

When a type II superconductor is in the vortex state, sufficiently large currents can cause the vortices to move perpendicular to the current. This vortex motion corresponds to a change in flux with time and produces resistance in the material. By adding impurities or other special inclusions, one can effectively pin the vortices and prevent their motion, to produce zero resistance in the vortex state. The critical current for type II superconductors is the current that, when multiplied by the flux in the vortices, gives a Lorentz force that overcomes the pinning force.

**EXAMPLE 12.4 A Type II Superconducting Solenoid**

A solenoid is to be constructed from wire made of the alloy $Nb_3Al$, which has an upper critical field of 32.0 T at $T = 0$ K and a critical temperature of 18.0 K. The wire has a radius of 1.00 mm, the solenoid is to be wound on a hollow cylinder of diameter 8.00 cm and length 90.0 cm, and there are to be 150 turns of wire per centimeter of length. (a) How much current is required to produce a magnetic field of 5.00 T at the center of the solenoid?

**Solution** The magnetic field at the center of a tightly wound solenoid is $B = \mu_0 nI$, where $n$ is the number of turns per unit length along the solenoid, and $I$ is the current in the solenoid wire. Taking $n = 150$ turns/cm $= 1.50 \times 10^4$ turns/m, and $B = 5.00$ T, we find

$$I = \frac{B}{\mu_0 n} = \frac{5.00 \text{ T}}{(4\pi \times 10^{-7} \text{ N/A}^2)(1.50 \times 10^4 \text{ m}^{-1})} = 265 \text{ A}$$

(b) What maximum current can the solenoid carry if its temperature is to be maintained at 15.0 K and it is to remain superconducting? (Note that $B$ near the solenoid windings is approximately equal to $B$ on its axis.)

**Solution** Using Equation 12.9, with $B_c(0) = 32.0$ T, we find $B_c = 9.78$ T at a temperature of 15.0 K. For this value of $B$, we find $I_{max} = 518$ A.

## 12.5 OTHER PROPERTIES OF SUPERCONDUCTORS

### Persistent Currents

Because the dc resistance of a superconductor is zero below the critical temperature, once a current is set up in the material, it persists *without any applied voltage* (which follows from Ohm's law and the fact that $R = 0$). These **persistent currents,** sometimes called supercurrents, have been observed to last for several years with no measurable losses. In one experiment conducted by S. S. Collins in Great Britain, a current was maintained in a superconducting ring for 2.5 years, stopping only because a trucking strike delayed delivery of the liquid helium that was necessary to maintain the ring below its critical temperature.[9]

To better understand the origin of persistent currents, consider a loop of wire made of a superconducting material. Suppose the loop is placed, in its normal state ($T > T_c$), in an external magnetic field, and then the temperature is lowered below $T_c$ so that the wire becomes superconducting, as in Figure 12.18a. As with a cylinder, the flux is excluded from the interior of the wire because of the induced surface currents. However, note that *flux lines still pass through the hole in the loop.* When the external field is turned off, as in Figure 12.18b, *the flux through this hole is trapped because the magnetic flux through the loop*

[9] This charming story was provided by Steve Van Wyk.

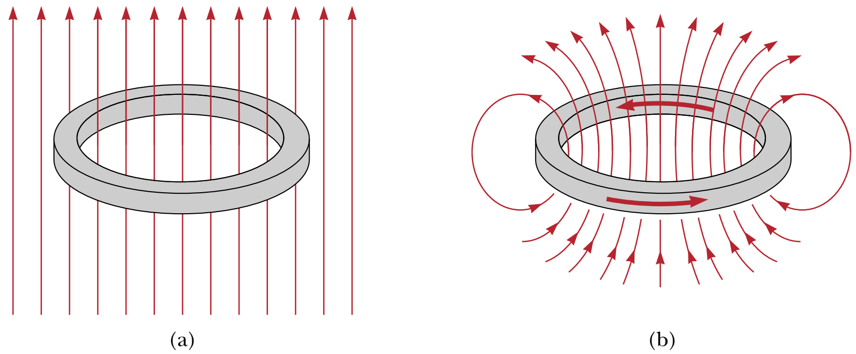



**Figure 12.18** (a) When a superconducting loop at $T > T_c$ is placed in an external magnetic field and the temperature is then lowered to $T < T_c$, flux passes through the hole in the loop even though it does not penetrate the interior of the material forming the loop. (b) After the external field is removed, the flux through the hole remains trapped, and an induced current appears in the material forming the loop.

*cannot change.*[10] The superconducting wire prevents the flux from going to zero through the advent of a large spontaneous current induced by the collapsing external magnetic field. If the dc resistance of the superconducting wire is truly zero, this current should persist forever. Experimental results using a technique known as nuclear magnetic resonance indicate that such currents will persist for more than $10^5$ years! The resistivity of a superconductor based on such measurements has been shown to be less than $10^{-26}\ \Omega\cdot\text{m}$. This reaffirms the fact that $R$ is zero for a superconductor. (See Problem 43 for a simple but convincing demonstration of zero resistance.)

Now consider what happens if the loop is cooled to a temperature $T < T_c$ before the external field is turned on. When the field is turned on while the loop is maintained at this temperature, *flux must be excluded from the entire loop, including the hole,* because the loop is in the superconducting state. Again, a current is induced in the loop to maintain zero flux through the loop and through the interior of the wire. In this case, the current disappears when the external field is turned off.

## Coherence Length

Another important parameter associated with superconductivity is the **coherence length,** $\xi$. The coherence length is the smallest dimension over which superconductivity can be established or destroyed. Table 12.6 lists typical values of the penetration depth, $\lambda$, and $\xi$ at 0 K for selected superconductors.

A superconductor is type I if $\xi < \lambda$; most of the pure metals that are superconductors fall into this category. An increase in the ratio $\lambda/\xi$ favors type II superconductivity. A detailed analysis shows that coherence length and penetration depth both depend on the mean free path of the electrons in the normal state. The mean free path of a metal can be reduced by the addition of impurities to the metal, which causes the penetration depth to increase while co-

[10] Alternatively, one can apply Equation 12.12, taking the line integral of the **E** field over the loop. Since $\rho = 0$ everywhere along a path on the superconductor, $\mathbf{E} = 0$, the integral is zero, and $d\Phi_B/dt = 0$.

**Table 12.6 Penetration Depths and Coherence Lengths of Selected Superconductors at $T = 0$ K[a]**

| Superconductor | $\lambda$ (nm) | $\xi$ (nm) |
|---|---|---|
| Al | 16 | 160 |
| Cd | 110 | 760 |
| Pb | 37 | 83 |
| Nb | 39 | 38 |
| Sn | 34 | 23 |

[a] These are calculated values from C. Kittel, *Introduction to Solid State Physics*, New York, John Wiley, 1986.

herence length decreases. Thus one can cause a metal to change from type I to type II by introducing an alloying element. For example, pure lead is a type I superconductor but changes to type II (with almost no change in $T_c$) when alloyed with 2% indium (by weight).

## Flux Quantization

The phenomenon of flux exclusion by a superconductor applies only to a simply connected object—that is, one with no holes or their topological equivalent. However, when a superconducting ring is placed in a magnetic field and the field is removed, flux lines are trapped and are maintained by a persistent circulating current, as shown in Figure 12.18b. Realizing that superconductivity is fundamentally a quantum phenomenon, Fritz London suggested that the trapped magnetic flux should be quantized in units of $h/e$.[11] (The electronic charge $e$ in the denominator arises because London assumed that the persistent current is carried by single electrons.) Subsequent delicate measurements on very small superconducting hollow cylinders showed that the flux quantum is one-half the value postulated by London.[12] That is, the magnetic flux $\Phi$ is quantized not in units of $h/e$ but in units of $h/2e$:

$$\Phi = \frac{nh}{2e} = n\Phi_0 \tag{12.15}$$

where $n$ is an integer and

$$\Phi_0 = \frac{h}{2e} = 2.0679 \times 10^{-15}\ \mathrm{T \cdot m^2} \tag{12.16}$$

Magnetic flux quantum

is the **magnetic flux quantum.**

[11] F. London, *Superfluids*, vol. I, New York, John Wiley, 1954.

[12] The effect was discovered by B. S. Deaver, Jr., and W. M. Fairbank, *Phys. Rev. Letters* 7:43, 1961, and independently by R. Doll and M. Nabauer, *Phys. Rev. Letters* 7:51, 1961.

## 12.6 ELECTRONIC SPECIFIC HEAT

The thermal properties of superconductors have been extensively studied and compared with those of the same materials in the normal state, and one very important measurement is specific heat. When a small amount of thermal energy is added to a normal metal, some of the energy is used to excite lattice vibrations, and the remainder is used to increase the kinetic energy of the conduction electrons. The electronic specific heat $C$ is defined as the ratio of the thermal energy absorbed by the electrons to the increase in temperature of the system.

Figure 12.19 shows how the electronic specific heat varies with temperature for both the normal state and the superconducting state of gallium, a type I superconductor. At low temperatures, the electronic specific heat of the material in the normal state, $C_n$, varies with temperature as $AT$, as explained in Chapter 11. The electronic specific heat of the material in the superconducting state, $C_s$, is substantially altered below the critical temperature. As the temperature is lowered starting from $T > T_c$, the specific heat first jumps to a very high value at $T_c$ and then falls below the value for the normal state at very low temperatures. Analyses of such data show that at temperatures well below $T_c$, the electronic part of the specific heat is dominated by a term that varies as $\exp(-\Delta/kT)$. A result of this form suggests the existence of an energy gap in the energy levels available to the electrons. We shall see that the energy gap is a measure of the thermal energy necessary to move electrons from a set of ground states (superconducting) to a set of excited states (normal), and that the energy gap is actually $2\Delta$.

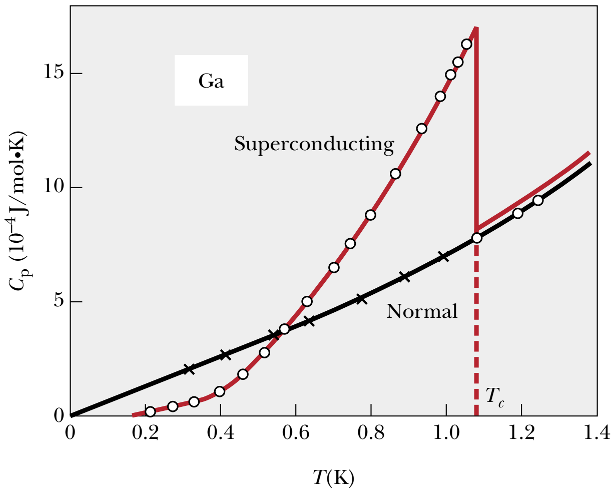



**Figure 12.19** Electronic specific heat versus temperature for superconducting gallium (in zero applied magnetic field) and normal gallium (in a 0.020-T magnetic field). For the superconducting state, note the discontinuity that occurs at $T_c$ and the exponential dependence on $1/T$ at low temperatures. *(Taken from N. Phillips,* Phys. Rev. *134: 385, 1964)*

## 12.7 BCS THEORY

According to classical physics, part of the resistivity of a metal is due to collisions between free electrons and thermally displaced ions of the metal lattice, and part is due to scattering of electrons from impurities or defects in the metal. Soon after the discovery of superconductivity, scientists recognized that this classical model could never explain the superconducting state, because the electrons in a material always suffer some collisions, and therefore resistivity can never be zero. Nor could superconductivity be understood through a simple microscopic quantum mechanical model, where one views an individual electron as an independent wave traveling through the material. Although many phenomenological theories based on the known properties of superconductors were proposed, none could explain why electrons enter the superconducting state and why electrons in this state are not scattered by impurities and lattice vibrations.

Several important developments in the 1950s led to better understanding of superconductivity. In particular, many research groups reported that the critical temperatures of isotopes of a metal decreased with increasing atomic mass. This observation, called the **isotope effect,** was early evidence that lattice motion played an important role in superconductivity. For example, in the case of mercury, $T_c = 4.161$ K for the isotope $^{199}$Hg, 4.153 K for $^{200}$Hg, and 4.126 K for $^{204}$Hg. The characteristic frequencies of the lattice vibrations are expected to change with the mass $M$ of the vibrating atoms. In fact, the lattice vibrational frequencies are expected to be proportional to $M^{-1/2}$ [analogous to the angular frequency $\omega$ of a mass-spring system, where $\omega = (k/M)^{1/2}$]. On this basis, it became apparent that any theory of superconductivity for metals must include electron-lattice interactions, which is somewhat surprising because electron-lattice interactions increase the resistance of normal metals.

The full microscopic theory of superconductivity presented in 1957 by Bardeen, Cooper, and Schrieffer has had good success in explaining the features of superconductors. The details of this theory, now known as the BCS theory, are beyond the scope of this text, but we can describe some of its main features and predictions.

