Universidade de São Paulo

Escola Superior de Agricultura "Luiz de Queiroz"

Departamento de Ciências Exatas

LCE 0220 - Cálculo II

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Lista de Exercício: Integral por Frações Parciais

1. Nos exercícios a seguir, calcular a integral indefinida:

(a) $\int \dfrac{2x^3}{x^2+x}\, dx$

(b) $\int \dfrac{2x+1}{2x^2+3x-2}\, dx$

(c) $\int \dfrac{x-1}{x^3+x^2-4x-4}\, dx$

(d) $\int \dfrac{3x^2}{2x^3-x^2-2x+1}\, dx$

(e) $\int \dfrac{x^2+5x+4}{x^2-2x+1}\, dx$

(f) $\int \dfrac{x-1}{(x-2)^2(x-3)^2}\, dx$

(g) $\int \dfrac{(x^2+1)}{x^4-7x^3+18x^2-20x+8}\, dx$

(h) $\int \dfrac{1}{x^3-4x^2}\, dx$

(i) $\int \dfrac{x^3+2x^2+4}{2x^2+2}\, dx$

(j) $\int \dfrac{5}{x^3+4x}\, dx$

(k) $\int \dfrac{3x-1}{x^2-x+1}\, dx$

(l) $\int \dfrac{1}{x^3+8}\, dx$

(m) $\int \dfrac{x-1}{(x^2+2x+3)^2}\, dx$

(n) $\int \dfrac{1}{x(x^2-x+1)^2}\, dx$

(o) $\int \dfrac{4x^4}{x^4-x^3-6x^2+4x+8}\, dx$

(p) $\int \dfrac{x^2}{3x^2-\frac{1}{2}x-\frac{1}{2}}\, dx$

(q) $\int \dfrac{1}{x^3+9x}\, dx$

(r) $\int \dfrac{1}{(x^2+1)(x^2+4)}\, dx$

(s) $\int \dfrac{x^3+x^2+2x+1}{x^3-1}\, dx$

(t) $\int \dfrac{x^3}{(x^2+2)^2}\, dx$

(u) $\int \dfrac{1}{x^4-3x^3+3x^2-x}\, dx$

(v) $\int \dfrac{x}{(x-1)^2(x+1)^2}\, dx$

(w) $\int \dfrac{x^2+2x-1}{(x-1)^2(x^2+1)}\, dx$

2. Calcular a área da região sob o gráfico de $y=\dfrac{1}{x^2+2x+5}$, de $x=-2$ até $x=2$.

Respostas:

1. Nos exercícios a seguir, calcular a integral indefinida:

(a) $x^2-2x+2\ln|x+1|+c$

(b) $\dfrac{2}{5}\ln|x-1/2|+\dfrac{3}{5}\ln|x+2|+c$

(c) $\dfrac{1}{12}\ln|x-2|+\dfrac{2}{3}\ln|x+1|-\dfrac{3}{4}\ln|x+2|+c$

(d) $\dfrac{3}{2}\ln|x-1|+\dfrac{1}{2}\ln|x+1|-\dfrac{1}{2}\ln|x-1/2|+c$

(e) $x+7\ln|x-1|-\dfrac{10}{x-1}+c$

(f) $3\ln|\dfrac{x-2}{x-3}|-\dfrac{1}{x-2}-\dfrac{2}{x-3}+c$

(g) $\ln\left(\dfrac{x-2}{x-1}\right)^2+\dfrac{1}{x-2}-\dfrac{5}{2(x-2)^2}+c$

(h) $\dfrac{1}{16}\ln|\dfrac{x-4}{x}|+\dfrac{1}{4x}+c$

(i) $\dfrac{x^2}{9} + x - \dfrac{1}{4}\ln(x^2+1) + \operatorname{arctg}(x) + c$

(j) $\dfrac{5}{4}\left[\ln|x| - \dfrac{1}{2}\ln(x^2+4)\right] + c$

(k) $\dfrac{3}{2}\ln|x^2-x+1| + \dfrac{1}{\sqrt{3}}\operatorname{arctg}\left(\dfrac{2x-1}{\sqrt{3}}\right) + c$

(l) $\dfrac{1}{12}\ln|x+2| - \dfrac{1}{24}\ln|x^2-2x+4| + \dfrac{1}{4\sqrt{3}}\operatorname{arctg}\left(\dfrac{x-1}{\sqrt{3}}\right) + c$

(m) $\dfrac{-x-2}{2(x^2+2x+3)} - \dfrac{1}{2\sqrt{2}}\operatorname{arctg}\left(\dfrac{x+1}{\sqrt{2}}\right) + c$

(n) $\ln|x| - \dfrac{1}{2}\ln|x^2-x+1| + \dfrac{5\sqrt{3}}{9}\operatorname{arctg}\left(\dfrac{2x-1}{\sqrt{3}}\right) + \dfrac{x+1}{3(x^2-x+1)} + c$

(o) $4x + \dfrac{4}{9}\ln|x+1| - 4\ln|x+2| + \dfrac{68}{9}\ln|x-2| - \dfrac{16}{3(x-2)} + c$

(p) $\dfrac{1}{3}x + \dfrac{1}{10}\ln|x-1/2| - \dfrac{2}{45}\ln|x+1/3| + c$

(q) $1/9\left[\ln|x| - \dfrac{1}{2}\ln(x^2+9)\right] + c$

(r) $\dfrac{1}{3}\operatorname{arctg}(x) - \dfrac{1}{6}\operatorname{arctg}(x/2) + c$

(s) $x + \dfrac{5}{3}\ln|x-1| - \dfrac{1}{3}\ln|x^2+x+1| + c$

(t) $\dfrac{1}{2}\ln(x^2+2) + \dfrac{1}{x^2+2} + c$

(u) $\ln\left|\dfrac{x-1}{x}\right| + \dfrac{1}{x-1} - \dfrac{1}{2(x-1)^2} + c$

(v) $\dfrac{1}{4}\left(\dfrac{1}{x+1} - \dfrac{1}{x-1}\right) + c$

(w) $\ln|x-1| - \dfrac{1}{x-1} - \dfrac{1}{2}\ln(x^2+1) - \operatorname{arctg}(x) + c$

2. $\dfrac{1}{2}\left[\operatorname{arctg}(3/2) - \operatorname{arctg}(-1/2)\right] u.a.$

FLEMMING, D.M.; GONÇALVES, M.B. **Cálculo A: Funções, limites, derivação e integração**. 6ª ed. São Paulo. Pearson, 2012. 448p.