

SEM0104 – Aula 8

Trens de Engrenagens

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USP - EESC - SEM

LabRoM

Prof. Dr. Marcelo Becker - SEM - EESC - USP

Sumário da Aula

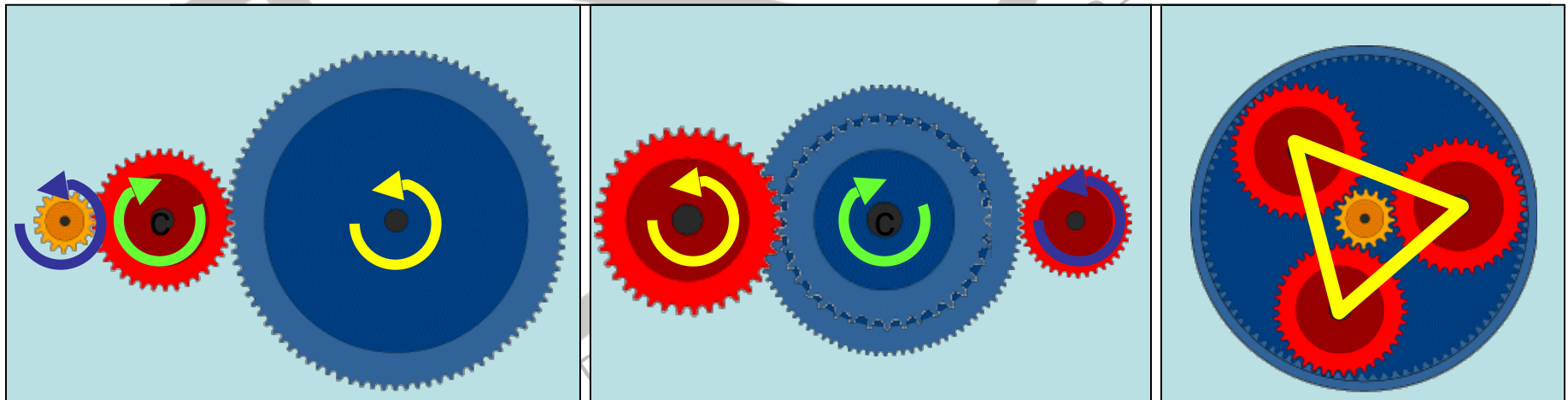
- **Introdução**
- Representações
- Montagens
- Análise Cinemática
- Análise de Torque
- Exemplos de Aplicação

Introdução

Trens de Engrenagens

Tipos:

- Simples: 1 engrenagem / eixo
- Compostos: 2 ou + engrenagens / eixo
- Planetários: elemento de suporte



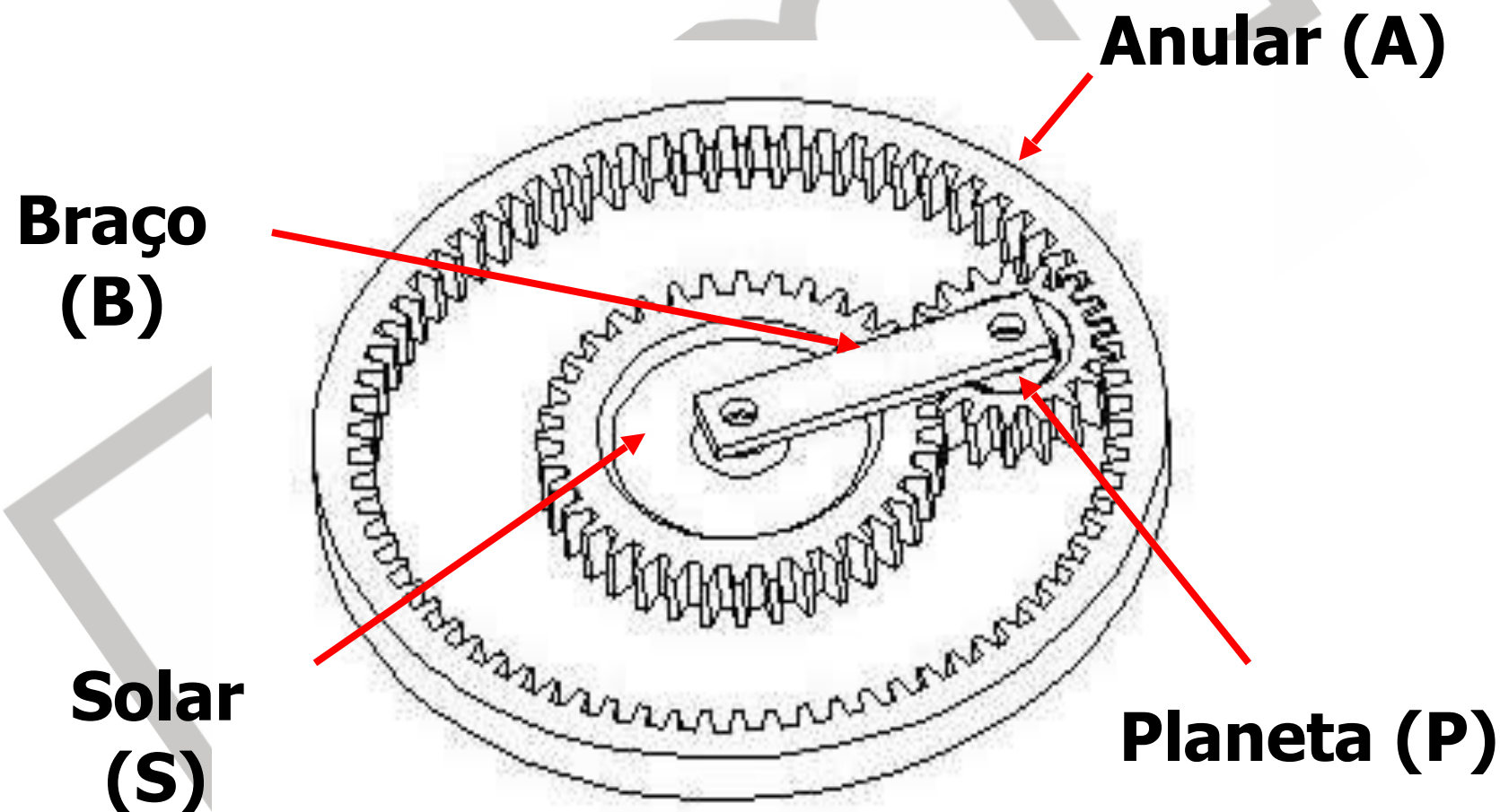
Simple

Composto

Planetário

Introdução

Trens de Engrenagens Planetárias



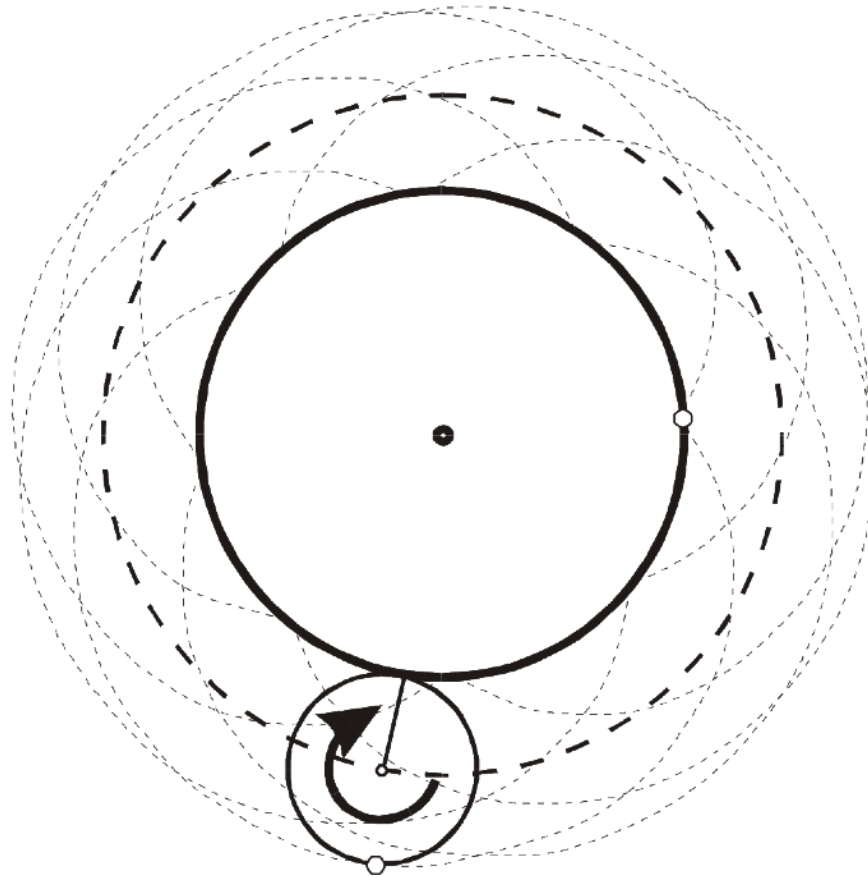
Equação para Número de Dentes:

$$Z_A = 2 \cdot Z_P + Z_S$$

Introdução

Trens de Engrenagens Planetárias

Curva Epicicloidal



<http://www.youtube.com/watch?v=FbDXZgqJOWQ>

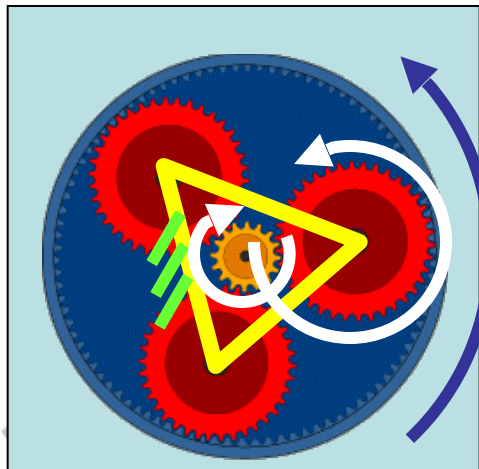
Introdução

Trens de Engrenagens Planetárias

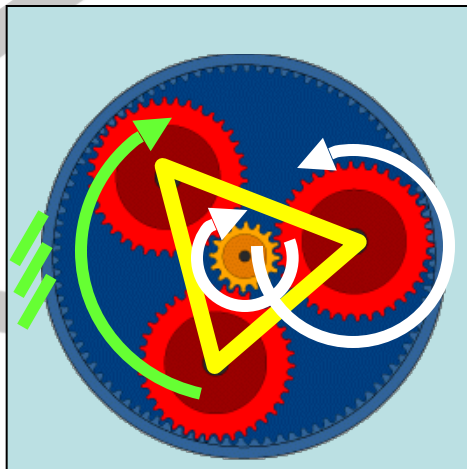
- 2 GDL
- Alta Complexidade Cinemática
- “Difícil de Entender”
- Permanentemente Engrenado
- Alta Confiabilidade
- Compacto
- Alta Redução / Multiplicação
- Adição ou Divisão de Torque
- Múltiplas RTs

Introdução

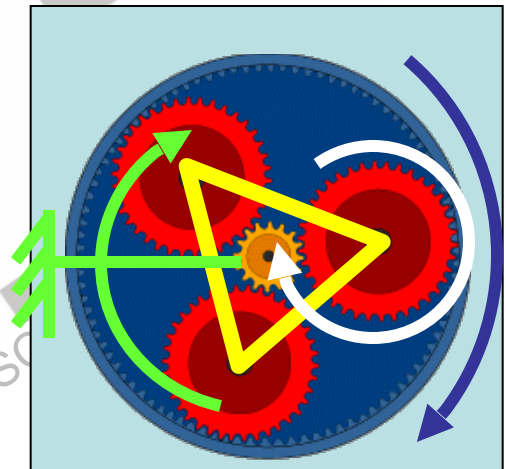
Trens de Engrenagens Planetárias



Elemento Fixo: Braço
Solar **SH**
Planeta **SAH**
Anular **SAH**



Elemento Fixo: Anular
Solar **SH**
Planeta **SAH**
Braço **SH**



Elemento Fixo: Solar
Braço **SH**
Planeta **SH**
Anular **SH**

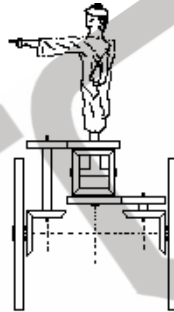
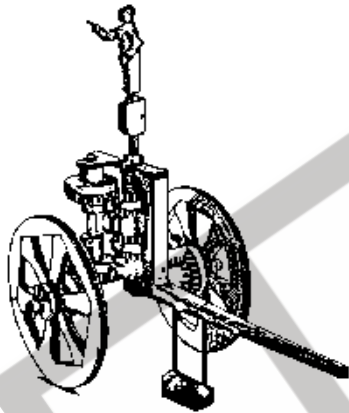


<http://www.youtube.com/watch?v=JBB1sC7LCuQ>

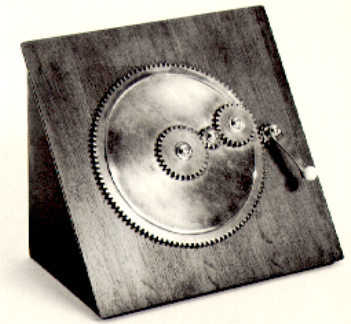
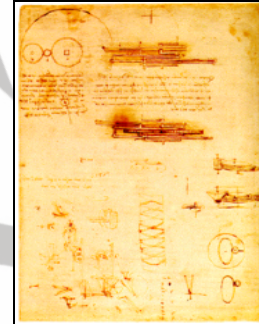
Introdução

Trens de Engrenagens Planetárias

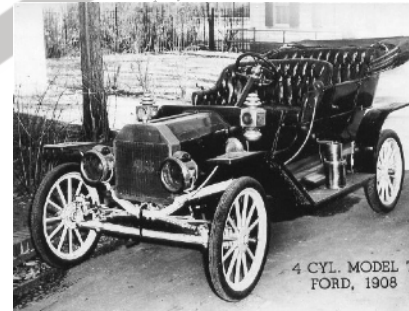
Histórico



- Carruagem Chinesa: 2.634 a.C.
- Arquimedes: 212 a.C.



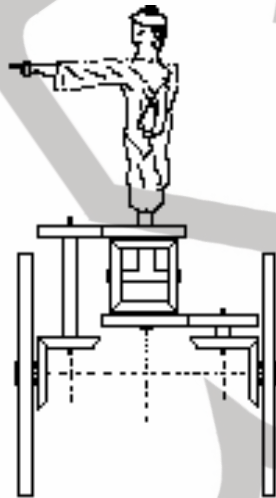
- Leonardo da Vinci: 1.500
- FORD T: 1908 a 1927



Introdução

Trens de Engrenagens Planetárias

Carruagem Chinesa (2.634 a.C.)

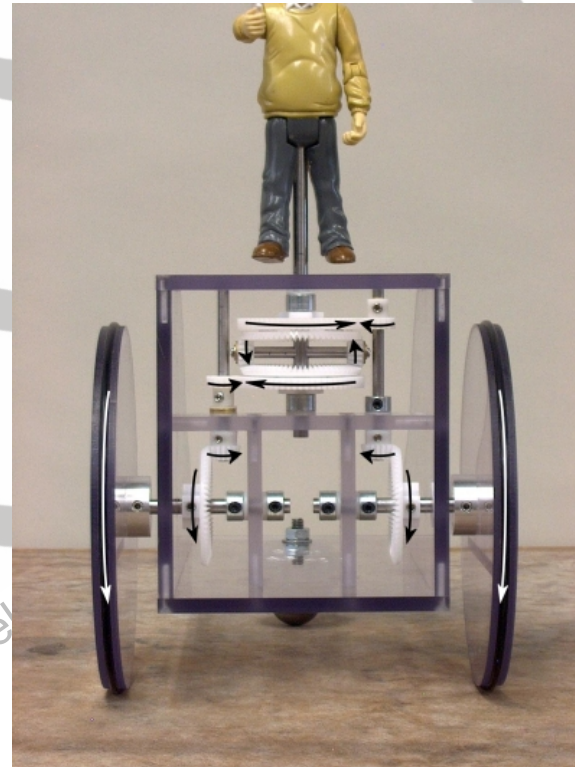


Imperador Huang Di (China)

Introdução

Trens de Engrenagens

Carruagem Chinesa (2.634 a.C.)

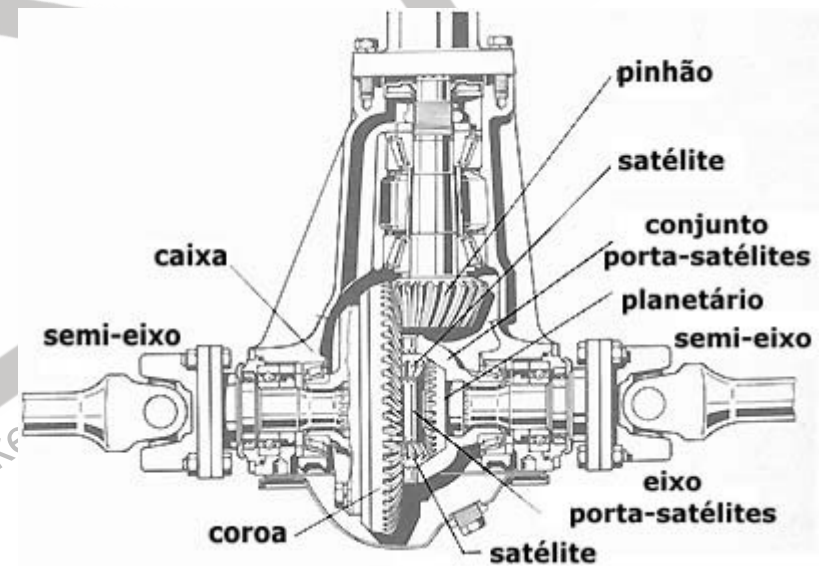


<http://web.physics.ucsb.edu/~lecturedemonstrations/Composer/Pages/92.36>

Introdução

Trens de Engrenagens

Carruagem Chinesa (2.634 a.C.)

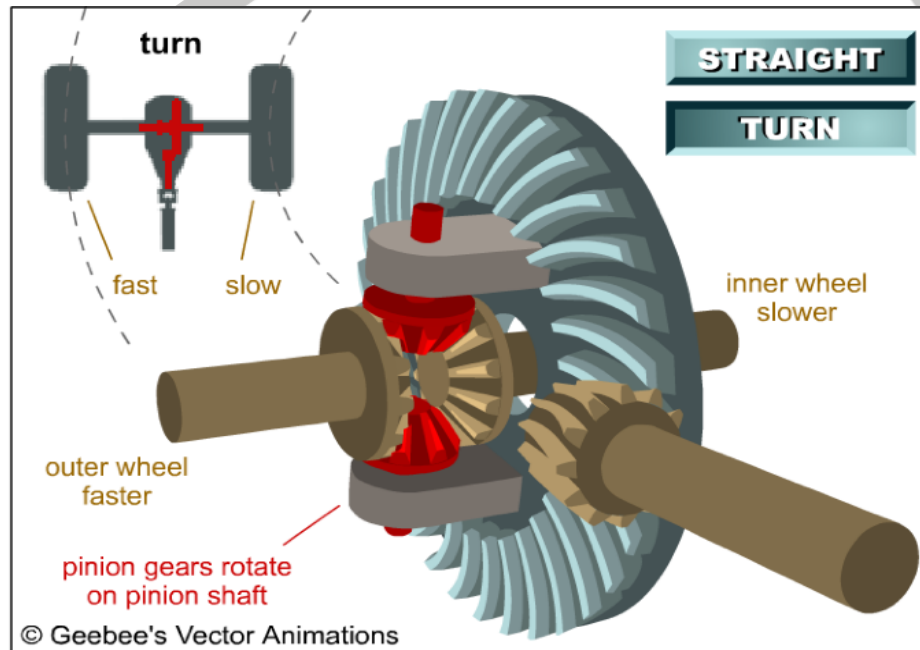


Diferencial Automotivo

Introdução

Trens de Engrenagens

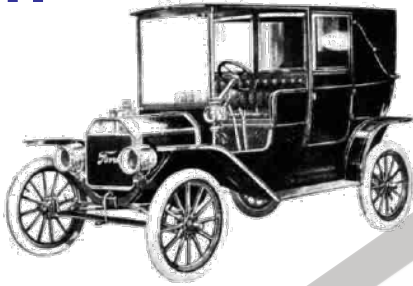
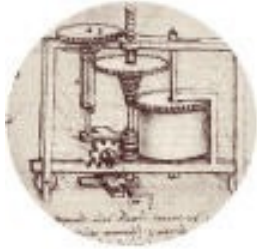
Diferencial Automotivo



<http://auto.howstuffworks.com/differential2.htm>

Transmissões Veiculares

Ontem



Ford Modelo T (1908 - 1928)
Transmissão Automática



Anel Sincronizador



BMW Série 7 (2001)
Transmissão Automática



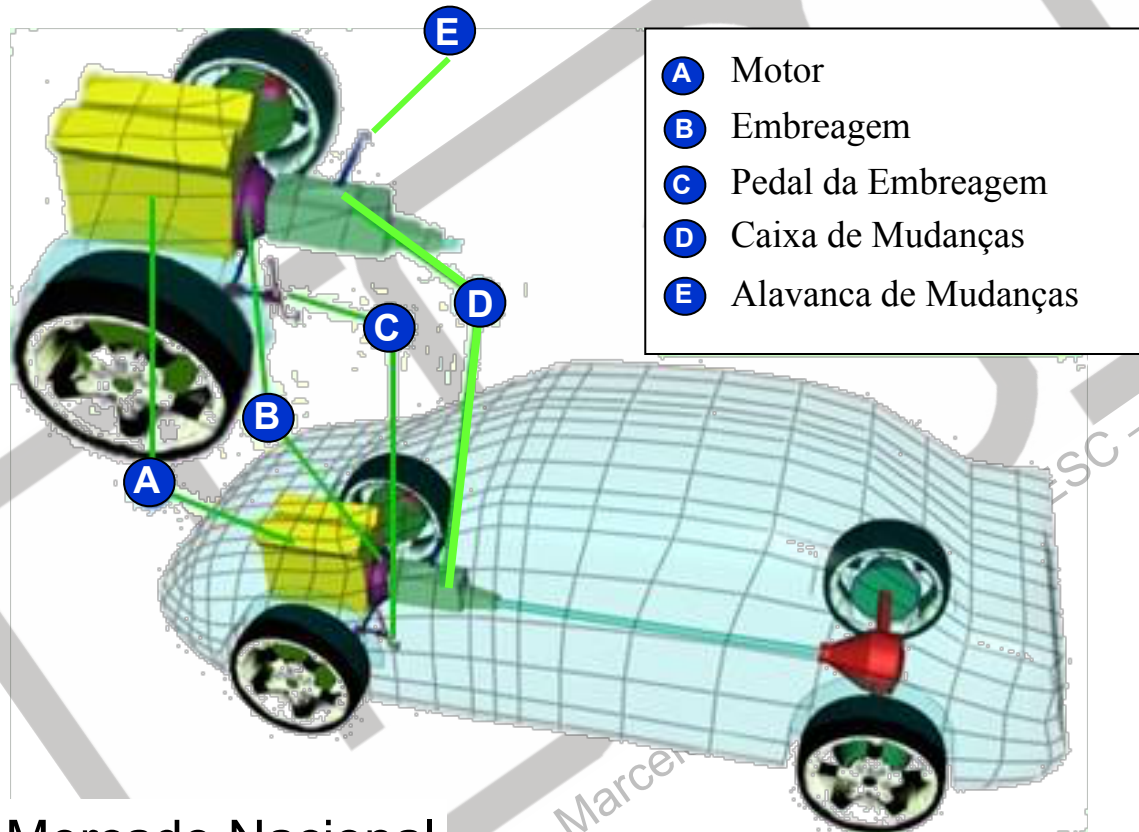
Fiat Stilo (2008)
Transmissão Dual-logic



Transmissões Manuais

Transmissões Veiculares

Hoje



Mercado Nacional

Transmissões Automobilísticas

Hoje

Mercado Brasileiro: supremacia da Transmissão Manual

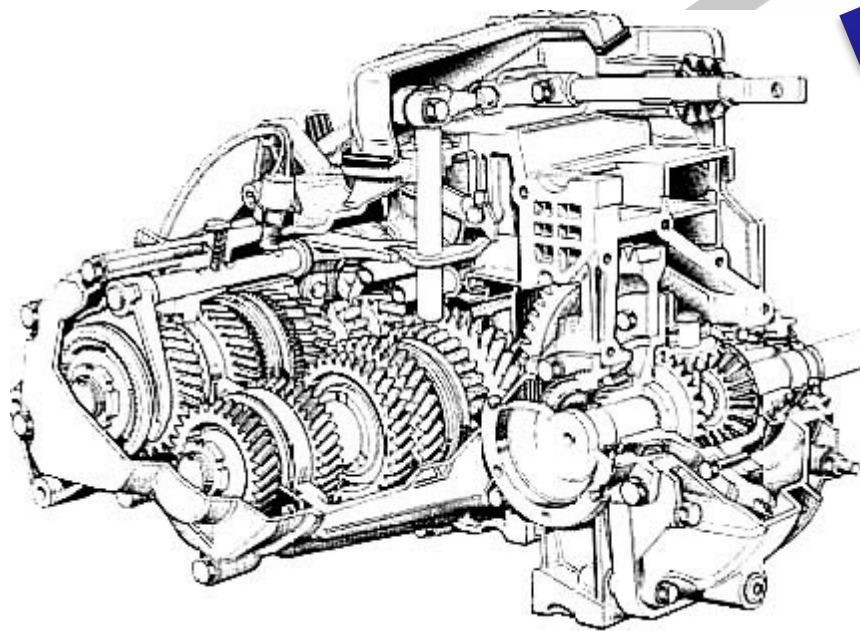
Mercado Americano: supremacia da Transmissão Automática

Mercado Europeu: supremacia da Trans. Automática e Servo-assistida

Mercado Asiático: supremacia da Transmissão Automática

Transmissões Automobilísticas

Amanhã?...