The central feature of the BCS theory is that two electrons in the superconductor are able to form a bound pair called a **Cooper pair** if they somehow experience an attractive interaction. This notion at first seems counterintuitive since electrons normally repel one another because of their like charges. However, a net attraction can be achieved if the electrons interact with each other via the motion of the crystal lattice as the lattice structure is momentarily deformed by a passing electron.[13] To illustrate this point, Figure 12.20 shows two electrons moving through the lattice. The passage of electron 1 causes nearby ions to move inward toward the electron, resulting in a slight increase in the

[13] For a lively description of this process, see D. Teplitz, ed., *Electromagnetism: Path to Research*, New York, Plenum Press, 1982. In particular, see Chapter 1, "Electromagnetic Properties of Superconductors," by Brian B. Schwartz and Sonia Frota-Pessoa. Note that the electron that causes the lattice to deform remains in that region for a very short time, $\approx 10^{-16}$ s, compared to the much longer time it takes the lattice to deform, $\approx 10^{-13}$ s. Thus the sluggish ions continue to move inward for a time interval about 1000 times longer than the response time of the electron, so the region is effectively positively charged between $10^{-16}$ s and $10^{-13}$ s.

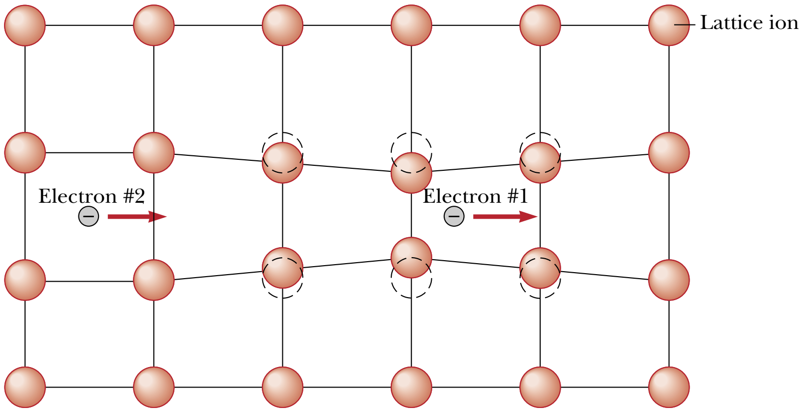
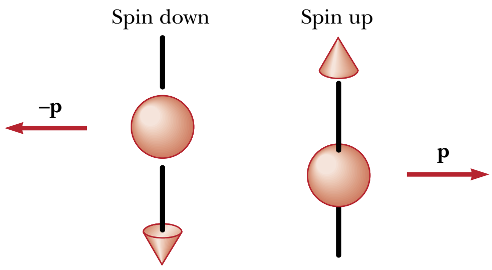



**Figure 12.20** The basis for the attractive interaction between two electrons via the lattice deformation. Electron 1 attracts the positive ions, which move inward from their equilibrium positions (dashed circles). This distorted region of the lattice has a net positive charge, and hence electron 2 is attracted to it.

concentration of positive charge in this region. Electron 2 (the second electron of the Cooper pair), approaching before the ions have had a chance to return to their equilibrium positions, is attracted to the distorted (positively charged) region. The net effect is a weak delayed attractive force between the two electrons, resulting from the motion of the positive ions. As one researcher has beautifully put it, "the following electron surfs on the virtual lattice wake of the leading electron."[14] In more technical terms, one can say that the attractive force between two Cooper electrons is an *electron-lattice-electron interaction,* where the crystal lattice serves as the mediator of the attractive force. Some scientists refer to this as a *phonon-mediated mechanism,* because quantized lattice vibrations are called *phonons.*

A Cooper pair in a superconductor consists of two electrons having opposite momenta and spin, as described schematically in Figure 12.21. In the superconducting state, the linear momenta can be equal and opposite, corresponding to no net current, or slightly different and opposite, corresponding to a

**Figure 12.21** A schematic diagram of a Cooper pair. The electron moving to the right has a momentum **p** and its spin is up, while the electron moving to the left has a momentum −**p** and its spin is down. Hence the total momentum of the system is zero and the total spin is zero.




[14] Many authors choose to refer to this cooperative state of affairs as a **collective state.** As an analogy, one author wrote that the electrons in the paired state "move like mountain-climbers tied together by a rope: should one of them leave the ranks due to the irregularities of the terrain (caused by the thermal vibrations of the lattice atoms) his neighbors would pull him back."

net superconducting current. Because Cooper pairs have zero spin, they can all be in the same state. This is in contrast with electrons, which are fermions (spin $\frac{1}{2}$) that must obey the Pauli exclusion principle. In the BCS theory, a ground state is constructed in which *all electrons form bound pairs.* In effect, all Cooper pairs are "locked" into the *same quantum state.* One can view this state of affairs as a condensation of all electrons into the same state. Also note that, because the Cooper pairs have zero spin (and hence zero angular momentum), their wavefunctions are spherically symmetric (like the *s*-states of the hydrogen atom.) In a "semiclassical" sense, the electrons are always undergoing head-on collisions and as such are always moving in each other's wakes. Because the two electrons are in a bound state, their trajectories always change directions in order to keep their separation within the coherence length.

The BCS theory has been very successful in explaining the characteristic superconducting properties of zero resistance and flux expulsion. From a qualitative point of view, one can say that in order to reduce the momentum of any single Cooper pair by scattering, it is necessary to simultaneously reduce the momenta of all the other pairs—in other words, it is an all-or-nothing situation. One cannot change the velocity of one Cooper pair without changing those of all of them.[15] Lattice imperfections and lattice vibrations, which effectively scatter electrons in normal metals, have no effect on Cooper pairs! In the absence of scattering, the resistivity is zero and the current persists forever. It is rather strange, and perhaps amazing, that the mechanism of lattice vibrations that is responsible (in part) for the resistivity of normal metals also provides the interaction that gives rise to their superconductivity. Thus, copper, silver, and gold, which exhibit small lattice scattering at room temperature, are not superconductors, whereas lead, tin, mercury, and other modest conductors have strong lattice scattering at room temperature and become superconductors at low temperatures.

As we mentioned earlier, the superconducting state is one in which the Cooper pairs act collectively rather than independently. The condensation of all pairs into the same quantum state makes the system behave as a giant quantum mechanical system or macromolecule that is quantized on the macroscopic level. *The condensed state of the Cooper pairs is represented by a single coherent wavefunction* $\Psi$ *that extends over the entire volume of the superconductor.*

The stability of the superconducting state is critically dependent on strong correlation between Cooper pairs. In fact, the theory explains superconducting behavior in terms of the energy levels of a kind of "macromolecule" and the existence of an energy gap $E_g$ between the ground and excited states of the system, as in Figure 12.22a. Note that in Figure 12.22b there is no energy gap for a normal conductor. In a normal conductor, the Fermi energy $E_F$ represents the largest kinetic energy the free electrons can have at 0 K.

The energy gap in a superconductor is very small, of the order of $k_B T_c$ ($\approx 10^{-3}$ eV) at 0 K, as compared with the energy gap in semiconductors ($\approx 1$ eV) or the Fermi energy of a metal ($\approx 5$ eV). The energy gap represents the energy needed to break apart a Cooper pair. The BCS theory predicts that

[15] A Cooper pair is somewhat analogous to a helium atom, $^4_2$He, in that both are bosons with zero spin. It is well known that the superfluidity of liquid helium may be viewed as a condensation of bosons in the ground state. Likewise, superconductivity may be viewed as a superfluid state of Cooper pairs, all in the same quantum state.

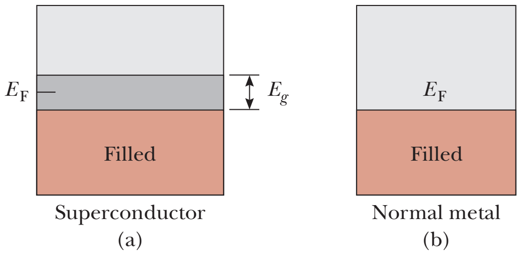



**Figure 12.22** (a) A simplified energy-band structure for a superconductor. Note the energy gap between the lower filled states and the upper empty states. (b) The energy-band structure for a normal conductor has no energy gap. At $T = 0$ K, all states below the Fermi energy $E_F$ are filled, and all states above it are empty.

at $T = 0$ K,

$$E_g = 3.53\ k_B T_c \tag{12.17}$$

Thus superconductors that have large energy gaps have relatively high critical temperatures. The exponential dependence of the electronic heat capacity discussed in the preceding section, $\exp(-\Delta/k_B T)$, contains an experimental factor, $\Delta = E_g/2$, that may be used to determine the value of $E_g$. Furthermore, the energy-gap values predicted by Equation 12.17 are in good agreement with the experimental values in Table 12.7. (The tunneling experiment used to obtain these values is described later.) As we noted earlier, the electronic heat capacity in zero magnetic field undergoes a discontinuity at the critical temperature. Furthermore, at finite temperatures, thermally excited individual electrons interact with the Cooper pairs and reduce the energy gap continuously from a peak value at 0 K to zero at the critical temperature, as shown in Figure 12.23 for several superconductors.

Because the two electrons of a Cooper pair have opposite spin angular momenta, an external magnetic field raises the energy of one electron and lowers

**Table 12.7 Energy Gaps for Some Superconductors at $T = 0$ K**

| Superconductor | $E_g$ (meV) |
|---|---|
| Al | 0.34 |
| Ga | 0.33 |
| Hg | 1.65 |
| In | 1.05 |
| Pb | 2.73 |
| Sn | 1.15 |
| Ta | 1.4 |
| Zn | 0.24 |
| La | 1.9 |
| Nb | 3.05 |

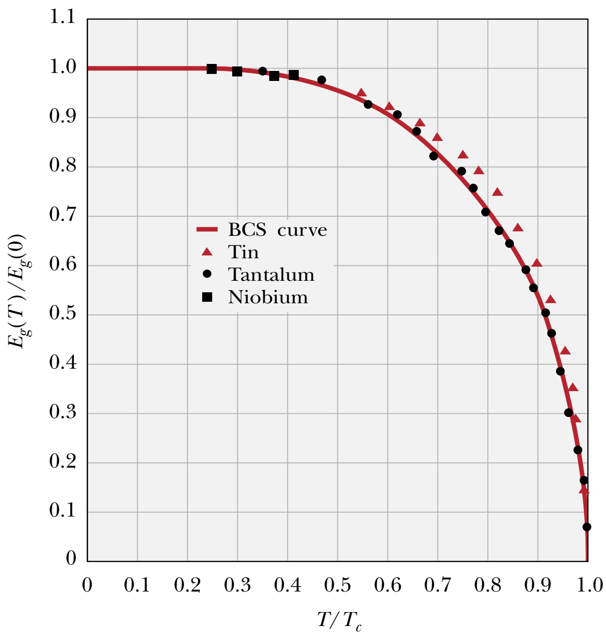



**Figure 12.23** The points on this graph represent reduced values of the observed energy gap $E_g(T)/E_g(0)$ as a function of the reduced temperature $T/T_c$ for tin, tantalum, and niobium. The solid curve gives the values predicted by the BCS theory. (*Data are from electron tunneling measurements by P. Townsend and J. Sutton,* Phys. Ref. *28*:591, 1962)

the energy of the other (Problem 29). If the magnetic field is made strong enough, it becomes energetically favorable for the pair to break up into a state where both spins point in the same direction and the superconducting state is destroyed. The value of the external field that causes the breakup corresponds to the critical field.

**EXAMPLE 12.5 The Energy Gap for Lead**

Use Equation 12.17 to calculate the energy gap for lead, and compare the answer with the experimental value in Table 12.7.

**Solution** Because $T_c = 7.193$ K for lead, Equation 12.17 gives

$$E_g = 3.53k_BT_c = (3.53)(1.38 \times 10^{-23} \text{ J/K})(7.193 \text{ K})$$
$$= 3.50 \times 10^{-22} \text{ J} = 0.00219 \text{ eV} = 2.19 \times 10^{-3} \text{ eV}$$

The experimental value is $2.73 \times 10^{-3}$ eV, corresponding to a difference of about 20%.

## 12.8 ENERGY GAP MEASUREMENTS

### Single-Particle Tunneling

The energy gaps in superconductors can be measured very precisely in single-particle tunneling experiments (those involving normal electrons), first reported by Giaever in 1960.[16] As described in Chapter 6, tunneling is a phenom-

[16] I. Giaever, *Phys. Rev Letters* 5:147, 464, 1960.

enon in quantum mechanics that enables a particle to penetrate and go through a barrier even though classically it has insufficient energy to go over the barrier. If two metals are separated by an insulator, the insulator normally acts as a barrier to the motion of electrons between the two metals. However, if the insulator is made sufficiently thin (less than about 2 nm), there is a small probability that electrons will tunnel from one metal to the other.

First consider two normal metals separated by a thin insulating barrier, as in Figure 12.24a. If a potential difference $V$ is applied between the two metals, electrons can pass from one metal to the other, and a current is set up. For small applied voltages, the current-voltage relationship is linear (the junction obeys Ohm's law). However, if one of the metals is replaced by a superconductor maintained at a temperature below $T_c$, as in Figure 12.24b, something quite unusual occurs. As $V$ increases, no current is observed until $V$ reaches a threshold value that satisfies the relationship $V_t = E_g/2e = \Delta/e$, where $\Delta$ is half the energy gap. (The voltage source provides the energy required to break a Cooper pair and free an electron to tunnel. The factor of one half comes from the fact that we are dealing with single-particle tunneling, and the energy required is one-half the binding energy of a pair, $2\Delta$.) That is, if $eV \geqslant 0.5E_g$, then tunneling can occur between the normal metal and the superconductor.

