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On the dynamics of SCARA robot

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Abstract

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Assembly automation using robots has proven to be very successful. It has been shown that use of robots improves the accuracy of assembly, and saves assembly time and cost as well. SCARA (Selective Compliance Assembly Robot Arm) type robots have been widely used in industry for assembly operations in both the mechanical and electronics domains of manufacturing. One of the important issues in designing an optimal manipulator is the dynamic behavior of the manipulator which is highly non-linear and strongly coupled due to the interaction of inertial, centripetal, coriolis and gravitational forces. Although extensive study involving optimal time study, dynamic study using the Lagrange method, path planning etc. have been made, no closed solution for the dynamics of this important robot has been reported.

This paper presents the study of kinematics and dynamics of the SCARA assembly robot. Closed solutions for the dynamics of this robot are derived. To study the variations of the torques of the links a computer code is developed. It is found that the torque is independent of the angular position, which makes the robot highly compliant.

Keywords: Dynamics; Robotics; Assembly robot.

Nomenclature

A_i^{i-1}	D-H transformation matrix for adjacent frames, i and $i-1$
C_i	cosine θ_i
C_{ijk}	cosine $\theta_{ijk} = \text{cosine}\{(\theta_i + \theta_j) + \theta_k\}$
d_i	distance from the origin of the $(i-1)$th coordinate frame to the intersection of the Z_{i-1} axis with the X_i axis along Z_{i-1} axis
e_i	position of the center of mass of link i from the origin of the coordinate system (X_i, Y_i, Z_i)
F_i	input force for ith joint
f_i	force exerted on link i by link $i-1$ at the coordinate frame $(X_{i-1}, Y_{i-1}, Z_{i-1})$ to support link i and the links above it
Γ_i	input torque for ith joint
I_i	inertia matrix of link i about its center of mass with reference to the coordinate system (X_0, Y_0, Z_0)
J_i	inertia matrix of link i about its center of mass referred to its own link coordinate system (X_i, Y_i, Z_i)
l_i	the shortest distance between Z_{i-1} and Z_i axes
m_{eff}	effective mass of the 3rd link
m_i	mass of the ith link
n_i	moment exerted on link-i by link $i-1$ at the coordinate frame $(X_{i-1}, Y_{i-1}, Z_{i-1})$
p_i^*	the origin of ith coordinate frame with respect to $(i$-1)th coordinate system

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θ_i the joint angle from X_{i-1} axis to the X_i axis about the Z_{i-1} axis (using the right hand rule)

R^{i-1} a 3×3 rotation matrix which transforms any vector with reference to coordinate frame (X_i, Y_i, Z_i) to the coordinate system $(X_{i-1}, Y_{i-1}, Z_{i-1})$

S_i sine θ_i

S_{ijk} sine $\theta_{ijk} = \text{sine}\{(\theta_i + \theta_j) + \theta_k\}$

V_i linear velocity of the coordinate system (X_i, Y_i, Z_i) with respect to base coordinate system (X_0, Y_0, Z_0)

ω_i angular velocity of the coordinate system (X_i, Y_i, Z_i) with respect to base coordinate system (X_0, Y_0, Z_0)

1. Introduction

In the manufacturing domain assembly plays a vital role. With the advent of modern computers and sophisticated robots, assembly automation is making rapid progress. SCARA is an open loop-type manipulator that is used for assembly in production industries successfully. It is composed of three revolute joints allowing it to move and orient in the x–y plane, and one prismatic joint allows the movement of the end effector in the vertical (z-direction) direction. This robot is best suited for planar tasks.

This robot has high compliance in the x–y plane in the sense that the arm moves freely to accomplish the assembly task accurately. A robot is said to be of high compliance if the arm moves a lot in response to a small force and the manipulator is then said to be spongy or springy. On the contrary, if it moves a little the compliance is low and the manipulator is said to be stiff.

Fig. 1 illustrates this 2R-P-R robot with 4 axes. The first two axes are actuated by DC servo motors through harmonic drives, whose backlash is almost zero. These two axes allow the robot to move in a plane parallel to the x–y plane within the work-space. The z-axis movement is made possible by the ball screw which converts the rotational motion of the motor to translational motion of the axis. The end effector is rotated by a DC servo motor through a gear box.

