PEF 2309 Fundamentos de Mecânica das Estruturas

Exercícios resolvidos extraídos de **Mechanics of Materials**, 4th edition, **GERE**, **J.M.** e **Timoshenko,** S.P.,PWS Publishing Company, 1997, Boston, USA, p.408-410, 580-583.

Example 8-4

The rotor shaft of a helicopter drives the rotor blades that provide the lifting force to support the helicopter in the air (Fig. 8-24a). As a consequence, the shaft is subjected to a combination of torsion and axial loading (Fig. 8-24b).

For a 50-mm diameter shaft transmitting a torque T = 2,4 kN.m and a tensile force P = 125kN, determine the maximum shear stress in the shaft.

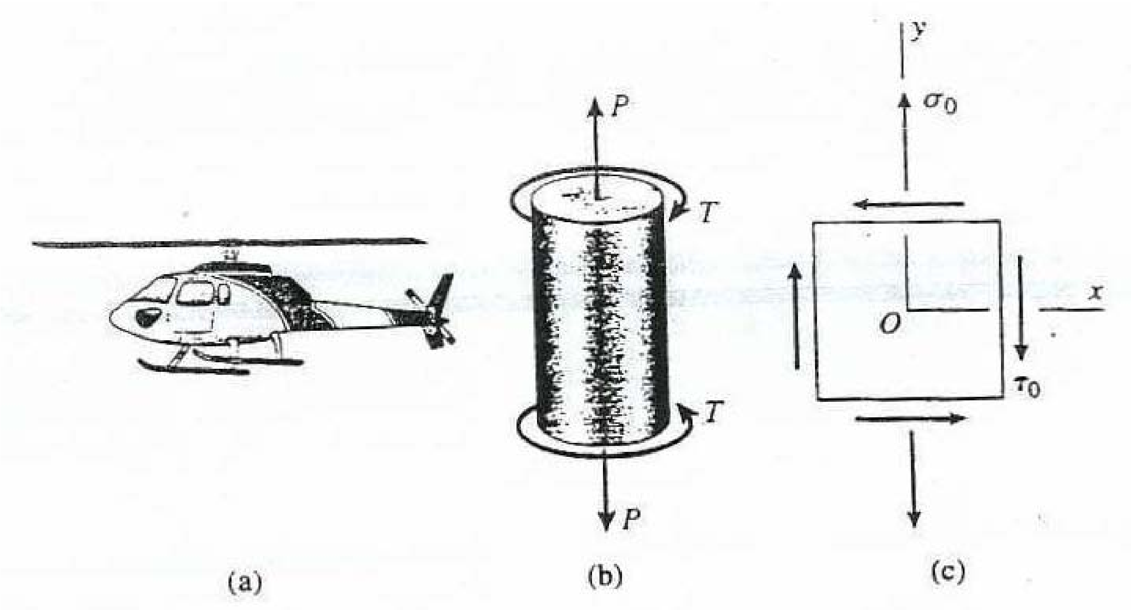


Fig. 8-24 Example 8-4. Rotor shaft of a helicopter (combined torsion and axial force).

Solution The stress in the rotor shaft are produced by the combined action of the axial force P and the torque T (Fig. 8-24b). Therefore, the stress at any point on the surface of the shaft consist of a tensile stress σ_0 and shear stresses τ_0 as shown on the stress element of Fig. 8-24c. Note that the y axis is parallel to the longitudinal axis of the shaft.

The tensile stress σ_0 equals the axial force divided by the cross-sectional area:

$$\sigma_0 = \frac{P}{A} = \frac{4P}{\pi d^2} = \frac{4(125\,kN)}{\pi(50\,mm)^2} = 63.66\,MPa$$

The shear stress τ_0 is obtained from the torsion formula (Eq. 3-11 of Section 3.3):

$$\tau_0 = \frac{Tr}{I_p} = \frac{16T}{\pi d^3} = \frac{16(2.4\,kN.m)}{\pi(50\,mm)^3} = 97.78\,MPa$$

The stress σ_0 and τ_0 act directly on cross sections of the shaft.

Knowing the stress σ_0 and τ_0, we can now obtain the principal stresses and maximum shear stresses by the methods described in section 7.3. The principal stresses are obtained from Eq. (7-17):

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (d)$$

Substituting $\sigma_x = 0$, $\sigma_y = 63.66$ MPa, and $\tau_{xy} = -\tau_0 = -97.78$ MPa, we get

$$\sigma_{1,2} = 32MPa \pm 103MPa$$

or $\sigma_1 = 135MPa \qquad \sigma_2 = -71MPa$

These are the maximum tensile and compressive stress in the rotor shaft.