Thus, single-particle tunneling provides a direct experimental measurement of the energy gap. The value of $\Delta$ obtained from such experiments is in good

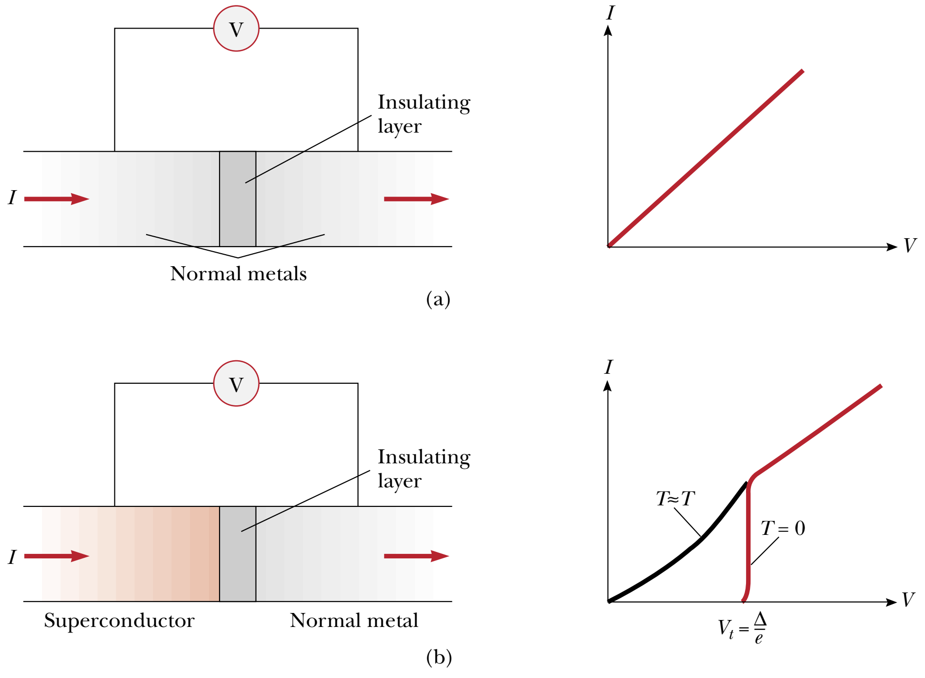



**Figure 12.24** (a) The current-voltage relationship for electron tunneling through a thin insulator between two normal metals. The relationship is linear for small currents and voltages. (b) The current-voltage relationship for electron tunneling through a thin insulator between a superconductor and a normal metal. The relationship is nonlinear and strongly temperature-dependent. (*Adapted from N. W. Ashcroft and N. D. Mermin,* Solid State Physics, *Philadelphia, Saunders College Publishing, 1975*)

agreement with the results of electronic heat capacity measurements. The *I-V* curve in Figure 12.24b shows the nonlinear relationship for this junction. Note that as the temperature increases toward $T_c$, some tunneling current occurs at voltages smaller than the energy-gap threshold voltage. This is due to a combination of thermally excited electrons and a decrease in the energy gap.

### Absorption of Electromagnetic Radiation

Another experiment used to measure the energy gaps of superconductors is the absorption of electromagnetic radiation. In superconductors, photons can be absorbed by the material when their energy is greater than the gap energy. Electrons in the valence band of the semiconductor absorb incident photons, exciting the electrons across the gap into the conduction band. In a similar manner, superconductors absorb photons if the photon energy exceeds the gap energy, $2\Delta$. If the photon energy is less than $2\Delta$, no absorption occurs. When photons are absorbed by the superconductor, Cooper pairs are broken apart. Photon absorption in superconductors typically occurs in the range between microwave and infrared frequencies, as the following example shows.

**EXAMPLE 12.6 Absorption of Radiation by Lead**

Find the minimum frequency of a photon that can be absorbed by lead at $T = 0$ K to break apart a Cooper pair.

**Solution** From Table 12.7 we see that the energy gap for lead is $2.73 \times 10^{-3}$ eV. Setting this equal to the photon energy $hf$, and noting that 1 eV $= 1.60 \times 10^{-19}$ J, we find

$$hf = 2\Delta = 2.73 \times 10^{-3} \text{ eV} = 4.37 \times 10^{-22} \text{ J}$$

$$f = \frac{4.37 \times 10^{-22} \text{ J}}{6.626 \times 10^{-34} \text{ J}\cdot\text{s}} = 6.60 \times 10^{11} \text{ Hz}$$

***Exercise*** What is the maximum wavelength of radiation that can be absorbed by lead at 0 K?

***Answer*** $\lambda = c/f = 0.455$ mm, which is in the far infrared region.

## 12.9 JOSEPHSON TUNNELING

In the preceding section we described single-particle tunneling from a normal metal through a thin insulating barrier into a superconductor. Now we consider tunneling between two superconductors separated by a thin insulator. In 1962 Brian Josephson proposed that, in addition to single particles, Cooper pairs can tunnel through such a junction. Josephson predicted that pair tunneling can occur without any resistance, producing a direct current when the applied voltage is zero and an alternating current when a dc voltage is applied across the junction.

At first, physicists were very skeptical about Josephson's proposal because it was believed that single-particle tunneling would mask pair tunneling. However, when the phase coherence of the pairs is taken into account, one finds that, under the appropriate conditions, the probability of tunneling of pairs across the junction is comparable to that of single-particle tunneling. In fact, when the insulating barrier separating the two superconductors is made sufficiently thin (say, $\approx 1$ nm) Josephson tunneling is as easy to observe as single-particle tunneling.

In the remainder of this section, we describe three remarkable effects associated with pair tunneling.

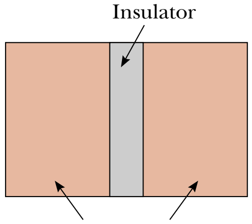



**Figure 12.25** A Josephson junction consists of two superconductors separated by a very thin insulator. Cooper pairs can tunnel through this insulating barrier in the absence of an applied voltage, setting up a direct current.

## The dc Josephson Effect

Consider two superconductors separated by a thin oxide layer, typically 1 to 2 nm thick, as in Figure 12.25. Such a configuration is known as a Josephson junction. In a given superconductor, the pairs could be represented by a wavefunction $\Psi = \Psi_0 e^{i\phi}$, where $\phi$ is the phase and is the same for every pair. If the superconductor to the left of the insulating layer has a phase $\phi_1$ and that to the right has a phase $\phi_2$, Josephson showed that at zero voltage there appears across the junction a supercurrent satisfying the relationship

$$I_s = I_{\text{max}} \sin(\phi_2 - \phi_1) = I_{\text{max}} \sin \delta \tag{12.18}$$

where $I_{\text{max}}$ is the maximum current across the junction under zero-voltage conditions. The value of $I_{\text{max}}$ depends on the surface area of each superconductor-oxide interface and decreases exponentially with the thickness of the oxide layer.

Rowell and Anderson made the first confirmation of the dc Josephson effect in 1963. Since then, all of Josephson's other theoretical predictions have been verified.

## The ac Josephson Effect

When a dc voltage $V$ is applied across the Josephson junction, a most remarkable effect occurs: *the dc voltage generates an alternating current I*, given by

$$I = I_{\text{max}} \sin(\delta + 2\pi f t) \tag{12.19}$$

where $\delta$ is a constant equal to the phase at $t = 0$ and $f$ is the frequency of the Josephson current:

$$f = \frac{2eV}{h} \tag{12.20}$$

A dc voltage of 1 $\mu$V results in a current frequency of 483.6 MHz. Precise measurements of the frequency and voltage have enabled physicists to obtain the ratio $e/h$ to unprecedented precision.

The ac Josephson effect can be demonstrated in various ways. One method is to apply a dc voltage and detect the electromagnetic radiation generated by the junction. Another method is to irradiate the junction with external radiation of frequency $f'$. With this method, a graph of direct current versus voltage has steps when the voltages correspond to Josephson frequencies $f$ that are integral multiples of the external frequency $f'$—that is, when $V = hf/2e = nhf'/2e$ (Fig. 12.26). Because the two sides of the junction are in different quantum states, the junction behaves as an atom undergoing transitions between these states as it absorbs or emits radiation. In effect, when a Cooper pair crosses the junction, a photon of frequency $f = 2eV/h$ is either emitted or absorbed by the system.

## Quantum Interference

Quantum interference is the behavior of the direct-tunneling current in the presence of an external magnetic field. When a Josephson junction is subjected to a magnetic field, the maximum critical current in the junction depends on

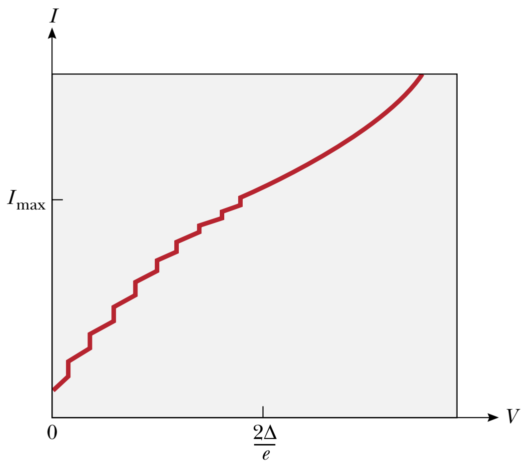
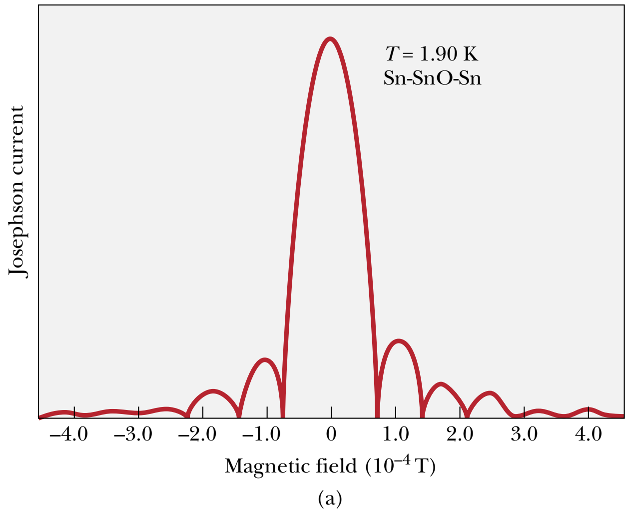



**Figure 12.26** A plot of dc current as a function of bias voltage for a Josephson junction placed in an electromagnetic field. At a frequency of 10 GHz, as many as 500 steps have been observed. (In the presence of an applied electromagnetic field, there is no clear onset of single-particle tunneling.)

the magnetic flux through the junction. The tunneling current under these conditions is predicted to be periodic in the number of flux quanta through the junction.

Figure 12.27a shows the maximum tunneling current as a function of magnetic field in an Sn-SnO-Sn junction at 1.90 K. Note that the current depends



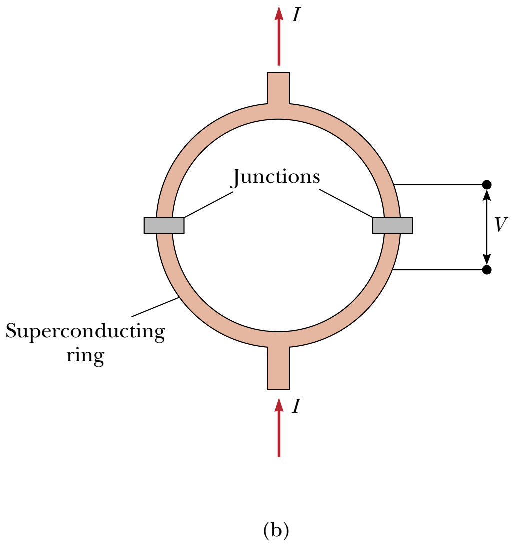




**Figure 12.27** (a) Maximum Josephson tunneling current as a function of applied magnetic field in an Sn-SnO-Sn junction. The zero current points are directly related to the number of flux quanta through the junction. (*From R. D. Parks, ed.,* Superconductivity, *vol. 1, New York, Dekker, 1969*) (b) A schematic diagram of a SQUID constructed from two Josephson junctions in parallel.

periodically on the magnetic flux. For typical junctions, the field periodicity is about $10^{-4}$ T. This is not surprising in view of the quantum nature of the magnetic flux.

If a superconducting circuit is constructed with two Josephson junctions in parallel, such as in the ring in Figure 12.27b, one can observe interference effects similar to the interference of light waves in Young's double-slit experiment. In this case, the total current depends periodically on the flux inside the ring. Because the ring can have an area much greater than that of a single junction, the magnetic field sensitivity is greatly increased.

A device that contains one or more Josephson junctions in a loop is called a SQUID, an acronym for *s*uperconducting *qu*antum *i*nterference *d*evice. SQUIDs are very useful for detecting very weak magnetic fields, of the order of $10^{-14}$ T, which is a very small fraction of the Earth's field ($B_{\text{Earth}} \approx 0.5 \times 10^{-4}$ T). Commercially available SQUIDs are capable of detecting a change in flux of about $10^{-5}\ \Phi_0 \approx 10^{-20}\ \text{T}\cdot\text{m}^2$ in a bandwidth of 1 Hz (where $\Phi_0$ is the magnetic flux quantum given by Eq. 12.16). For example, they are being used to scan "brain waves," a medical term for fields generated by current-carrying neurons. The essay by Clark Hamilton following this chapter contains a more detailed discussion of SQUIDs and other superconducting devices.