2. Previous work

Roth [1] studied this robot to achieve the optimal design for the links to minimize the pick-and-place motion with digital computer simulation. The goal was to obtain a robot design that will perform the task in the shortest time. The links were idealized as rigid bodies and secondary

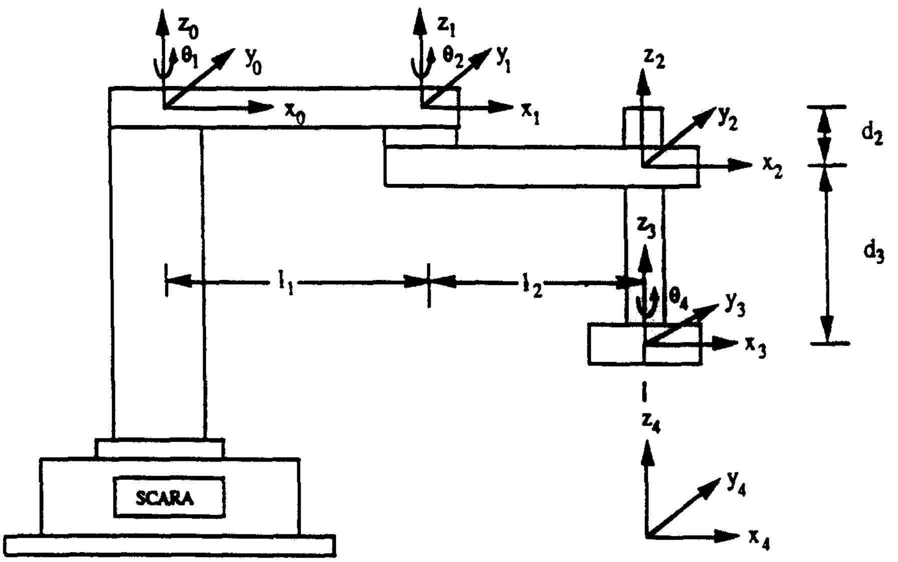

Fig. 1. Schematic figure of the SCARA robot.

effects such as blacklash were ignored. Development of an integrated simulation package has been reported from Gold Star Company of South Korea, by Lee et al. [2]. The fourth order Rungga-Kutta method is employed as the integration technique. The approach is based on the state equations that are integrated to find the velocity and positions. The acceleration is computed as a function of motor torque. Callaja et al. [3] studied this robot to design the servos for it. Their study was primarily focussed on the control aspects of this robot. The dynamics of this robot is studied by Ibrahim et al. [4] using polynomial and NC2 trajectories. Their work included a software development in FORTRAN 77 to study the dynamic behavior with links subjected to different velocity trajectories. They used the Lagrangian model taking advantage of recursive formulation. However, there is no closed solution reported for the dynamics of this robot. The dynamic study can provide both qualitative and quantitative analysis of this robot as well as can guide for a better design of this robot.

3. Present work

The present analysis of this robot is carried out to study the dynamic behavior of this robot. The kinematics is considered first to find the relationship between the angular displacement and the position of the end effector, and to achieve this inverse kinematics is employed. The dynamics study is done using the Newton-Euler method. The significance of this study lies in the fact that it gives insight into the dynamic behavior of this robot.

3.1. Kinematic analysis

Robot kinematics deals with the analytical study of motion of the robot arm with respect to a fixed reference coordinate system as a function of the time and without regard to the force or torque that causes motion. The kinematics can be of two types: direct kinematics or inverse kinematics. In the direct kinematics, the joint and link parameters are given and the end effector position is calculated. In the inverse kinematics the joint angles are calculated for a given end effector positon and link parameters. In this work, the inverse kinematics of the SCARA robot is discussed.

