

Computational Fluid and Solid Mechanics

Miguel Luiz Bucelem
Klaus-Jürgen Bathe

**The Mechanics of
Solids and Structures –
Hierarchical Modeling and
the Finite Element Solution**

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Miguel Luiz Bucalem · Klaus-Jürgen Bathe

The Mechanics of Solids and Structures – Hierarchical Modeling and the Finite Element Solution

Prof. Miguel Luiz Bucalem
Escola Politécnica da Universidade
de São Paulo
São Paulo, Brazil
mlbucale@usp.br

Prof. Klaus-Jürgen Bathe
Massachusetts Institute of Technology
Cambridge, MA
USA
kjb@mit.edu

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Preface

Our main objective in this book is to provide a rational, structured and modern framework for the modeling and analysis of engineering structures.

Although engineering structures have been modeled and analyzed for centuries, the mathematical models that could actually be solved were relatively simple. This situation has dramatically changed during the last decades. Today, with powerful computers and reliable finite element procedures widely available, very complex models of solids and structures can be solved, and consequently the range and complexity of analyses has drastically increased.

In any finite element analysis, the first step for an analyst is to choose an *appropriate mathematical model*, and the second step is to solve that model using *finite element procedures*. In almost all analyses, the first step is most important and also most difficult. In order to choose an appropriate model, the analyst must be familiar with the basic mathematical models that are available, and in particular know the hierarchy of such models. Only if the analyst is deeply familiar with the various mathematical models available, their hierarchy, and reliable finite element procedures, can the analyst choose the most effective model, perform an efficient analysis, and properly interpret the analysis results.

Many books on finite element methods have been published; however, our aim in this book is broader. We aim to present in one treatise – both – the basic mathematical models of solid and structural mechanics and modern reliable finite element procedures for the solution of these models. The book can be used for teaching, in a modern way, structural and solid mechanics, finite element methods, and for self-study – from elementary to quite advanced material.

We draw in this treatise heavily from the material published in the books *Theory of Elasticity*, by S. P. Timoshenko and J. N. Goodier (1970), and *Theory of Plates and Shells*, by S. P. Timoshenko and S. Woinowsky-Krieger (1959), regarding the mechanics of solids and structures, and from the material in the book *Finite Element Procedures*, by K. J. Bathe (1996), regarding finite element formulations and solution techniques. In essence, we try to synthesize the presentation of the models and methods of classical mechanics with the procedures of finite element analysis, add new insights, and show

how to solve the classical general models of elasticity in a modern – effective and reliable – manner.

Towards that aim, we first develop from elementary concepts the basic mathematical models of solids and structures, then we present modern finite element methods for solution, and finally we give examples of applications using the finite element program ADINA. Emphasis is given to the hierarchical nature of the mathematical models, from simple to complex, and on choosing the simplest reliable and effective model for analysis, see Chapter 1. The process of hierarchical modeling and finite element solution, with the benefits reached, is finally illustrated in the examples of Chapter 7. These broadly indicate how we recommend modern analysis to be conducted.

To perform an effective analysis is an art. In many cases, the analyst tries to look into the future by asking how the design of a structure will perform; in other cases, the analyst tries to understand a phenomenon of nature to be able to predict when and how this phenomenon will occur, how it can be affected, and possibly be remedied. All these analyses are based on present knowledge and the analyst endeavors to predict the future by means of computational modeling and simulations. This clearly cannot be an easy task.

Since any analysis depends on the knowledge of the analyst, performing in the *art of analysis* is most stimulating and requires constant learning. We hope that this book will be valuable in this learning process.

While we focus on the linear elastic static analysis of solids and structures, with only a short introduction to nonlinear analysis in Chapter 8, the general concepts that we present regarding modeling and analysis are equally applicable in much more complex cases, including fluids and multi-physics phenomena. Hence, this book not only gives valuable information regarding the linear analysis of solids, but also demonstrates universal concepts of analysis that are generally applicable.

The writing of this book required a very large effort, and we would like to thank all of those who have supported us in this endeavor.

Miguel Luiz Bucalem is very thankful to the Department of Structural and Geotechnical Engineering of the University of São Paulo. He was very fortunate to first learn structural engineering from great engineers and professors, specifically Decio Leal de Zagottis and Henrique Lindenberg Neto. Decio Leal de Zagottis was his first mentor whose brilliant teaching and encouragement attracted him to the field. Since he joined the Faculty of the Department in 1985, he has had an excellent environment for his teaching and research and for writing this book. The interactions with his colleagues Carlos Eduardo Nigro Mazzilli, João Cyro André and Sergio Cifu in teaching subjects related to this book were very rich and helped him shape his thoughts about some of the contents in this book. He is also very grateful to have been the doctoral student of Klaus-Jürgen Bathe at M.I.T. Finally, he would like to thank his students who have been a source of inspiration

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Klaus-Jürgen Bathe is very thankful to the Department of Mechanical Engineering, M.I.T. for having provided to him an excellent environment for his teaching, research and scholarly writing. He has been fortunate to work with many brilliant students and colleagues, including Miguel Luiz Bucalem, who have much contributed to his research and teaching. The idea to write this book was born when he was asked to teach the undergraduate course in solid mechanics and he interacted with Miguel. It became apparent that new, exciting and modern ways to teach mechanics and analysis are needed, and we believe that this book offers one such option. K. J. Bathe is also thankful to his students Daniel Payen and Seounghyun Ham for their help in the proof-reading of the manuscript, and would like to acknowledge that for his teaching and research, and work on this book, his involvement in ADINA R & D, Inc. has been very valuable.

Finally, we both would like to express our unbounded thanks to our wives Tamara and Zorka, and our children, for their encouragement and unconditional support over many years in all our work endeavors, including the writing of this book.

M. L. Bucalem and K. J. Bathe

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