The maximum in-plane shear stresses (Eq. 7-5) are

$$\tau_{max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (e)$$

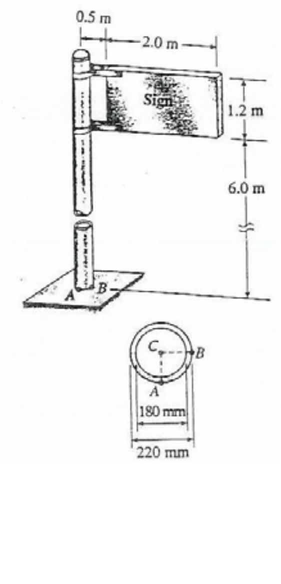
This term was evaluated above, so we see immediately that

$$\tau_{max} = 103MPa$$

Because the principal stresses σ_1 and σ_2 have opposite signs, the maximum in-plane shear stresses are larger than the maximum out-of-plane shear stresses (see Eqs. 7-28a, b, and c and the accompanying discussion). Therefore, the maximum shear stress in the shaft is 103 MPa.

Example 8-6

A sign of dimensions 2.0 m x 1.2 m is supported by a hollow circular pole having outer diameter 220 mm and inner diameter 180 mm (Fig. 8-20). The sign is offset 0.5 m from the centerline of the pole and its lower edge is 6.0 m above the ground.



Determine the principal stresses and maximum shear stresses at points A and B at the bases of the pole due to a wind pressure of 2.0 kPa against the sign.

Fig. 8-26 Example 8-6. Wind pressure against a sign (combined bending, torsion, and shear of the pole).

Solution *Stress resultants*. The wind pressure against the sign produces a resultant force W that acts at the midpoint of yhe sign (Fig. 8-27a) and is equal to the pressure *p* times the area A over which it acts:

$$W = pA = (2.0\,kPa)(2.0\,m \times 1.2\,m) = 4.8\,kN$$

The line of action of this force is at height *h* = 6.6 m above the ground and at distance *b* = 1.5 m from the centerline of the pole.

The wind force acting on the sign is statically equivalent to a lateral force W and a torque T acting on the pole (Fig. 8-27b). The torque is equal to the force W times the distance *b*:

$$T = Wb = (4.8\,kN)(1.5\,m) = 7.2\,kN \cdot m$$

The stress reultants at the base of the pole (Fig. 8-27c) consist of a bending moment M, a torque T, and a shear force V. Their magnetudesn are

$$M = Wh = (4.8\,kN)(6.6\,m) = 31.68\,kN \cdot m$$

$$T = 7.2\,kN \cdot m \qquad V = W = 4.8\,kN$$

Examination of these stress resultants shows that maximum bending stresses occur at point A and maximum shear stresses at point B. Therefore, A and B are critical points where the stresses should be determined. (Another critical points is diametrically opposite point A, as explained im the *Note* below).

Stresses at points A and B. The bending moment M produces a tensile stress σ_A at point A (Fig. 8-27d) but no stress at point B (which is located on the neutral axis). The stress σ_A is obtained from the formula:

$$\sigma_A = \frac{M(d_2/2)}{I}$$

in which d_2 is the outer diameter (220 mm) and *I* is moment of inertia of the cross section. The moment of inertia is

$$I = \frac{\pi}{64}\left(d_2^4 - d_1^4\right) = \frac{\pi}{64}\left[(220\,mm)^4 - (180\,mm)^4\right] = 63.46 \times 10^{-6}\,m^4$$

in which d_1 is the inner diameter. Therefore,

$$\sigma_A = \frac{Md_2}{2I} = \frac{(31.68\,kN \cdot m)(220\,mm)}{2(63.46 \times 10^{-6}\,m^4)} = 54.91\,MPa$$