The translational and rotational relationship between the links of this robot are described by attaching coordinate frames according to the Denavit-Hartenberg (D-H) notation. The expression for the end effector frame relative to the base frame is given by the arm matrix (T) as:

$$T = A_4^0 = A_1^0 A_2^1 A_3^2 A_4^3,$$

where

$$A_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 & l_1C_1 \\ S_1 & C_1 & 0 & l_1S_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 & l_2C_2 \\ S_2 & C_2 & 0 & l_2S_2 \\ 0 & 0 & 1 & -d_2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_3^2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^3 = \begin{bmatrix} C_4 & -S_4 & 0 & 0 \\ S_4 & C_4 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$A_4^0 = \begin{bmatrix} C_{124} & -S_{124} & 0 & l_1C_1 + l_2C_{12} \\ S_{124} & C_{124} & 0 & l_1S_1 + l_2S_{12} \\ 0 & 0 & 1 & -d_2 - d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

The arm matrix (T) can also be expressed in terms of a hand coordinate system using ***n, s, a, p*** notations where ***n*** is the normal vector, ***s*** is the sliding vector, ***a*** is the approach vector and ***p*** is the position vector of the hand.

$$T = \begin{bmatrix} n_x & s_x & a_x & p_x \\ n_y & s_y & a_y & p_y \\ n_z & s_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Comparing the T and A_4^0 matrices we have:

$$p_x = l_1C_1 + l_2C_{12}$$

$$p_y = l_1S_1 + l_2S_{12}$$

$$p_z = -d_2 - d_3.$$

Squaring the terms and by algebraic manipulation the equations become:

$C_2 = \left[\left(r^2 - l_1^2 - l_2^2\right)/2l_1l_2\right]$ where $r^2 = p_x^2 + p_y^2$

$\theta_2 = \tan^{-1}\left[\pm\sqrt{\left(1 - C_2^2\right)/C_2}\right]$

$\theta_1 = \tan^{-1}\left[\left(-Mp_x + Np_y\right)/\left(Mp_y + Np_x\right]\right.$

$d_3 = -d_2 - p_z,$

where $M = l_2S_2$, $N = l_1 + l_2C_2$.

3.2. Dynamic analysis

Dynamics is the science of motion which treats motion with regard to the torques applied by the actuators or by external forces applied to the manipulator. There are two problems related to the dynamics of a manipulator. The first one is direct dynamics where the torque is calculated for given values of angular displacement, angular velocity and acceleration. The second problem is of indirect dynamics where the angular velocity, angular displacements and acceleration are calculated for a given torque.

The dynamic study of a manipulator can be done by several methods such as: Lagrange Euler (L-E) method, Newton-Euler (N-E) method etc. However, the L-E method is not commonly used for real time control as it needs large amount of computation time and space. The N-E method involves a set of forward and backward recursive equations. The forward recursion propagates the kinematics information such as velocities and accelerations at the center of mass of each link. The backward recursive formulations propagates the forces and moments exerted on each link from the end effector to the base of the robot. It has observed that the computation time is linearly proportional to the number of joints and independent of the robot arm configuration. It is feasible to implement this method in real-time control of the robot. In this paper, the mathematical formulations of the equation of motion are derived using the N-E method.

The rotation matrices are as follows:

$$R_1^0 = \begin{bmatrix} C_1 & -S_1 & 0 \\ S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^1 = \begin{bmatrix} C_2 & -S_2 & 0 \\ S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_4^3 = \begin{bmatrix} C_4 & -S_4 & 0 \\ S_4 & C_4 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_2^0 = \begin{bmatrix} C_{12} & -S_{12} & 0 \\ S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_3^0 = R_2^0$$

$$R_4^0 = \begin{bmatrix} C_{124} & -S_{124} & 0 \\ S_{124} & C_{124} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^1 = \begin{bmatrix} C_1 & S_1 & 0 \\ -S_1 & C_1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_1^2 = \begin{bmatrix} C_2 & S_2 & 0 \\ -S_2 & C_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^2 = \begin{bmatrix} C_{12} & S_{12} & 0 \\ -S_{12} & C_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$R_0^3 = R_0^2.$$