Transmissões CVT
(Continuously Variable Transmission)

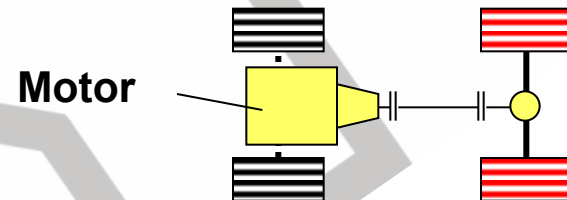
Transmissões Automáticas

Transmissões Servo - Assistidas

Tendência Atual

- Atualmente a tendência na Indústria Automobilística é, em função dos Torques (Gerhard Wagner - ZF):

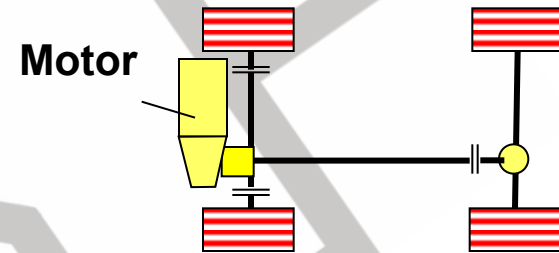
Motor Longitudinal Frontal
Tração Traseira



Transmissão Longitudinal Frontal e Traseira		
Torque	Transmissão Manual	Transmissão Automática
< 180 N.m	5 marchas	CVT
180-350 N.m	6 marchas	CVT
> 350 N.m	6 marchas	6 marchas

Tendência Atual

Motor Transversal Frontal
Tração 4-Rodas



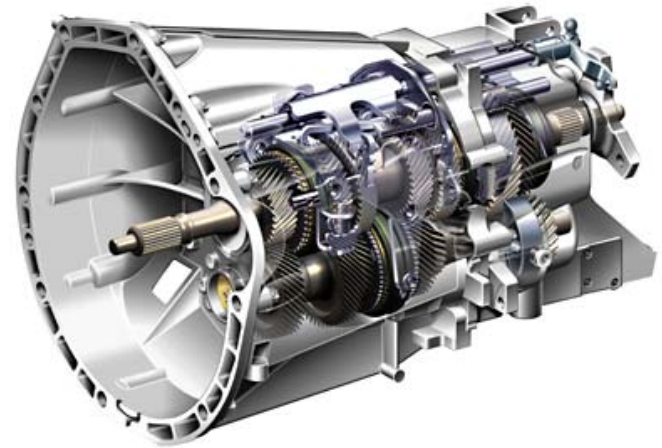
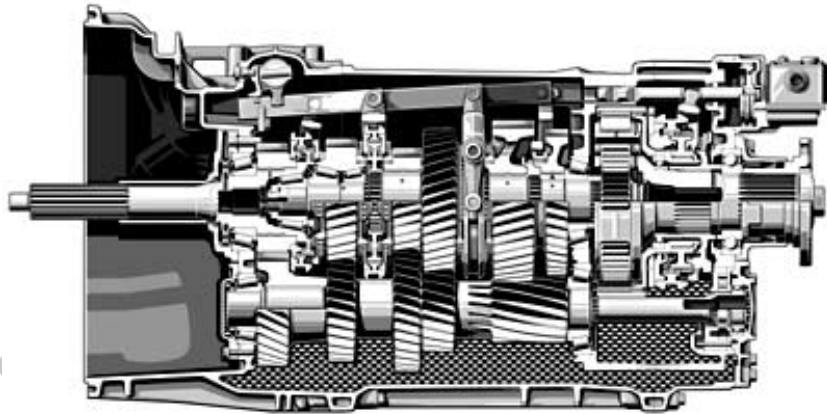
Transmissão Transversal Frontal e Traseira

Torque	Transmissão Manual	Transmissão servo-assistida	Transmissão Automática
< 180 N.m	5 marchas	5 marchas	CVT
180-350 N.m	5 marchas	-	CVT
> 350 N.m	-	-	5 e 6 marchas

Introdução

Trens de Engrenagens

Transmissão Manual



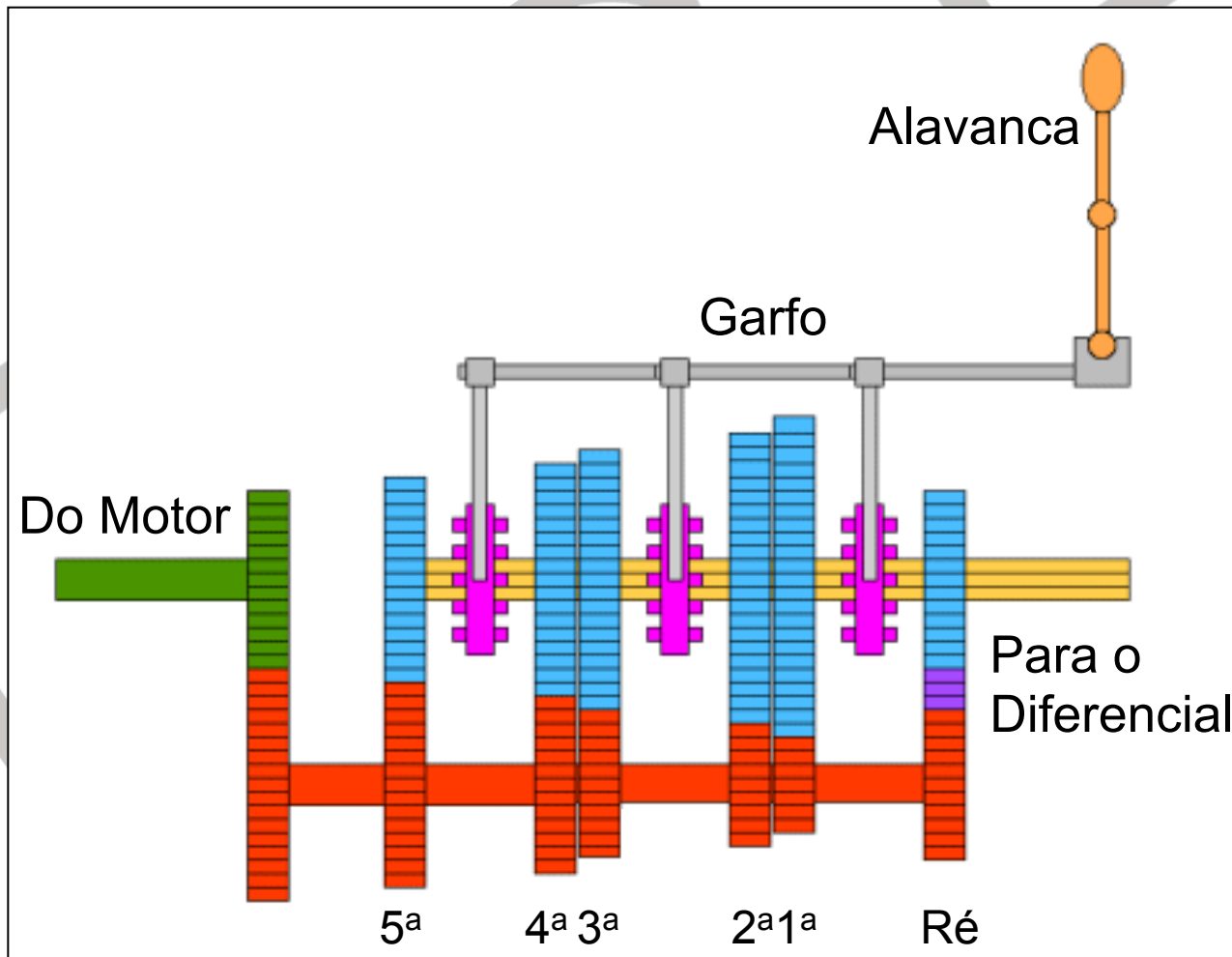
<http://auto.howstuffworks.com/transmission.htm>

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Transmissão Manual

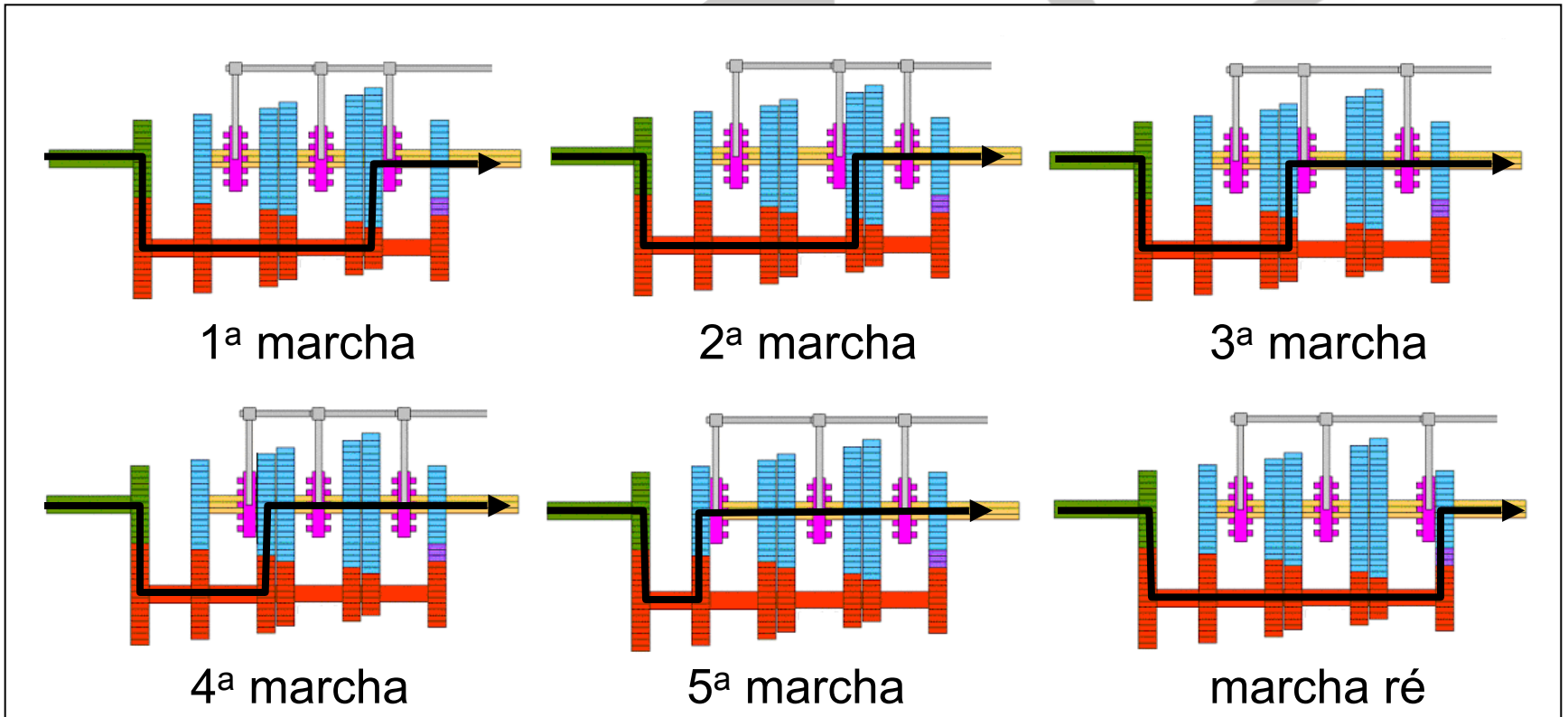
Funcionamento Simplificado

- Caixa de Transmissão Manual de 5 marchas + Ré



Transmissão Manual

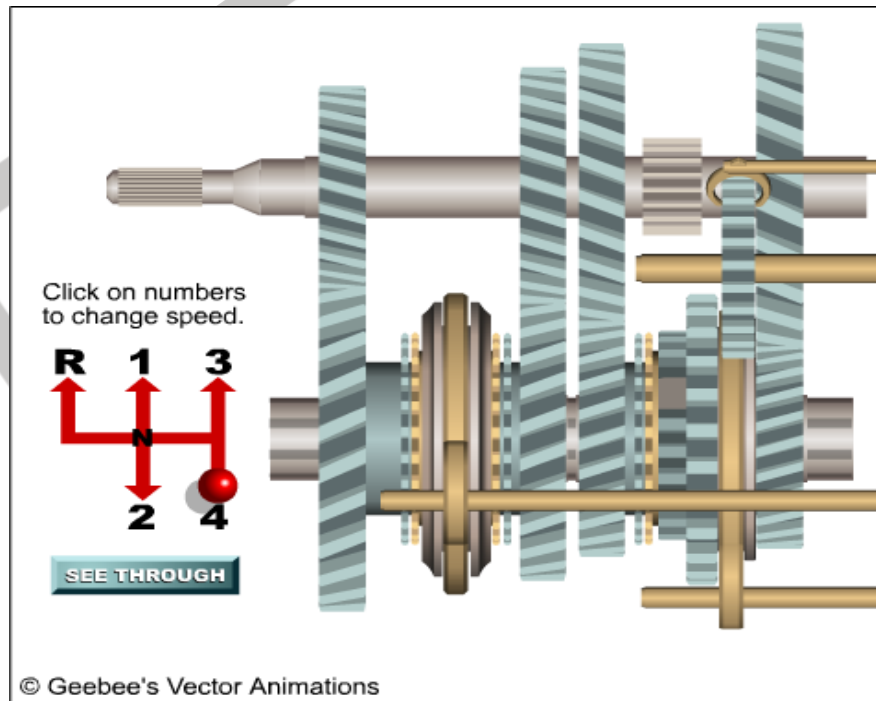
Funcionamento Simplificado



Introdução

Trens de Engrenagens

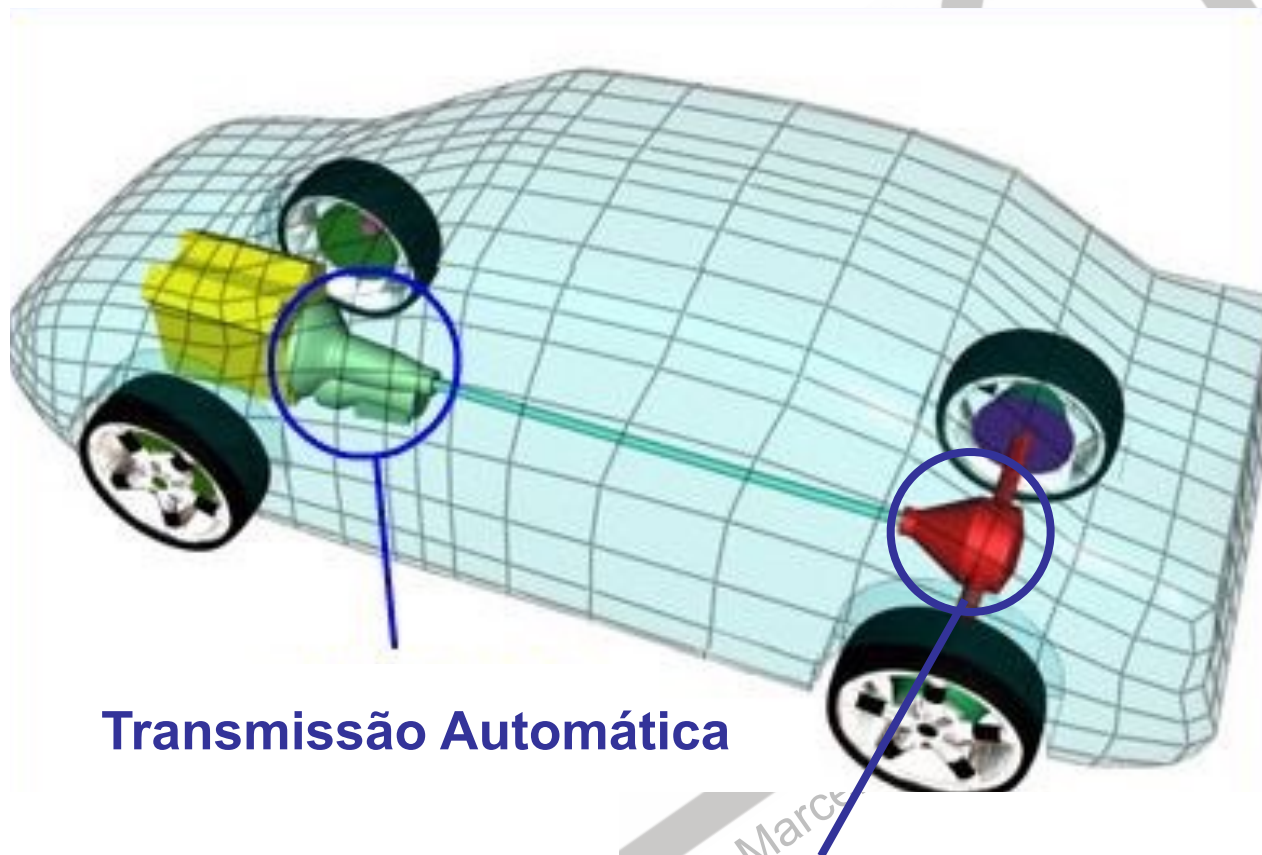
Transmissão Manual



<http://auto.howstuffworks.com/transmission4.htm>

Transmissões Veiculares

Transmissão Automática



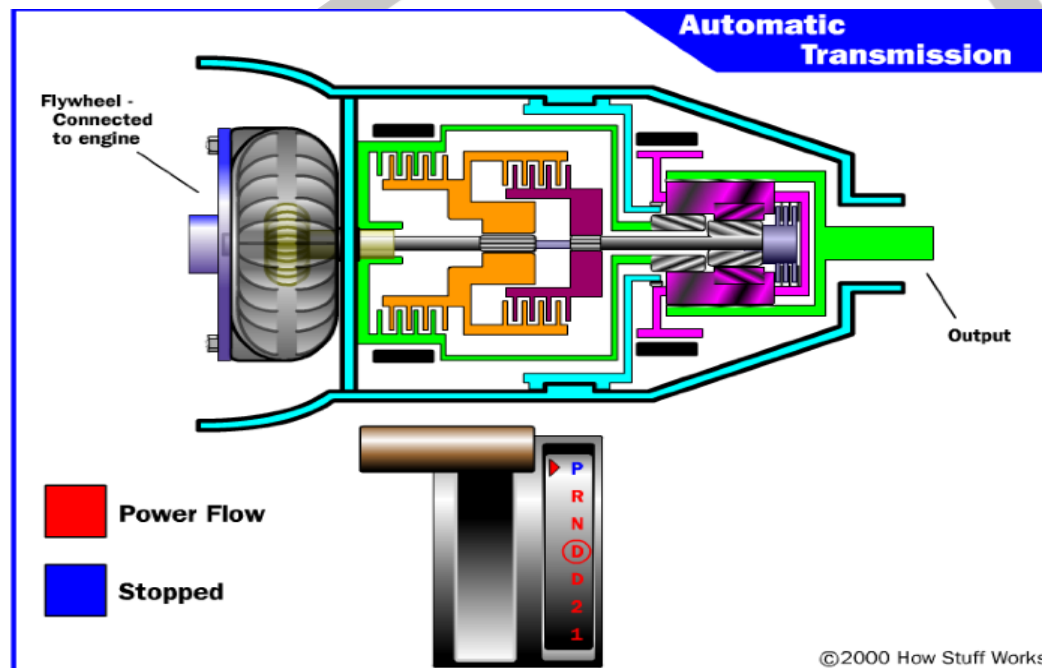
Transmissão Automática

Diferencial



Transmissões Veiculares

Transmissão Automática

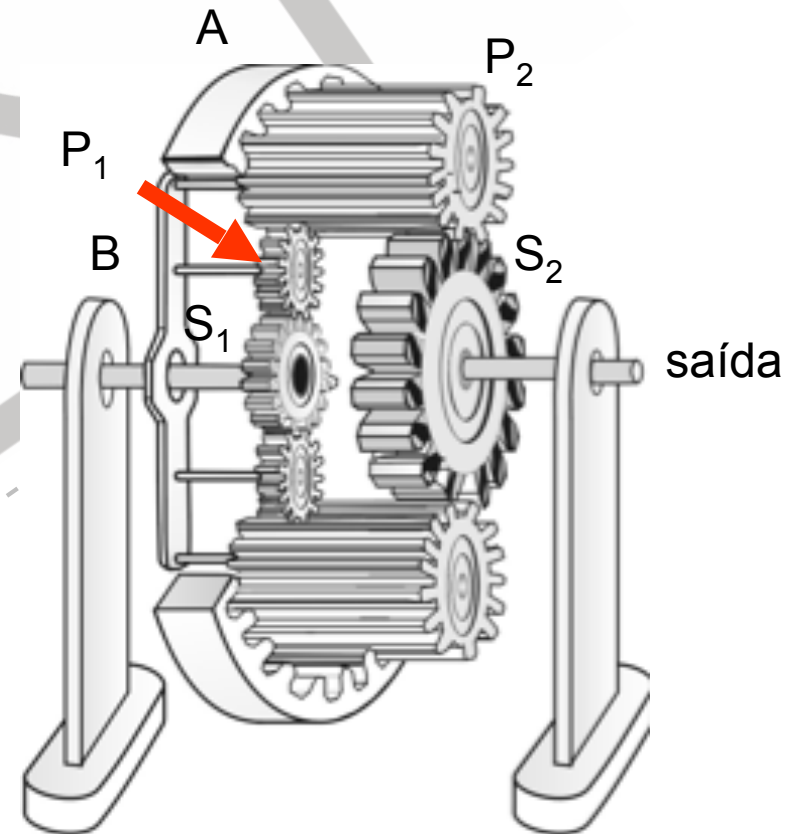
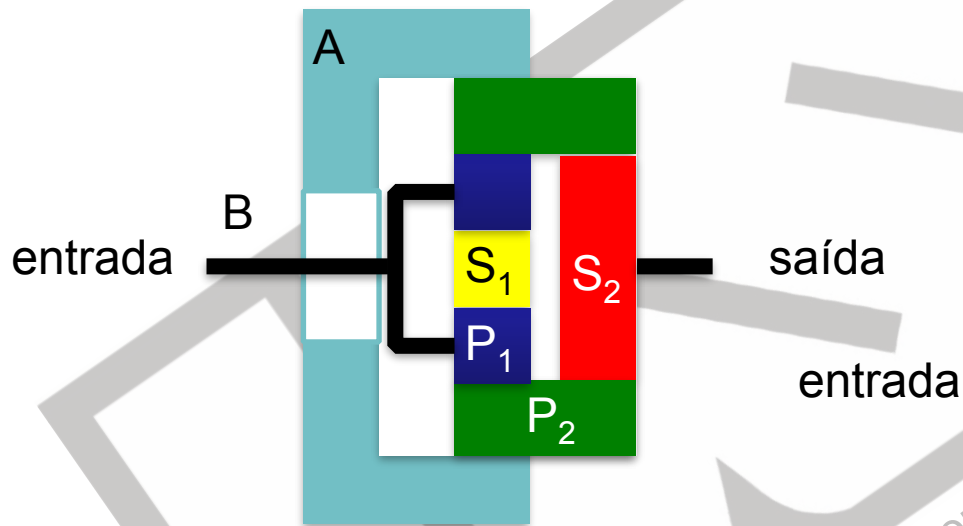


<http://auto.howstuffworks.com/automatic-transmission4.htm>

Introdução

Trens de Engrenagens

TEP tipo Ravigneaux



<http://www.youtube.com/watch?v=7iTn8OWxVFU>

Pol Ravigneaux, 1949

Sumário da Aula

- Introdução
- **Representações**
- Montagens
- Análise Cinemática
- Análise de Torque
- Exemplos de Aplicação

Representações

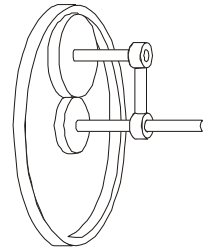
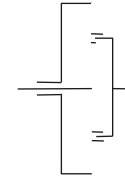
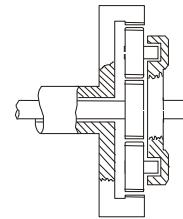
Representações
de TEPs

Funcional

Convencional

Corte

3D

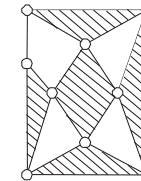
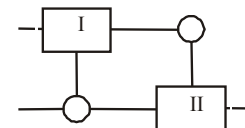


Esquemática

Juntas e barras

Diagrama de Blocos

Diagrama de Torque

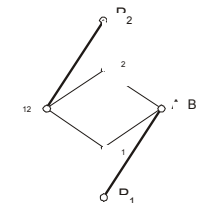
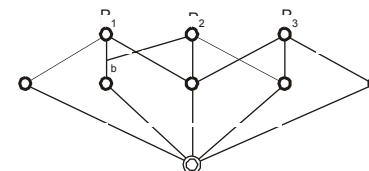
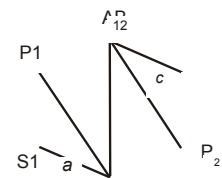
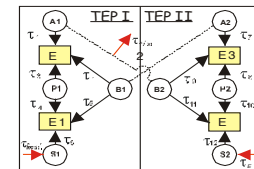


Grafo

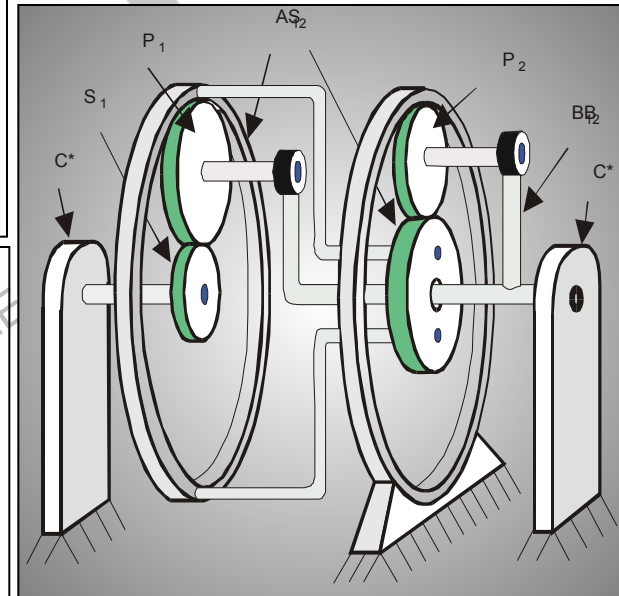
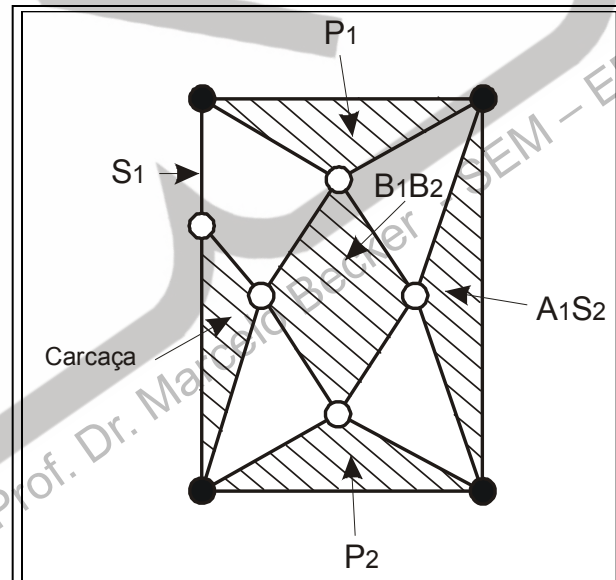
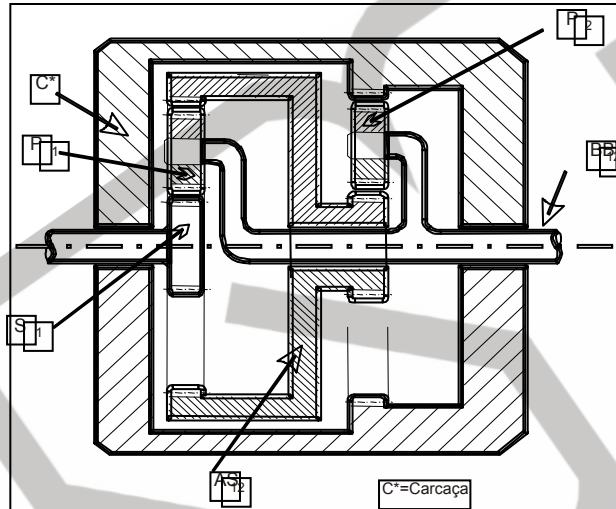
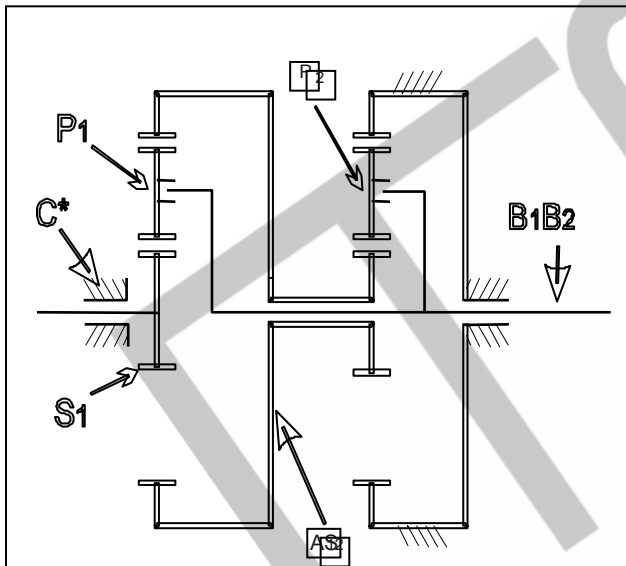
Convencional

Nova

Canônica

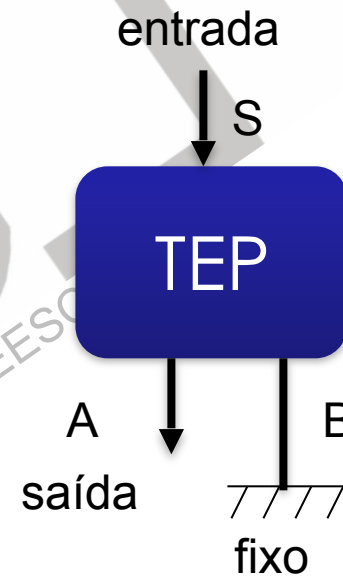
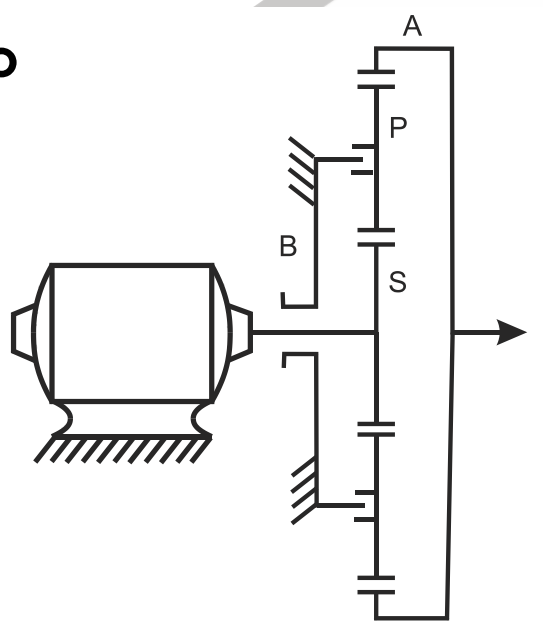