## 12.10 HIGH-TEMPERATURE SUPERCONDUCTIVITY

For many years, scientists have searched for materials that are superconductors at higher temperatures, and until 1986 the alloy $Nb_3Ge$ had the highest known critical temperature, 23.2 K. Theorists made various predictions that the maximum critical temperature for a superconductor in which the electron-lattice interaction was important would be in the neighborhood of 30 K. Early in 1986 J. Georg Bednorz and Karl Alex Müller of IBM Research Laboratory in Zurich made a remarkable discovery that has revolutionized the field of superconductivity. They found that an oxide of lanthanum, barium, and copper became superconducting at about 30 K. Figure 12.28 shows the temperature dependence of the resistivity of their samples. A portion of the abstract from this unpretentious paper reads as follows:

> Upon cooling, the samples show a linear decrease in resistivity, then an approximately logarithmic increase, interpreted as a beginning of localization. Finally, an abrupt decrease by up to three orders of magnitude occurs, reminiscent of the onset of percolative superconductivity. The highest onset temperature is observed in the 30 K range. It is markedly reduced by high current densities.[8]

The superconducting phase was soon identified at other laboratories as the compound $La_{2-x}Ba_xCuO_4$, where $x \approx 0.2$. By replacing barium with strontium, researchers soon raised the value of $T_c$ to about 36 K. Inspired by these developments, scientists worldwide worked feverishly to discover materials with even higher $T_c$ values, and research in the superconducting behavior of metallic oxides accelerated at a tremendous pace. The year 1986 marked the beginning of a new era of high-temperature superconductivity. Bednorz and Müller were awarded the Nobel prize in 1987 (the fastest-ever recognition by the Nobel committee) for their very important discovery.

Early in 1987 research groups at the University of Alabama at Huntsville and

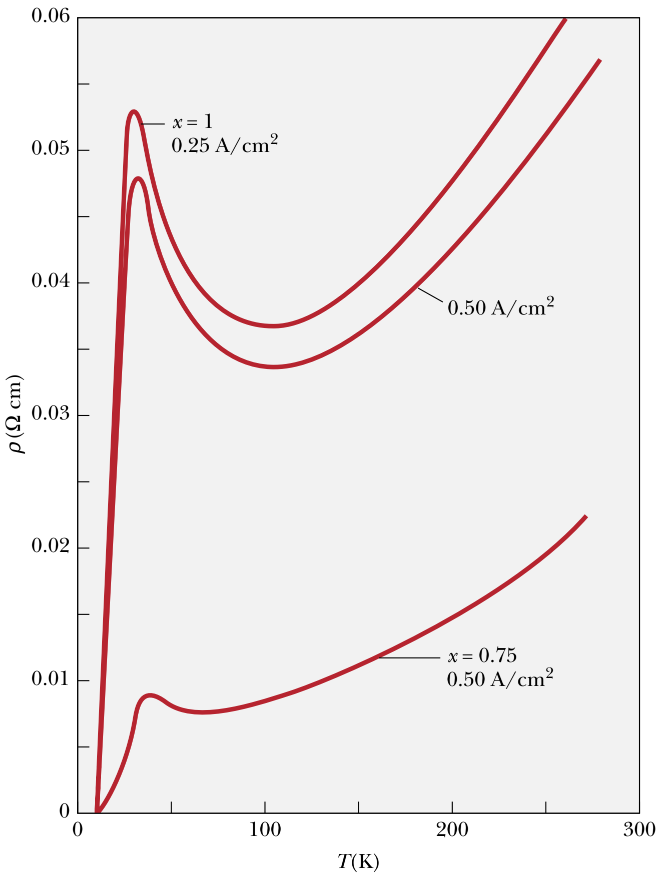



**Figure 12.28** Temperature dependence of the resistivity of Ba-La-Cu-O for samples containing different concentrations of barium and lanthanum. The influence of current density is shown in the upper two curves. (*Taken from J. G. Bednorz and K. A. Müller,* Z. Phys. B, *64:189, 1986*)

the University of Houston announced the discovery of superconductivity near 92 K in a mixed-phase sample containing yttrium, barium, copper, and oxygen.[17] The discovery was confirmed in other laboratories around the world, and the superconducting phase was soon identified to be the compound $YBa_2Cu_3O_{7-\delta}$. Figure 12.29 shows a plot of resistance versus temperature for this compound. This was an important milestone in high-temperature superconductivity, because the transition temperature of this compound is above the boiling point of liquid nitrogen (77 K), a coolant that is readily available, inexpensive, and much easier to handle than liquid helium.

At an American Physical Society meeting on March 18, 1987, a special panel discussion on high-temperature superconductivity introduced the world to the newly discovered novel superconductors. This all-night session, which attracted a standing-room-only crowd of about 3000, produced great excitement in the

[17] M. K. Wu, J. R. Ashburn, C. J. Torng, P. H. Hor, R. L. Meng, L. Gao, Z. J. Huang, Y. Q. Wang, and C. W. Chu, *Phys. Rev. Letters* 58:908, 1987.

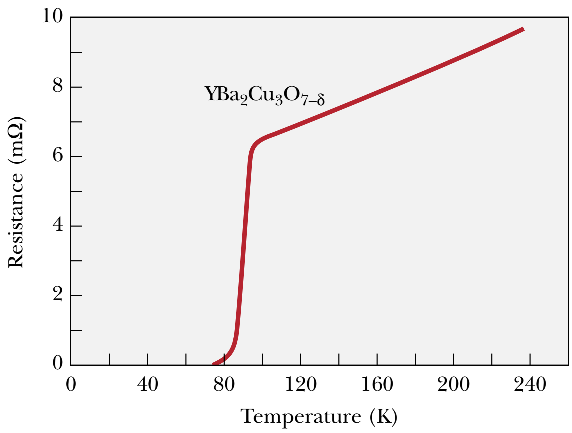



**Figure 12.29** Temperature dependence of the resistance of a sample of $YBa_2Cu_3O_{7-\delta}$ showing the transition temperature near 92 K. (*Taken from M. W. Wu et al.*, Phys. Rev. Letters *58:908, 1987*)

scientific community and has been referred to as "the Woodstock of physics." Realizing the possibility of routinely operating superconducting devices at liquid-nitrogen temperature and perhaps eventually at room temperature, thousands of scientists from a variety of disciplines entered the arena of superconductivity research. The exceptional interest in these novel materials arises from at least four factors:

- The metallic oxides are relatively easy to fabricate and hence can be investigated at smaller laboratories and universities.
- They have very high $T_c$ values and very high upper critical magnetic fields, estimated to be greater than 100 T in several materials (Table 12.8).

**Table 12.8 Some Properties of High-$T_c$ Superconductors in the Form of Bulk Ceramics**

| **Superconductor** | **$T_c$ (K)** | **$B_{c2}(0)$ (T)**[a] |
|---|---|---|
| La-Ba-Cu-O | 30 | |
| $La_{1.85}Sr_{0.15}CuO_5$ | 36.2 | $>36$ |
| $La_2CuO_4$ | 40 | |
| $YBa_2Cu_3O_{7-\delta}$ | 92 | $\approx 160$ |
| $ErBa_2Cu_3O_{9-\delta}$ | 94 | $>28$ |
| $DyBa_2Cu_3O_7$ | 92.5 | |
| Bi-Sr-Ca-Cu-O | 120 | |
| Tl-Ba-Ca-Cu-O | 125 | |
| $HgBa_2Ca_2Cu_3O_8$[b] | 134 | |

[a] These are projected extrapolations based on data up to about 30 T.

[b] The compound $HgBa_2Ca_2Cu_3O_8$ has a critical temperature of 164 K under a hydrostatic pressure of 45 GPa.

- Their properties and the mechanisms responsible for their superconducting behavior represent a great challenge to theoreticians.
- They may be of considerable technological importance.

Since 1986, several complex metallic oxides in the form of ceramics have been investigated, and critical temperatures above 100 K (triple-digit superconductivity) have been observed. Early in 1988 researchers reported superconductivity at 120 K in a Bi-Sr-Ca-Cu-O compound and at 125 K in a Tl-Ba-Ca-Cu-O compound. As of early 1995, the record high critical temperature was 134 K, in the compound $HgBa_2Ca_2Cu_3O_8$. The increase in $T_c$ since 1986 is dramatized in Figure 12.30. As you can see from this graph, the new high-$T_c$ materials are all copper oxides of one form or another.

The various superconducting compounds that have been extensively studied to date can be classified in terms of what are called perovskite crystal structures. The first class (Fig. 12.31a) consists of the cubic perovskites ($a = b = c$), such as $BaPb_{1-x}Bi_xO_3$, one of the original "high-$T_c$ materials" ($T_c \approx 10$ K). The second class (Fig. 12.31b) is single-layer perovskites, which have a tetragonal distortion ($a = b \neq c$); $La_2CuO_4$ is an example ($T_c \approx 38$ K). Note that the lattice parameters $a$ and $b$ are measured in the copper-oxygen planes, while $c$ is perpendicular to these planes. The third class is multilayer perovskites (Fig.

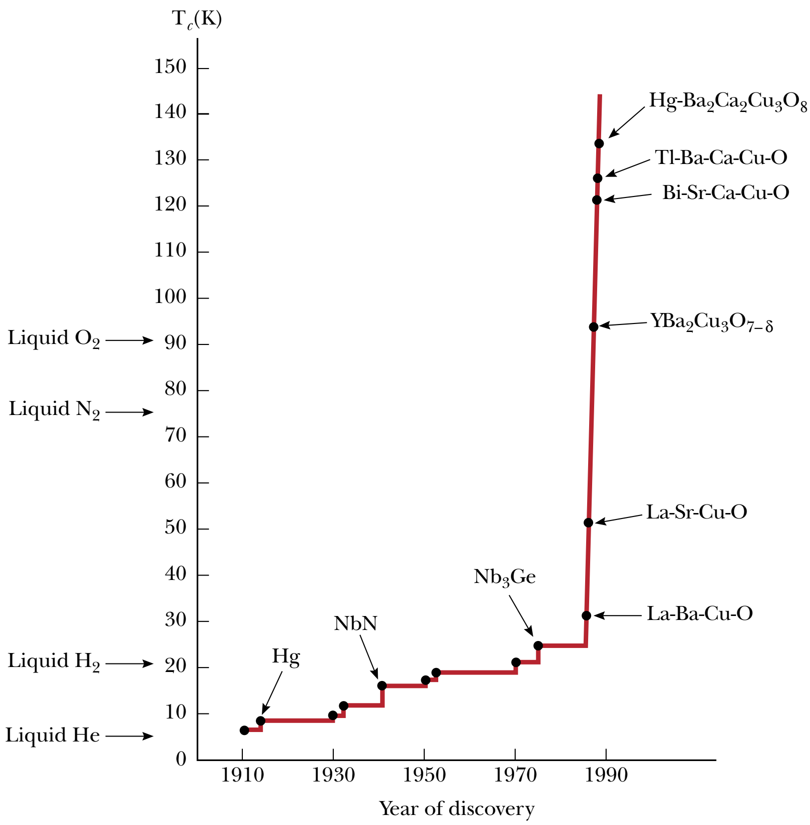



**Figure 12.30** Evolution of the superconducting critical temperature since the discovery of the phenomenon.

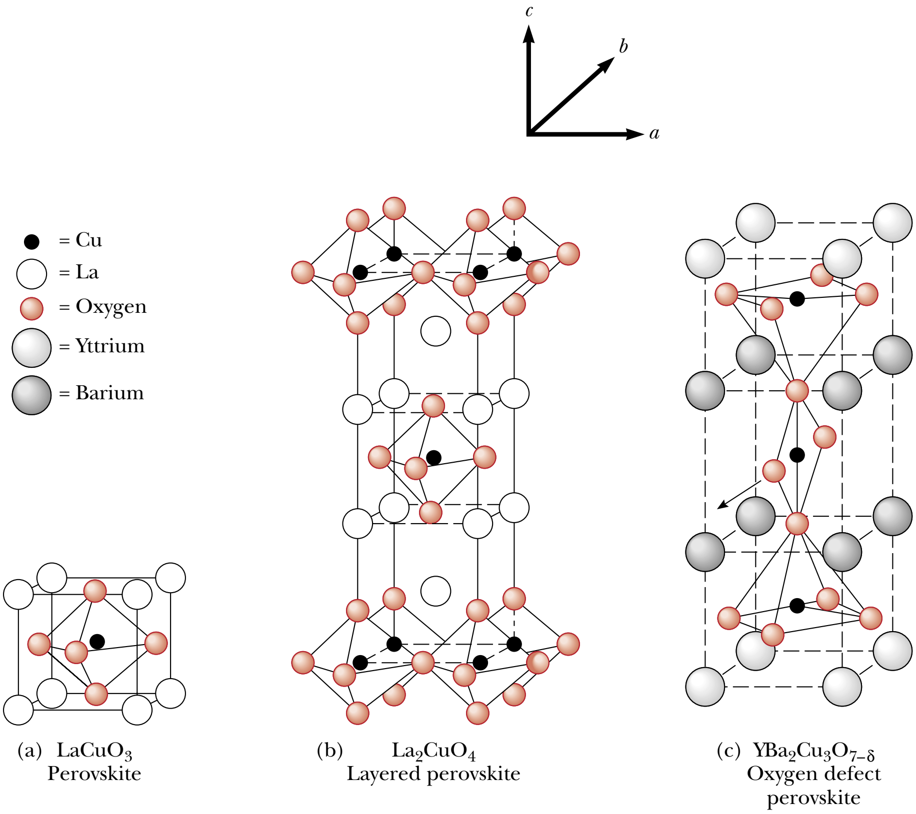



**Figure 12.31** Crystal structures for some of the new superconducting materials, the general class structure of which is perovskite. (a) The fundamental perovskite unit. (b) The single-layered perovskite having a tetragonal distortion ($a = b \neq c$). The first high-temperature superconductor ($T_c = 30$ K) falls in this class. (c) The double-layered perovskite with its orthorhombic structure ($a \neq b \neq c$). This structure is also related to the fundamental perovskite cube but has missing oxygen atoms.