The vectors representing the distance from $(i-1)$th co-ordinate system to ith co-ordinate system are given as follows:

$$P_1^* = [l_1C_1, l_1S_1, 0]^{\mathrm{T}}$$

$$P_2^* = [l_1C_{12}, l_1S_{12}, -d_2]^{\mathrm{T}}$$

$$p_3^* = [0, 0, -d_3]^{\mathrm{T}}$$

$$p_4^* = [0, 0, 0]^{\mathrm{T}}.$$

The following initial conditions are assumed for the present analysis:

$$\omega_0 = \dot{\omega}_0 = V_0 = 0.$$

$$\dot{V}_0 = (0, 0, g)^{\mathrm{T}}.$$

Forward recursive equations

The forward equations for this robot are evaluated in the following section.

$$R_0^1\omega_1 = R_0^1(\omega_0 + Z_0\dot{\theta}_1) = [0 \quad 0 \quad 1]^{\mathrm{T}}\dot{\theta}_1$$

$$R_0^2\omega_2 = R_1^2(R_0^1\omega_1 + Z_0\dot{\theta}_2) = [0 \quad 0 \quad 1]^{\mathrm{T}}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$R_0^3\omega_3 = R_2^3(R_0^2\omega_2) = [0 \quad 0 \quad 1]^{\mathrm{T}}(\dot{\theta}_1 + \dot{\theta}_2)$$

$$R_0^4\omega_4 = R_3^4(R_0^3\omega_3 + Z_0\dot{\theta}_4) = [0 \quad 0 \quad 1]^{\mathrm{T}}(\dot{\theta}_1 + \dot{\theta}_2 + \dot{\theta}_4)$$

$$R_0^1\dot{\omega}_1 = R_0^1[\dot{\omega}_0 + Z_0\ddot{\theta}_1 + \omega_0 \times Z_0\theta_1] = [0 \quad 0 \quad 1]^{\mathrm{T}}\ddot{\theta}_1$$

$$R_0^2\dot{\omega}_2 = R_1^2[R_0^1\dot{\omega}_1 + Z_0\ddot{\theta}_2 + (R_0^1\omega_1) \times Z_0\dot{\theta}_2] = [0 \quad 0 \quad 1]^{\mathrm{T}}(\ddot{\theta}_1 + \ddot{\theta}_2)$$

$$R_0^3\dot{\omega}_3 = R_2^3[R_0^2\dot{\omega}_2] = [0 \quad 0 \quad 1]^{\mathrm{T}}(\ddot{\theta}_1 + \ddot{\theta}_2)$$

$$R_0^4\dot{\omega}_4 = R_3^4[R_0^3\dot{\omega}_3 + Z_0\ddot{\theta}_4 + (R_0^3\omega_3) \times Z_0\dot{\theta}_4] = [0 \quad 0 \quad 1]^{\mathrm{T}}(\ddot{\theta}_1 + \ddot{\theta}_2 + \ddot{\theta}_4)$$

$$R_0^1V_1 = (R_0^1\dot{\omega}_1) \times (R_0^1p_1^*) + (R_0^1\omega_1) \times [(R_0^1\omega_1) \times (R_0^1p_1^*)] + R_0^1(R_0^1\dot{V}_0) = \left[-l_1\dot{\theta}_1^2,\ l_1\ddot{\theta}_1,\ g\right]^{\mathrm{T}}$$

$$R_0^2V_2 = (R_0^2\dot{\omega}_2) \times (R_0^2p_2^*) + (R_0^2\omega_2) \times [(R_0^2\omega_2) \times (R_0^2p_2^*)] + R_1^2(R_0^1\dot{V}_1)$$
$$= \Big[l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2(\dot{\theta}_1 + \dot{\theta}_2)^2,\ l_2(\ddot{\theta}_1 + \ddot{\theta}_2) + l_1\dot{\theta}_1^2S_2 + l_1\ddot{\theta}_1C_2,\ g\Big]^{\mathrm{T}}$$

$$R_0^3V_3 = R_2^3(Z_0\ddot{\theta}_3 + R_0^2\dot{V}_2) + (R_0^3\dot{\omega}_3) \times (R_0^3p_3^*) + [2(R_0^3\omega_3) \times (R_2^3Z_0\dot{\theta}_3)] + R_0^3\omega_2 \times [(R_0^3\omega_3) \times (R_0^3p_3^*)]$$
$$= R_2^3(R_0^2\dot{V}_2) + (R_0^3\dot{\omega}_3) \times (R_0^3p_3^*) + R_0^3\omega_3 \times [(R_o^3\omega_3) \times (R_0^3p_3^*)]$$
$$= \Big[l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2(\dot{\theta}_1 + \dot{\theta}_2)^2,\ l_1\ddot{\theta}_1C_2 + l_1\dot{\theta}_1^2S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2),\ g\Big]^{\mathrm{T}}$$