Representações



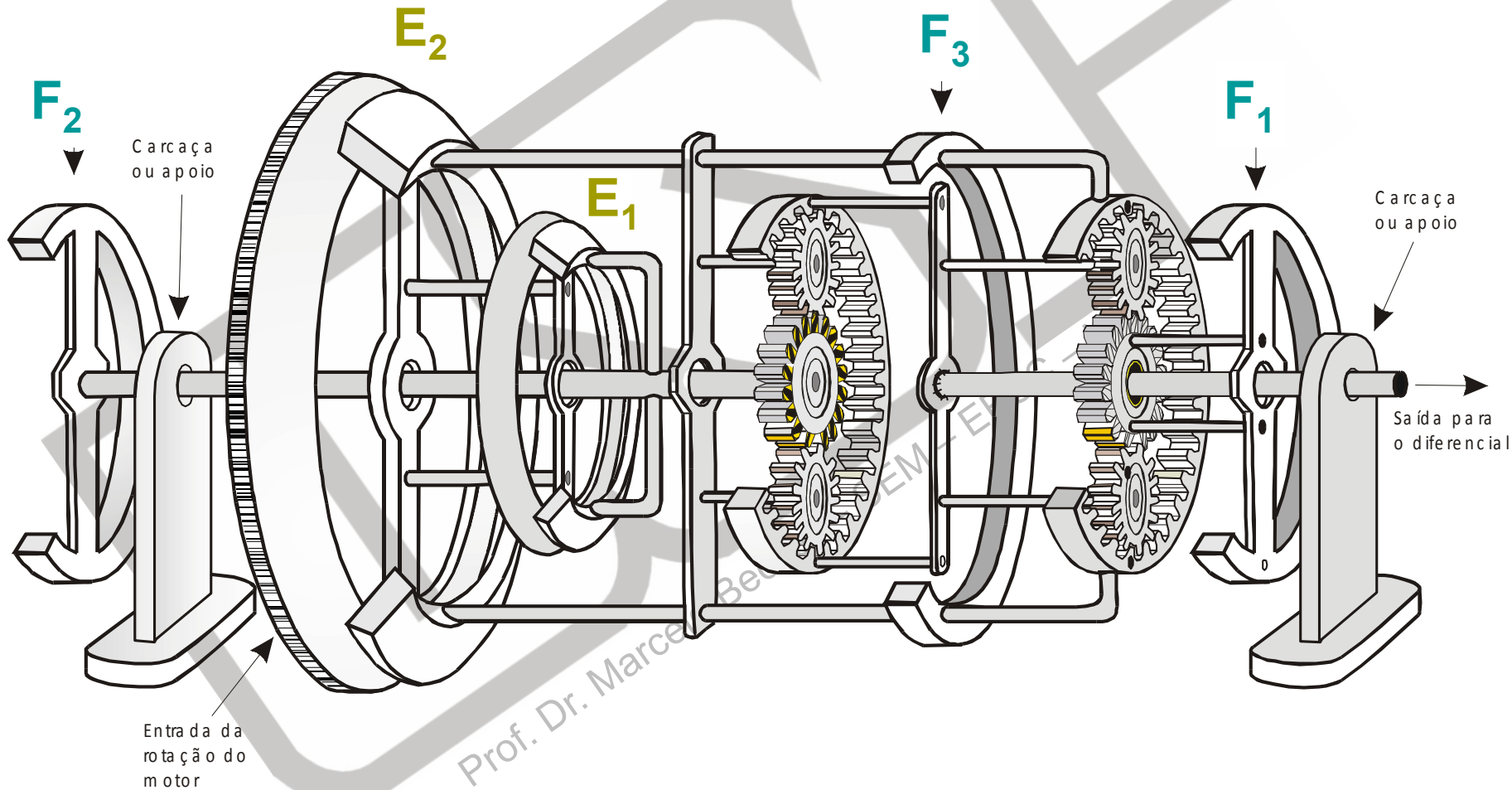
Representação: 1 TEP

- 1 Entrada
- 1 Saída
- 1 Elem. Fixo



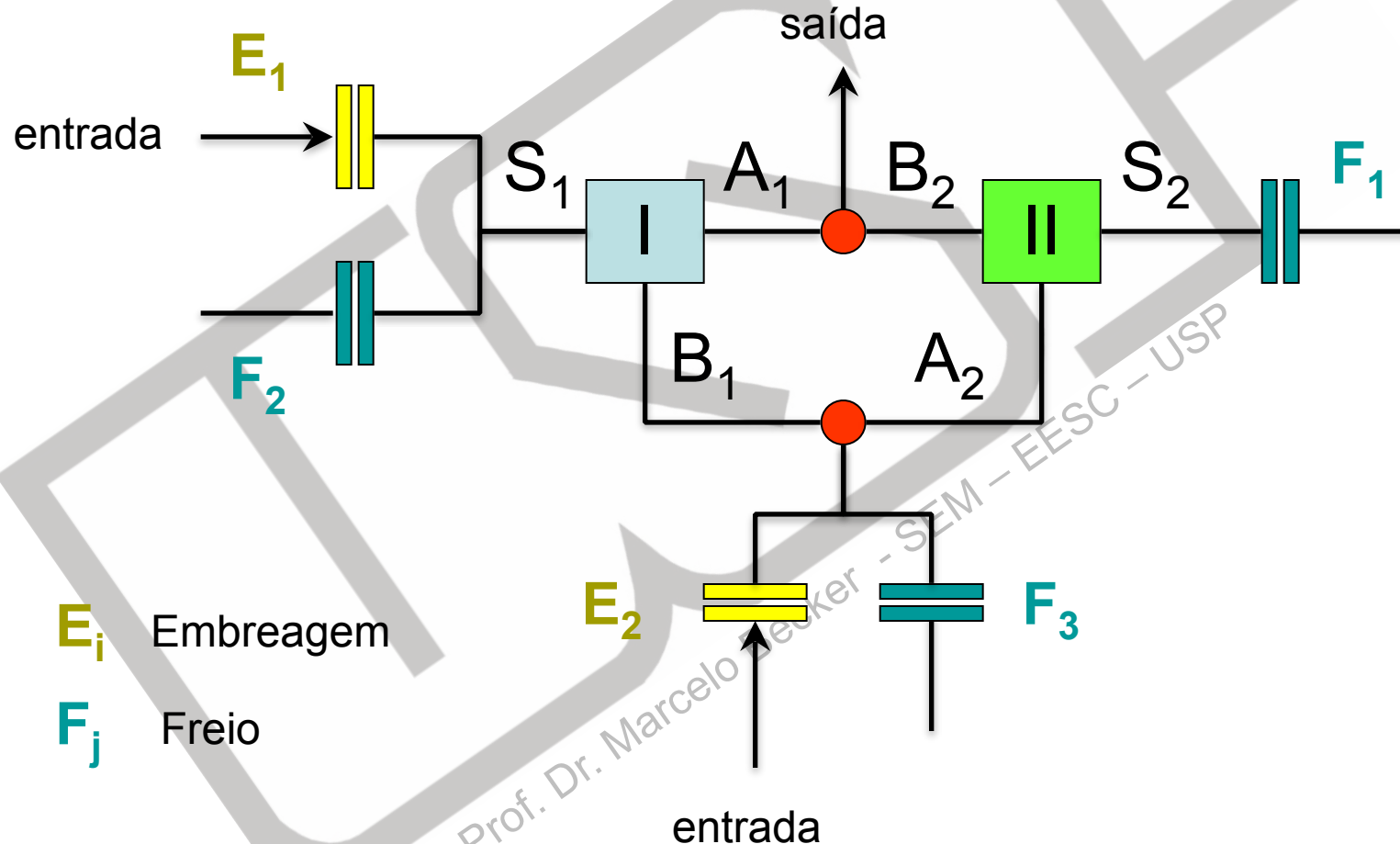
Transmissões Veiculares

Caixa GM HIDRA-MATIC THM – 440 PGT



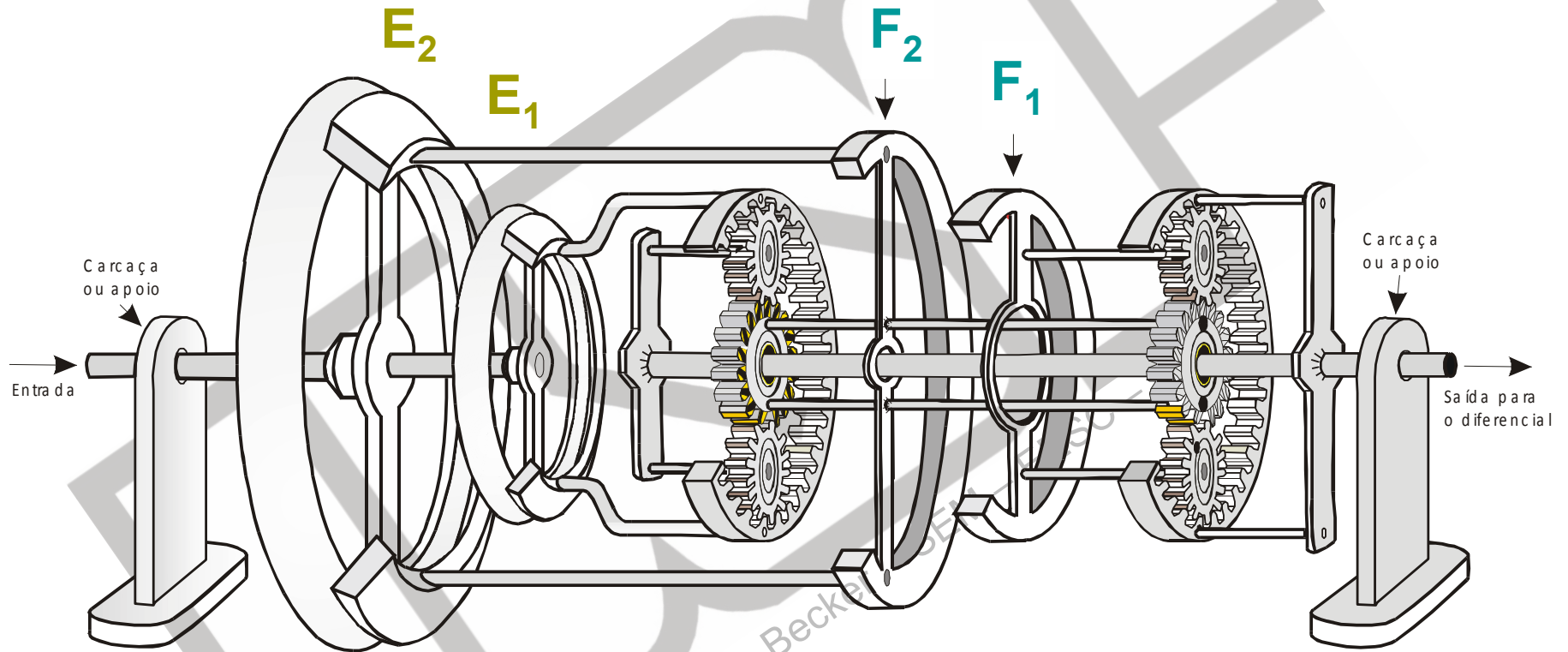
Transmissões Veiculares

Caixa GM HIDRA-MATIC THM – 440 PGT



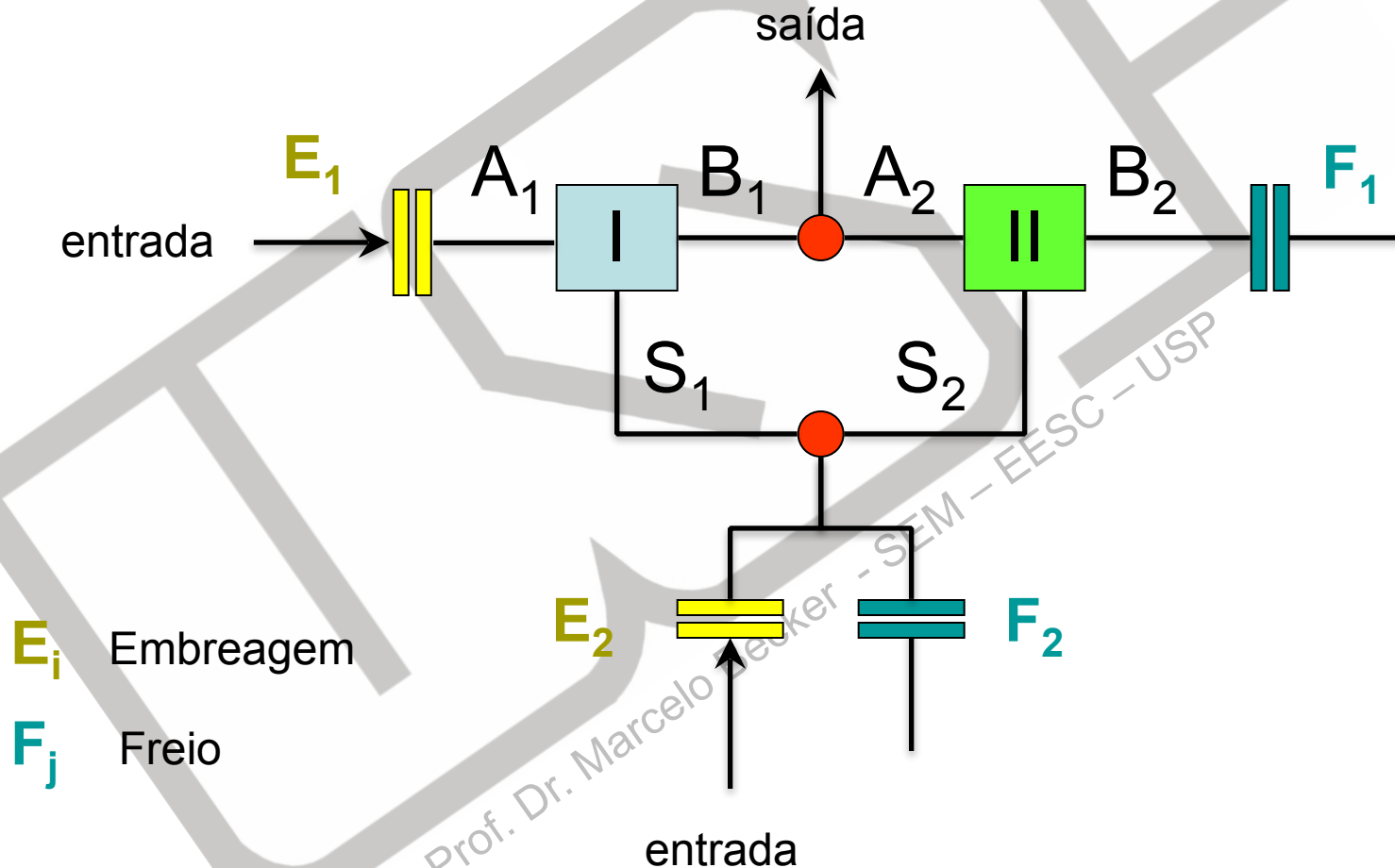
Transmissões Veiculares

Caixa Simpson



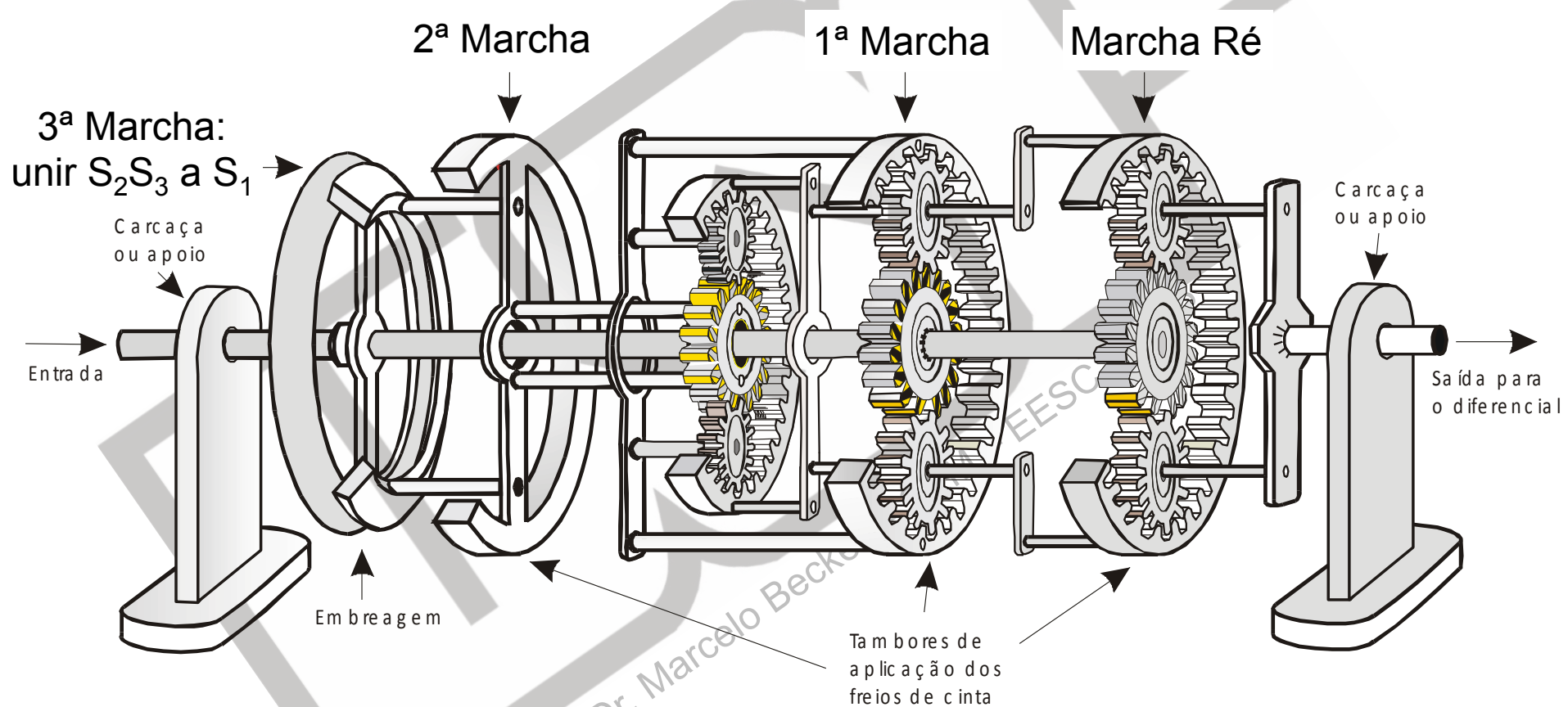
Transmissões Veiculares

Caixa Simpson



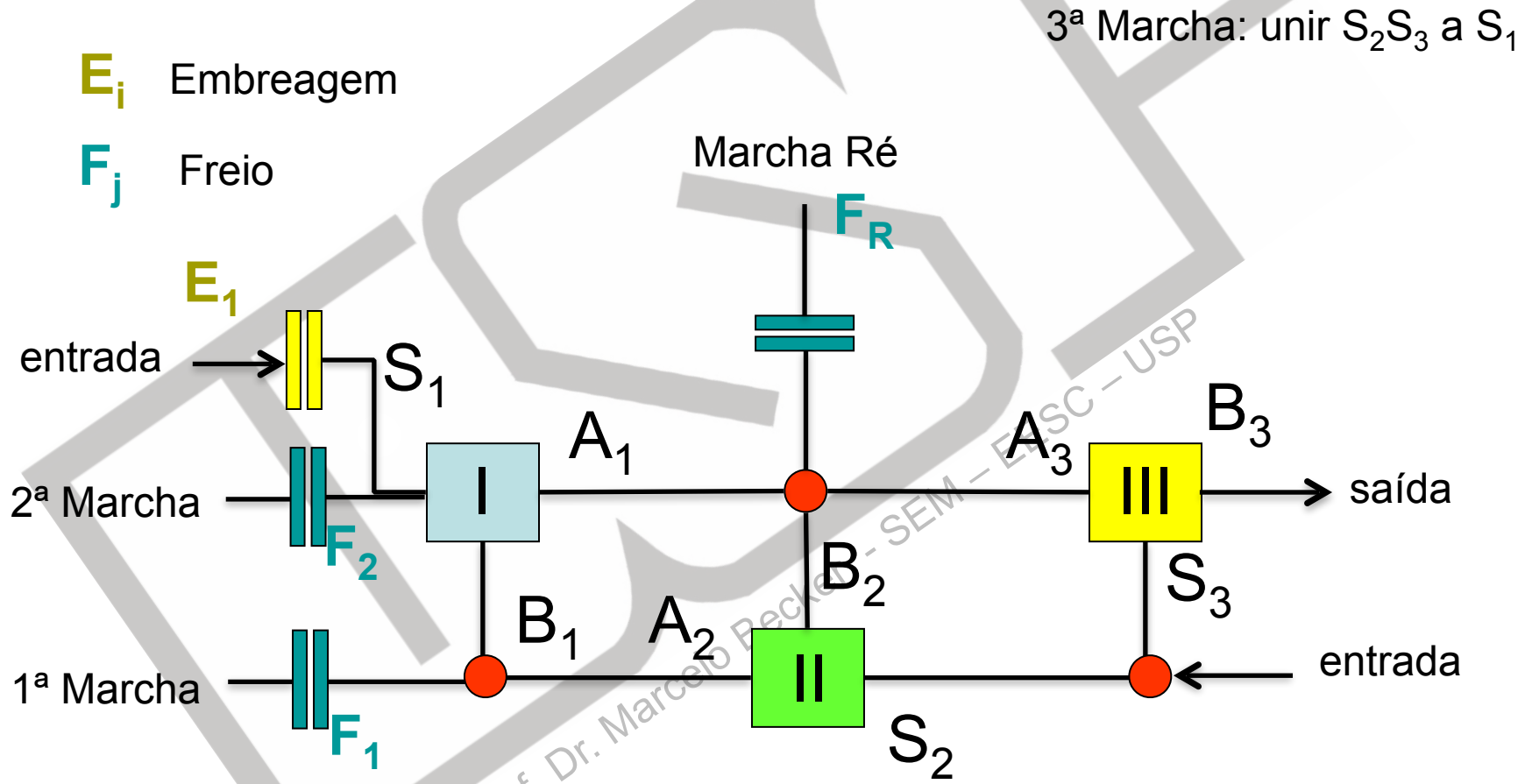
Transmissões Veiculares

Caixa Wilson de 3 Marchas



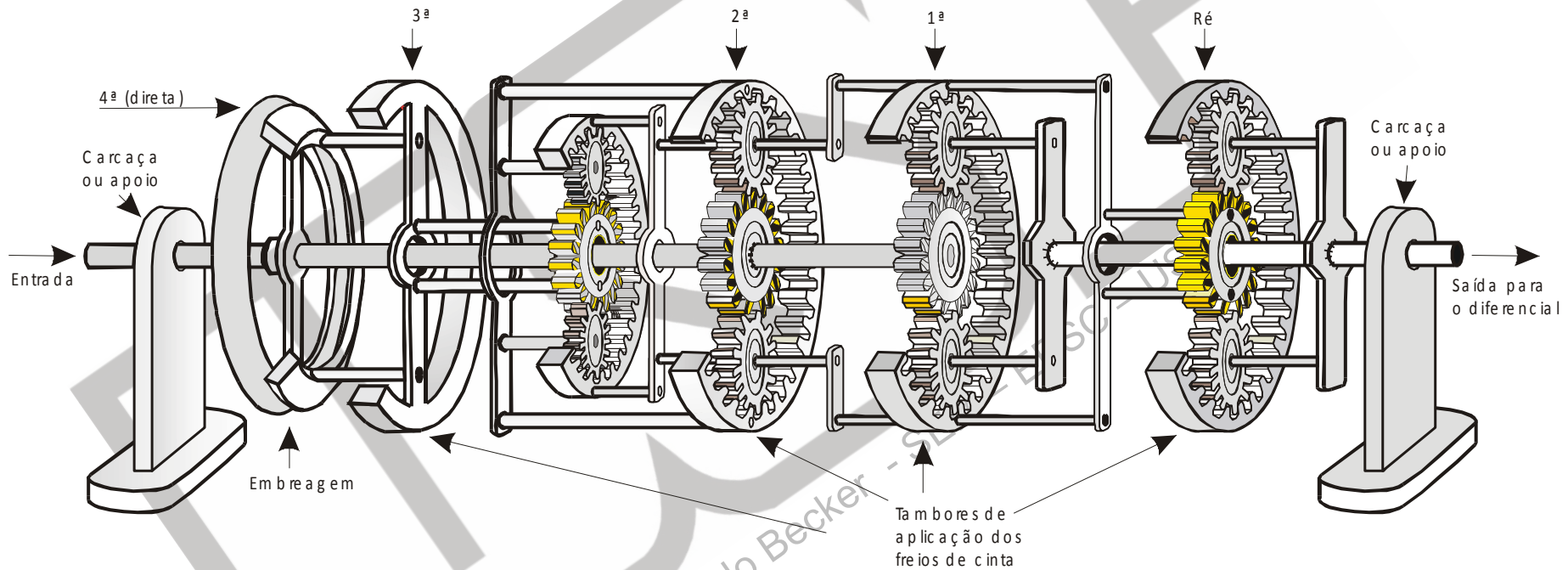
Transmissões Veiculares

Caixa Wilson de 3 Marchas



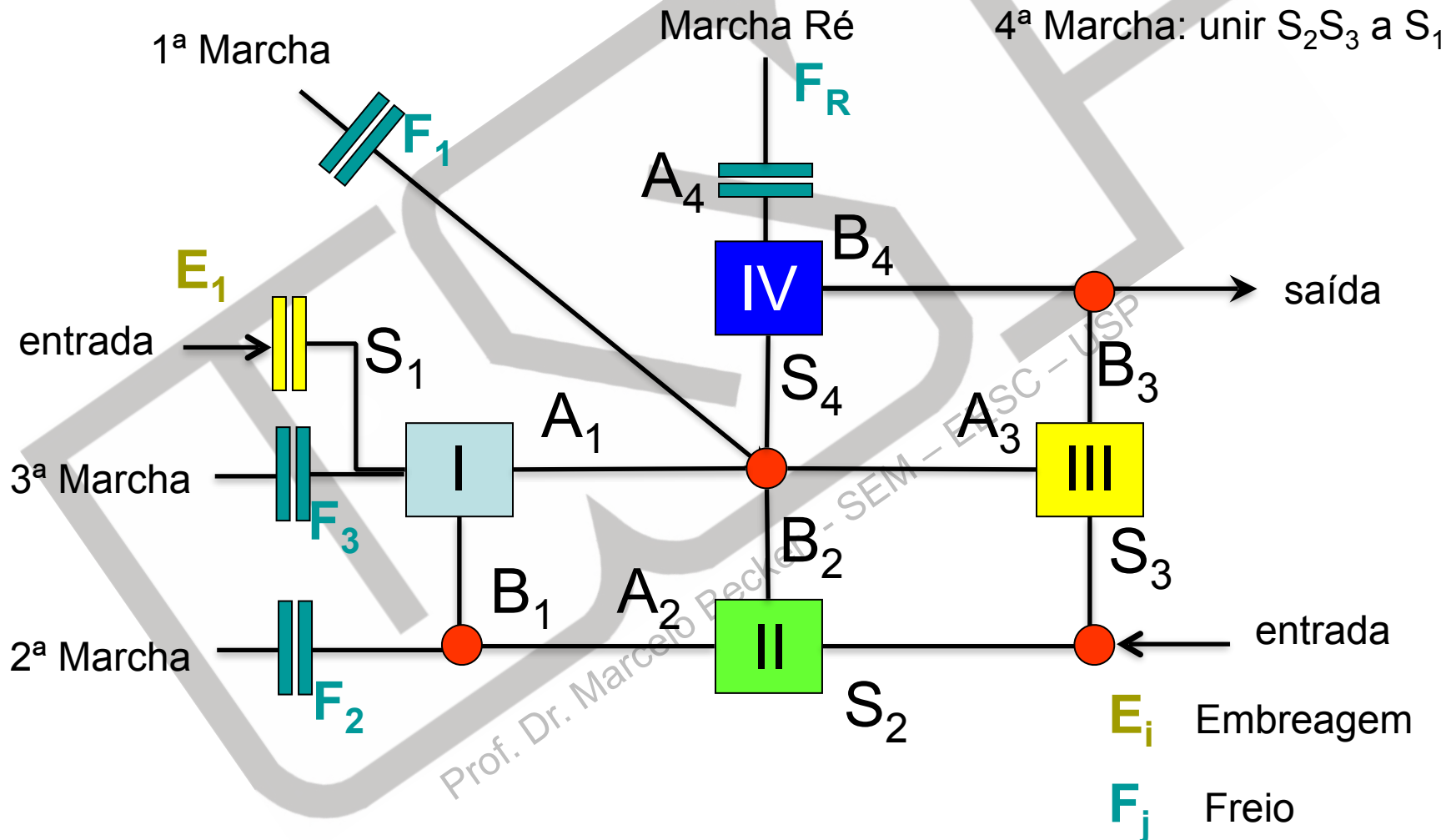
Transmissões Veiculares

Caixa Wilson de 4 Marchas



Transmissões Veiculares

Caixa Wilson de 4 Marchas



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- **Montagens**
- Análise Cinemática
- Análise de Torque
- Exemplos de Aplicação

Introdução

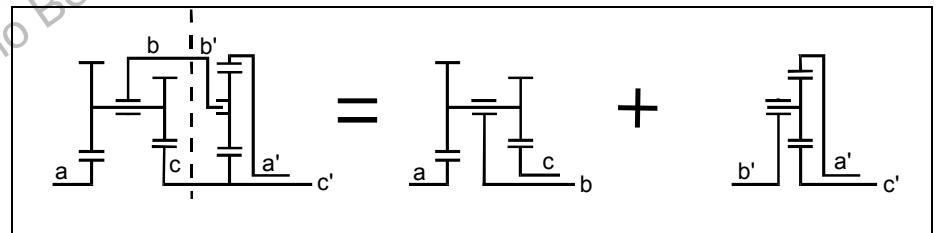
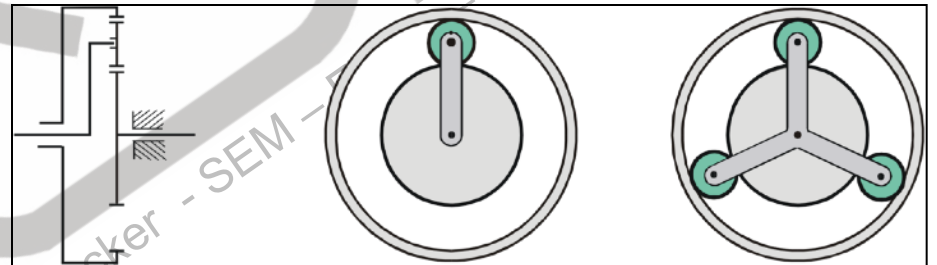
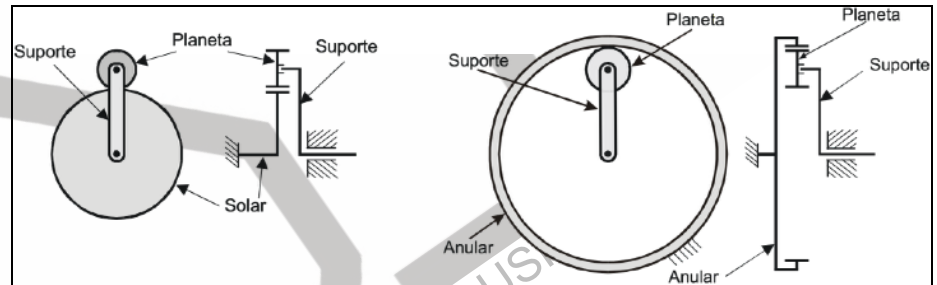
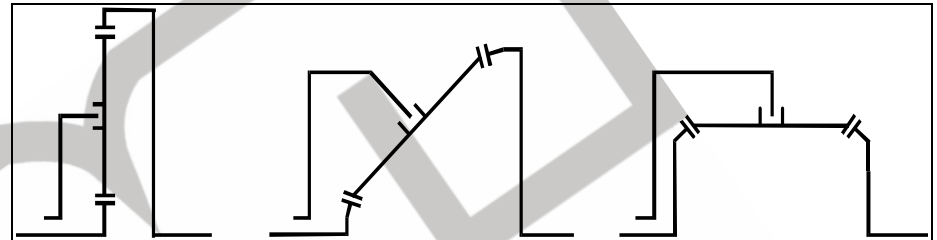
TEPs