12.31c), such as $YBa_2Cu_3O_7$ ($T_c \approx 92$ K), with an orthorhombic structure ($a \neq b \neq c$). Compounds in this class are sometimes called 1-2-3 materials because of their relative metallic composition.

The structures of the more complex copper oxides are not illustrated in Figure 12.31, but a most interesting observation can be made about them. *There appears to be a direct correlation between the number of copper-oxygen layers and critical temperature*, with critical temperature clearly increasing as copper-oxygen layers are added. There is some reason to expect that adding even more copper-oxygen layers to these complex oxides will raise the critical temperature even higher. On the basis of such results, some experts in the field are predicting $T_c$ values above 200 K.

The maximum supercurrents in these structures are high in the copper-oxygen planes and much lower in the direction perpendicular to those planes. In effect, conduction can be viewed as being two-dimensional. Recently the Los Alamos Superconductivity Technology Center manufactured flexible superconducting tape having a critical-current density of more than $10^{10}$ A/m$^2$ at 77 K with a critical temperature of about 90 K. The high critical-current density was achieved when a metallurgical technique known as texturing was used to achieve grain alignment. The tape consists of three layers: a substrate of flexible nickel, a second layer of textured cubic zirconia, and a third layer of superconducting $YBa_2Cu_3O_7$.

### Mechanisms of High-$T_c$ Superconductivity

Although the framework of the conventional BCS theory has been quite successful in explaining the behavior and properties of the "old-generation" superconductors, theoreticians are still trying to understand the nature of superconductivity in the "new-generation" high-$T_c$ cuprate oxides. The various models and mechanisms that have been proposed are far too technical to describe here.

Recently, researchers at IBM measured the symmetry of the wavefunction that describes Cooper pairs in $YBa_2Cu_3O_7$ to be a *d*-wave ($L = 2$).[18] They made their measurements on a sample consisting of a ring of $YBa_2Cu_3O_7$ deposited on a tricrystal substrate of $SrTiO_3$. By forming an odd number of Josephson junctions at the grain boundaries of the substrate, they observed a spontaneous magnetic flux equal to an odd integer times the magnetic flux quantum (that is, $\Phi_0/2$, $3\Phi_0/2$, and so on). This experiment provides strong evidence that antiferromagnetism in the cuprate oxides plays an important role in the establishment of the superconducting state in these high-$T_c$ compounds and the formation of corresponding Cooper pairs.

## 12.11 APPLICATIONS OF SUPERCONDUCTIVITY

OPTIONAL

High-temperature superconductors may lead to many important technological advances, such as highly efficient, lightweight superconducting motors. However, many significant materials-science problems must be overcome before such applications become reality. Perhaps the most difficult technical challenge is to mold the brittle ceramic materials into useful shapes, such as wires and ribbons for large-scale applications and thin films for small devices (e.g., SQUIDs). Another major problem is the relatively low current density that bulk ceramic compounds can carry. Assuming that such problems will be overcome, it is interesting to speculate on some of the future applications of these newly discovered materials.

An obvious application using the property of zero resistance to direct currents is low-loss electrical power transmission. A significant fraction of electrical power is lost as heat when current is passed through normal conductors. If power transmission lines could be made superconducting, these dc losses could be eliminated and substantial savings in energy costs would result.

The new superconductors could have a major impact in the field of electronics. Because of its switching properties, the Josephson junction can be used as a computer element. In addition, if one could use superconducting films to interconnect computer chips, chip size could be reduced and consequently speeds would be enhanced. Thus information could be transmitted more rapidly and more chips could be contained on a circuit board with far less heat generation.

[18] C. C. Tsuei and J. R. Kirtley, *Phys. Rev. Lett.* 73:593, 1994.

This prototype train, constructed in Japan, has superconducting magnets built into its base. A powerful magnetic field both levitates the train a few inches above the track and propels it smoothly at speeds of 300 mph or more. *(Joseph Brignolo/The Image Bank)*

The phenomenon of magnetic levitation can be exploited in the field of transportation. In fact, a prototype train that runs on superconducting magnets has been constructed in Japan. The moving train levitates above a normal conducting metal track through eddy-current repulsion. One can envision a future society of vehicles of all sorts gliding above a freeway, making use of superconducting magnets. Some scientists are speculating that the first major market for levitating devices will be the toy industry.

Another very important application of superconductivity is the superconducting magnet, a crucial component in particle accelerators. Currently, all high-energy particle accelerators use liquid-helium-based superconducting technology. Again, significant savings in cooling and operating costs would result if a liquid-nitrogen-based technology were developed.

An important application of superconducting magnets is a diagnostic tool called magnetic resonance imaging (MRI). This technique has played a prominent role in diagnostic medicine because it uses relatively safe radio-frequency radiation, rather than x-rays, to produce images of body sections. Because the technique relies on intense magnetic fields generated by superconducting magnets, the initial and operating costs of MRI systems are high. A liquid-nitrogen-cooled magnet could reduce such costs significantly.

In the field of power generation, companies and government laboratories have worked for years to develop superconducting motors and generators. In fact, a small-scale superconducting motor using the newly discovered ceramic superconductors has already been constructed at Argonne National Laboratory in Illinois.

We have already mentioned some small-scale applications of superconductivity—namely SQUIDs and magnetometers that make use of the Josephson effect and quantum interference. Josephson junctions can also be used as voltage standards and as infrared detectors. SQUIDs have been fabricated from films of $YBa_2Cu_3O_{7-\delta}$, and Josephson junctions using this compound have been operated at liquid-nitrogen temperature and above.

## SUMMARY

**Paramagnetic** and **ferromagnetic** materials are those that have atoms with permanent magnetic dipole moments that tend to align in the direction of an external magnetic field impressed on these materials. **Diamagnetic** materials are those whose atoms have no permanent magnetic dipole moments. When a diamagnetic material is placed in an external magnetic field, weak dipole moments opposing the field are induced in the material.

Superconductors are materials that have zero dc resistance below a certain temperature $T_c$, called the **critical temperature.** A second characteristic of a type I superconductor is that it behaves as a perfect diamagnet. At temperatures below $T_c$, any applied magnetic flux is expelled from the interior of a type I superconductor. This phenomenon of flux expulsion is known as the **Meissner effect.** The superconductivity of a type I superconductor is destroyed when an applied magnetic field exceeds the **critical magnetic field,** $B_c$, which is less than 0.2 T for the elemental superconductors.

A type II superconductor is characterized by two critical fields. When an applied field is less than the lower critical field, $B_{c1}$, the material is entirely superconducting and there is no flux penetration. When the applied field exceeds the upper critical field, $B_{c2}$, the superconducting state is destroyed and the flux penetrates the material completely. However, when the applied field lies between $B_{c1}$ and $B_{c2}$, the material is in a vortex state that is a combination of superconducting regions threaded by regions of normal resistance.

Once set up in a superconducting ring, **persistent currents** (also called supercurrents) circulate for several years with no measurable losses and with zero applied voltage. This is a direct consequence of the fact that the dc resistance is truly zero in the superconducting state.

A central feature of the BCS theory of superconductivity for metals is the formation of a bound state called a **Cooper pair,** consisting of two electrons having opposite momenta and opposite spins. The two electrons can form a bound state through a weak attractive interaction in which the crystal lattice of the metal serves as a mediator. In effect, one electron is weakly attracted to the other after the lattice is momentarily deformed by the first electron. In the ground state of the superconducting system, all electrons form Cooper pairs and all Cooper pairs are in the same quantum state. Thus the superconducting state is represented by a single coherent wavefunction that extends over the entire volume of the sample.

The BCS model predicts an energy gap $E_g = 3.53kT_c$, in contrast to a normal conductor, which has no energy gap. This value represents the energy needed to break up a Cooper pair and is of the order of 1 meV for the elemental superconductors.

High-temperature superconductors have critical temperatures as high as 150 K. They are all copper oxides, and the critical temperatures appear to be linked to the number of copper-oxygen layers in the structures. The new-generation materials, known to be type II superconductors, have highly anisotropic resistivities and high upper critical fields. However, in the form of bulk ceramic samples, the materials have limited critical currents and are quite brittle. Although the BCS model appears to be consistent with most empirical observations on high-temperature superconductors, the basic mechanisms giving rise to the superconducting state remain unknown.

## QUESTIONS

1. Why is $\mathbf{M} = 0$ in a vacuum? What is the relationship between **B** and **H** in a vacuum?
2. Explain why some atoms have permanent magnetic dipole moments and others do not.
3. Explain the significance of the Curie temperature for a ferromagnetic substance.
4. Explain why it is desirable to use hard ferromagnetic materials to make permanent magnets.
5. Describe how you would measure the two major characteristics of a superconductor.
6. Why is it not possible to explain zero resistance using a classical model of charge transport through a solid?
7. Discuss the differences between type I and type II superconductors. Discuss their similarities.
8. The specific heat of a superconductor in the absence of a magnetic field undergoes an anomaly at the critical temperature and decays exponentially toward zero below this temperature. What information does this behavior provide?
9. What is the isotope effect, and why does it play an important role in testing the validity of the BCS theory?
10. What are Cooper pairs? Discuss their essential properties, such as their momentum, spin, binding energy, and so on.
11. How would you explain the fact that lattice imperfections and lattice vibrations can scatter electrons in normal metals but have no effect on Cooper pairs?
12. Define single-particle tunneling and the conditions under which it can be observed. What information can one obtain from a tunneling experiment?
13. Define Josephson tunneling and the conditions under which it can be observed. What is the difference between Josephson tunneling and single-particle tunneling?
14. What are the features of high-temperature superconductors that limit their possible applications?
15. Discuss four features of high-$T_c$ superconductors that make them superior to the old-generation superconductors. In what way are the new superconductors inferior to the old ones?

The following questions deal with the Meissner effect. A small permanent magnet is placed on top of a high-temperature superconductor (usually $YBa_2Cu_3O_{7-\delta}$), starting at room temperature. As the superconductor is cooled with liquid nitrogen (77 K), the permanent magnet levitates—a most dazzling phenomenon. Assume, for simplicity, that the superconductor is type I.

16. Because of the Meissner effect, a magnetic field is expelled from the superconductor below its critical temperature. How does this explain the levitation of the permanent magnet? What must happen to the superconductor in order to account for this behavior?
17. As soon as the permanent magnet has levitated, it gains potential energy. What accounts for this increase in mechanical energy? (This is a tricky one.)
18. If the experiment is repeated by first cooling the superconductor below its critical temperature and then placing the permanent magnet on top of it, will the magnet still levitate? If so, will there be any difference in its elevation compared to the previous case?
19. If a calibrated thermocouple is attached to the superconductor to measure its temperature, describe how you could use this demonstration to obtain a value for $T_c$. (*Hint:* Start the observation below $T_c$ with a levitated permanent magnet.)