$$R_0^4V_4 = (R_0^4\dot{\omega}_4) \times (R_0^4p_4^*) + (R_0^4\omega_4) \times [(R_0^4\omega_4) \times (R_0^4p_4^*)] + R_3^4(R_0^3\dot{V}_3)$$
$$= \Big[l_1\ddot{\theta}_1S_{24} - l_1\dot{\theta}_1^2C_{24} - l_2\{(\dot{\theta}_1 + \dot{\theta}_2)^2C_4 - (\ddot{\theta}_1 + \ddot{\theta}_2)S_4\},\ l_1\ddot{\theta}_1C_{24} + l_1\dot{\theta}_1^2S_{24} + l_2\{(\dot{\theta}_1 + \dot{\theta}_1)^2S_4 - (\ddot{\theta}_1 - \ddot{\theta}_2)C_4\},\ g\Big]^{\mathrm{T}}.$$

The position of the center of mass of link i from the origin of the co-ordinate system (X_i, Y_i, Z_i) is denoted as e_i. For this robot, we have:

$$e_1 = [-l_1C_1/2,\ -l_1S_1/2,\ 0]^{\mathrm{T}}$$
$$e_2 = [-l_2C_{12}/2,\ -l_2S_{12}/2,\ d_2]^{\mathrm{T}}$$
$$e_3 = [0,\ 0,\ l_3/2]^{\mathrm{T}}$$
$$e_4 = [0,\ 0,\ 0]^{\mathrm{T}}.$$

Now, computing the linear accelerations at the center of the mass for the four links we have:

$$R_0^1a_1 = (R_0^1\dot{\omega}_1) \times (R_0^1e_1) + (R_0^1\omega_1) \times [(R_0^1\omega_1) \times (R_0^1e_1)] + (R_0^1\dot{V}_1) = \left[-l_1\dot{\theta}_1^2/2,\ l_1\ddot{\theta}_1/2,\ g\right]^{\mathrm{T}}$$

$$R_0^2a_2 = (R_0^2\omega_2) \times (R_0^2e_2) + (R_0^2\omega_2) \times [(R_0^2\omega_2) \times (R_0^2e_2] + (R_0^2\dot{V}_2)$$
$$= \Big[l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2(\dot{\theta}_1 + \dot{\theta}_2)^2/2,\ l_1\ddot{\theta}_1C_2 + l_1\dot{\theta}_1^2S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2)/2,\ g\Big]^{\mathrm{T}}$$

$$R_0^3a_3 = (R_0^3\dot{\omega}_3) \times (R_0^3e_3) + (R_0^3\omega_3) \times [(R_0^3\omega_3) \times (R_0^3e_3] + (R_0^3\dot{V}_3) = R_0^3\dot{V}_3$$
$$= \Big[l_1\ddot{\theta}_1S_2 - l_1\dot{\theta}_1^2C_2 - l_2(\dot{\theta}_1 + \dot{\theta}_2)^2,$$

$$l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2),\ g\Big]^{\mathrm{T}}$$

$$\begin{aligned} R_0^4 a_4 &= (R_0^4\dot{\omega}_4) \times (R_0^4 e_4) + (R_0^4\omega_4) \\ &\quad \times [(R_0^4\omega_4) \times (R_0^4 e_4] + (R_0^4 V_4) \\ &= R_0^4 \dot{V}_4 \\ &= \Big[l_1\ddot{\theta}_1 S_{24} - l_1\dot{\theta}_1^2 C_{24} \\ &\quad -l_2\Big\{(\dot{\theta}_1 + \dot{\theta}_2)^2 C_4 - (\ddot{\theta}_1 + \ddot{\theta}_2) S_4\Big\}, \\ &\quad l_1\ddot{\theta}_1 C_{24} + l_1\dot{\theta}_1^2 S_{24} \\ &\quad +l_2\Big\{(\dot{\theta}_1 + \dot{\theta}_2)^2 S_4 + (\ddot{\theta}_1 + \ddot{\theta}_2) C_4\Big\},\ g\Big]^{\mathrm{T}}. \end{aligned}$$