Engrenagens Cilíndricas ou Cônicas

TEP Básico

TEP Simples

TEP Ligado



Montagens

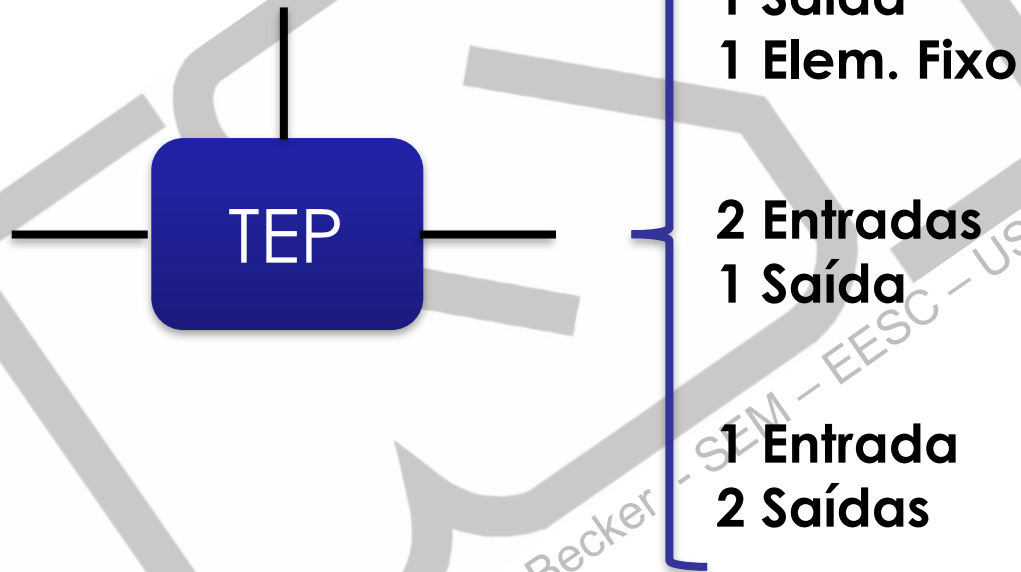
Número de Eixos Freiados \swarrow GDL
 Número de Conexões \swarrow

	Simpson	GM Hydra Matic 440 PGT	Perrigneaux	Hydra Matic 3L 80 PGT
1 st	2,786	2,920	2,846	2,857
2 nd	1,529	1,567	1,532	1,517
Diret-Drive	1,000	1,000	1,000	1,000
"Over Drive"	0,703	0,705	0,711	0,703
Reverse	-2,375	-2,384	-2,466	-2,590
# of PGTs	2	2	2*	3

N	Família	c	I	GL	Grupo
1	nº. 1	0	0	2	3
			1	1	
2	nº. 2	1	0	3	41
			1	2	
			2	1	
2	nº. 3	2	0	2	22
			1	1	
			0	2	
2	nº. 4	2	0	4	520
			1	3	
			2	2	
			3	1	
3	nº. 5	3	0	3	330A
			1	2	330B
					2
3	nº. 6	3	1	2	330B
			2	1	411
					0
4	nº. 7	4	0	2	140
			1	1	221
					1

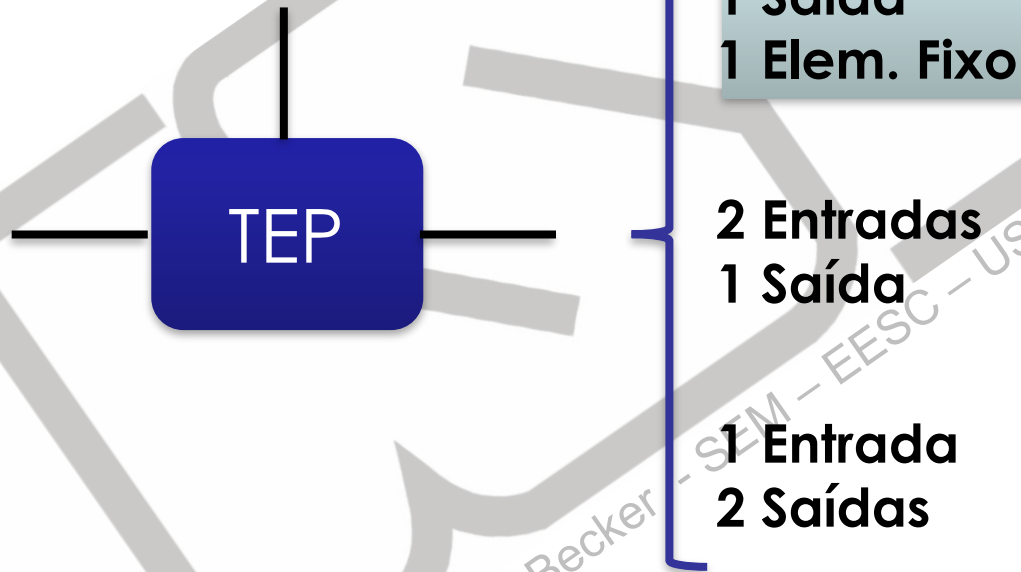
Montagens

1 TEP



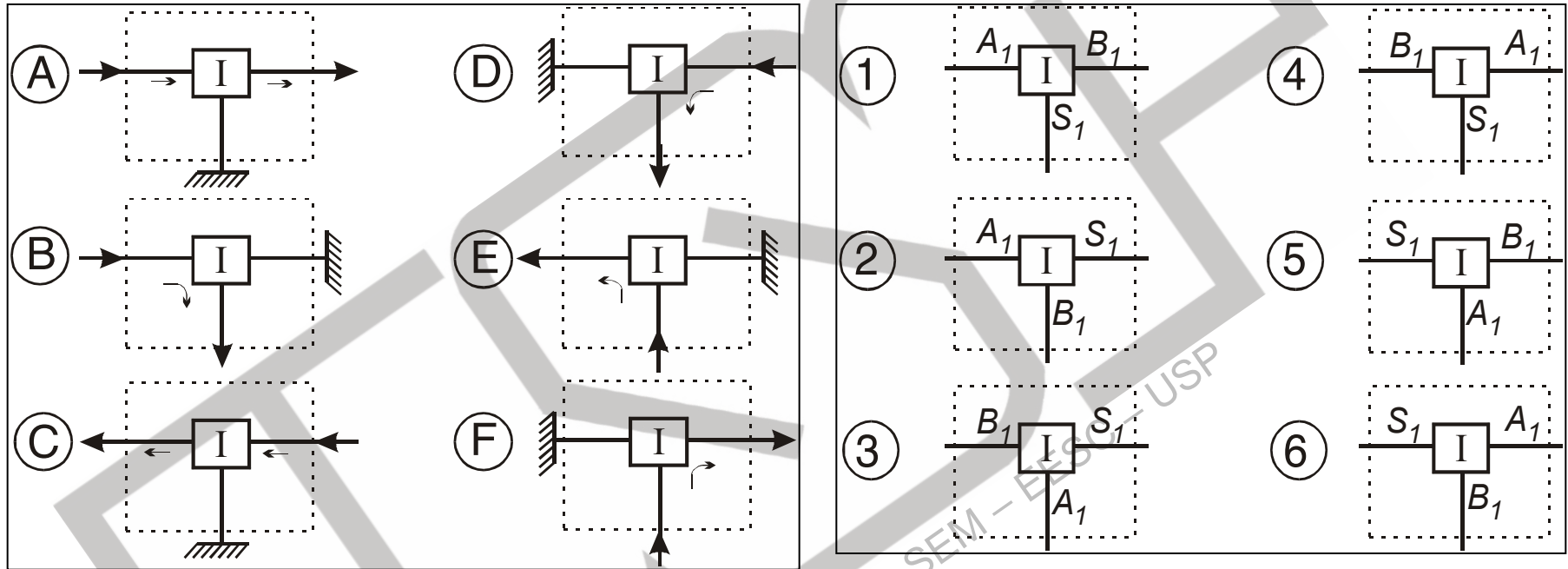
Montagens

1 TEP



Montagens

1 TEP



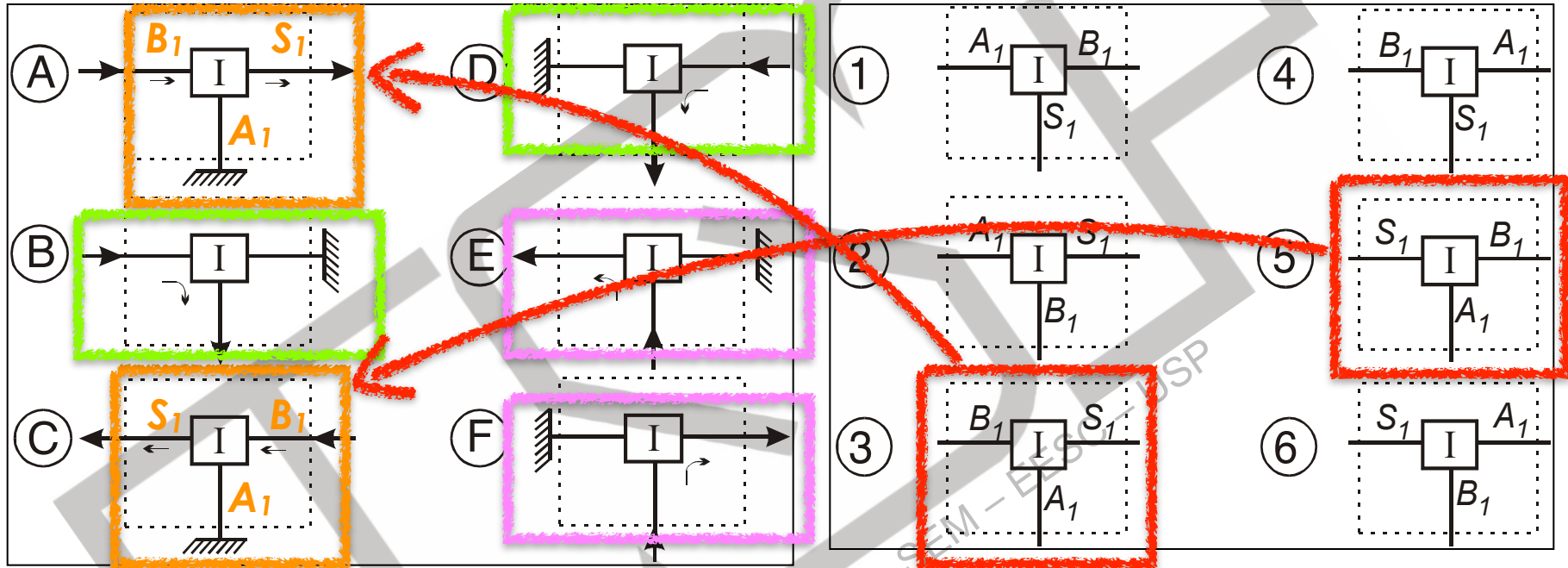
6 possíveis combinações (A a F)

6 possíveis posições (1 a 6)

Total: $6 \times 6 = 36$ Montagens

Montagens

1 TEP



6 possíveis combinações (A a F)

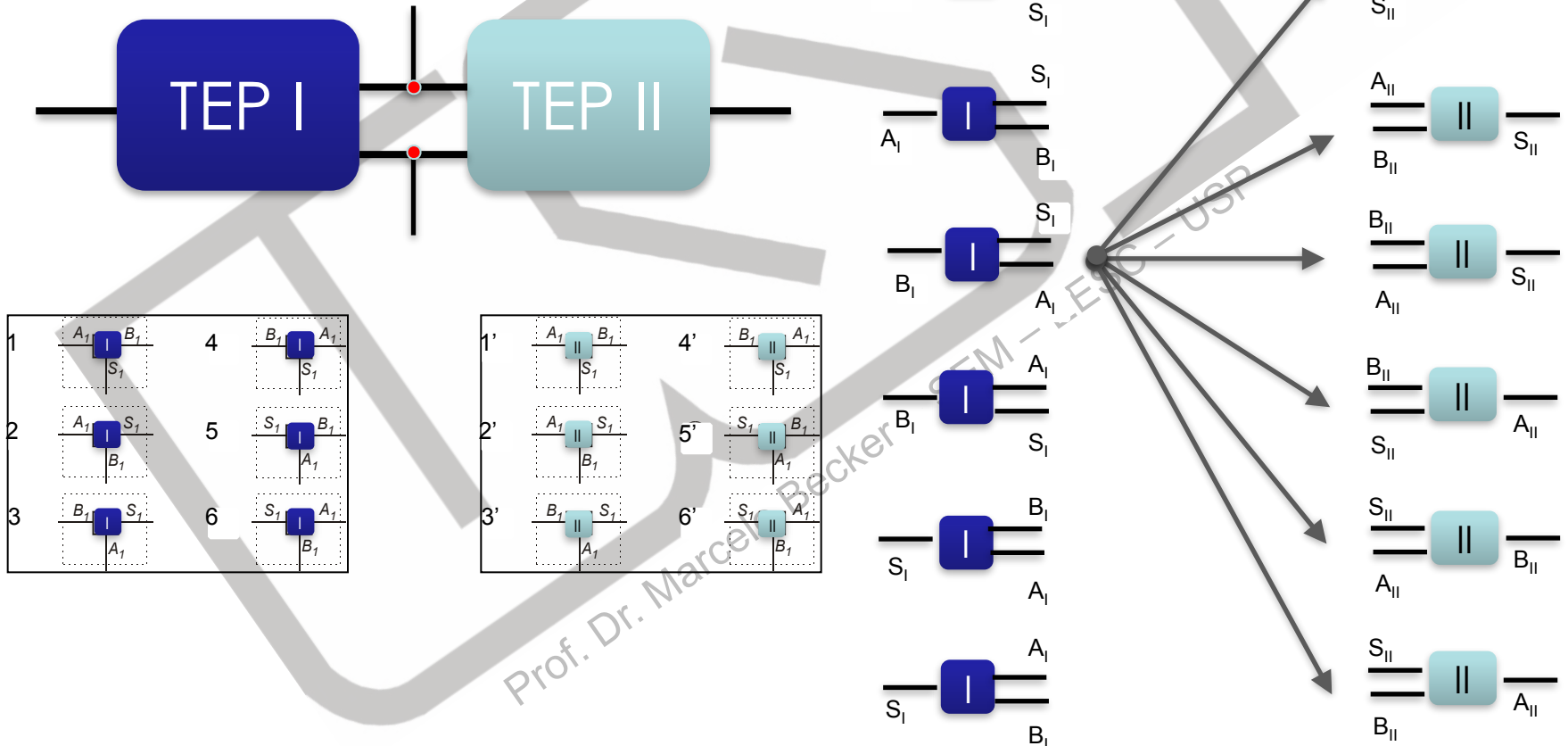
6 possíveis posições (1 a 6)

Total: $6 \times 6 = 36 \div 2 = 18$ Montagens!!

Montagens

2 TEPs

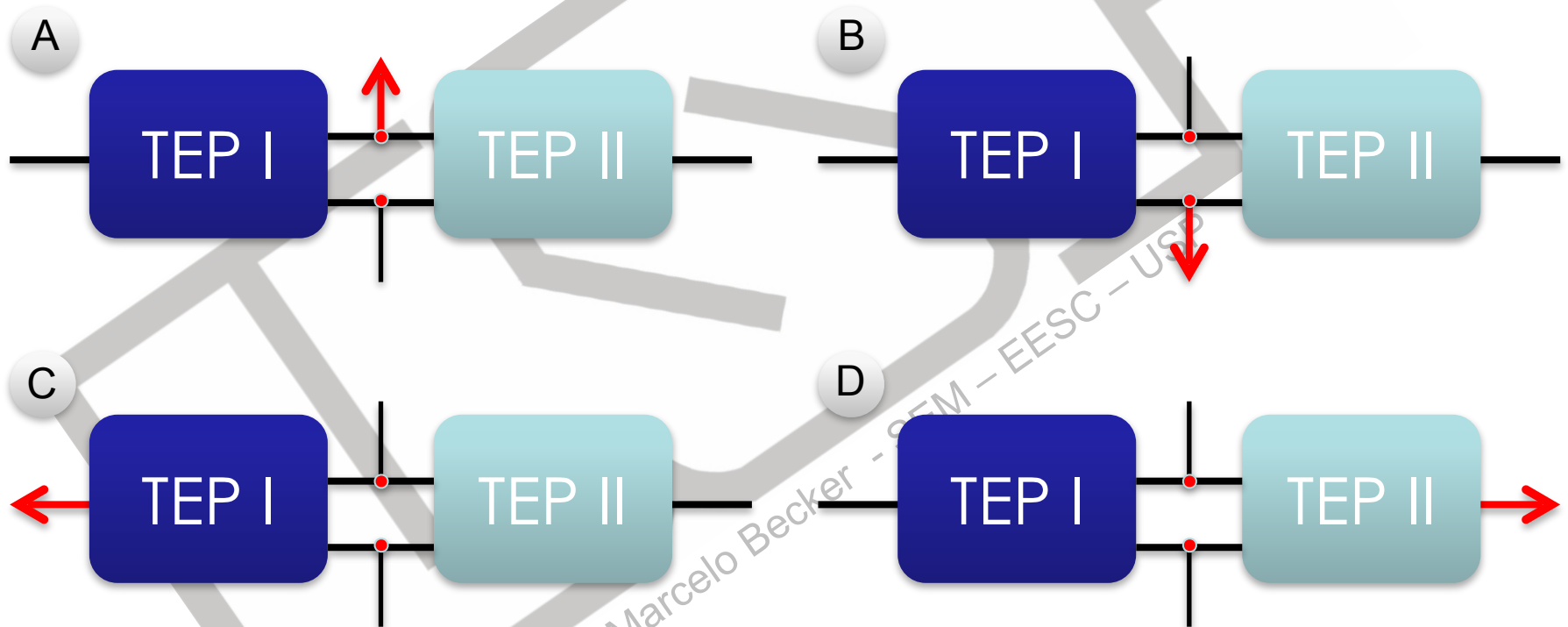
Posições Possíveis:



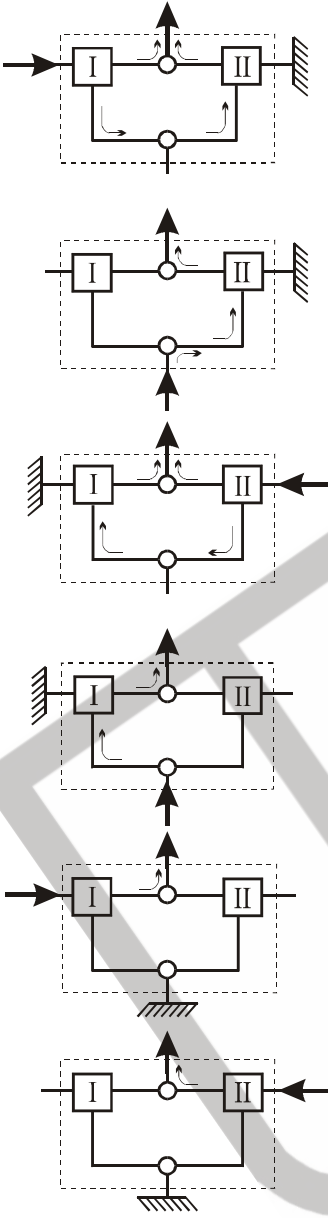
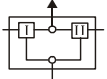
Montagens

2 TEPs

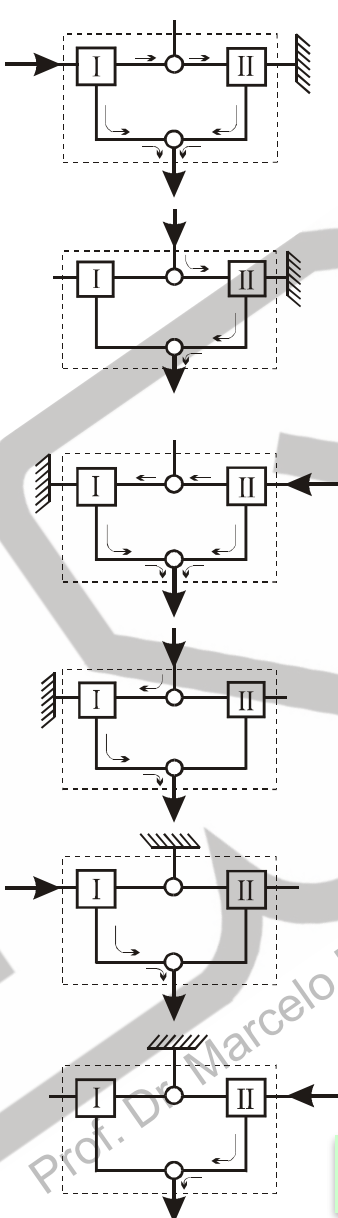
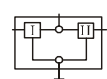
Combinações são uma fç da posição do eixo de saída!!



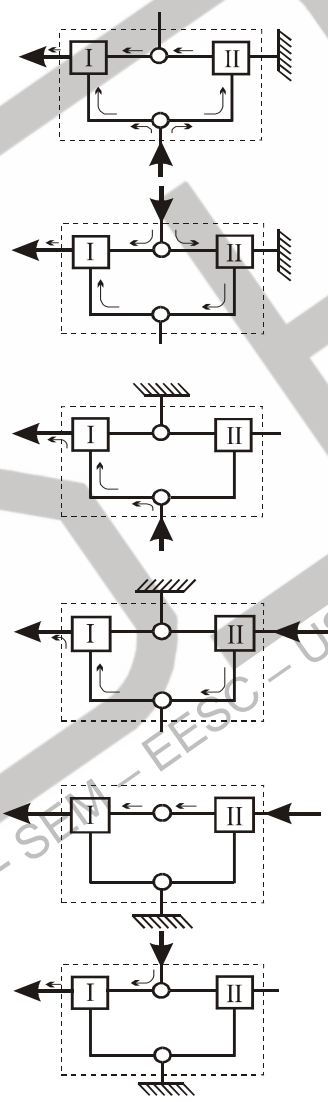
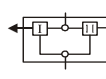
(A)



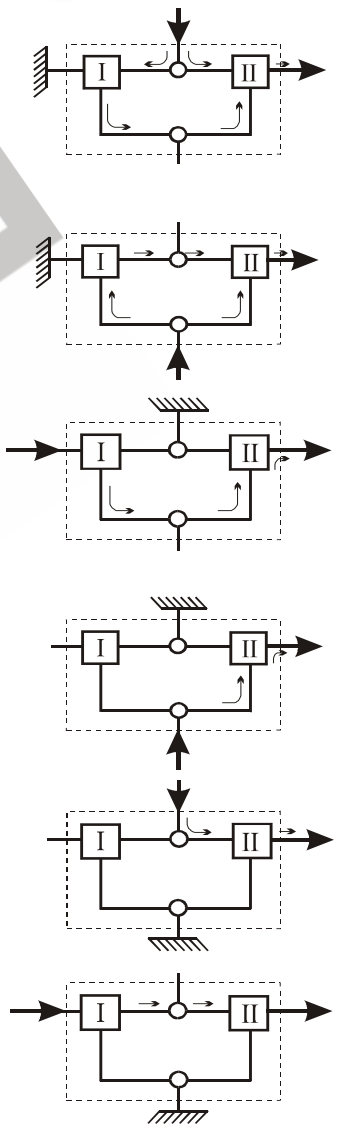
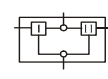
(B)



(C)



(D)

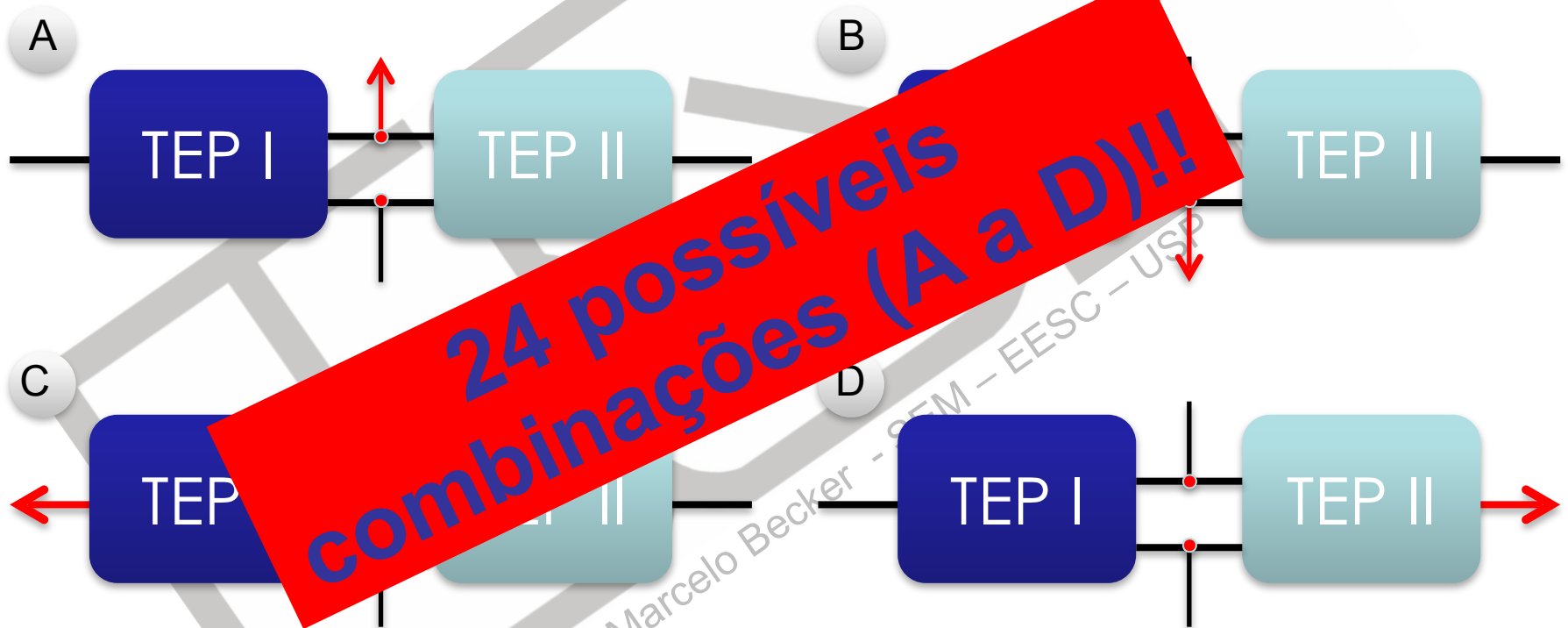


Total: 6 x 4 = 24 Possíveis Combinações

Montagens

2 TEPs

Combinações são uma fç da posição do eixo de saída!!



Montagens

2 TEPs

24 possíveis combinações (A a D)

6 possíveis posições (TEP I)

6 possíveis posições (TEP II)

Total: $24 \times 6 \times 6 = 864$ Possíveis Montagens

Mas, quantos dentes cada engrenagem deve ter??

Montagens

2 TEPs

Baseado na Equação: $Z_A = 2 \cdot Z_P + Z_S$, obtêm-se:

	Z_{\min}	Z_{\max}
Planeta (Z_P)	12	119
Solar (Z_S)	12	226
Anular (Z_A)	36	250

11.664 possíveis combinações entre “trincas” de dentes para um TEP

Montagens

2 TEPs

Dessa forma, obtêm-se:

11.664 possíveis trincas para TEP I

11.664 possíveis trincas para TEP II

864 possíveis montagens

Total: 117.546.246.144 Possíveis Soluções

~ 117,5 bilhões de possíveis soluções...

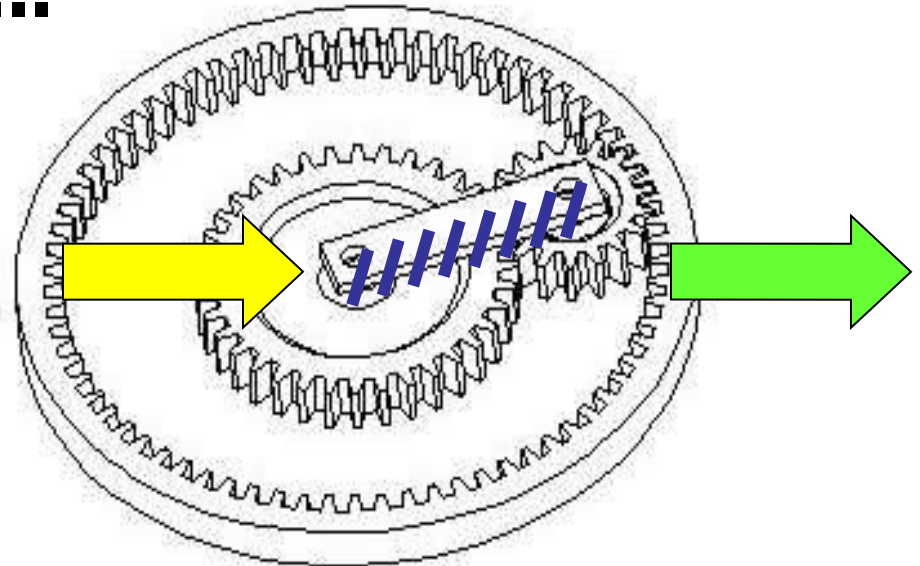
Sumário da Aula

- Introdução
- Representações
- Montagens
- **Análise Cinemática**
- Análise de Torque
- Exemplos de Aplicação

Cinemática

Determinação da Rotação de Saída: Velocidade Relativa...

$$\frac{w_{A/B}}{w_{S/B}} = \frac{w_A - w_B}{w_S - w_B}$$



Fixo

$$R^B_{AS} = \frac{w_A - w_B}{w_S - w_B} = -\frac{Z_S}{Z_A} = b$$

Saída

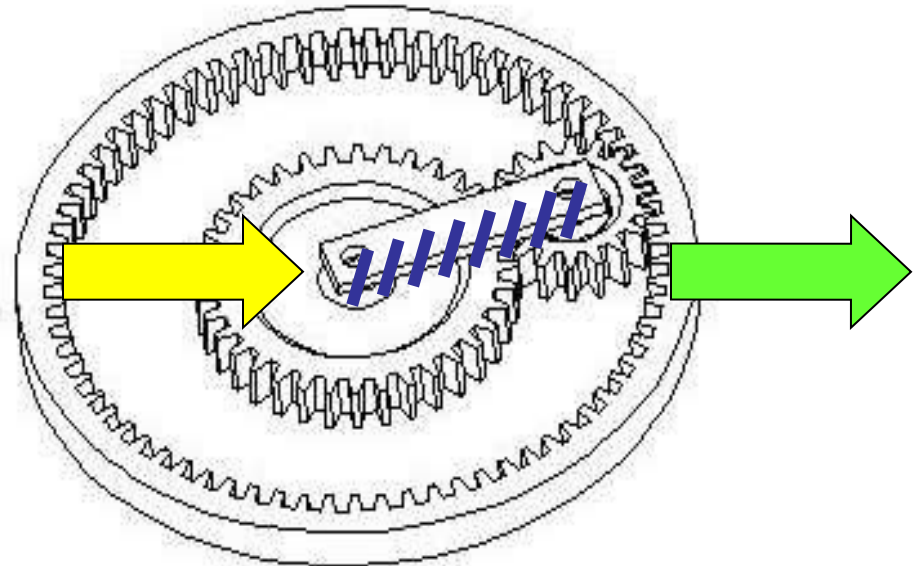
Entrada

Razão Básica

Cinemática

Determinação da Rotação de Saída: Produtórios...