## PROBLEMS

### 12.1 Magnetism in Matter

1. What is the relative permeability of a material that has a magnetic susceptibility of $1.00 \times 10^{-4}$?
2. An iron-core toroid is wrapped with 250 turns of wire per meter of its length. The current in the winding is 8.00 A. Taking the magnetic permeability of iron to be $\mu_m = 5000\mu_0$, calculate (a) the magnetic field strength **H** and (b) the magnetic flux density **B**. (*Note:* **B** is frequently called the magnetic field as well, but its units [tesla] clearly distinguish it from **H**, which has units of amperes per meter.)
3. A toroid with a mean radius of 20 cm and 630 turns of wire is filled with powdered steel whose magnetic susceptibility $\chi$ is 100. If the current in the windings is 3.00 A, find $B$ (assumed uniform) inside the toroid.
4. A toroid has an average radius of 9.0 cm and carries a current of 0.50 A. How many turns are required to produce a magnetic field strength of 700 A·turns/m within the toroid?
5. A magnetic field of magnitude 1.3 T is to be set up in an iron-core toroid. The toroid has a mean radius of 10 cm and magnetic permeability of $5000\mu_0$. What current is required if there are 470 turns of wire in the winding?
6. A toroid has an average radius of 10 cm and a cross-sectional area of 1.0 cm$^2$. There are 400 turns of wire on the soft iron core, which has a permeability of $800\mu_0$. Calculate the current necessary to produce a magnetic flux of $5.0 \times 10^{-4}$ Wb through a cross-section of the core.
7. A coil of 500 turns is wound on an iron ring ($\mu_m = 750\mu_0$) of 20-cm mean radius and 8.0-cm$^2$ cross-sectional area. Calculate the magnetic flux $\Phi_B$ in this Rowland ring when the current in the coil is 0.50 A.
8. A uniform ring of radius $R$ and total charge $Q$ rotates with constant angular speed $\omega$ about an axis perpendicular to the plane of the ring and passing through its center. What is the magnetic moment of the rotating ring?
9. In the text, we found that an alternative description for the magnetic field **B** in terms of magnetic field strength **H** and magnetization **M** is $\mathbf{B} = \mu_0\mathbf{H} + \mu_0\mathbf{M}$. Relate the magnetic susceptibility $\chi$ to $|\mathbf{H}|$ and $|\mathbf{M}|$ for paramagnetic or diamagnetic materials.
10. A magnetized cylinder of iron has a magnetic field $B =$ 0.040 T in its interior. The magnet is 3.0 cm in diameter and 20 cm long. If the same magnetic field is to be produced by a 5.0-A current carried by an air-core solenoid with the same dimensions as the cylindrical magnet, how many turns of wire must be on the solenoid?
11. In Bohr's 1913 model of the hydrogen atom, the electron is in a circular orbit of radius $5.3 \times 10^{-11}$ m, and its speed is $2.2 \times 10^6$ m/s. (a) What is the magnitude of the magnetic moment due to the electron's motion? (b) If the electron orbits counterclockwise in a horizontal circle, what is the direction of this magnetic moment vector?
12. At saturation, the alignment of spins in iron can contribute as much as 2.0 T to the total magnetic field $B$. If each electron contributes a magnetic moment of $9.27 \times 10^{-24}$ A·m$^2$ (one Bohr magneton), how many electrons per atom contribute to the saturated field of iron? (*Hint:* There are $8.5 \times 10^{28}$ iron atoms in a cubic meter.)
13. *Curie's law of paramagnetism.* Curie's law, Equation 12.8, may be derived on the basis of a few assumptions. Recall that paramagnetism is produced by the net alignment of electronic magnetic moments of size $\cong\mu_B$ along the direction of an applied magnetic field, **B**. The paramagnetic magnetization $M = \mu_B(n_1 - n_2)$, where $n_1$ is the number of moments per unit volume parallel to the applied field and $n_2$ is the number of moments antiparallel. (a) Show that the ratio

$$\frac{n_1}{n_2} = e^{2\mu_B B/k_B T} \cong 1 + \frac{2\mu_B B}{k_B T}$$

by using the Maxwell-Boltzmann distribution and assuming that dipoles oriented parallel to **B** have energy $-\mu_B B$ and those antiparallel to **B** have energy $+\mu_B B$. (b) Show that the exponent $2\mu_B B/k_B T \ll 1$ for $B = 1.0$ T and $T = 300$ K. Thus

$$\frac{n_1 - n_2}{n_2} \cong \frac{2\mu_B B}{k_B T}$$

(c) Show that $n_1 - n_2 \cong n\mu_B B/k_B T$, where $n$ is the total number of magnetic moments, and show that the paramagnetic magnetization is given by Equation 12.8, $M = C(B/T)$, with $C = n\mu_B{}^2/k_B$.

### 12.3 Some Properties of Type I Superconductors
### 12.4 Type II Superconductors

14. A wire made of $Nb_3Al$ has a radius of 2.0 mm and is maintained at 4.2 K. Using the data in Table 12.5, find (a) the upper critical field for this wire at this temperature, (b) the maximum current that can pass through the wire before its superconductivity is destroyed, and (c) the magnetic field 6.0 mm from the wire surface when the current has its maximum value.
15. A superconducting solenoid is to be designed to generate a 10-T magnetic field. (a) If the winding has 2000 turns/m, what is the required current? (b) What force per meter does the magnetic field exert on the inner windings?
16. Determine the current generated in a superconducting ring of niobium metal 2.0 cm in diameter if a 0.020-T magnetic field directed perpendicular to the ring is suddenly decreased to zero. The inductance of the ring is $L = 3.1 \times 10^{-8}$ H.
17. Determine the magnetic field energy, in joules, that needs to be added to destroy superconductivity in 1.0 cm$^3$ of lead near 0 K. Use the fact that $B_c(0)$ for lead is 0.080 T.
18. The penetration depth for lead at 0.0 K is 39 nm. Find the penetration depth in lead at (a) 1.0 K, (b) 4.2 K, and (c) 7.0 K.
19. Find the critical magnetic field in mercury at (a) 1.0 K and (b) 4.0 K.

### 12.5 Other Properties of Superconductors

20. *Persistent currents.* In an experiment carried out by S. C. Collins between 1955 and 1958, a current was maintained in a superconducting lead ring for 2.5 years with no observed loss. If the inductance of the ring was $3.14 \times 10^{-8}$ H, and the sensitivity of the experiment was 1 part in $10^9$, determine the maximum resistance of the ring. (*Hint:* Treat this as a decaying current in an *RL* circuit, and recall that $e^{-x} \cong 1 - x$ for small $x$.)
21. *Speed of electron flow.* Current is carried throughout the body of niobium-tin, a type II superconductor. If a niobium-tin wire of cross-section 2.0 mm$^2$ can carry a maximum supercurrent of $1.0 \times 10^5$ A, estimate the average speed of the superconducting electrons. (Assume that the density of conducting electrons is $n_s = 5.0 \times 10^{27}/\text{m}^3$.)
22. *Diamagnetism.* When a superconducting material is placed in a magnetic field, the surface currents established make the magnetic field inside the material truly zero. (That is, the material is perfectly diamagnetic.) Suppose a circular disk, 2.0 cm in diameter, is placed in a 0.020-T magnetic field with the plane of the disk perpendicular to the field lines. Find the equivalent surface current if it all lies at the circumference of the disk.
23. *Surface currents.* A rod of a superconducting material 2.5 cm long is placed in a 0.54-T magnetic field with its cylindrical axis along the magnetic field lines. (a) Sketch the directions of the applied field and the induced surface current. (b) Estimate the magnitude of the surface current.

### 12.7 BCS Theory
### 12.8 Energy Gap Measurements

24. Calculate the energy gap for each superconductor in Table 12.4 as predicted by the BCS theory. Compare your answers with the experimental values in Table 12.7.
25. Calculate the energy gap for each superconductor in Table 12.5 as predicted by the BCS theory. Compare your values with those found for type I superconductors.
26. *High-temperature superconductor.* Estimate the energy gap $E_g$ for the high-temperature superconductor $YBa_2Cu_3O_{7-\delta}$, which has a critical temperature of 92 K, assuming BCS theory holds.
27. *Isotope effect.* Because of the isotope effect, $T_c \propto M^{-\alpha}$. Use these data for mercury to determine the value of the constant $\alpha$. Is your result close to what you might expect on the basis of a simple model?

| Isotope | $T_c$ (K) |
|---|---|
| $^{199}$Hg | 4.161 |
| $^{200}$Hg | 4.153 |
| $^{204}$Hg | 4.126 |

28. *Cooper pairs.* A Cooper pair in a type I superconductor has an average separation of about $1.0 \times 10^{-4}$ cm. If these two electrons can interact within a volume of this diameter, how many other Cooper pairs have their centers within the volume occupied by one pair? Use the appropriate data for lead, which has $n_s = 2.0 \times 10^{22}$ electrons/cm$^3$.
29. *Dipole in a magnetic field.* The potential energy of a magnetic dipole of moment $\boldsymbol{\mu}$ in the presence of a magnetic field **B** is $U = -\boldsymbol{\mu}\cdot\mathbf{B}$. When an electron is placed in a magnetic field, its magnetic moment can be aligned either with or against the field. The energy separation between these two states is $\Delta E = 2\mu B$, where the magnetic moment of the electron is $\mu = 5.79 \times 10^{-5}$ eV/T.

(a) If a Cooper pair is subjected to a 38-T magnetic field (the critical field for $Nb_3Ge$), calculate the energy separation between the spin-up and spin-down electrons. (b) Calculate the energy gap for $Nb_3Ge$ as predicted by the BCS theory at 0 K, using the fact that $T_c = 23$ K. (c) How do your answers to (a) and (b) compare? What does this result suggest, based on what you have learned about critical fields?

### 12.9 Josephson Tunneling

30. Estimate the area of a ring that would fit one of your fingers, and calculate the magnetic flux through the ring due to the Earth's magnetic field ($5.8 \times 10^{-5}$ T). If this flux were quantized, how many fluxons would the ring enclose?
31. A Josephson junction is fabricated using indium for the superconducting layers. If the junction is biased with a dc voltage of 0.50 mV, find the frequency of the alternating current generated. (For comparison, note that the energy gap of indium at 0 K is 1.05 meV.)
32. If a magnetic flux of $1.0 \times 10^{-4}\Phi_0$ ($\frac{1}{10\,000}$ of the flux quantum) can be measured with a SQUID (Fig. P12.32), what is the smallest magnetic field change $\Delta B$ that can be detected with this device for a ring having a radius of 2.0 mm?

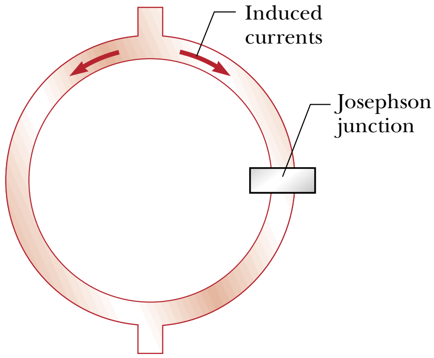



**Figure P12.32** A *s*uperconducting *qu*antum *i*nterference *d*evice (SQUID).

33. A superconducting circular loop of very fine wire has a diameter of 2.0 mm and a self-inductance of 5.0 nH. The flux through the loop, $\Phi$, is the sum of the applied flux, $\Phi_{app}$, and the flux due to the supercurrent, $\Phi_{sc} = LI$, where $L$ is the self-inductance of the loop. Because the flux through the loop is quantized, we have

$$n\Phi_0 = \Phi_{app} + \Phi_{sc} = \Phi_{app} + LI$$

where $\Phi_0$ is the flux quantum. (a) If the applied flux is zero, what is the smallest current that can meet this quantization condition? (b) If the applied field is perpendicular to the plane of the loop and has a magnitude of $3.0 \times 10^{-9}$ T, find the smallest current that circulates around the loop.

### Additional Problems

34. A solenoid of diameter 8.0 cm and length 1 m is wound with 2000 turns of superconducting wire. If the solenoid carries a current of 150 A, find (a) the magnetic field at the center, (b) the magnetic flux through the center cross-section, and (c) the number of flux quanta through the center.
35. *Energy storage.* A novel method has been proposed to store electrical energy. A huge underground superconducting coil, 1.0 km in diameter, carries a maximum current of 50 kA through each winding of a 150-turn $Nb_3Sn$ solenoid. (a) If the inductance of this huge coil is 50 H, what is the total energy stored? (b) What is the compressive force per meter acting between two adjacent windings 0.25 m apart?
36. *Superconducting power transmission.* Superconductors have been proposed for power transmission lines. A single coaxial cable (Fig. P12.36) could carry $1.0 \times 10^3$ MW (the output of a large power plant) at 200 kV, dc, over a distance of 1000 km without loss. The superconductor would be a 2.0-cm-radius inner $Nb_3Sn$ cable carrying the current $I$ in one direction, while the outer surrounding superconductor of 5.0-cm radius would carry the return current $I$. In such a system, what is the magnetic field (a) at the surface of the inner conductor and (b) at the inner surface of the outer conductor? (c) How much energy would be stored in the space between the conductors in a 1000-km superconducting line? (d) What is the force per meter length exerted on the outer conductor?

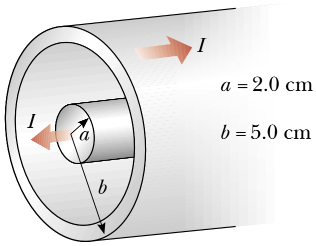



**Figure P12.36**

37. *Penetration depth.* The penetration depth of a magnetic field into a superconductor is found, from London's equations, to be

$$\lambda = \sqrt{\frac{m_e}{\mu_0 n_s e^2}}$$

where $m_e$ is the mass of the electron, $e$ is the charge of the electron, and $n_s$ is the number of superconducting electrons per unit volume. (a) Estimate $n_s$ in lead at 0 K if the magnetic penetration depth near 0 K is $\lambda_0 = 4.0 \times 10^{-8}$ m. (b) Starting with Equation 12.12, determine $n_s$ in lead as a function of temperature for $T \leqslant T_c$.

38. *"Floating" a wire.* Is it possible to "float" a superconducting lead wire of radius 1.0 mm in the magnetic field of the Earth? Assume that the horizontal component of the Earth's magnetic field is $4.0 \times 10^{-5}$ T.

39. *Magnetic field inside a wire.* A type II superconducting wire of radius $R$ carries current uniformly distributed through its cross-section. If the total current carried by the wire is $I$, show that the magnetic energy per unit length inside the wire is $\mu_0 I^2/16\pi$.