Backward recursive equations

Backward equations for the links are dealt with in the following section. Assuming no load condition f_5 and n_5 are zero.

$$\begin{aligned} R_0^4 f_4 &= R_5^4(R_0^5 f_5) + m_4 R_0^4 a_4 \\ &= m_4 R_0^4 a_4 \\ &= m_4\Big[l_1\ddot{\theta}_1 S_{24} - l_1\dot{\theta}_1^2 C_{24} \\ &\quad -l_2\Big\{(\dot{\theta}_1 + \dot{\theta}_2)^2 C_4 - (\ddot{\theta}_1 + \ddot{\theta}_2) S_4\Big\}, \\ &\quad l_1\ddot{\theta}_1 C_{24} + l_1\dot{\theta}_1^2 S_{24} \\ &\quad +l_2\Big\{(\dot{\theta}_1 + \dot{\theta}_2)^2 S_4 + (\ddot{\theta}_1 + \ddot{\theta}_2) C_4\Big\},\ g\Big]^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} R_0^3 f_3 &= R_4^3(R_0^4 f_4) + m_3 R_0^3 a_3 \\ &= (m_3 + m_4)\Big[l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2 - l_2(\dot{\theta}_1 + \dot{\theta}_2)^2, \\ &\qquad l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2),\ g\Big]^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} R_0^2 f_2 &= R_3^2(R_0^3 f_3) + m_2 R_0^2 a_2 \\ &= \Big[\Big\{x(l_1\ddot{\theta}_1 S_2 - l_1\dot{\theta}_1^2 C_2) - y l_2(\dot{\theta}_1 + \dot{\theta}_2)^2\Big\}, \\ &\qquad \Big\{x(l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2) + y l_2(\ddot{\theta}_1 + \ddot{\theta}_2)\Big\},\ xg\Big]^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} R_0^1 f_1 &= R_2^1(R_0^2 f_2) + m_1 R_0^1 a_1 \\ &= \Big[\Big\{-l_1\dot{\theta}_1^2(x + m_1/2) \\ &\quad -y l_2\Big((\dot{\theta}_1 + \dot{\theta}_2)^2 C_2 + (\ddot{\theta}_1 + \ddot{\theta}_2) S_2\Big)\Big\}, \\ &\quad \Big\{l_1\ddot{\theta}_1(x + m_1/2) \\ &\quad -y l_2\Big((\dot{\theta}_1 + \dot{\theta}_2)^2 S_2 - (\ddot{\theta}_1 + \ddot{\theta}_2) C_2\Big)\Big\}, \\ &\quad (x + m_1) g\Big]^{\mathrm{T}}, \end{aligned}$$

where: $x = m_2 + m_3 + m_4$,
$y = m_2/2 + m_3 + m_4$.

The moments exerted on the links are calculated as follows:

$$\begin{aligned} R_0^4 n_4 &= R_5^4[R_0^5 n_5 + (R_0^5 p_4^*) \times (R_0^5 f_5)] \\ &\quad + (R_0^4 p_4^* + R_0^4 e_4) \times (m_4 R_0^4 a_4) \\ &\quad + J_4 R_0^4 \dot{\omega}_4 + [(R_0^4\omega_4) \times J_4(R_0^4\omega_4)], \end{aligned}$$

where $J_i = R_0^i I_i R_i^0$, $i = 1, 2, 3, 4$.