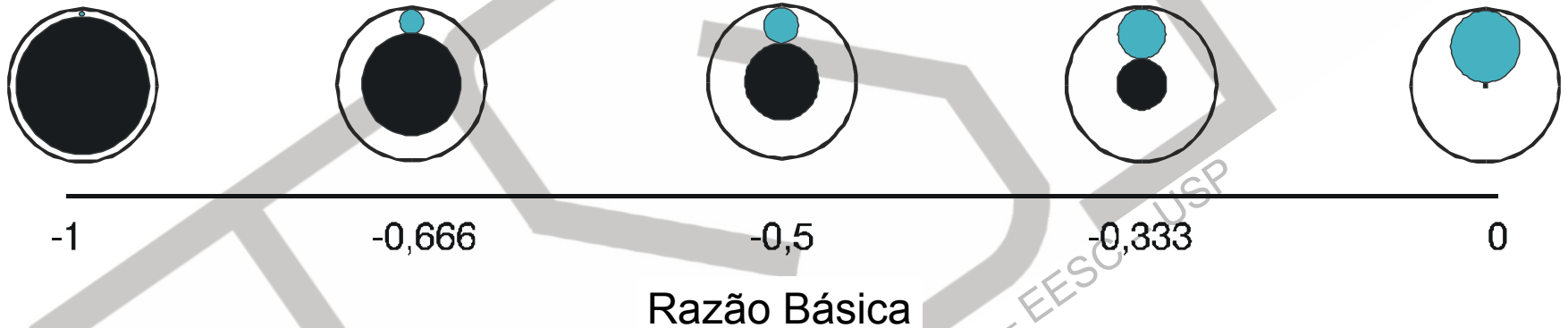
Razão Básica



$$b = \pm \frac{\prod Z_{\text{motoras}}}{\prod Z_{\text{movidas}}} = - \frac{\cancel{Z_S} \cdot \cancel{Z_P}}{\cancel{Z_P} \cdot Z_A} = - \frac{Z_S}{Z_A}$$

Cinemática

Razão Básica

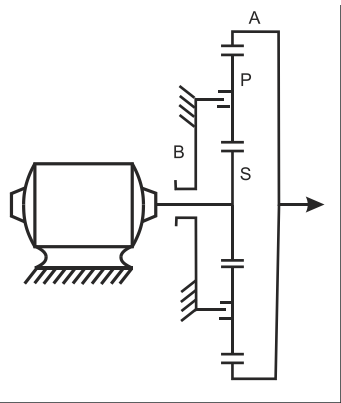


Equação Geral para 1 TEP simples

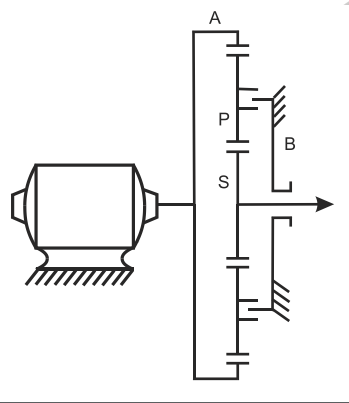
$$w_A + (b - 1) \cdot w_B - b \cdot w_S = 0$$

Cinemática

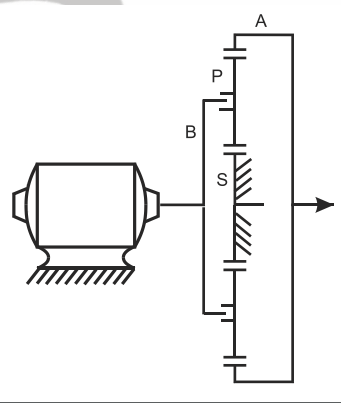
1 TEP



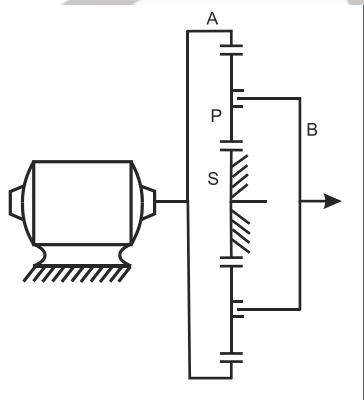
(a)



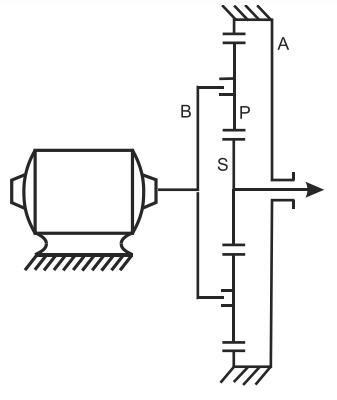
(b)



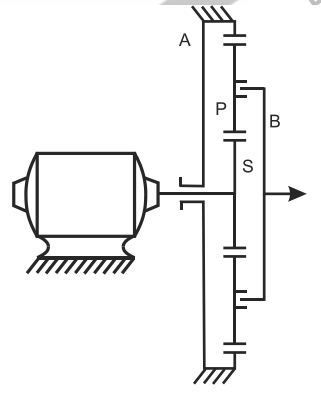
(c)



(d)



(e)



(f)

entrada



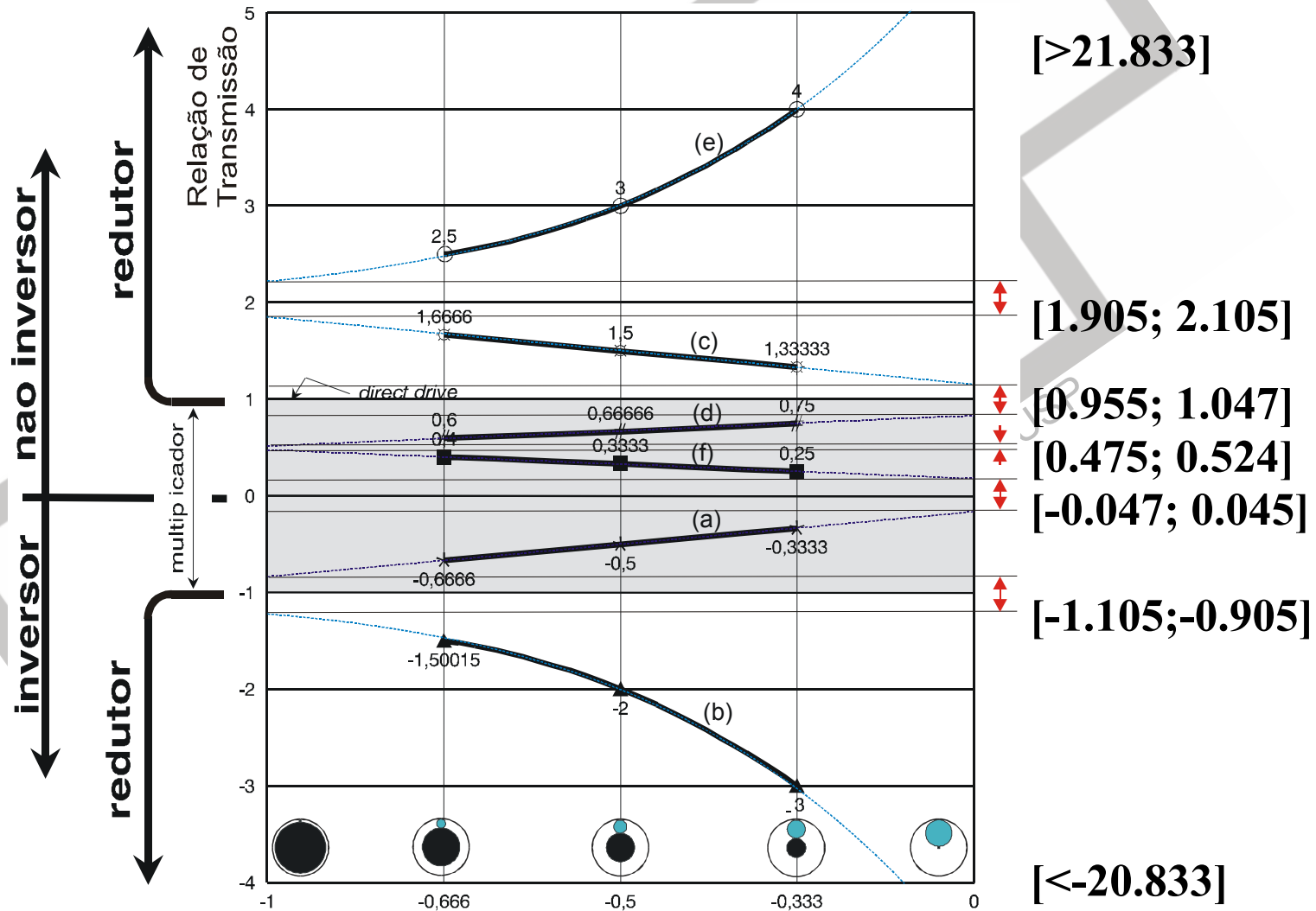
saída

fixo

1 Entrada
1 Saída
1 Elem. Fixo

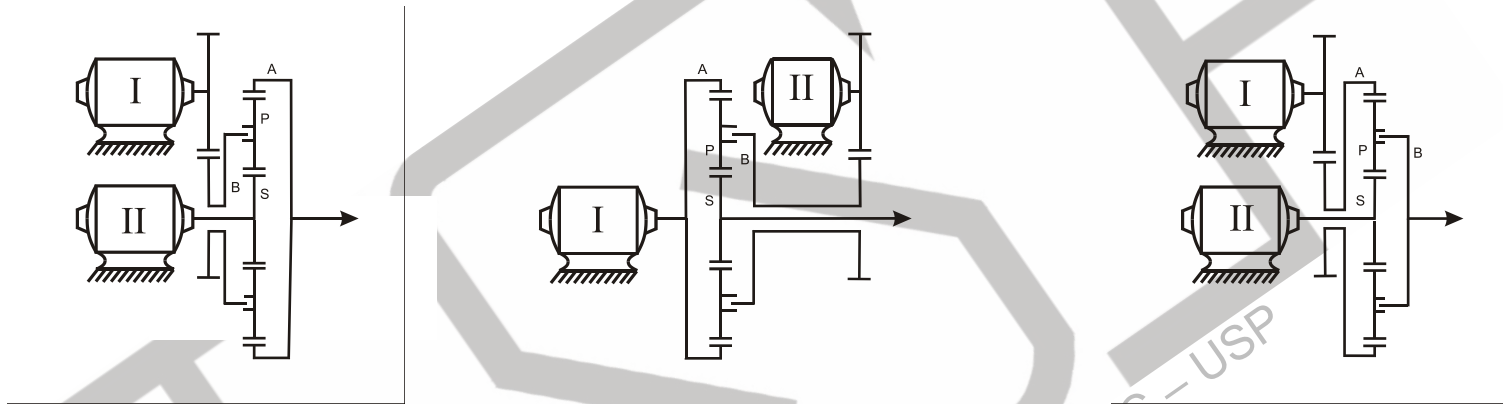
Cinemática

1 TEP



Cinemática

1 TEP



entrada



TEP

entrada

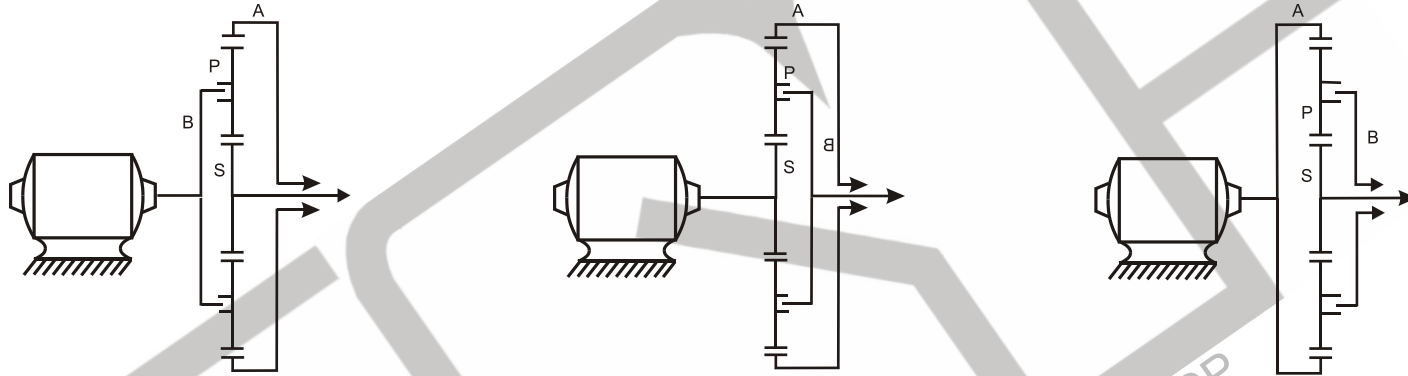


saída

2 Entradas
1 Saída

Cinemática

1 TEP



saída

saída

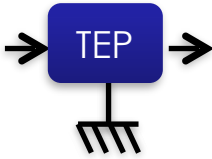
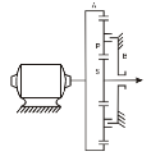
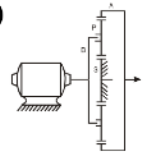
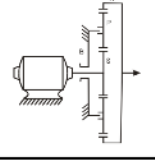
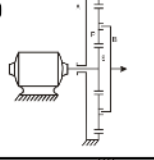
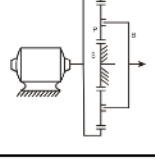
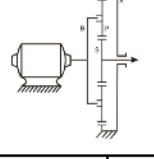

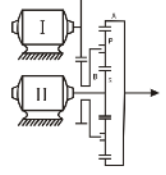
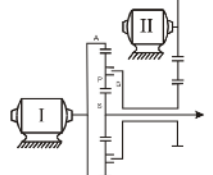
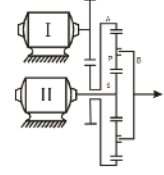
TEP

entrada

**1 Entrada
2 Saídas**

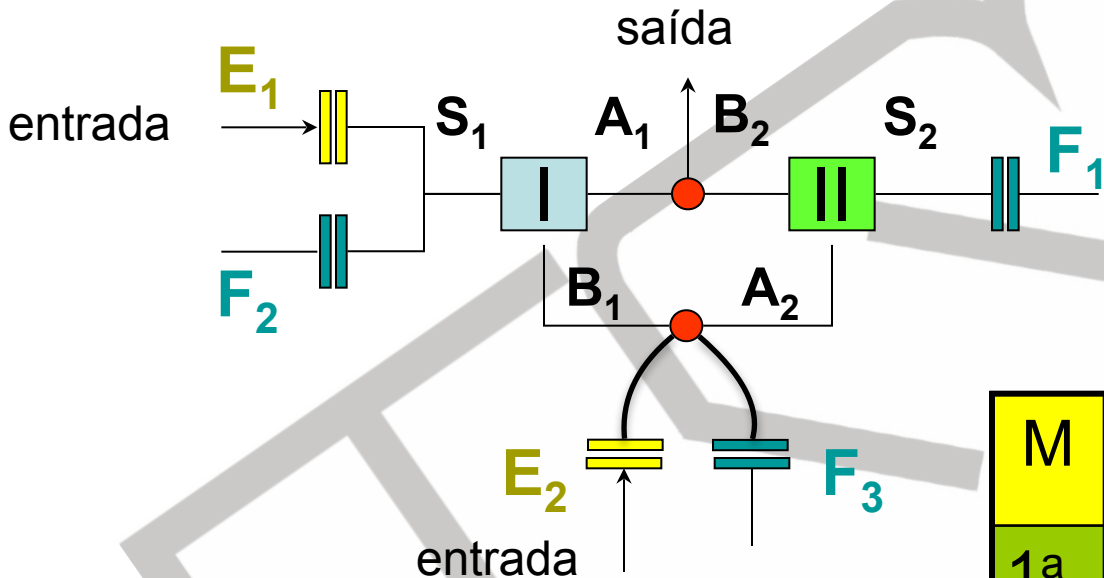
Cinemática

1 TEP

Relações cinemáticas dos TEPs			
	(a)  $w_S = \frac{1}{b} \cdot w_A$	(d)  $w_A = (1-b) \cdot w_B$	
	(b)  $w_A = b \cdot w_S$	(e)  $w_B = \frac{b}{b-1} \cdot w_S$	
	(c)  $w_B = \frac{1}{1-b} \cdot w_A$	(f)  $w_S = \frac{b-1}{b} \cdot w_B$	
	(a) 	(b) 	(c) 
	$w_A = (1-b) \cdot w_B + b \cdot w_S$	$w_S = \frac{1}{b} \cdot w_A + \frac{b-1}{b} \cdot w_B$	$w_B = \frac{1}{1-b} \cdot w_A + \frac{b}{b-1} \cdot w_S$
	$w_A + (b-1) \cdot w_B - b \cdot w_S = 0$		

Transmissões Veiculares

Caixa GM HIDRA-MATIC THM – 440 PGT

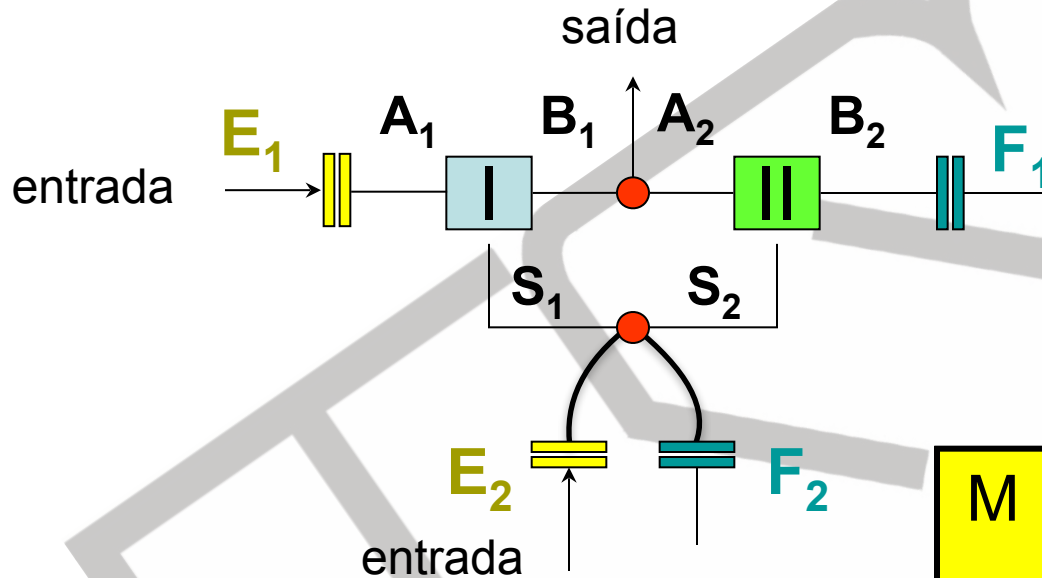


	Z_S	Z_P	Z_A
TEP I	26	18	62
TEP II	42	16	74

M	RT:1	E_1	E_2	F_1	F_2	F_3
1 ^a	2,92	X		X		
2 ^a	1,57		X	X		
3 ^a	1,00	X	X			
4 ^a	0,71		X		X	
ré	-2,38	X				X

Transmissões Veiculares

Caixa Simpson



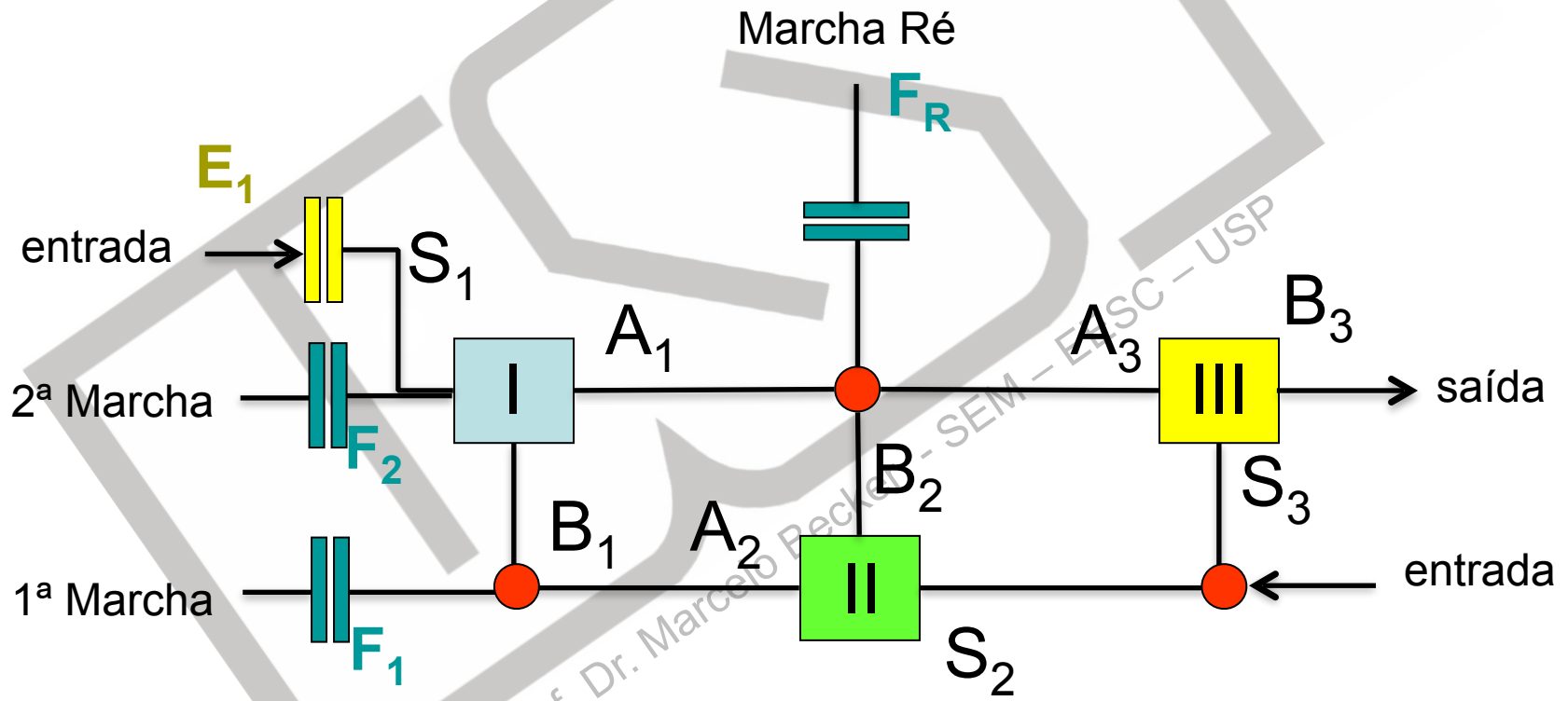
	Z_S	Z_P	Z_A
TEP I	36	16	68
TEP II	32	22	76

M	RT:1	E_1	E_2	F_1	F_2
1 ^a	2,79	X		X	
2 ^a	1,53	X			X
3 ^a	1,00	X	X		
ré	-2,38		X	X	

Transmissões Veiculares

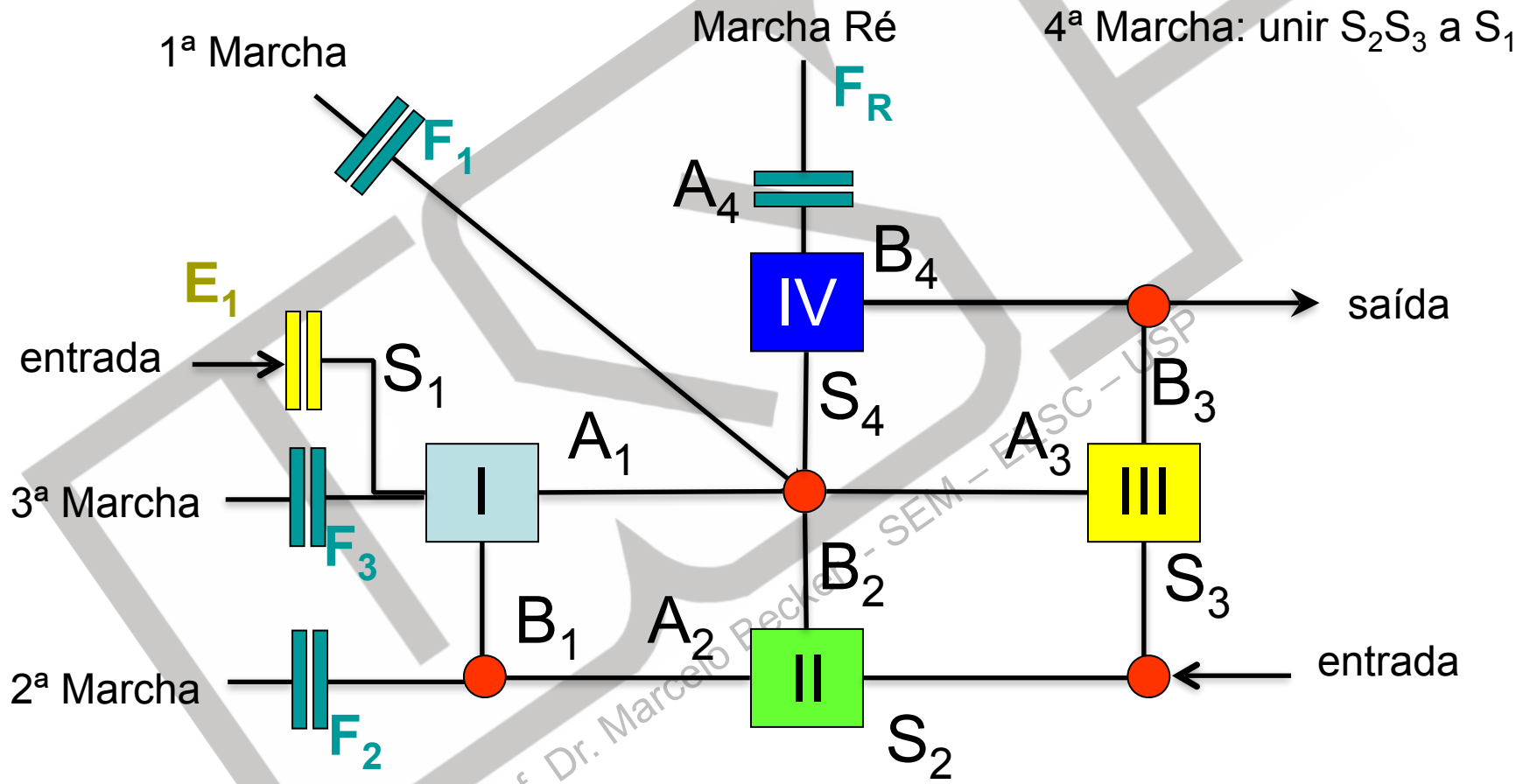
Caixa Wilson de 3 Marchas

3ª Marcha: unir S_2S_3 a S_1



Transmissões Veiculares

Caixa Wilson de 4 Marchas



Sumário da Aula

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- Exemplos de Aplicação

Fluxo de Torque

Segundo Hsieh (1997): “O torque flui para os engrenamentos ...”



Em equilíbrio estático: $\tau_{E_a,k} + \tau_{E_b,k} + \tau_{B,k} = 0$

Para ω^{cte} , sem atrito: $P_{E_a,k} + P_{E_b,k} + P_{B,k} = 0$

Holmes (1977)

Fluxo de Torque

Reescrevendo a equação: $P_{E_a,k} + P_{E_b,k} + P_{B,k} = 0$

$$\tau_{E_a,k} \cdot W_{E_a} + \tau_{E_b,k} W_{E_b} + \tau_{B,k} W_B = 0$$

Resolvendo o sistema de equações:

$$\left\{ \begin{array}{l} \tau_{E_a,k} + \tau_{E_b,k} + \tau_{B,k} = 0 \\ \tau_{E_a,k} \cdot W_{E_a} + \tau_{E_b,k} W_{E_b} + \tau_{B,k} W_{B,k} = 0 \\ \frac{W_{E_a,k} - W_{B,k}}{W_{E_b,k} - W_{B,k}} = r_k = \pm \frac{Z_{E_b}}{Z_{E_a}} \end{array} \right.$$

Fluxo de Torque

Para os Engrenamentos:

$$\begin{cases} \tau_{E_b,k} + r_k \cdot \tau_{E_a,k} = 0 \\ \tau_{B,k} + (1 - r_k) \cdot \tau_{E_a,k} = 0 \end{cases}$$

Onde: a e b são as engrenagens, B é o braço e k é o engrenamento

$$r_k = \pm \frac{Z_{E_b}}{Z_{E_a}} \begin{cases} \text{Se “-”, ambas engrenagens possuem dentes externos} \\ \text{Se “+”, uma engrenagem tem dentes internos} \end{cases}$$

Fluxo de Torque

Segundo Hsieh: “O torque flui para os engrenamentos ...”