40. *Magnetic levitation.* If a very small but very strong permanent magnet is lowered toward a flat-bottomed type I superconducting dish, at some point the magnet will levitate above the superconductor because the superconductor is a perfect diamagnet and expels all magnetic flux. Therefore the superconductor acts like an identical magnet lying an equal distance below the surface (Fig. P12.40). At what height does the magnet levitate if its mass is 4.0 g and its magnetic moment $\mu = 0.25$ A·m$^2$? (*Hint:* The potential energy between two dipole magnets a distance $r$ apart is $\mu_0\mu^2/4\pi r^3$).

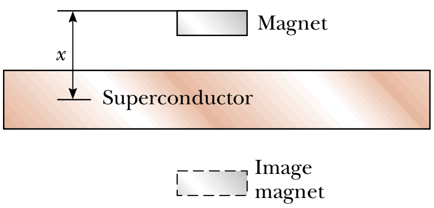



**Figure P12.40** A magnetic levitation experiment.

41. *Designing a superconducting solenoid.* A superconducting solenoid was made with $Nb_3Zr$ wire wound on a 10-cm-diameter tube. The solenoid winding consisted of a double layer of 0.50-mm-diameter wire with 984 turns (corresponding to a coil length of 25 cm). (a) Calculate the inductance, $L$, of the solenoid, assuming its length is great relative to its diameter. (b) The magnitude of the persistent current in this solenoid in the superconducting state has been reported to decrease by 1 part in $10^9$ per hour. If the resistance of the solenoid is $R$, then the current in the circuit should decay according to $I = I_0 e^{-Rt/L}$. This resistance, although small, is due to magnetic flux migration in the superconductor. Determine an upper limit of the coil's resistance in the superconducting state. (These data were reported by J. File and R. G. Mills, *Phys. Rev. Letters* 10:93, 1963.)

42. *Entropy difference.* The entropy difference per unit volume between the normal and superconducting states is

$$\frac{\Delta S}{V} = -\frac{\partial}{\partial T}\left(\frac{B^2}{2\mu_0}\right)$$

where $B^2/2\mu_0$ is the magnetic energy per unit volume required to destroy superconductivity. Determine the entropy difference between the normal and superconducting states in 1.0 mol of lead at 4.0 K if the critical magnetic field $B_c(0) = 0.080$ T and $T_c = 7.2$ K.

43. *A convincing demonstration of zero resistance.* A direct and relatively simple demonstration of zero dc resistance can be carried out using the four-point probe method. The experimental set-up, shown in Figure P12.43, consists of a disc of $YBa_2Cu_3O_{7-\delta}$ (a high-$T_c$ superconductor) to which four wires are attached by indium solder (or some other suitable contact material). A constant current, maintained through the sample by a dc voltage between points $a$ and $b$, is measured with a dc ammeter. (The current is varied with the variable resistance $R$.) The potential difference $V_{cd}$ between $c$ and $d$ is measured with a digital voltmeter. When the disc is immersed in liquid nitrogen, it cools quickly to 77 K, which is below its critical temperature (92 K); the current remains approximately constant, but $V_{cd}$ *drops abruptly to zero.* (a) Explain this observation on the basis of what you know about superconductors and your understanding of Ohm's law. (b) The data in Table 12.9 represent actual values of $V_{cd}$ for different values of $I$ taken on a sample at room temperature. Make an $I$-$V$ plot of the data, and determine whether the sample behaves in a linear

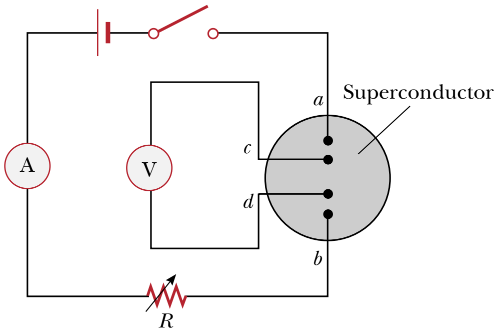



**Figure P12.43** A circuit diagram used in the four-point probe measurement of the dc resistance of a sample. A dc digital ammeter is used to measure the current, and the potential difference between $c$ and $d$ is measured with a dc digital voltmeter. Note that there is no voltage source in the inner loop circuit where $V_{cd}$ is measured.

manner. From the data, obtain a value of the dc resistance of the sample at room temperature. (c) At room temperature, it is found that $V_{cd} = 2.234$ mV for $I = 100.3$ mA, but after the sample is cooled to 77 K, $V_{cd} = 0$ and $I = 98.1$ mA. What do you think might explain the slight decrease in current?

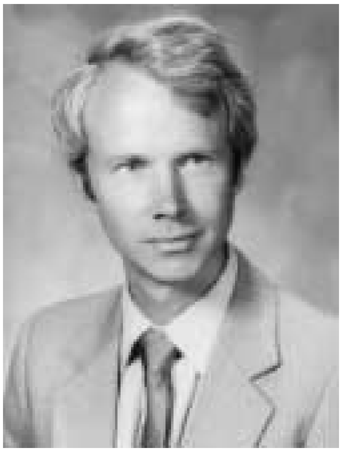
**Table 12.9 Current Versus Potential Difference $V_{cd}$ in a Bulk Ceramic Sample of $YBa_2Cu_3O_{7-\delta}$ at Room Temperature[a]**

| $I$ (mA) | $V_{cd}$ (mV) |
|---|---|
| 57.8 | 1.356 |
| 61.5 | 1.441 |
| 68.3 | 1.602 |
| 76.8 | 1.802 |
| 87.5 | 2.053 |
| 102.2 | 2.398 |
| 123.7 | 2.904 |
| 155 | 3.61 |

[a] The current was supplied by a 6-V battery in series with a variable resistor $R$. The values of $R$ ranged from 10 Ω to 100 Ω. The data are from the author's laboratory.

# Essay

## SUPERCONDUCTING DEVICES*



Clark A. Hamilton

*National Institute of Standards and Technology Electromagnetic Technology Division*

Superconductivity was discovered in 1911 by Dutch physicist Heike Kamerlingh Onnes. In experiments carried out to measure the resistance of frozen mercury, he made the startling discovery that resistance vanished completely at a temperature a few degrees above absolute zero. Since that time, superconductivity has been observed in a large number of elemental metals and alloys. The first proposed application of this new phenomenon was the construction of powerful magnets, since superconductors can carry current with no heat generation. Unfortunately it was soon discovered that modest magnetic fields destroy the superconductivity in materials such as mercury, tin, and lead. In fact, the conditions under which superconductivity exists in any material can be described by a three-dimensional diagram such as that shown in Figure 1a. The axes are temperature, magnetic field, and current density. The curved surface in this figure represents the transition between normal and superconducting behavior, and its intersections with the three axes are the critical current density ($J_c$), critical field ($H_c$), and critical temperature ($T_c$). The primary objective of research in superconducting materials since 1911 has been to expand the volume of superconducting behavior in this diagram. This effort has generated a wide variety of what are now called conventional superconductors, with critical temperatures up to 23 K and critical fields up to 40 K (the Earth's magnetic field is 0.00005 T). The primary drawback of these conventional superconductors is the expense and inconvenience of their refrigeration systems. Nevertheless, the fabrication of superconducting magnets for magnetic resonance imaging, accelerator magnets, fusion research, and other applications is a $200 million/year business.

In 1987 Karl Alex Müller and J. Georg Bednorz received the Nobel prize in physics for their discovery in 1986 of a new class of higher-temperature superconducting ceramics consisting of oxygen, copper, barium, and lanthanum. Expanding on their work, researchers at the University of Alabama at Huntsville and the University of Houston developed the compound $YBa_2Cu_3O_x$, which has a transition temperature above 90 K. For the first time, superconductivity became possible at liquid-nitrogen temperatures.

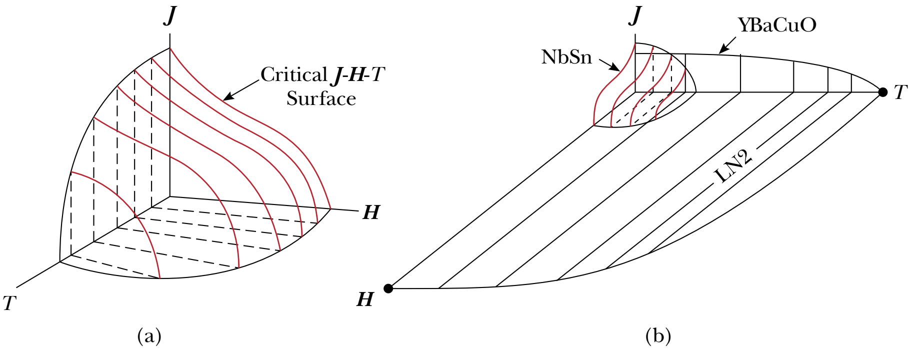



**Figure 1** (a) A three-dimensional diagram illustrating the surface separating normal and superconducting behavior as a function of temperature ($T$), magnetic field ($H$), and current density ($J$). (b) A comparison of the $J$-$H$-$T$ diagrams for $Nb_3Sn$ and $YBa_2Cu_3O_x$. *(Courtesy of Alan Clark, National Institute of Standards and Technology.)*

An explosion of research on the new materials has resulted in the discovery of many materials with transition temperatures near 100 K and hints of superconductivity at much higher temperatures. Figure 1b compares the transition parameters of the old and new materials and illustrates the potential of the new superconductors. With their high transition temperatures and critical fields projected to hundreds of tesla, the parameter space where superconductivity exists has been greatly expanded. Unfortunately, the critical-current density of the new materials over most of the *H-T* plane is very low. It is expected that this problem will be solved when the new materials can be made in sufficiently pure form.

## SUPERCONDUCTIVITY

In 1957 John Bardeen, L. N. Cooper, and J. R. Schrieffer published a theory which shows that superconductivity results from an interaction between electrons and the lattice in which they flow.[1,2,3] Coulomb attraction between electrons and lattice ions produces a lattice distortion that can attract other electrons. At sufficiently low temperatures, the attractive force overcomes thermal agitation, and the electrons begin to condense into pairs. As the temperature is reduced further, more and more electrons condense into the paired state. In so doing, each electron pair gives up a condensation energy $2\Delta$. The electron pairs are all described by a common quantum mechanical wavefunction, and thus a long-range order exists among all of the electron pairs in the superconductor. As a result of this long-range order, the pairs can flow through the lattice without suffering the collisions that lead to resistance in normal conductors.

We can model a superconductor as a material in which current is carried by two different electron populations. The first population consists of normal electrons, which behave as the electrons in any normal metal do. The second population consists of electron pairs, which behave coherently and are described by a common wavefunction $\Psi(x, t)$. The amplitude of $\Psi(x, t)$ is usually constant throughout the superconductor, but its phase may vary, depending on the current. The condensation energy of the electron pairs produces an energy gap between normal and superconducting electrons, which is $2\Delta/e \approx 2$ meV for conventional superconductors. This model is known as the two-fluid model.

## JOSEPHSON JUNCTIONS

In addition to the work on large-scale applications of superconductivity (primarily magnets), considerable research has been done on small-scale superconducting devices. Most of these devices take advantage of the unusual properties of a Josephson junction, which is simply two superconductors separated by an insulating barrier, as shown in Figure 2. When the barrier is thin enough, electron pairs can tunnel from one superconductor to the other. Brian Josephson shared the Nobel prize in physics in 1973 for his prediction of the behavior of such a device. The remainder of this essay will discuss the operation of Josephson devices and circuits.

The equations for the current associated with electron pairs across a Josephson junction can be derived by solving two coupled wave equations with a potential energy difference $eV$ that depends on the voltage $V$ across the junction.[1] The result is the Josephson equations:

$$I_J = I_0 \sin \phi \tag{1}$$

$$\frac{d\phi}{dt} = \frac{2eV}{\hbar} \tag{2}$$

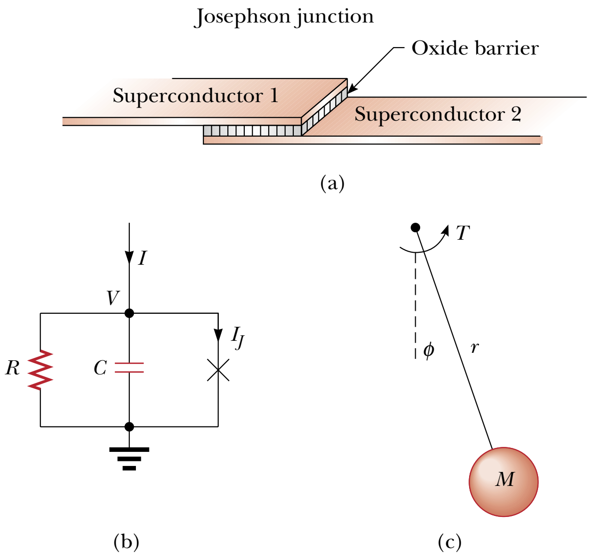



**Figure 2** The geometry of a Josephson junction (a), its equivalent circuit (b), and a mechanical analog for the junction behavior (c).

where $I_J$ is the electron-pair current through the junction, $\phi$ is the phase difference between the superconducting wavefunctions on the two sides of the barrier, and $I_0$ is a constant related to the barrier thickness. The full behavior of the junction can be modeled by the equivalent circuit of Figure 2b. The resistance $R$ describes the flow of normal electrons; X represents a pure Josephson element having a flow of electron pairs, $I_J$, given by Equations 1 and 2; and $C$ is the normal geometrical capacitance. Thus the total junction current is

$$I = \frac{V}{R} + C\frac{dV}{dt} + I_0 \sin \phi \tag{3}$$

Substituting Equation 2, this becomes

$$I = \frac{C\hbar}{2e}\frac{d^2\phi}{dt^2} + \frac{\hbar}{2eR}\frac{d\phi}{dt} + I_0 \sin \phi \tag{4}$$

Under different drive and parameter conditions, Equation 4 leads to an almost infinite variety of behavior and is the subject of more than 100 papers.