Assuming the J_4 is very small in comparison to other moment of inertia terms, the $R_0^4 n_4$ term is evaluated to be:

$$R_0^4 n_4 = 0$$

Although the robot links are not always cylindrical, in this analysis they are considered to be cylindrical for the sake of simplicity. A link with other shapes can be transformed into an equivalent cylindrical shape to use the derived results in this paper.

$$\begin{aligned} R^3 n_3 &= R_4^3[R_0^4 n_4 + (R_0^4 p_3^*) \times (R_0^4 f_4)] \\ &\quad + (R_0^3 p_3^* + R_0^3 e_3) \times (m_3 R_0^3 a_3) \\ &\quad + J_3 R_0^3 \dot{\omega}_3 + [(R_0^3\omega_3) \times J_3(R_0^3\omega_3)] \\ &= \{d_3(m_2 + m_4) - m_3 l_3/2\} \\ &\quad \times \Big[l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2), \\ &\qquad -l_1\ddot{\theta}_2 S_2 + l_1\dot{\theta}_1^2 C_2 + l_2(\dot{\theta}_1 + \dot{\theta}_2)^2,\ 0\Big]^{\mathrm{T}} \end{aligned}$$

$$\begin{aligned} R_0^2 n_2 &= R_3^2[R_0^3 n_3 + (R_0^3 p_2^*) \times (R_0^3 f_3)] \\ &\quad + (R_0^2 p_2^* + R_0^2 e_2) \times (m_2 R_0^2 a_2) \\ &\quad + J_2 R_0^2 \dot{\omega}_2 + [(R_0^2\omega_2) \times J_2(R_0^2\omega_2)] \\ &= \Big[\Omega\Big\{l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2 + l_2(\ddot{\theta}_1 + \ddot{\theta}_2)\Big\}, \end{aligned}$$

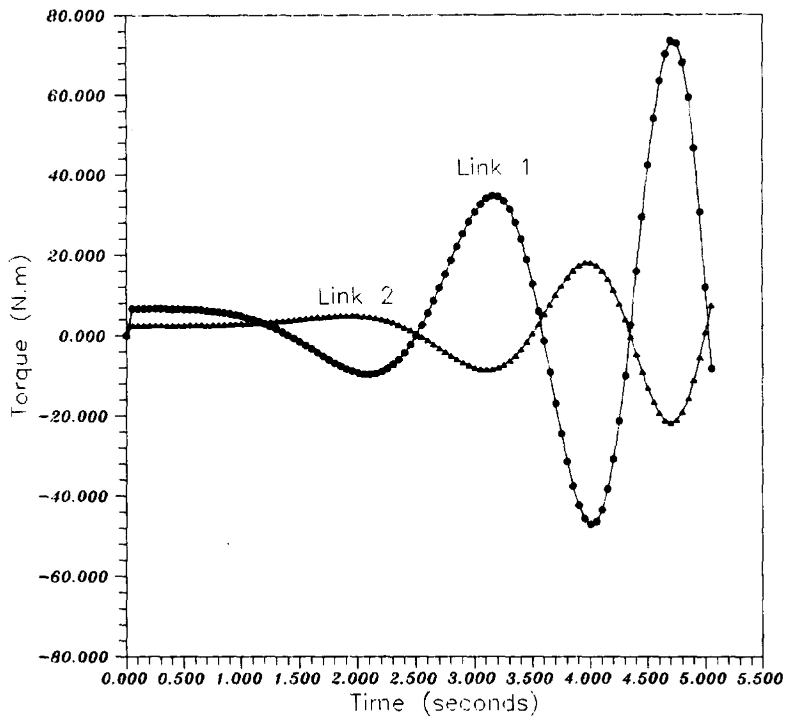

Fig. 2. Torque variations with time.

$$\Omega\left\{-l_1\ddot{\theta}_1 S_2 + l_1\dot{\theta}_1^2 C_2 + l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2\right\}$$

$$-l_2 gy,\ l_2 y\left(l_1\ddot{\theta}_1 C_2 + l_1\dot{\theta}_1^2 S_2\right) + l_2^2\Lambda\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)\Big]^{\mathrm{T}},$$

where $\Omega = d_2(m_3 + m_4) + d_3(m_3 + m_4) - m_3 l_3/2$
$\Lambda = m_2/3 + m_3 + m_4$