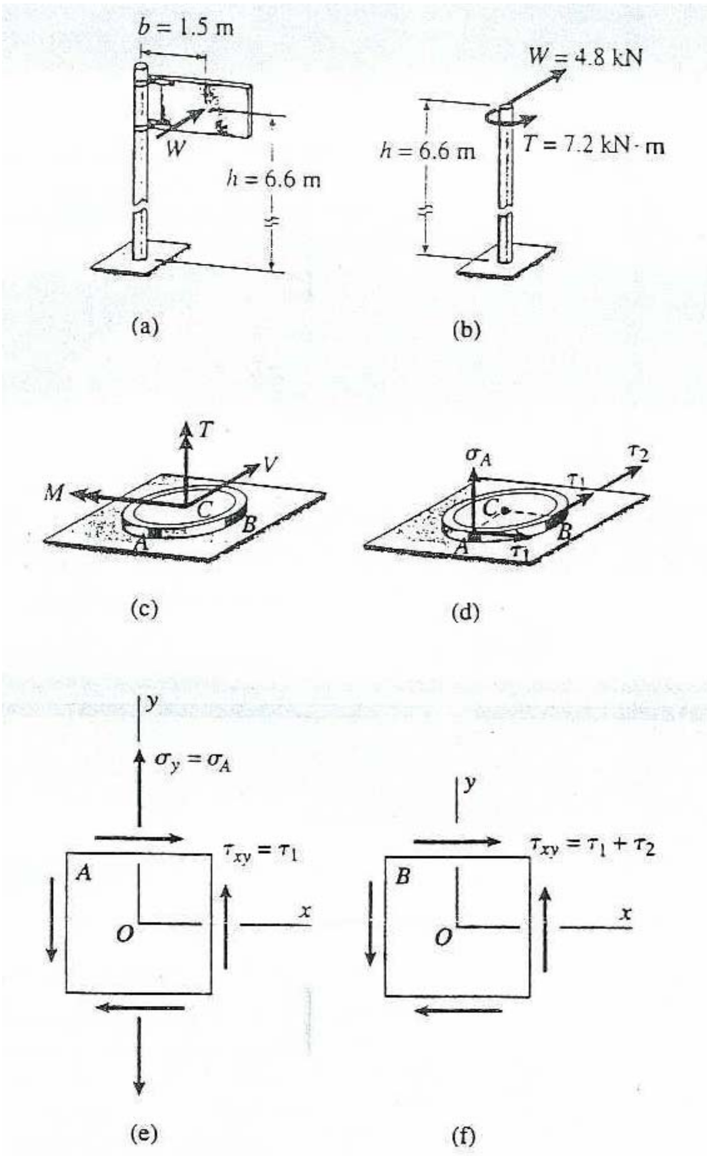


Fig. 8-27 Solution to Example 8-6.

The torque *T* produces shear stresses τ_1 at points A and B (Fig. 8-27d). We can calculate these stresses from the torsion formula:

$$\tau_1 = \frac{T(d_2/2)}{I_p}$$

in which I_p is the polar moment of inertia:

$$I_p = \frac{\pi}{32}\left(d_2^4 - d_1^4\right) = 2I = 126.92 \times 10^{-6}\, m^4$$

Thus,

$$\tau_1 = \frac{Td_2}{2I_p} = \frac{(7.2\,kN\cdot m)(220\,mm)}{2(126.92\times10^{-6}\,m^4)} = 6.24\,MPa$$

Finally, we calculate the shear stress at point A and B due to the shear force V. The shear stress at point A is zero, and the shear stress at point B (denoted τ_2 in Fig. 8-27d) is obtained from the shear formula for a circular tube (Eq. 5-44 of Section 5.9):

$$\tau_2 = \frac{4V}{3A} = \left(\frac{r_2^2 + r_2 r_1 + r_1^2}{r_2^2 + r_1^2}\right) \qquad (j)$$

in which r_2 and r_1 are the outer and innewhich r2 and r1 are the outer and inne radii, respectively, and A is the cross-sectional area:

$$r_2 = \frac{d_2}{2} = 110\,mm \qquad r_1 = \frac{d_1}{2} = 90\,mm$$
$$A = \pi(r_2^2 - r_1^2) = 12{,}570\,mm^2$$

Substituting numerical values into Eq. (j), we obtain

$$\tau_2 = 0.76\,MPa$$

All stresses acting at points A and B have now been calculed.

Stress elements. The next step is to show these stresses on stress elements (Figs. 8-27e and f). For both elements, the y axis is parallel to the longitudinal axis of the pole and the x axis is horizontal. At point A the stresses acting on the element are

$$\sigma_x = 0 \qquad \sigma_y = \sigma_A = 54.91MPa \qquad \tau_{xy} = \tau_1 = 6.24MPa$$

At point B the stresses are

$$\sigma_x = \sigma_y = 0 \qquad \tau_{xy} = \tau_1 + \tau_2 = 6.24\,MPa + 0.76\,MPa = 7.00\,MPa$$

Since there are no normal stresses acting on the element, point B is in pure shear.

Now that all stresses acting on the stress elements (Figs. 8-27e and f) are known, we can use the equations given in Section 7.3 to determine the principal stresses and maximum shear stresses.

Principal stresses and maximum shear stresses at point A. The principal stresses are obtained from Eq. (7-17), which is repeated here:

$$\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2} \qquad (k)$$

Substituting σ_x = 0, σ_y = 54.91 MPa, and τ_{xy} = 6.24 MPa, we get

$$\sigma_{1,2} = 27.5\,MPa \pm 28.2\,MPa$$

or $\sigma_1 = 55.7\,MPa$ $\sigma_2 = -0.7\,MPa$

The maximum in-plane shear stresses may be obtained from Eq. (7-25)

$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$

This term was evaluated above, so we see immediately that

$$\tau_{\max} = 28.2\,MPa$$