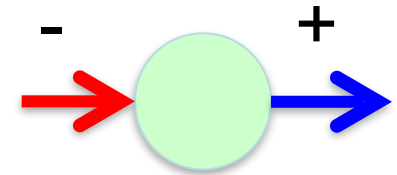
Para os Engrenamentos:

$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases}$$

Onde: x e y são as engrenagens, z é o braço e k é o engrenamento

$$r_k = \pm \frac{Z_y}{Z_x}$$

Para os Elementos: $\sum \tau = 0$

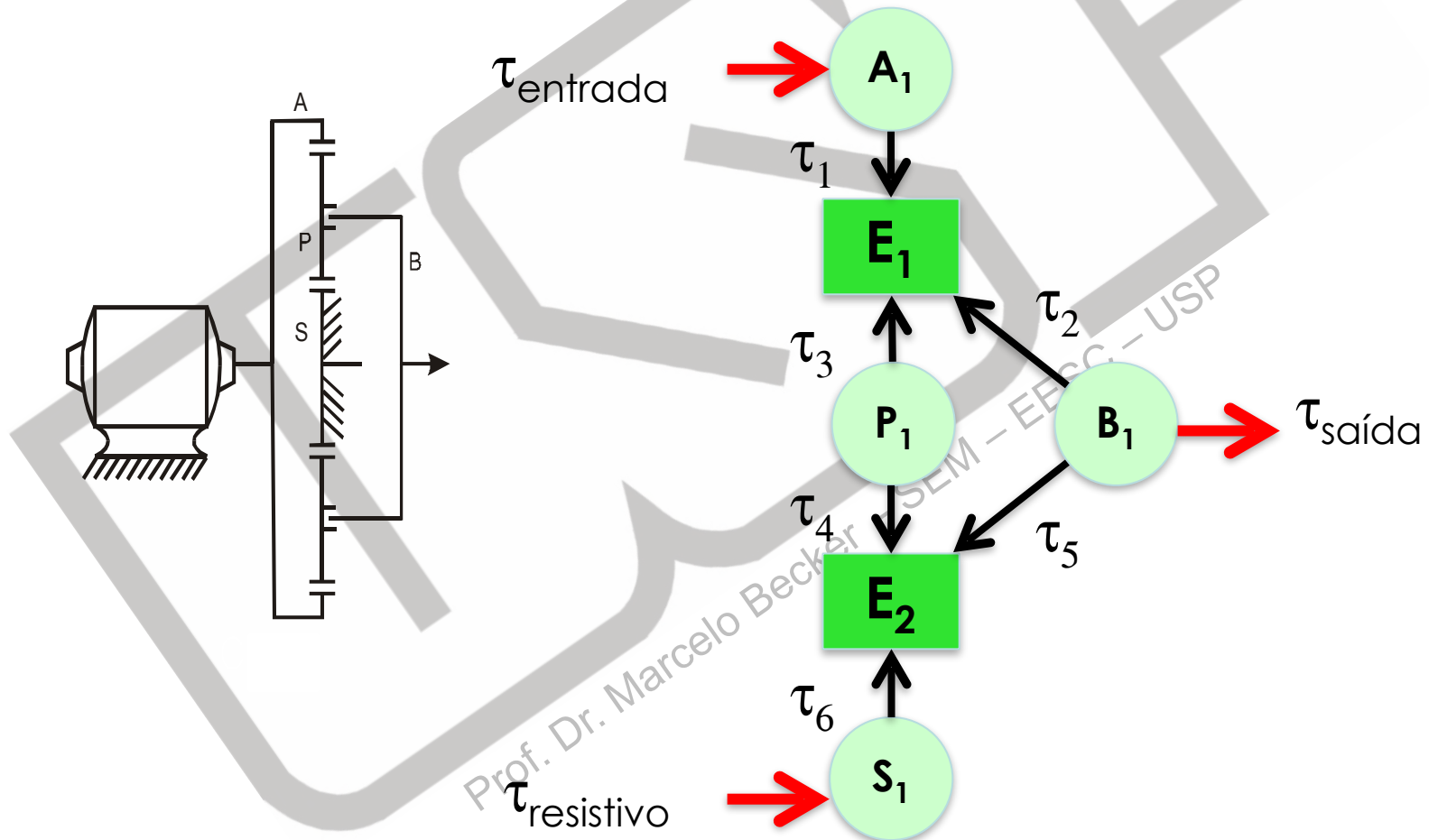


Equação Final: $[B] \cdot \{\tau\} = \{0\}$

Fluxo de Torque

1 TEP

Considerar: $Z_A=72$, $Z_P=12$ e $Z_S=48$



Fluxo de Torque

1 TEP

Para o Engrenamento 1:

$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases}$$

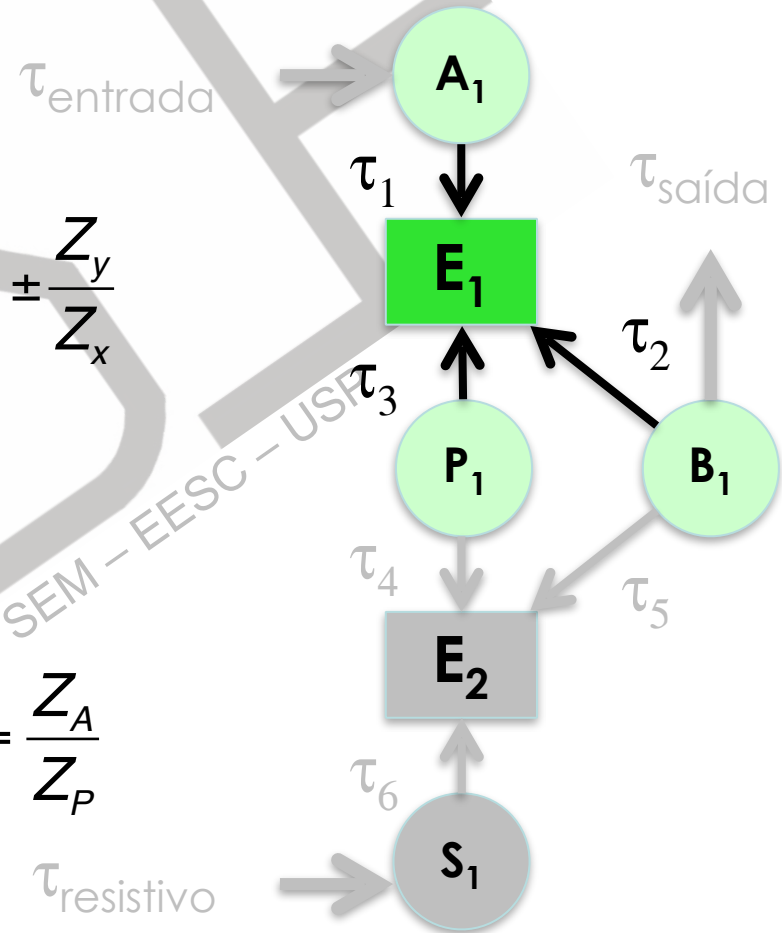
$$r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E_1} \begin{cases} \tau_1 + r_1 \cdot \tau_3 = 0 \\ \tau_2 + (1 - r_1) \cdot \tau_3 = 0 \end{cases}$$

$$r_1 = \frac{Z_A}{Z_P}$$

$\tau_{\text{resistivo}}$



Fluxo de Torque

1 TEP

Para o Engrenamento 2:

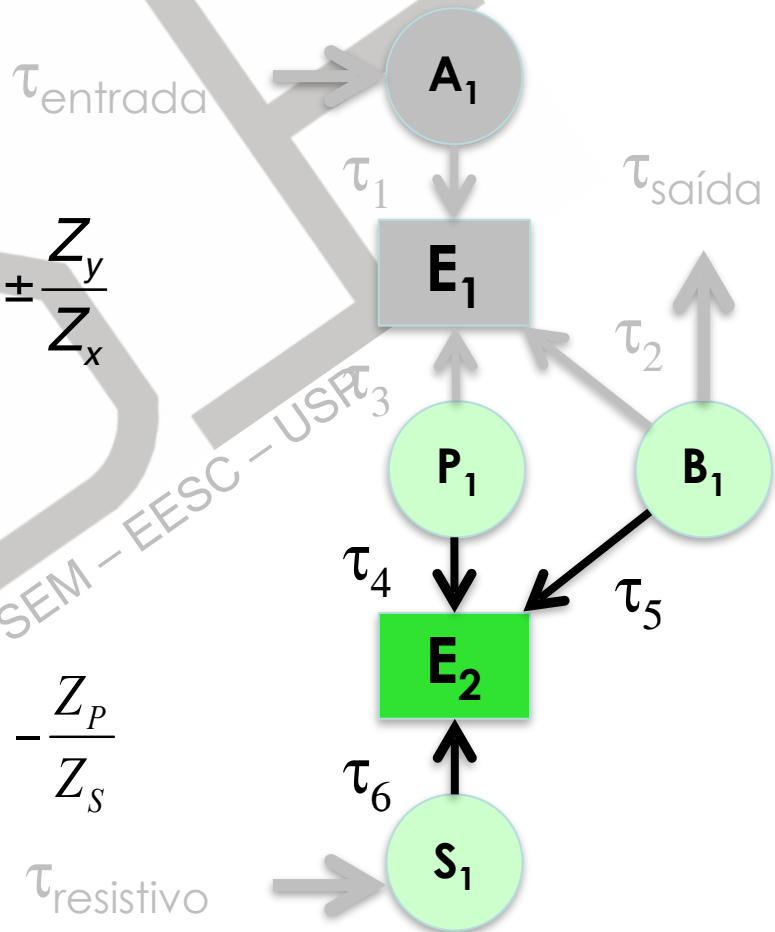
$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases}$$

$$r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E}_2 \begin{cases} \tau_4 + r_2 \cdot \tau_6 = 0 \\ \tau_5 + (1 - r_2) \cdot \tau_6 = 0 \end{cases}$$

$$r_2 = -\frac{Z_P}{Z_S}$$



Fluxo de Torque

1 TEP

Para os Engrenamentos:

$$\mathbf{E}_1 \begin{cases} \tau_1 + r_1 \cdot \tau_3 = 0 \\ \tau_2 + (1 - r_1) \cdot \tau_3 = 0 \end{cases} \quad r_1 = \frac{Z_A}{Z_P} = 6,0$$

$$\mathbf{E}_2 \begin{cases} \tau_4 + r_2 \cdot \tau_6 = 0 \\ \tau_5 + (1 - r_2) \cdot \tau_6 = 0 \end{cases} \quad r_2 = -\frac{Z_P}{Z_S} = -0,25$$

Lembrando que: $Z_A=72$, $Z_P=12$ e $Z_S=48$

Fluxo de Torque

1 TEP

Para os Elementos:

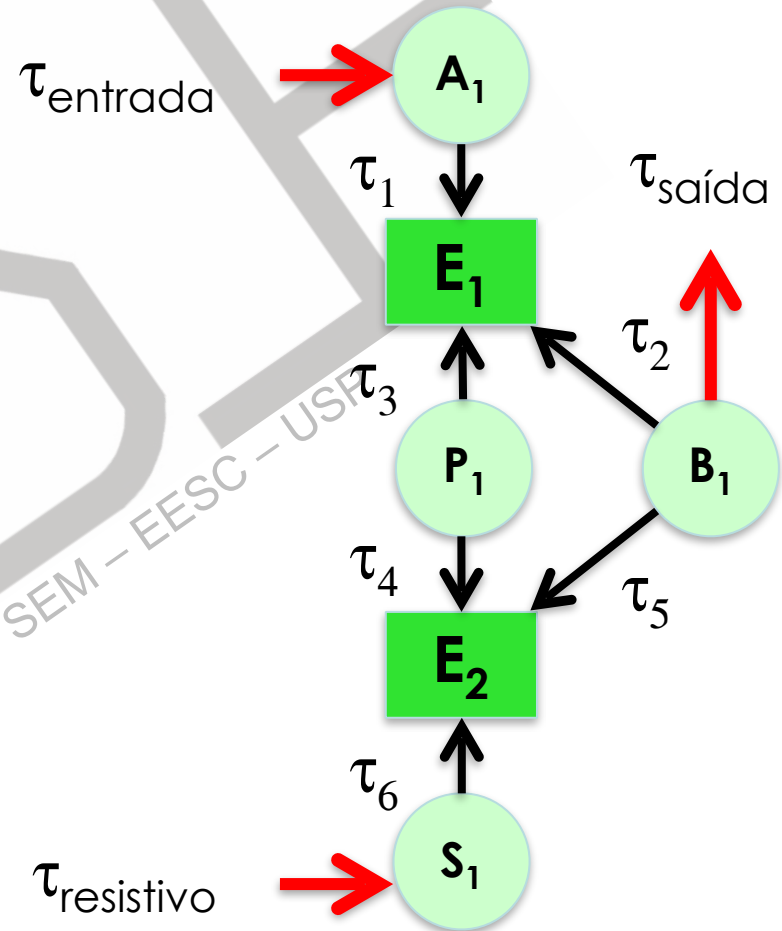
$$\sum \tau = 0$$

$$S_1: \tau_6 - \tau_{resistivo} = 0$$

$$P_1: \tau_3 + \tau_4 = 0$$

$$A_1: \tau_1 - \tau_{entrada} = 0$$

$$B_1: \tau_2 + \tau_5 + \tau_{saída} = 0$$



Fluxo de Torque

1 TEP

Na forma de Equação Matricial:

$$[B] \cdot \{\tau\} = \{0\}$$

(8 x 8)

(8 x 1)

$$\begin{bmatrix} 1 & 0 & 0 & 6 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & -0,25 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1,25 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \tau_1 \\ \tau_2 \\ \tau_5 \\ \tau_3 \\ \tau_4 \\ \tau_6 \\ \tau_{resistivo} \\ \tau_{saída} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \tau_{entrada} \\ 0 \end{bmatrix}$$

Fluxo de Torque

1 TEP

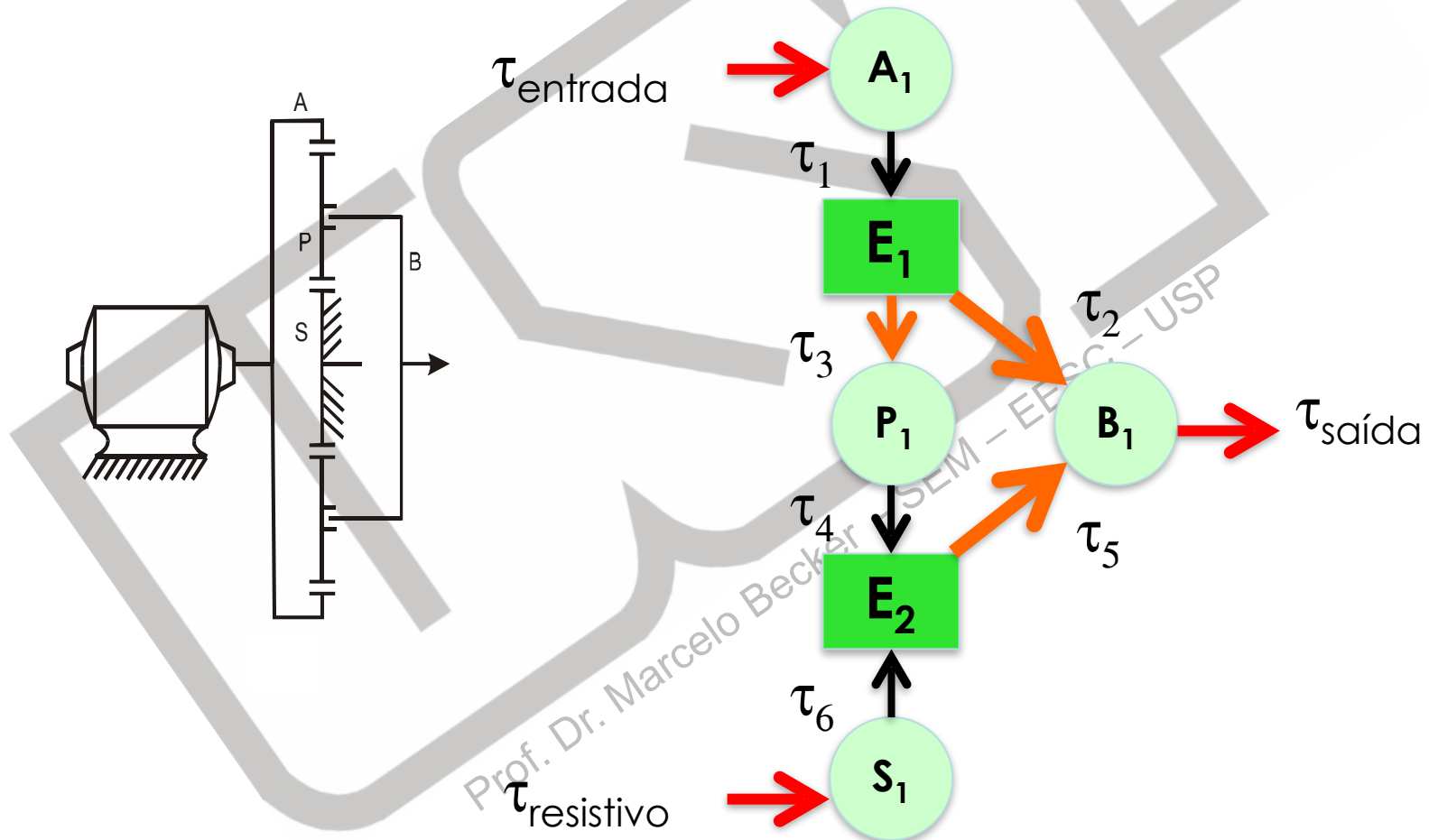
Resultados: aplicando o Método de Gauss

Torque	Torque
$\tau_1 = \tau_{entrada}$	$\tau_5 = -0,83 \tau_{entrada}$
$\tau_2 = -0,83 \tau_{entrada}$	$\tau_6 = 0,66 \tau_{entrada}$
$\tau_3 = -0,17 \tau_{entrada}$	$\tau_{resistivo} = 0,66 \tau_{entrada}$
$\tau_4 = 0,17 \tau_{entrada}$	$\tau_{saída} = 1,66 \tau_{entrada}$

Prof. Dr. Marcelo Becker

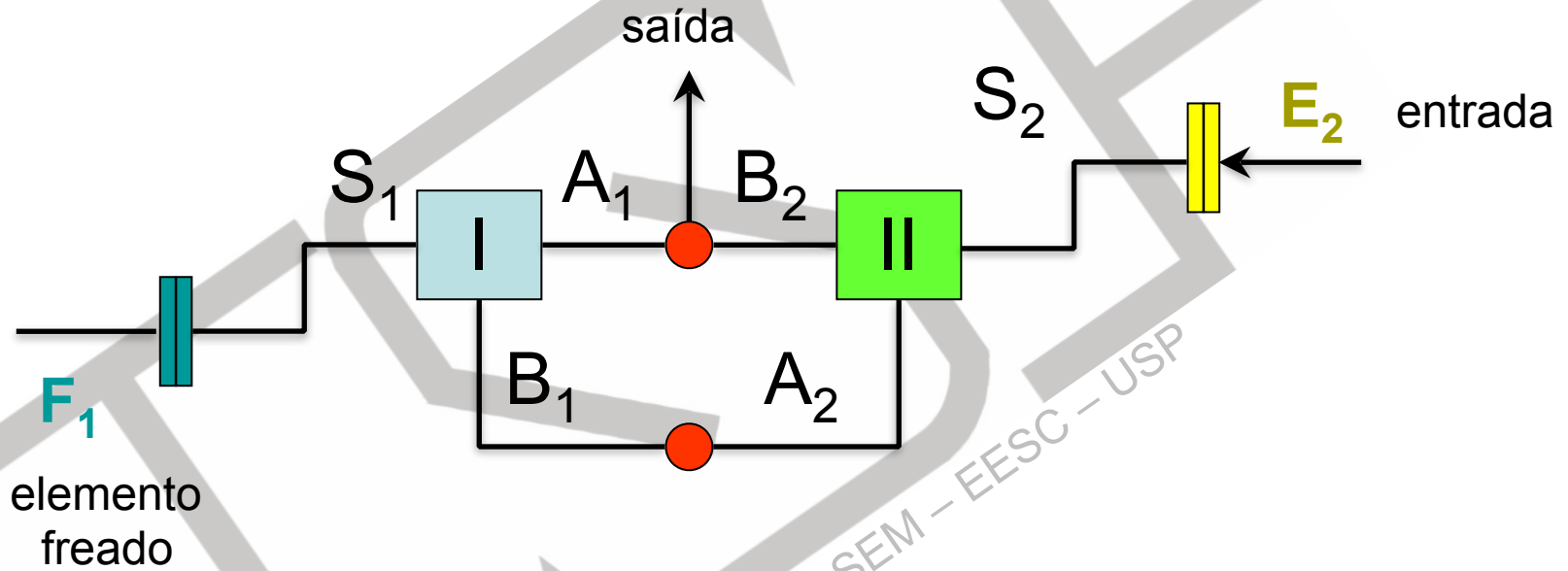
Fluxo de Torque

1 TEP



Fluxo de Torque

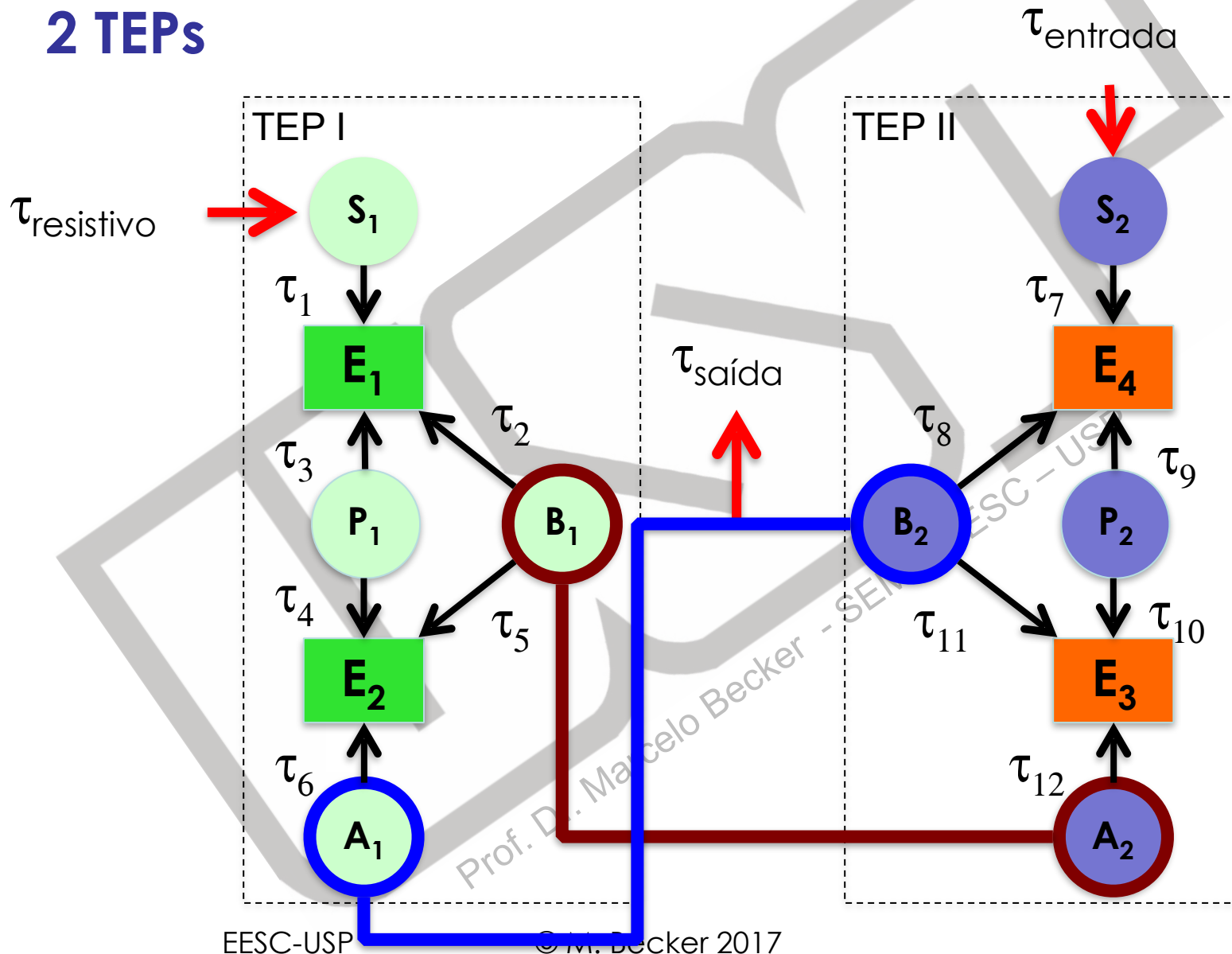
2 TEPs



	Z_S	Z_P	Z_A
TEP I	26	18	62
TEP II	42	16	74

Fluxo de Torque

2 TEPs



Fluxo de Torque

1 TEP

Para o Engrenamento 1:

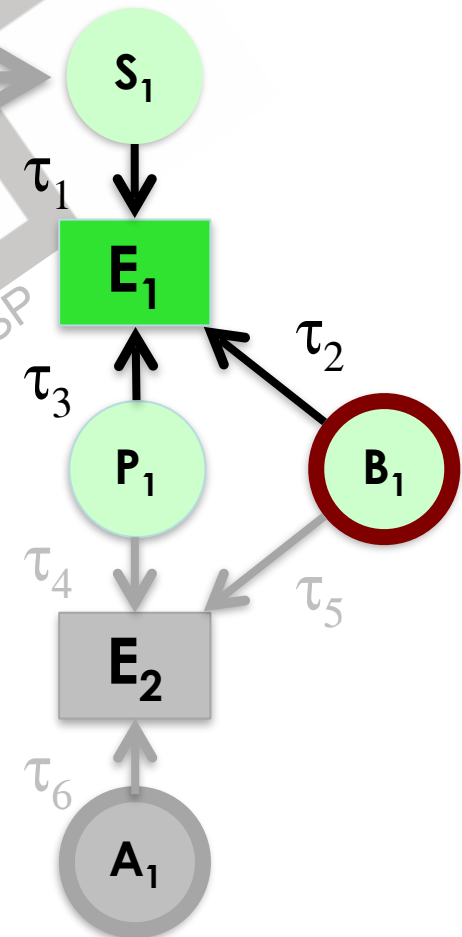
$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases}$$

$$r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E}_1 \begin{cases} \tau_3 + r_1 \cdot \tau_1 = 0 \\ \tau_2 + (1 - r_1) \cdot \tau_1 = 0 \end{cases} \quad r_1 = -\frac{Z_{P_1}}{Z_{S_1}}$$

TEP I



Fluxo de Torque

1 TEP

Para o Engrenamento 2:

$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases}$$

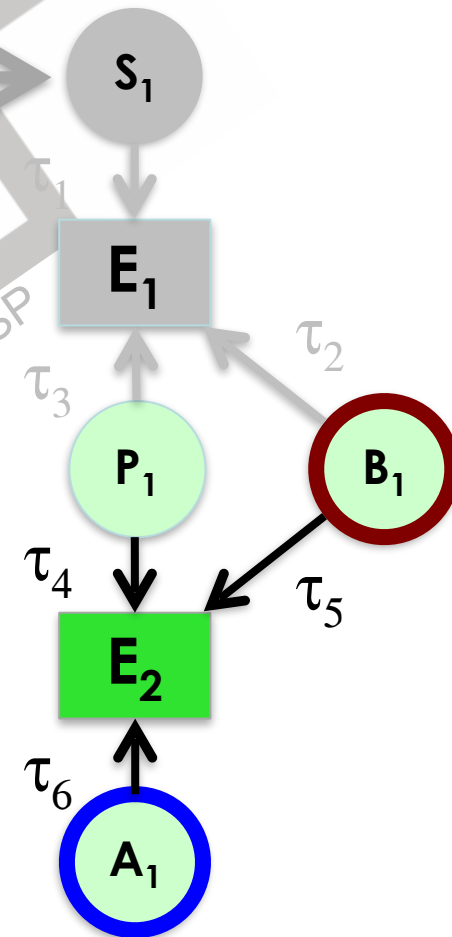
$$r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E2} \begin{cases} \tau_6 + r_2 \cdot \tau_4 = 0 \\ \tau_5 + (1 - r_2) \cdot \tau_4 = 0 \end{cases}$$

$$r_2 = \frac{Z_{A_1}}{Z_{P_1}}$$

TEP I



Fluxo de Torque

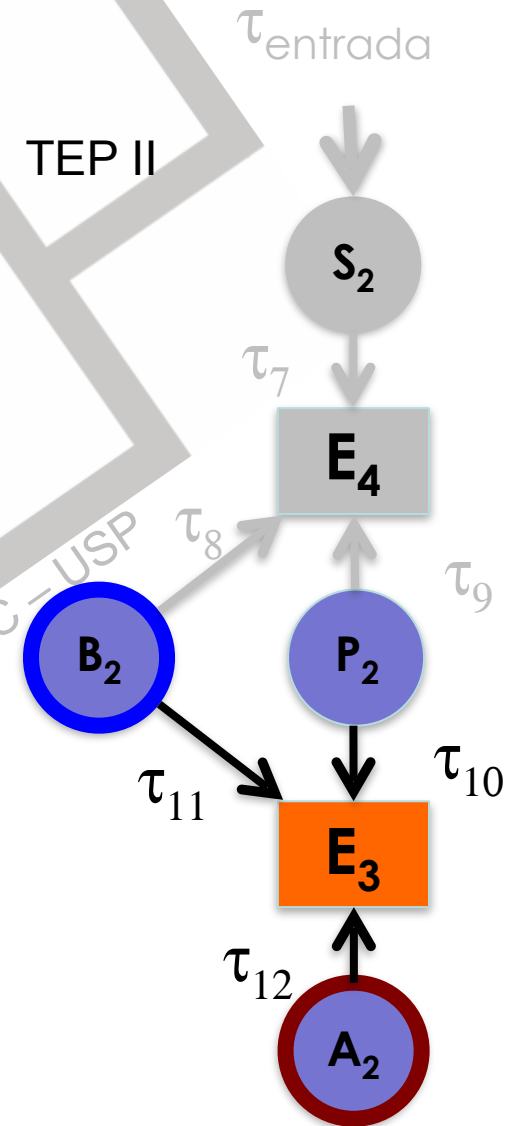
1 TEP

Para o Engrenamento 3:

$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases} \quad r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E}_3 \begin{cases} \tau_{10} + r_3 \cdot \tau_{12} = 0 \\ \tau_{11} + (1 - r_3) \cdot \tau_{12} = 0 \end{cases} \quad r_3 = \frac{Z_{P_2}}{Z_{A_2}}$$



Fluxo de Torque

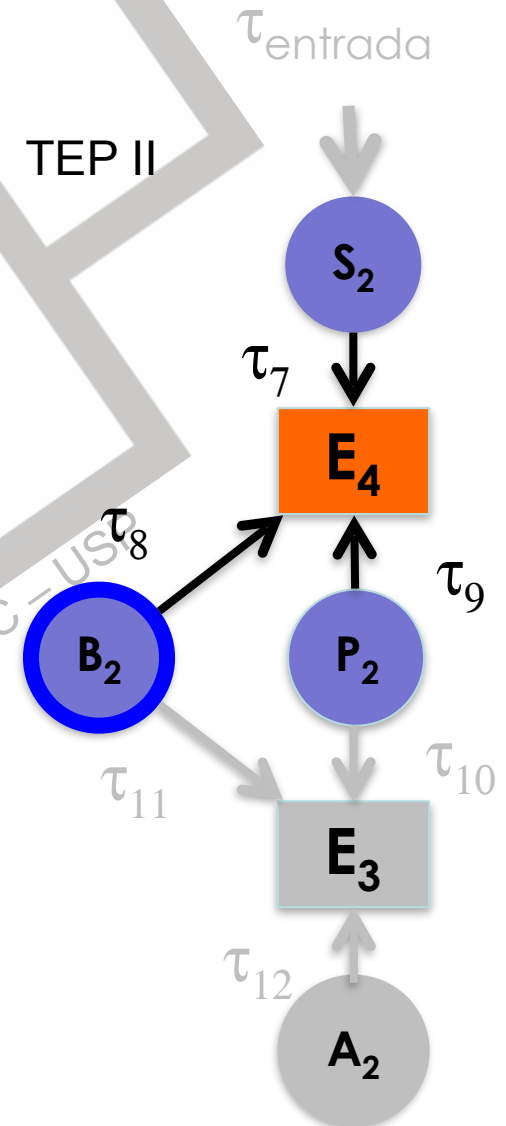
1 TEP

Para o Engrenamento 4:

$$\begin{cases} \tau_{y,k} + r_k \cdot \tau_{x,k} = 0 \\ \tau_{z,k} + (1 - r_k) \cdot \tau_{x,k} = 0 \end{cases} \quad r_k = \pm \frac{Z_y}{Z_x}$$

Onde: x e y são as engrenagens
 z é o braço e k é o engrenamento

$$\mathbf{E}_4 \begin{cases} \tau_7 + r_4 \cdot \tau_9 = 0 \\ \tau_8 + (1 - r_4) \cdot \tau_9 = 0 \end{cases} \quad r_4 = -\frac{Z_{S_2}}{Z_{P_2}}$$



Fluxo de Torque

2 TEPs

Assim, para os Engrenamentos:

$$\mathbf{E1} \begin{cases} \tau_3 + r_1 \cdot \tau_1 = 0 \\ \tau_2 + (1 - r_1) \cdot \tau_1 = 0 \end{cases}$$

$$\mathbf{E3} \begin{cases} \tau_{10} + r_3 \cdot \tau_{12} = 0 \\ \tau_{11} + (1 - r_3) \cdot \tau_{12} = 0 \end{cases}$$

$$\mathbf{E2} \begin{cases} \tau_6 + r_2 \cdot \tau_4 = 0 \\ \tau_5 + (1 - r_2) \cdot \tau_4 = 0 \end{cases}$$

$$\mathbf{E4} \begin{cases} \tau_7 + r_4 \cdot \tau_9 = 0 \\ \tau_8 + (1 - r_4) \cdot \tau_9 = 0 \end{cases}$$

Fluxo de Torque

2 TEPs

Onde:

	Z_S	Z_P	Z_A
TEP I	26	18	62
TEP II	42	16	74

$$r_1 = -\frac{Z_{P_1}}{Z_{S_1}} = -0,6923$$

$$r_3 = \frac{Z_{P_2}}{Z_{A_2}} = 0,2162$$

$$r_2 = \frac{Z_{A_1}}{Z_{P_1}} = 3,4444$$

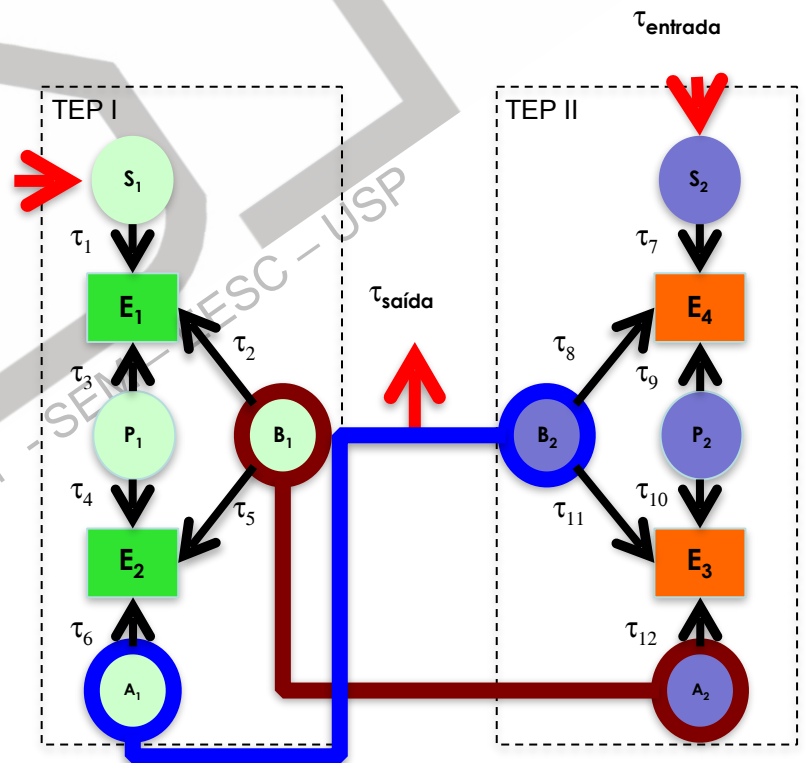
$$r_4 = -\frac{Z_{S_2}}{Z_{P_2}} = -2,6250$$

Fluxo de Torque

2 TEPs

Para os Elementos: $\sum \tau = 0$

$$\left\{ \begin{array}{l} \mathbf{S}_1: \quad \tau_1 - \tau_{resistivo} = 0 \\ \mathbf{P}_1: \quad \tau_3 + \tau_4 = 0 \\ \mathbf{A}_1\mathbf{B}_2: \quad \tau_6 + \tau_8 + \tau_{11} + \tau_{saída} = 0 \\ \mathbf{A}_2\mathbf{B}_1: \quad \tau_2 + \tau_5 + \tau_{12} = 0 \\ \mathbf{P}_2: \quad \tau_9 + \tau_{10} = 0 \\ \mathbf{S}_2: \quad \tau_7 - \tau_{entrada} = 0 \end{array} \right.$$



Fluxo de Torque

2 TEPs

(14 x 14)

(14 x 1)

$$[B] \cdot \{\tau\} = \{0\}$$

$$\begin{bmatrix}
 -0,6923 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
 1,6923 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3,444 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2,444 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0,2162 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
 0 & 0 & 1 & 0 & 0 & 0 & 0,7838 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -2,625 & 0 & 0 \\
 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3,625 & 0 & 0 \\
 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\
 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0
 \end{bmatrix} \cdot \begin{bmatrix}
 \tau_1 \\
 \tau_6 \\
 \tau_{11} \\
 \tau_8 \\
 \tau_2 \\
 \tau_5 \\
 \tau_{12} \\
 \tau_7 \\
 \tau_3 \\
 \tau_4 \\
 \tau_{10} \\
 \tau_9 \\
 \tau_{resistivo} \\
 \tau_{saída}
 \end{bmatrix} = \begin{bmatrix}
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0 \\
 0
 \end{bmatrix} \tau_{entrada}$$

Fluxo de Torque

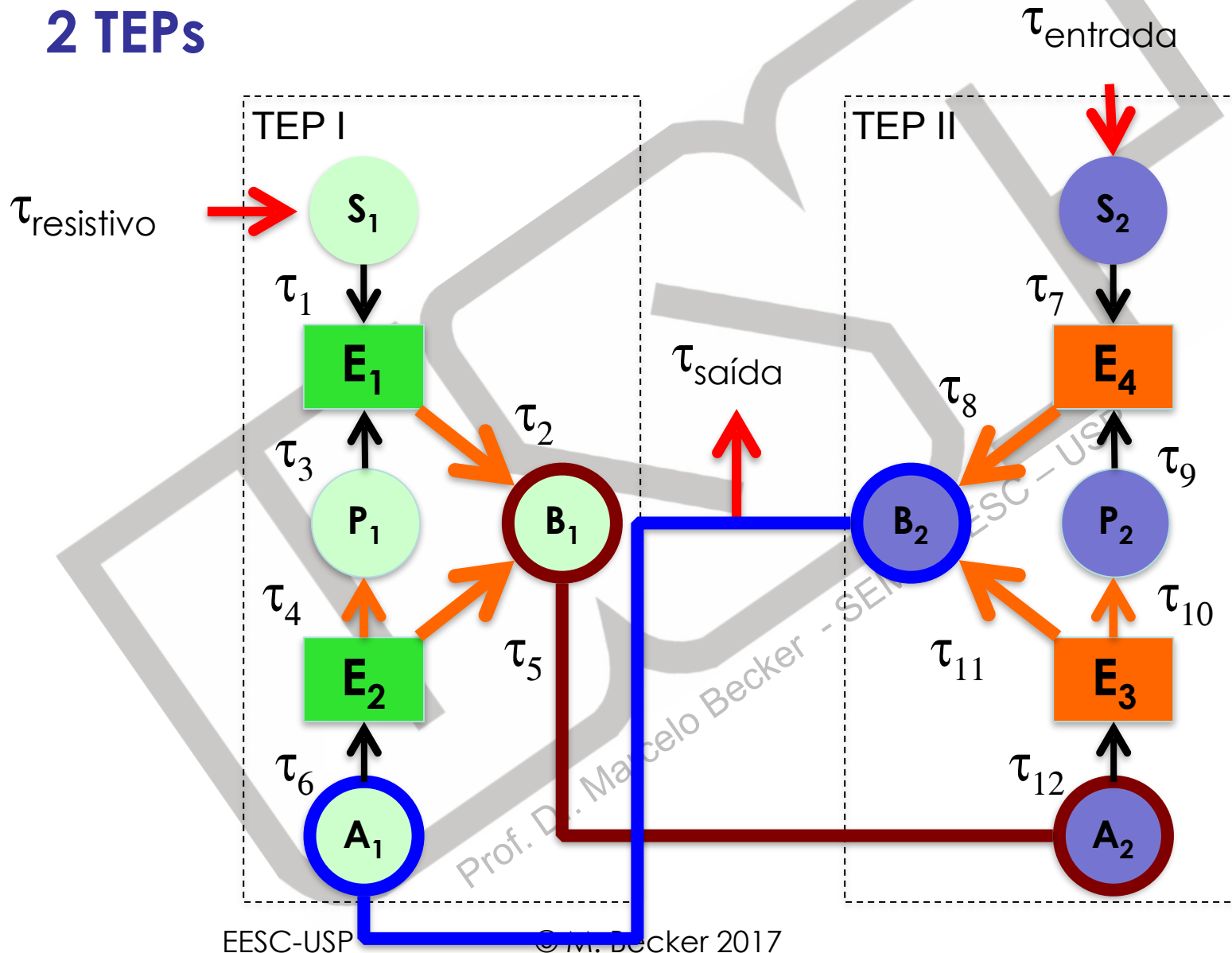
2 TEPs

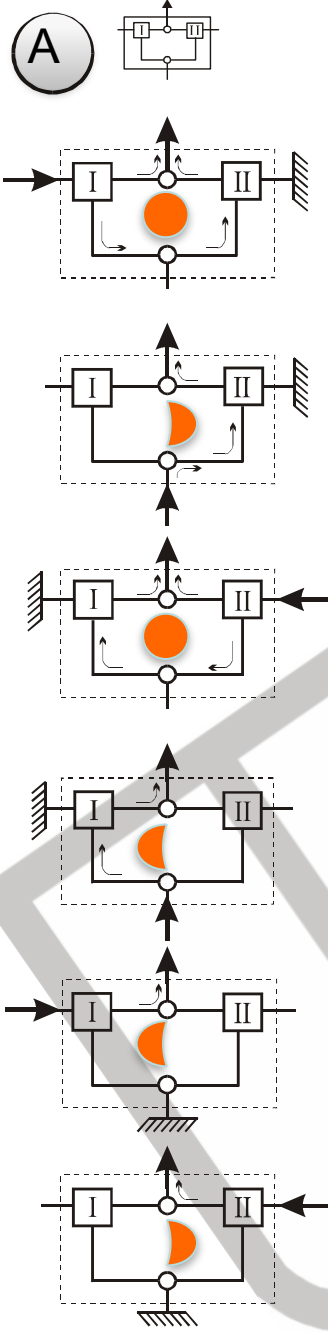
Resultados: aplicando o Método de Gauss

Torques	Torques	Torques
$\tau_1 = 0,5206 \tau_{entrada}$	$\tau_5 = - 0,8810 \tau_{entrada}$	$\tau_{10} = - 0,3810 \tau_{entrada}$
$\tau_6 = 1,2416 \tau_{entrada}$	$\tau_{12} = 1,7620 \tau_{entrada}$	$\tau_9 = 0,3810 \tau_{entrada}$
$\tau_{11} = - 1,3811 \tau_{entrada}$	$\tau_7 = \tau_{entrada}$	$\tau_{resistivo} = 0,5206 \tau_{entrada}$
$\tau_8 = - 1,3811 \tau_{entrada}$	$\tau_3 = 0,3605 \tau_{entrada}$	$\tau_{saída} = 1,5206 \tau_{entrada}$
$\tau_2 = - 0,8810 \tau_{entrada}$	$\tau_4 = - 0,3605 \tau_{entrada}$	

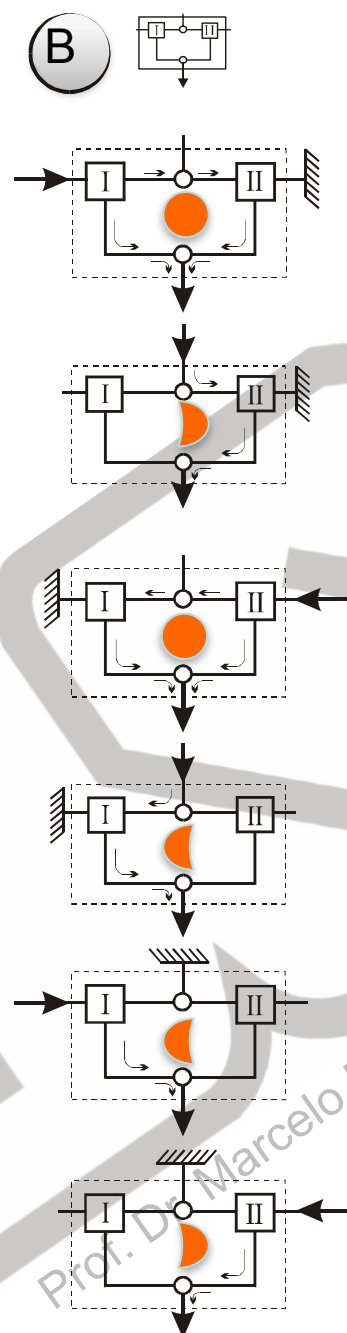
Fluxo de Torque

2 TEPs

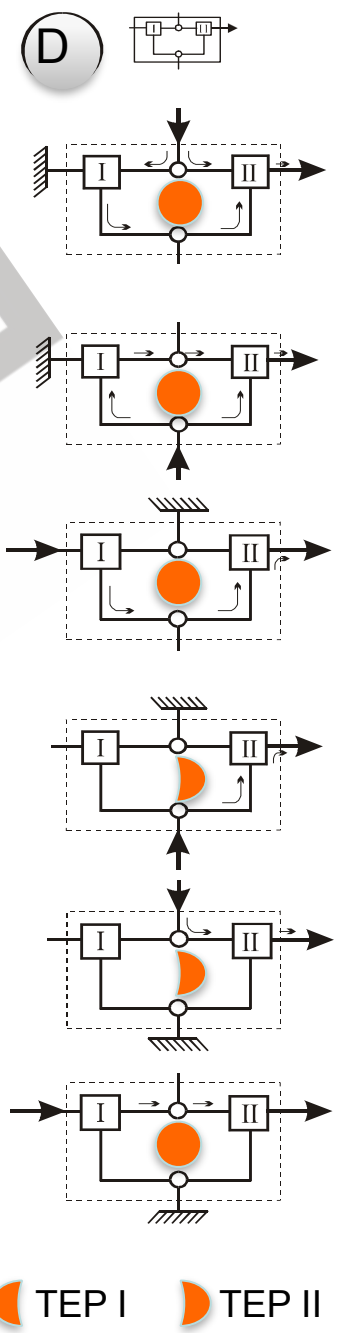
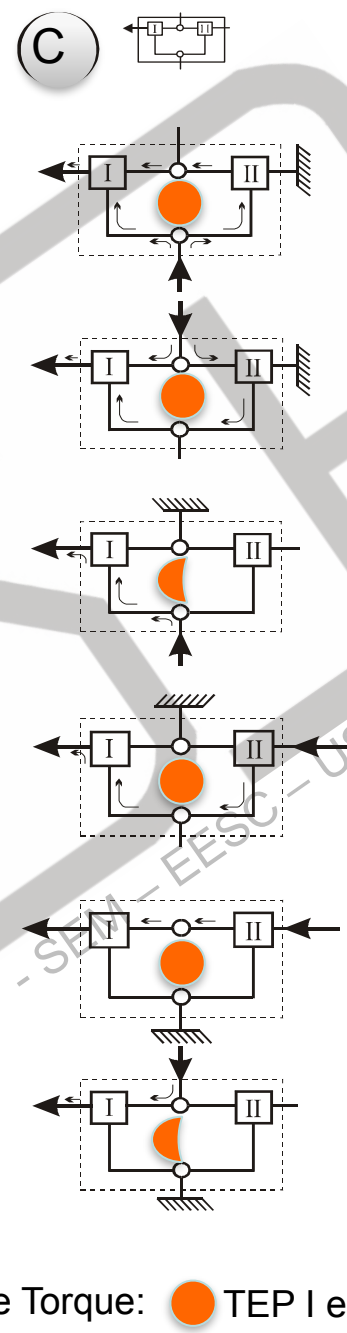




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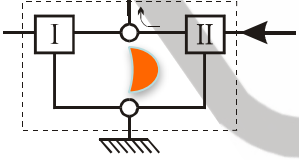
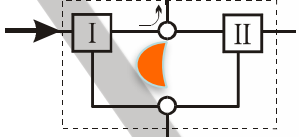
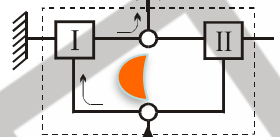
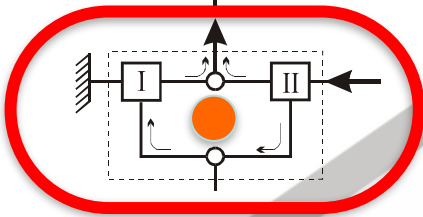
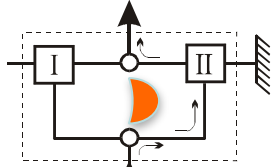
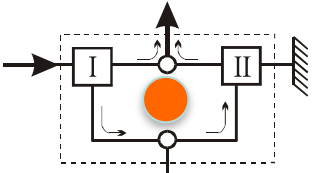
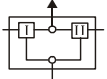


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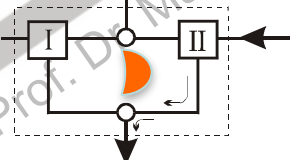
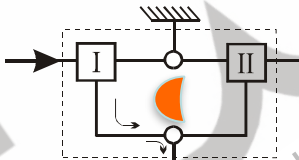
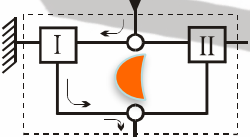
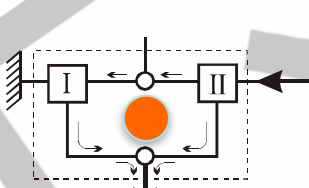
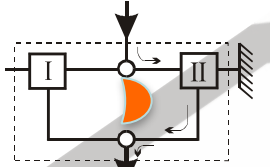
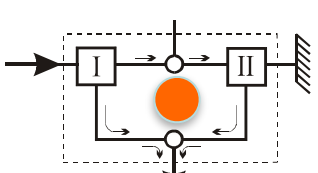
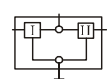


Fluxo de Torque: ● TEP I e II ◐ TEP I ◑ TEP II

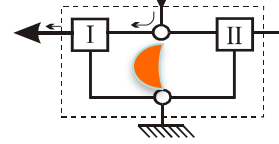
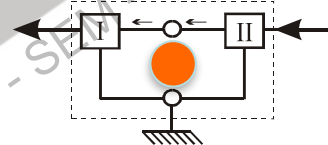
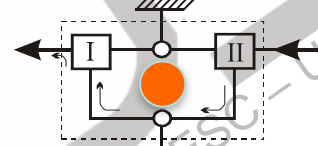
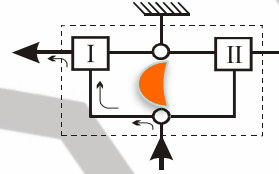
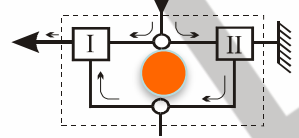
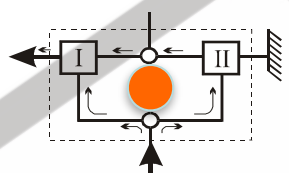
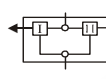
(A)



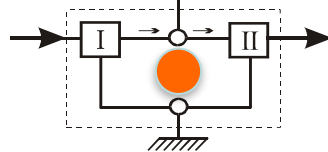
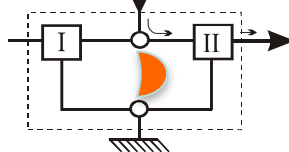
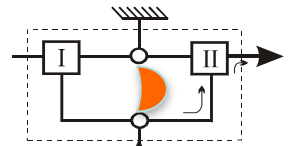
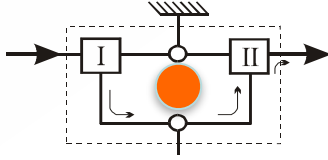
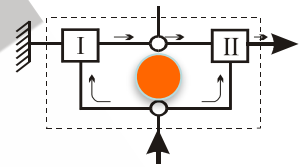
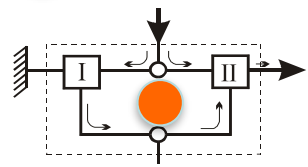
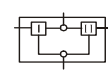
(B)



(C)



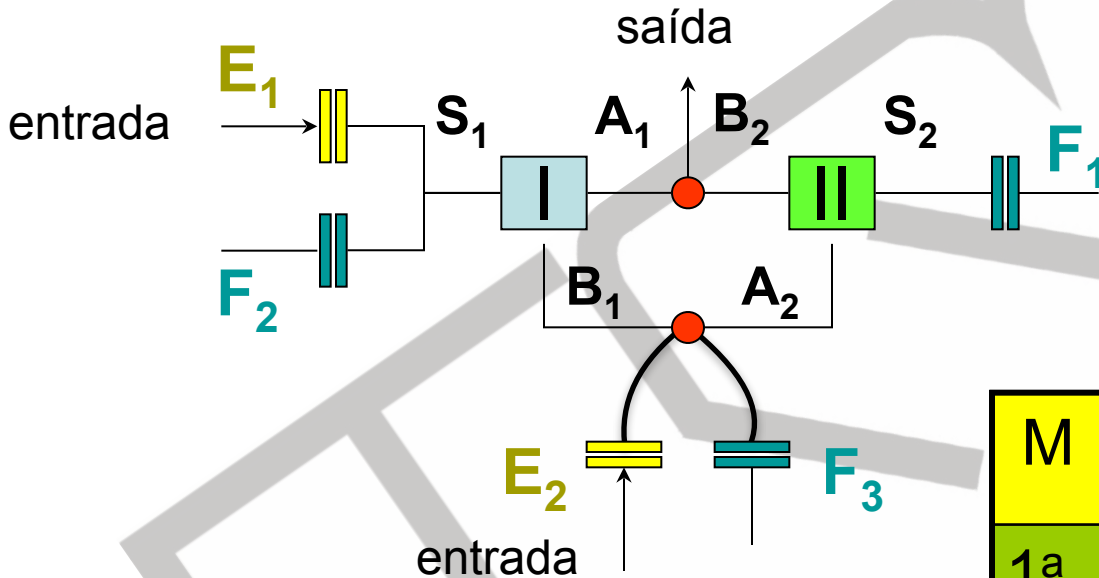
(D)



Fluxo de Torque: ● TEP I e II ◐ TEP I ◑ TEP II

Transmissões Veiculares

Caixa GM HIDRA-MATIC THM – 440 PGT

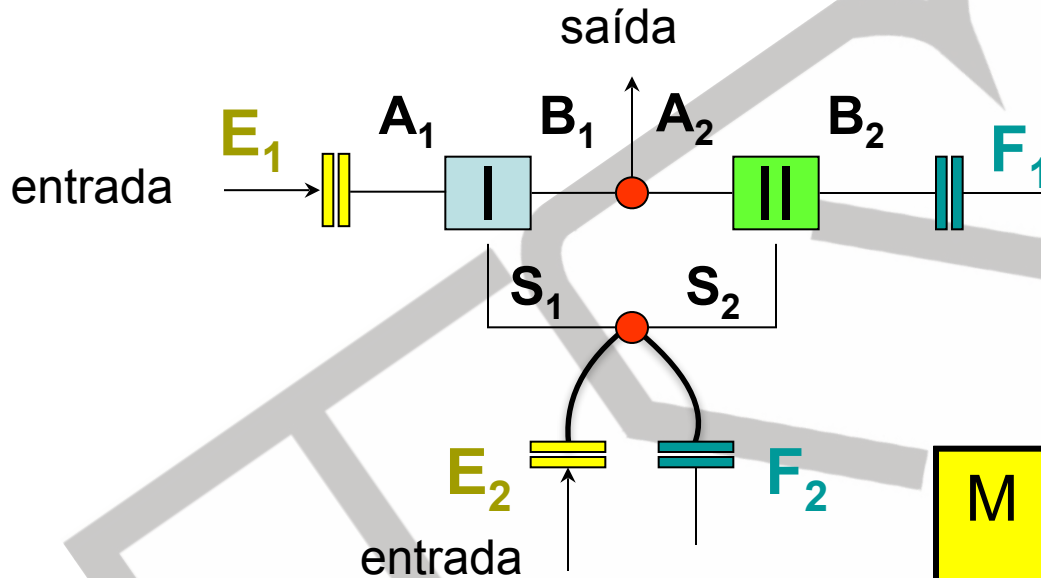


	Z_S	Z_P	Z_A
TEP I	26	18	62
TEP II	42	16	74

M	RT:1	E_1	E_2	F_1	F_2	F_3
1 ^a	2,92	X		X		
2 ^a	1,57		X	X		
3 ^a	1,00	X	X			
4 ^a	0,71		X		X	
ré	-2,38	X				X