$$\begin{aligned}
R^1 n_1 = {} & R_2^1\left[R_0^2 n_2 + \left(R_0^2 p_1^*\right)\times\left(R_0^2 f_2\right)\right] \\
& + \left(R_0^1 p_1^* + R_0^1 e_1\right)\times\left(m_1 R_0^1 a_1\right) \\
& + J_1 R_0^1\dot{\omega}_1 + \left[\left(R_0^1\omega_1\right)\times J_1\left(R_0^1\omega_1\right)\right] \\
= {} & \Big[\Omega\left\{l_1\ddot{\theta}_1 + l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)C_2 - l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 S_2\right\} \\
& + l_2 gyS_2, \\
& \Omega\left\{l_1\dot{\theta}_1^2 + l_2\left(\ddot{\theta}_1 + \ddot{\theta}_2\right)S_2 + l_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 C_2\right\} \\
& - l_2 g(x + yC_2 + m_1/2), \\
& \Big\{\left(l_1^2(x + m_1/3) + l_2^2\Lambda + 2l_1 l_2 C_2 y\right)\ddot{\theta}_1 \\
& + \left(l_2^2\Lambda + l_1 l_2 C_2 y\right)\ddot{\theta}_2 \\
& - l_1 l_2 S_2 y\dot{\theta}_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right)\Big\}\Big]^{\mathrm{T}}.
\end{aligned}$$

In calculating the joint torques, the viscous effect at the joints is neglected. The torque at joint 1 is:

$$\begin{aligned}
\Gamma_1 = {} & \left[R_0^1 n_1\right]^{\mathrm{T}}\left(R_0^1 Z_0\right) \\
= {} & \left(l_1^2(x + m_1/3) + l_2^2\Lambda + 2l_1 l_2 C_2 y\right)\ddot{\theta}_1 \\
& + \left(l_2^2\Lambda + l_1 l_2 C_2 y\right)\ddot{\theta}_2 - l_1 l_2 S_2 y\dot{\theta}_2\left(2\dot{\theta}_1 + \dot{\theta}_2\right).
\end{aligned}$$

Similarly the torque at the joint 2 is given as:

$$\begin{aligned}
\Gamma_2 = {} & \left[R^2 n_2\right]^{\mathrm{T}}\left(R_1^2 Z_0\right) \\
= {} & \left(l_1 l_2 yC_2 + l_2^2\Lambda\right)\ddot{\theta}_1 + l_2^2\Lambda\ddot{\theta}_2 + l_1 l_2 yS_2\dot{\theta}_1^2.
\end{aligned}$$

There is no torque on link 3; rather, a force acts to move the link 3 in a vertical direction.

$$\begin{aligned}
F_3 & \left[R_0^3 f_3\right]^{\mathrm{T}}\left(R_2^3 Z_0\right) \\
& = (m_3 + m_4)g \\
& = m_{\mathrm{eff}}\, g.
\end{aligned}$$

4. Discussion

The configuration of the SCARA robot shows no coupling between the second and third link. For this reason, the effective mass of the third link can be added to that of second link while determining the torques. This fact is clear from the torque equations. The third link has motion in the vertical direction and no horizontal components. It is evident that there is no torque required for driving this link. Rather a force is required to achieve the vertical motion.

the torque-time analysis is carried out taking the link lengths of 0.35 and 0.3 meters for link 1 and link 2 respectively; the initial position for both link 1 and link 2 is 0 radian; masses of link 1, link 2, link 3 and link 4 are 10 kg, 5 kg and 0.5 kg respectively. The angular accelerations of link 1 and link 2 are assigned to be 1 rad/sec^2. *Fig. 2* illustrates the variation of the torques with time. It is found that the torque of link 1 and link 2 are out of phase by 180 degrees, and the magnitude of the torque of link 1 is higher than the torque requirement for link 2. There is an increasing difference in the two torques as time increased.

5. Conclusion

The kinematics and dynamics of the SCARA assembly robot are studied, and the dynamic equations of motions are derived using Newton-Euler method. From the dynamic equations it is found that the mass if the fourth link can be added to the mass of the link-3 for all analysis purposes and there is no coupling between the second and third link. Another fact that is revealed is that the torques are independent of angular positions and this makes the robot very compliant. With same initial angular positions, and angular accelerations for link 1 and link 2, it is found that the torque of link 1 and link 2 are out of phase by 180 degrees. With increased time, there is a trend of increasing difference between these two torques.

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