## THE PENDULUM ANALOGY

In understanding Equation 4, it is useful to make the analogy between the current in a Josephson junction and the torque applied to a pendulum that is free to rotate 360° in a viscous medium, as shown in Figure 2c. The applied torque, $\tau$, on such a pendulum can be written

$$\tau = Mr^2\frac{d^2\phi}{dt^2} + k\frac{d\phi}{dt} + Mgr \sin \phi \tag{5}$$

where $k\frac{d\phi}{dt}$ is the damping torque, $Mr^2\frac{d^2\phi}{dt^2}$ is the inertial term, and $Mgr \sin \phi$ is the gravitational restoring torque. Since Equations 4 and 5 have identical form, we can say

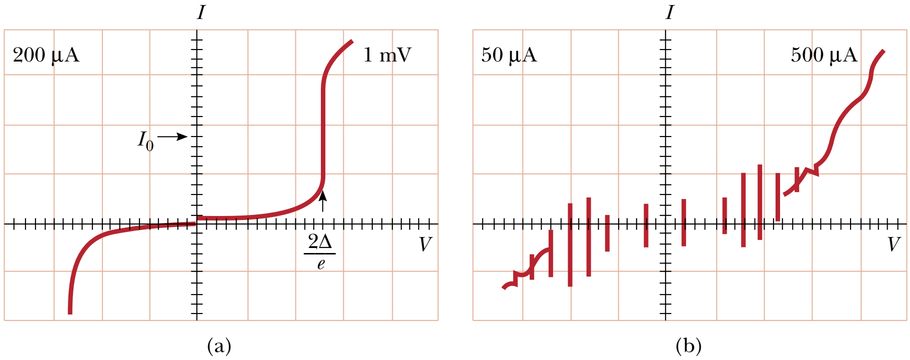



**Figure 3** (a) The *I-V* curve of a Josephson junction. (b) The *I-V* curve when the junction is exposed to 96 GHz radiation. The quantized voltage levels occur at the values $V_n = nhf/2e$.

that the torque on the pendulum is the analog of current in a Josephson junction, and the rate of rotation of the pendulum is the analog of voltage across the junction.

We are now in a position to understand the Josephson junction *I-V* curve shown in Figure 3a. For currents less than $I_0$, the junction phase assumes a value such that all of the current is carried by the Josephson element, as described by Equation 1. Except for a small transient, the voltage remains zero. In the analog, the pendulum assumes a fixed angle that balances the torque and the gravitational restoring force. When the torque exceeds the critical value $Mgr$, the pendulum rotation accelerates until the viscous damping term can absorb the applied torque. Similarly, when the current applied to a Josephson junction exceeds the critical value $I_0$, the junction switches to a voltage state in which the junction current oscillates and has an average value determined by the normal state resistance. As the current decreases, the voltage follows the normal state curve until the current reaches a value $I_{min}$. Below $I_{min}$ the voltage falls abruptly to zero as the junction switches back into the superconducting state. In the analog, with decreasing torque, the pendulum rotation slows down to the point where the applied torque is insufficient to push it over the top, and it then settles into a nonrotating state.

The nonlinearity in the normal state curve for the junction can be explained as follows. For small voltages, the resistance is high because there are comparatively few normal-state electrons to tunnel across the barrier. At voltages greater than the superconducting energy gap voltage, $2\Delta/e$, there is sufficient energy from the bias source to break superconducting pairs, and the substantial increase in the number of normal carriers causes an abrupt increase in current.

## JOSEPHSON VOLTAGE STANDARD

One of the first applications of the Josephson effect comes from the fact that the Josephson current oscillates at a frequency proportional to the applied voltage, where the frequency is $f = 2eV/h$. Since this relation is independent of all other parameters and has no known corrections, it forms the basis of voltage standards throughout the world. In practice, these standards operate by applying a microwave current through the junction and allowing the Josephson frequency to phase-lock to some harmonic of the applied frequency. With the correct choice of design parameters, the only stable operating points of the junction will be at one of the voltages $V = nhf/2e$, where $n$ is an integer. These voltages are very accurately known, because frequency can be measured with great accuracy and $2e/h = 483.594$ GHz/mV is a defined value for the purpose of voltage standards. Figure 3b is the *I-V* curve of a junction that is driven at 96 GHz. About ten quantized voltage levels span the range from −1 to 1 mV. Each of these levels represents a phase lock between a harmonic of the applied frequency at 96 GHz and the quantum phase difference, $\phi$, across the junction. Practical voltage standards use up to 20 000 junctions in series to generate reference voltages up to 10 V.

## FLUX QUANTIZATION

Consider a closed superconducting loop surrounding magnetic flux $\Phi$. We know from Faraday's law that any change in $\Phi$ must induce a voltage around the loop. Since the voltage across a superconductor is zero, the flux cannot change and is therefore trapped in the loop. There is a further restriction in that the phase of the quantum wavefunction around the loop must be continuous. It can be shown that the variation of the quantum phase along a superconducting wire is proportional to the product of the current and the inductance $L$ of the wire,

$$\Delta\phi = \frac{2\pi LI}{h/2e} \tag{6}$$

In order for $\phi$ to be continuous around the loop, $\Delta\phi$ must be a multiple of $2\pi$. It is thus a simple matter to show that the product $LI$ must be quantized in units of $h/2e$. Since $LI$ is just the magnetic flux through the loop, the flux is quantized; the quantity $\Phi_0 = h/2e = 2.07 \times 10^{-15}\ \mathrm{T \cdot m^2}$ is the flux quantum.

## SQUIDs

One of the most interesting of all superconducting devices is formed by inserting two Josephson junctions in a superconducting loop, as shown in Figure 4a.[2,3] From a circuit viewpoint, this looks like two junctions in parallel, and it therefore has an *I-V* curve like that of a single junction (Fig. 4b). The current through the loop is just the sum of the currents in the two junctions.

$$I = I_{01}\sin\phi_1 + I_{02}\sin\phi_2 \tag{7}$$

The equation for continuity of phase around the loop is

$$\Delta\phi = 2\pi n = \phi_1 - \phi_2 + 2\pi\frac{L(I_1 - I_2)}{\Phi_0} + 2\pi\frac{\Phi}{\Phi_0} \tag{8}$$

When Equations 7 and 8 are solved to maximize $I$, the total critical current $I_{max}$ is found to be a periodic function of the magnetic flux $\Phi$ applied to the loop as shown in Figure 4c. This pattern can be interpreted as the consequence of interference between the superconducting wavefunctions on the two sides of the junction. These devices are therefore called *s*uperconducting *qu*antum *i*nterference *d*evices, or SQUIDs.

The most important application of SQUIDs is the measurement of small magnetic fields. This is done by using a large pick-up loop to inductively couple flux into the SQUID loop. A sensitive electronic circuit senses the changes in critical current $I_{max}$ caused by changing flux in the SQUID loop. This arrangement can resolve flux changes of as little as $10^{-6}\ \Phi_0$. For a typical SQUID with an inductance $L = 100$ pH, this corresponds to an energy resolution of $\Delta E = \Delta\Phi^2/2L = 2 \times 10^{-32}$ J, which approaches the theoretical limit set by Planck's constant.

A rapidly expanding application of SQUIDs is the measurement of magnetic fields caused by the small currents in the heart and brain. Systems with as many as seven SQUIDs are being used to map these biomagnetic fields. This new nonintrusive tool may one day play an important role in locating the source of epilepsy and other brain dis-

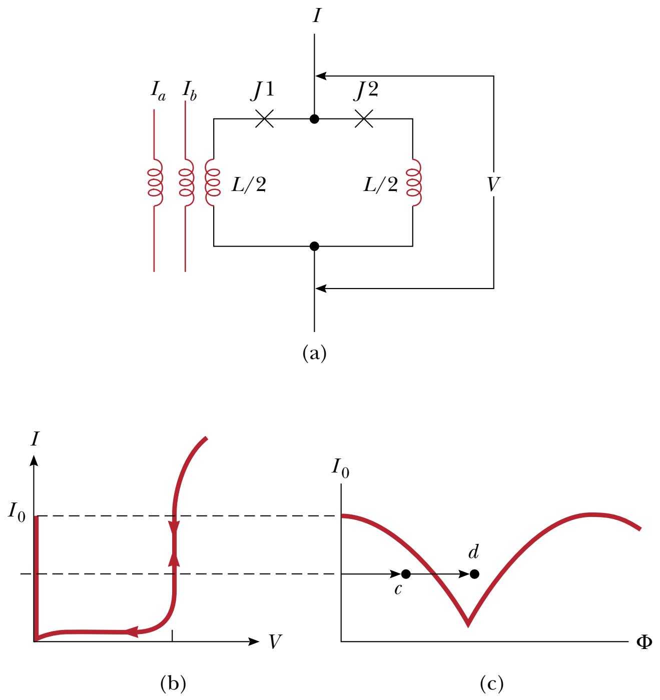



**Figure 4** (a) The circuit diagram for a SQUID. (b) Its *I-V* curve. (c) The dependence of critical current on applied magnetic flux.

orders. In such applications it is important to discriminate against distant sources of magnetic noise. This is done by using two pick-up coils wound in opposite directions, so that the SQUID is sensitive only to the *gradient* of the magnetic field.

## DIGITAL APPLICATIONS

The possibility of digital applications for Josephson junctions is suggested immediately by the bistable nature of the *I-V* curve in Figure 3a. At a fixed-bias current, the junction can be in either the zero-voltage state or the finite-voltage state, representing a binary 0 or 1. In the mid-1970s a major effort was mounted to develop large-scale computing systems that used Josephson devices.[4] The motivation for this is twofold. First, Josephson junctions can switch states in only a few picoseconds; second, the power dissipation of Josephson devices is nearly three orders of magnitude less than that of semiconductor devices. The low power dissipation results from the low logic levels (0 and 2 mV) of Josephson devices. Dissipation becomes increasingly important in ultra-high-speed computers because the need to limit the propagation delay forces the computer into a small volume. As the computer shrinks, heat removal becomes increasingly difficult.

A variety of Josephson logic circuits have been proposed. We will consider just one, based on the SQUID circuit of Figure 4a. Suppose that the SQUID carries a current at the level of the lower dashed line in Figure 4. Current in the two input lines ($I_a$, $I_b$) moves the bias point horizontally along this line in Figure 4c. For example, a current in either line moves the operating point to (c), while a current in both lines moves the operating point across the threshold curve to (d). When the threshold curve is crossed, the SQUID switches into the voltage state. The output of the device thus is the “AND” function of the two inputs. Other logic combinations can be achieved by adjusting the

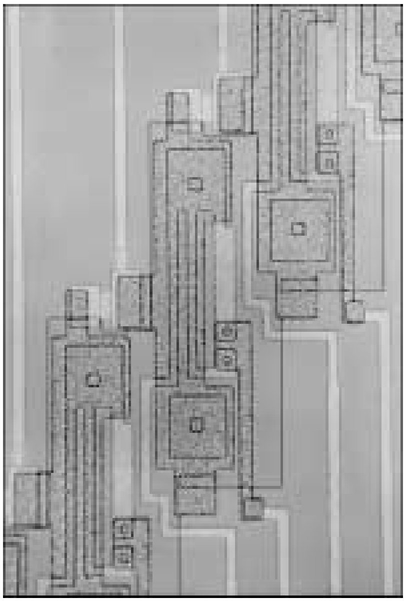


**Figure 5** A photograph of a Josephson integrated circuit consisting of two coupled SQUIDs. The minimum feature size is 4 microns.

coupling of the input lines. Memory can be made by using coupled SQUIDs, where one SQUID senses the presence or absence of flux in a second SQUID. Figure 5 is a photograph of two coupled thin-film SQUIDs that form a flip-flop circuit. These devices are made by photolithographic processes similar to those used in the semiconductor industry.

Josephson logic and memory chips with thousands of junctions have been built and successfully operated. However, there are still significant problems to be overcome in developing a complete Josephson computer. The greatest obstacle is the dependence of Josephson logic on threshold switching. This results in circuits that operate over only a small range of component parameters. Such circuits are said to have small *margins.* Until a way is found to improve the margins or until fabrication processes with very tight control are developed, the Josephson computer will remain on the drawing board.

## Suggestions for Further Reading

1. R. P. Feynman, R. B. Leighton, and M. Sands, "The Feynman Lectures on Physics," vol. 3, Addison Wesley, Reading, Mass., 1966, 21.1–21.18.
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3. D. G. McDonald, "Superconducting Electronics," *Physics Today*, February 1981.
4. J. Matisoo, "The Superconducting Computer," *Sci. American*, May 1980.