Transmissões Veiculares

Caixa Simpson



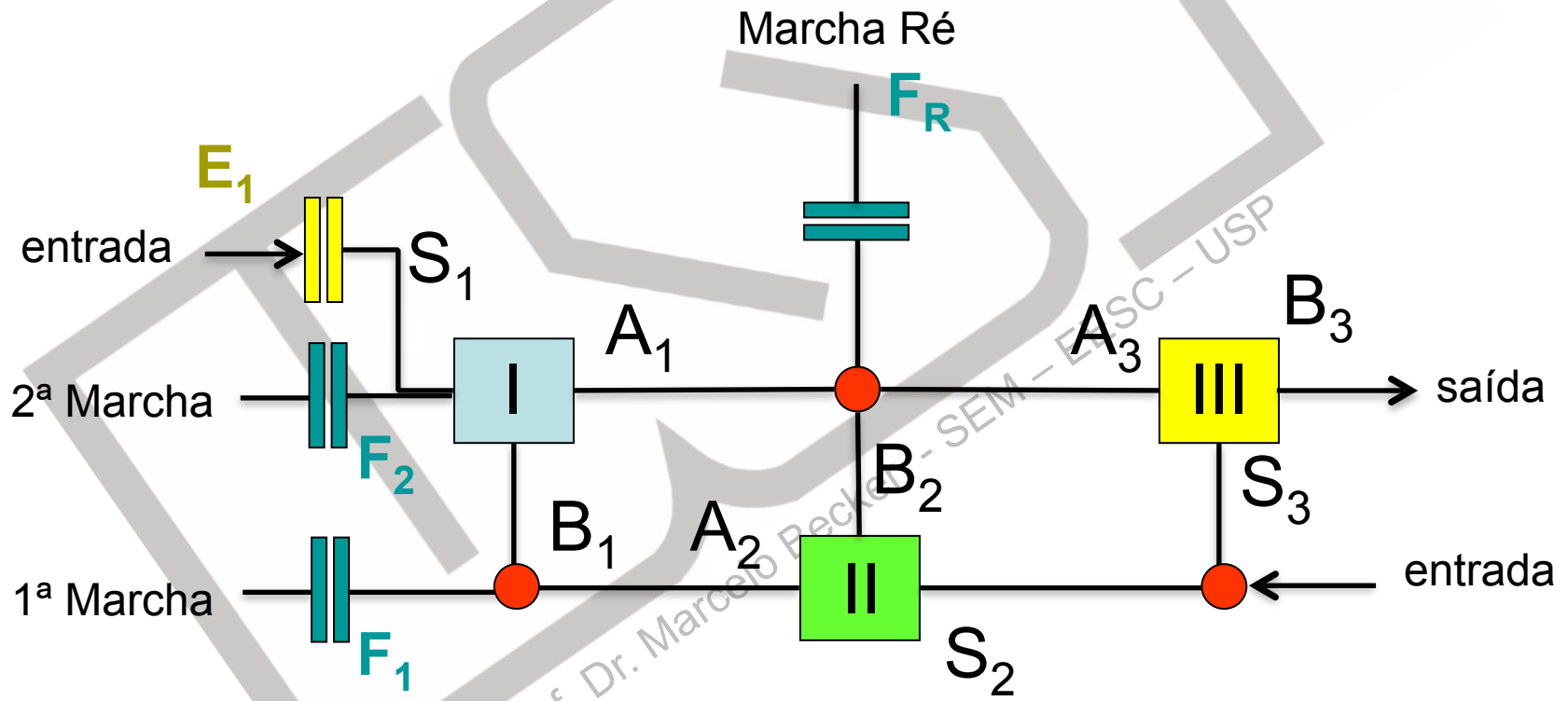
	Z_S	Z_P	Z_A
TEP I	36	16	68
TEP II	32	22	76

M	RT:1	E_1	E_2	F_1	F_2
1 ^a	2,79	X		X	
2 ^a	1,53	X			X
3 ^a	1,00	X	X		
ré	-2,38		X	X	

Transmissões Veiculares

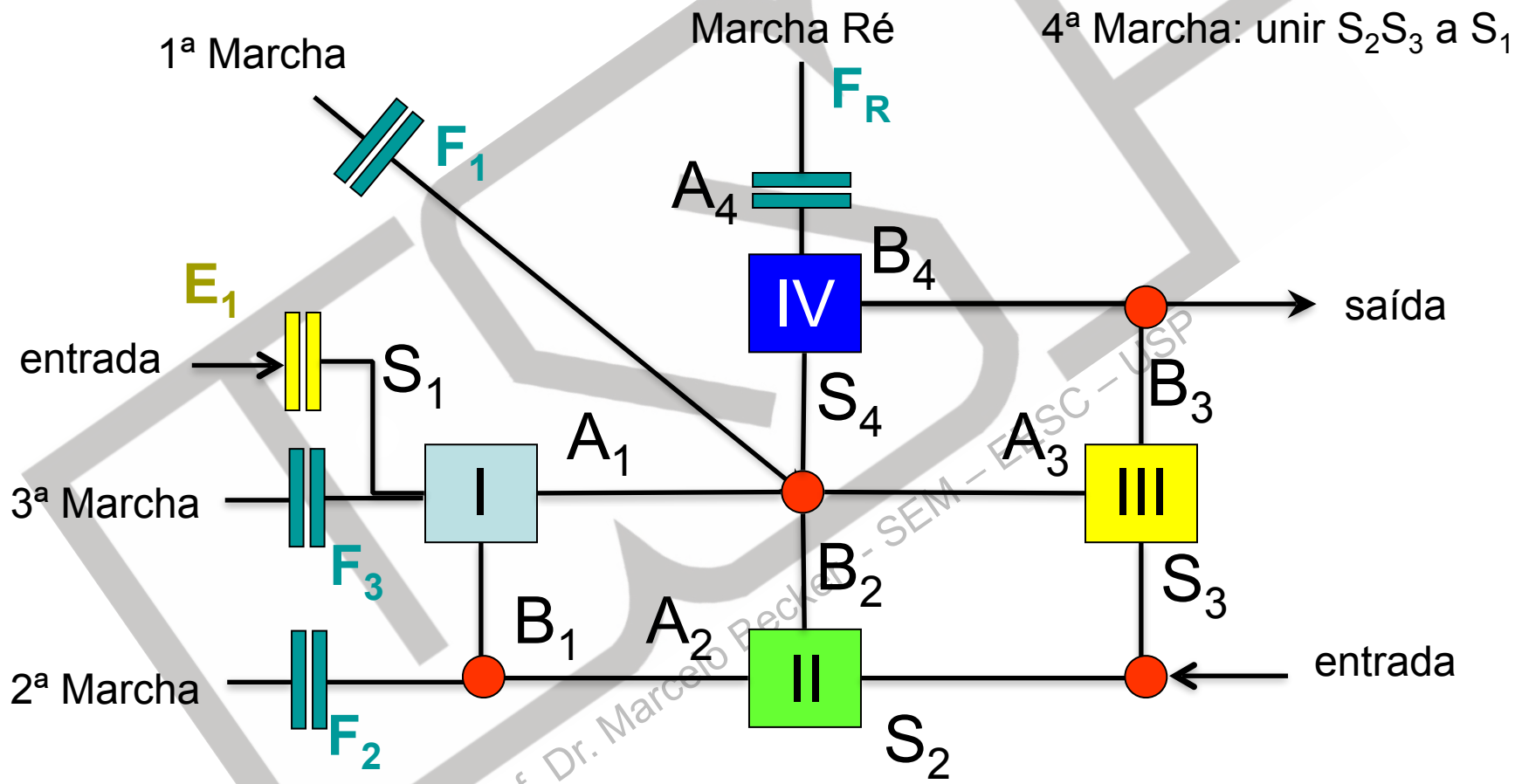
Caixa Wilson de 3 Marchas

3ª Marcha: unir S_2S_3 a S_1



Transmissões Veiculares

Caixa Wilson de 4 Marchas

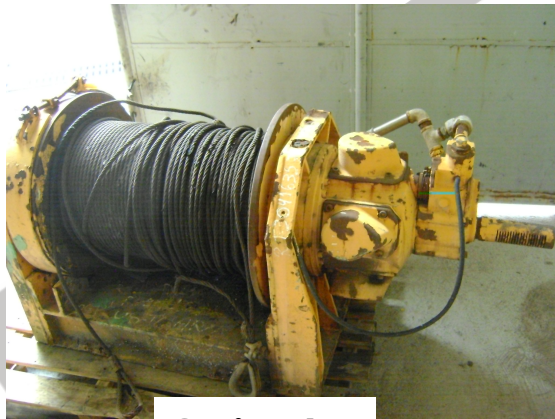


Sumário da Aula

- Introdução
- Representações
- Montagens
- Análise Cinemática
- Análise de Torque
- **Exemplos de Aplicação**

Case #1 – Guincho

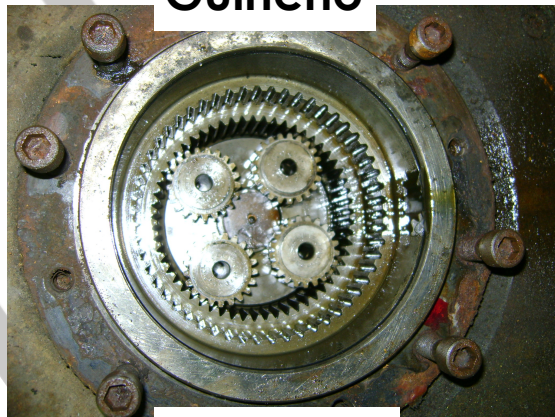
- Peças do Guindaste montado em Plataforma no mar.



Guincho



Retirada do Redutor

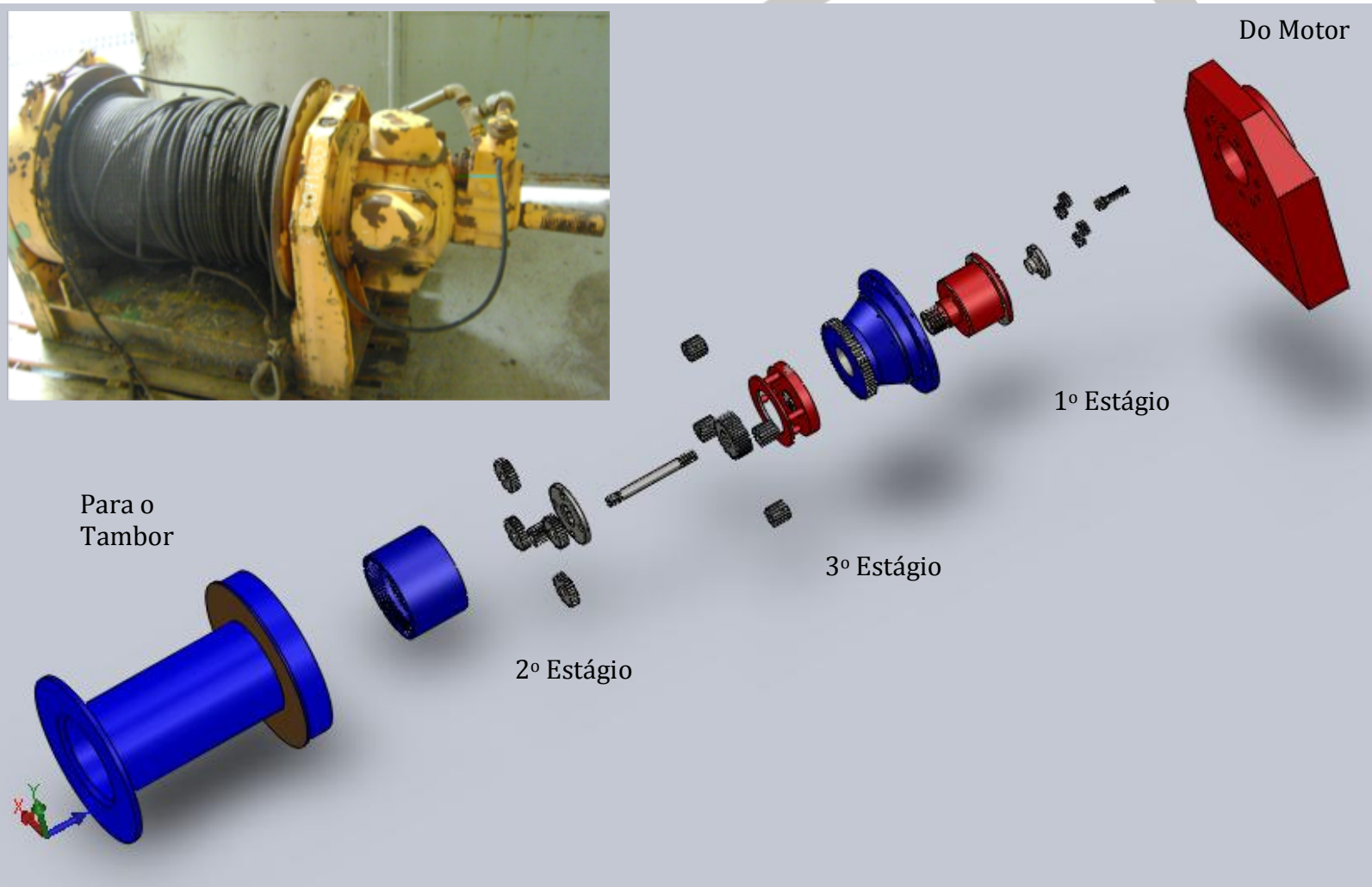


1º Estágio

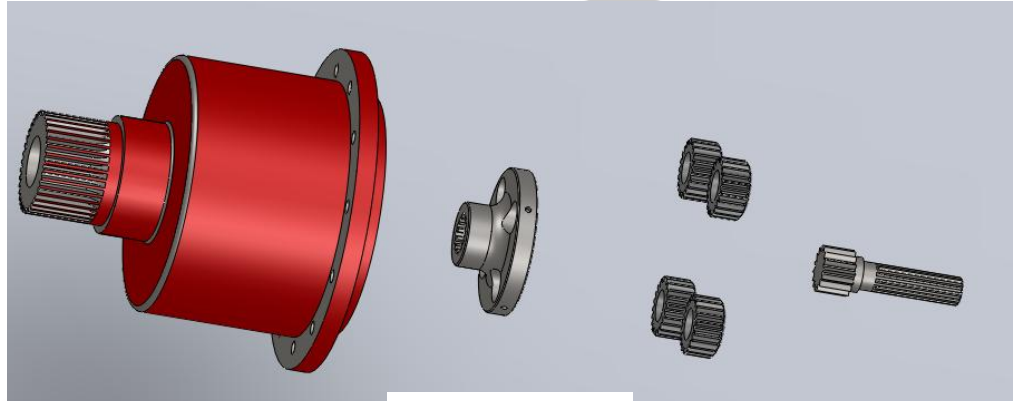


3º Estágio

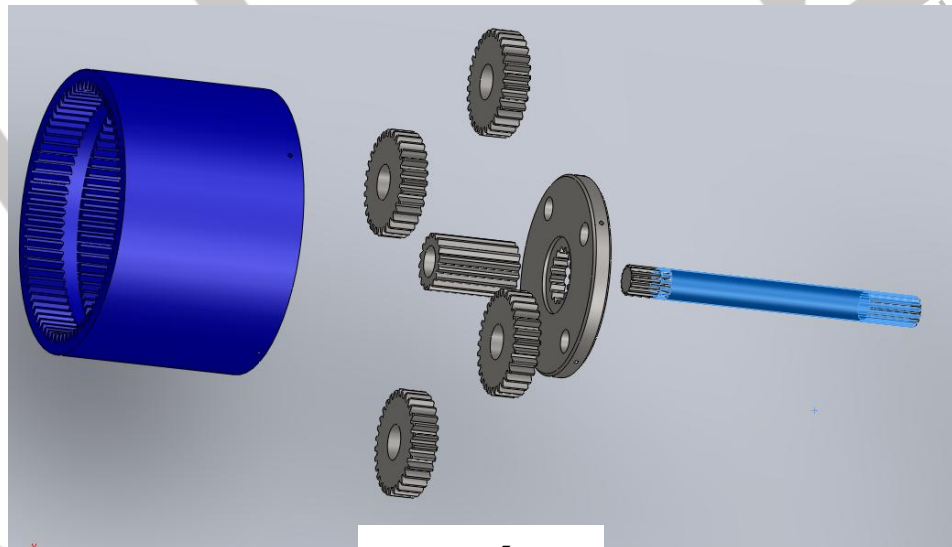
Case #1 – Guincho



Case #1 - Guincho

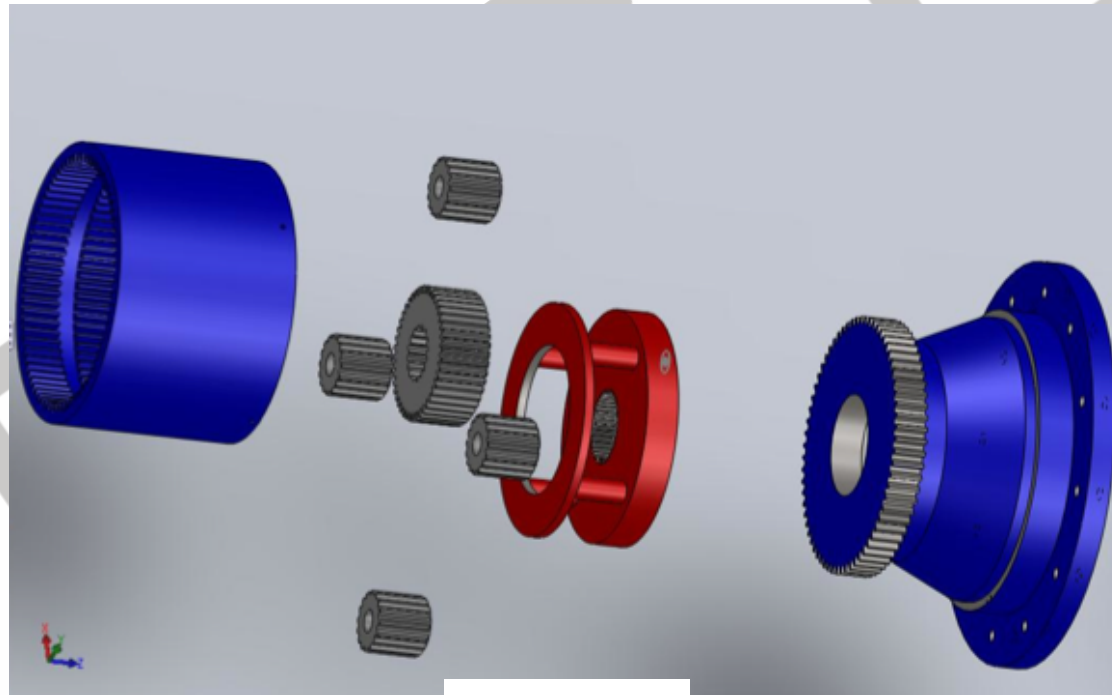


1º Estágio



2º Estágio

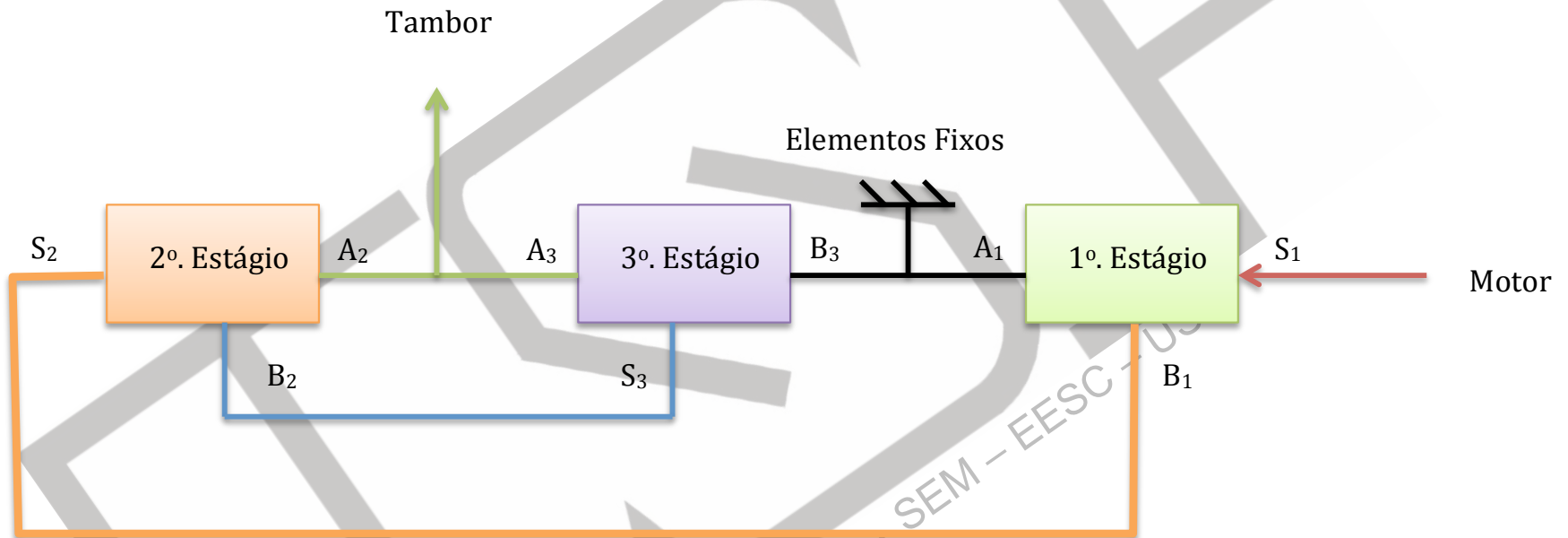
Case #1 - Guincho



3º Estágio

Prof. Dr. Marcei

Case #1 - Guincho



Case #1 - Guincho

Equações:

Equação Geral para cada Estágio:

Onde:

b : razão básica do Planetário e Z : número de dentes da engrenagem

Assim, para cada Estágio, temos:

$$b_1 = -15/57 = -0,263158$$

$$b_2 = -16/76 = -0,210526$$

$$b_3 = -44/76 = -0,578947$$

Com relação às velocidades em cada estágio, temos:

$$\omega_{A1} + (b_1 - 1) \cdot \omega_{B1} - b_1 \omega_{s1} = 0$$

$$\omega_{A2} + (b_2 - 1) \cdot \omega_{B2} - b_2 \omega_{s2} = 0$$

$$\omega_{A3} + (b_3 - 1) \cdot \omega_{B3} - b_3 \omega_{s3} = 0$$

$$\omega_A + (b - 1) \cdot \omega_B - b \cdot \omega_S = 0$$

$$b = \pm \frac{\prod Z_{motoras}}{\prod Z_{movidas}} = -\frac{Z_S \cdot Z_P}{Z_P \cdot Z_A} = -\frac{Z_S}{Z_A}$$

Case #1 - Guincho

Onde:

ω_{s1} = Rotação do Motor (ω_{motor})

ω_{B1} = ω_{s2}

$\omega_{A1} = \omega_{B3} = 0$ (elementos fixos)

$\omega_{A2} = \omega_{A3}$ = Rotação do Tambor (ω_{Tambor})

$\omega_{S3} = \omega_{B2}$

Assim:

$$0 + (b_1 - 1) \omega_{B1} - b_1 \omega_{motor} = 0$$

$$\omega_{Tambor} + (b_2 - 1) \omega_{B2} - b_2 \omega_{B1} = 0$$

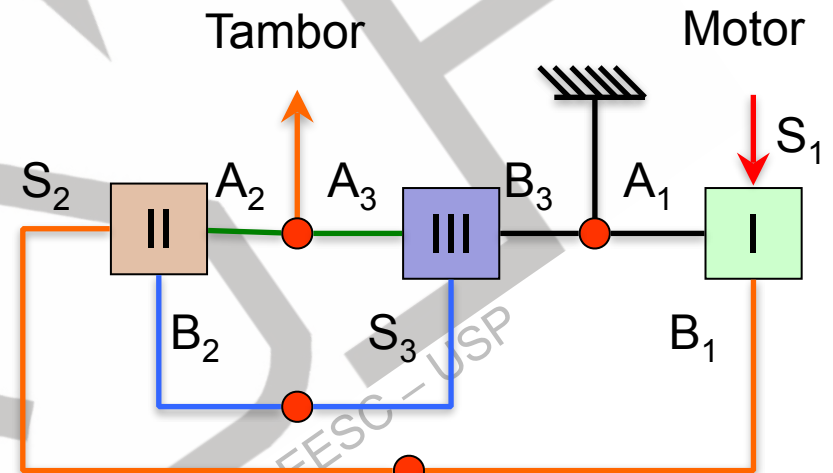
$$\omega_{Tambor} + (b_3 - 1) \cdot 0 - b_3 \omega_{B2} = 0$$

Simplificando (elementos conhecidos em verde!):

$$(b_1 - 1) \omega_{B1} - b_1 \omega_{motor} = 0$$

$$\omega_{Tambor} + (b_2 - 1) \omega_{B2} - b_2 \omega_{B1} = 0$$

$$\omega_{Tambor} - b_3 \omega_{B2} = 0$$



Case #1 - Guincho

$$(b_1 - 1) \omega_{B1} - b_1 \omega_{motor} = 0$$

$$\omega_{Tambor} + (b_2 - 1) \omega_{B2} - b_2 \omega_{B1} = 0$$

$$\omega_{Tambor} = b_3 \omega_{B2}$$

$$\omega_{Tambor} / b_3 = \omega_{B2}$$

$$(b_1 - 1) \omega_{B1} - b_1 \omega_{motor} = 0$$

$$\omega_{Tambor} + (b_2 - 1) \cdot \omega_{Tambor} / b_3 - b_2 \omega_{B1} = 0$$

$$\omega_{Tambor} + (b_2 - 1) \cdot \omega_{Tambor} / b_3 = \omega_{B1}$$

$$(b_1 - 1) \cdot [\omega_{Tambor} + (b_2 - 1) \cdot \omega_{Tambor} / b_3] - b_1 \omega_{motor} = 0$$

b_2

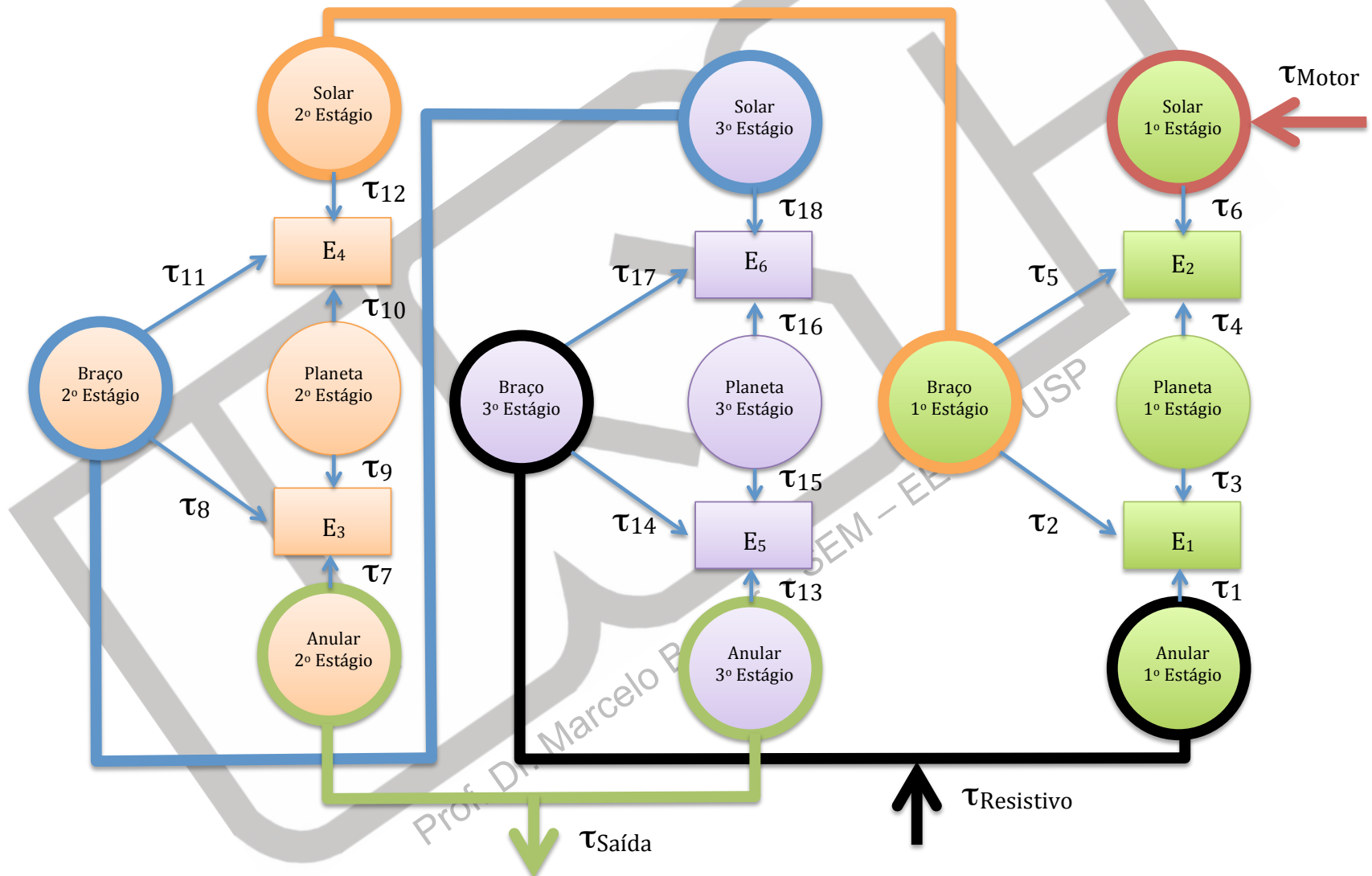
Isolando os termos ω_{Tambor} e ω_{Motor} e trabalhando as equações, obtemos a equação final:

$$\frac{\omega_{Motor}}{\omega_{Tambor}} = \frac{(b_1 - 1) \cdot (b_3 + b_2 - 1)}{b_1 b_2 b_3}$$

Substituindo os valores de b_i , temos:

$$\frac{\omega_{Motor}}{\omega_{Tambor}} = -70,47283 \text{ (reductor inversor)}$$

Case #1 - Guincho



Case #1 - Guincho

Para cada elemento do redutor, nos 3 estágios, temos:

$$S_1: \quad -\tau_{\text{Motor}} + \tau_6 = 0$$

$$P_1: \quad \tau_4 + \tau_3 = 0$$

$$A_1B_3: \quad \tau_1 - \tau_{\text{Resistivo}} + \tau_{17} + \tau_{14} = 0$$

$$B_1S_2: \quad \tau_5 + \tau_2 + \tau_{12} = 0$$

$$P_2: \quad \tau_{10} + \tau_9 = 0$$

$$A_2A_3: \quad \tau_7 + \tau_{13} + \tau_{\text{Saída}} = 0$$

$$B_2S_3: \quad \tau_{11} + \tau_8 + \tau_{18} = 0$$

$$P_3: \quad \tau_{16} + \tau_{15} = 0$$

...

Bibliografia Recomendada

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- Amaral, Danilo, 2000, “Metodologia, sistematização e seleção de projetos de transmissões epicicloidais com um e dois TEPs ligados” Tese de Doutorado – UNICAMP – FEM – DPM
- Hsieh, H-I, 1997, “Enumeration and Selection of Clutching Sequences Associated with Epicyclic-type Transmission Mechanisms”, PhD Thesis, University of Maryland, USA.
- Notas de Aula