

Exercise 5B: Single-slit diffraction

Equipment:

rulers

computer spreadsheet or computer program for adding sine waves

The following page is a diagram of a single slit and a screen onto which its diffraction pattern is projected. The class will make a numerical prediction of the intensity of the pattern at the different points on the screen. Each group will be responsible for calculating the intensity at one of the points. (Either 11 groups or six will work nicely – in the latter case, only points a, c, e, g, i, and k are used.) The idea is to break up the wavefront in the mouth of the slit into nine parts, each of which is assumed to radiate semicircular ripples as in Huygens' principle. The wavelength of the wave is 1 cm, and we assume for simplicity that each set of ripples has an amplitude of 1 unit when it reaches the screen.

1. For simplicity, let's imagine that we were only to use two sets of ripples rather than nine. You could measure the distance from each of the two points inside the slit to your point on the screen. Suppose the distances were both 25.0 cm. What would be the amplitude of the superimposed waves at this point on the screen?

Suppose one distance was 24.0 cm and the other was 25.0 cm. What would happen?

What if one was 24.0 cm and the other was 26.0 cm?

What if one was 24.5 cm and the other was 25.0 cm?

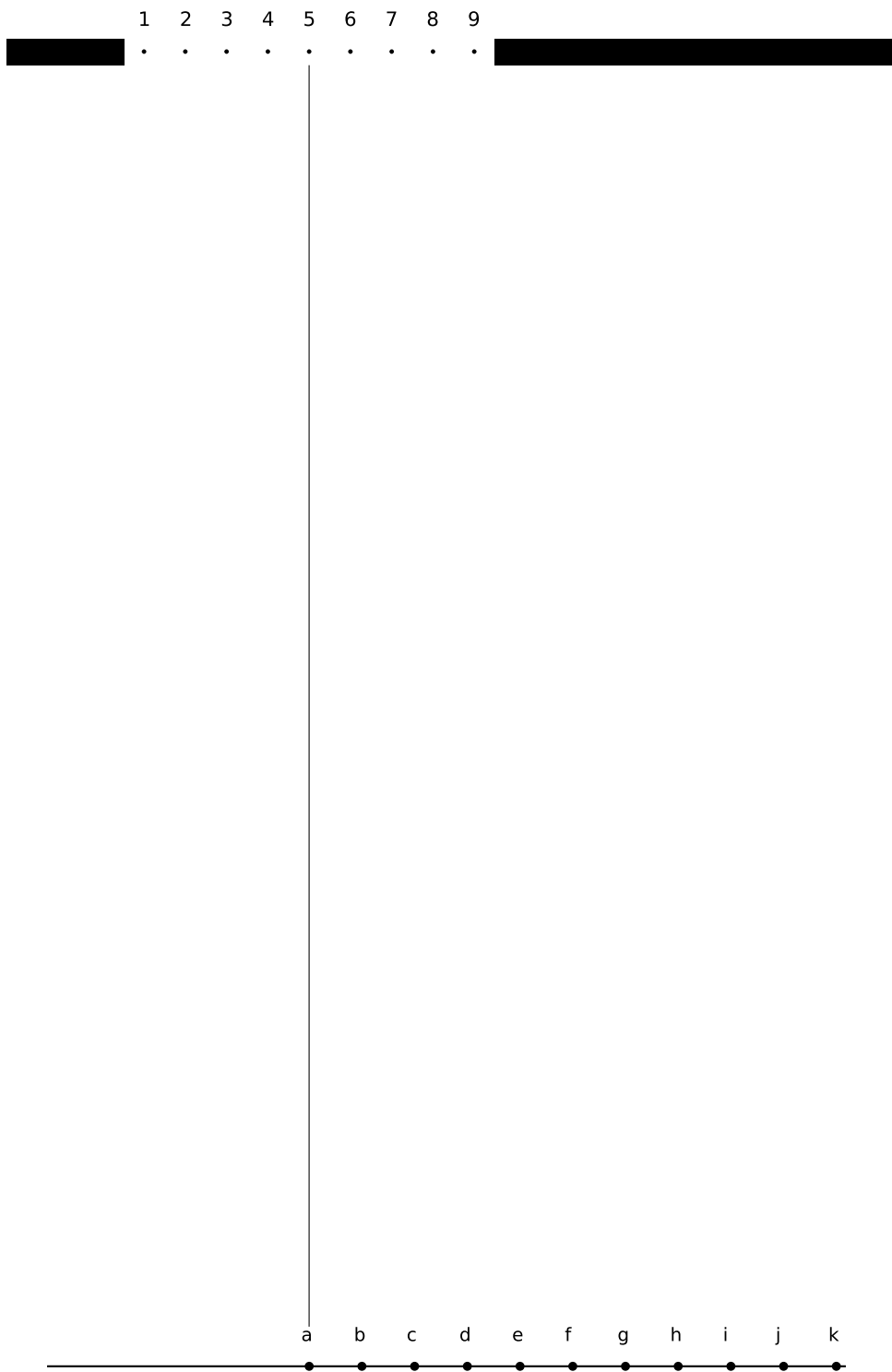
In general, what combinations of distances will lead to completely destructive and completely constructive interference?

Can you estimate the answer in the case where the distances are 24.7 and 25.0 cm?

2. Although it is possible to calculate mathematically the amplitude of the sine wave that results from superimposing two sine waves with an arbitrary phase difference between them, the algebra is rather laborious, and it become even more tedious when we have more than two waves to superimpose. Instead, one can simply use a computer spreadsheet or some other computer program to add up the sine waves numerically at a series of points covering one complete cycle. This is what we will actually do. You just need to enter the relevant data into the computer, then examine the results and pick off the amplitude from the resulting list of numbers.

3. Measure all nine distances to your group's point on the screen, and write them on the board - that way everyone can see everyone else's data, and the class can try to make sense of why the results came out the way they did. Determine the amplitude of the combined wave, and write it on the board as well.

The class will discuss why the results came out the way they did.



Exercise 5C: Diffraction of Light

Equipment:

slit patterns, lasers, straight-filament bulbs

station 1

You have a mask with a bunch of different double slits cut out of it. The values of w and d are as follows:

pattern A $w=0.04$ mm $d=.250$ mm

pattern B $w=0.04$ mm $d=.500$ mm

pattern C $w=0.08$ mm $d=.250$ mm

pattern D $w=0.08$ mm $d=.500$ mm

Predict how the patterns will look different, and test your prediction. The easiest way to get the laser to point at different sets of slits is to stick folded up pieces of paper in one side or the other of the holders.

station 2

This is just like station 1, but with single slits:

pattern A $w=0.02$ mm

pattern B $w=0.04$ mm

pattern C $w=0.08$ mm

pattern D $w=0.16$ mm

Predict what will happen, and test your predictions. If you have time, check the actual numerical ratios of the w values against the ratios of the sizes of the diffraction patterns

station 3

This is like station 1, but the only difference among the sets of slits is how many slits there are:

pattern A double slit

pattern B 3 slits

pattern C 4 slits

pattern D 5 slits

station 4

Hold the diffraction grating up to your eye, and look through it at the straight-filament light bulb. If you orient the grating correctly, you should be able to see the $m = 1$ and $m = -1$ diffraction patterns off the left and right. If you have it oriented the wrong way, they'll be above and below the bulb instead, which is inconvenient because the bulb's filament is vertical. Where is the $m = 0$ fringe? Can you see $m = 2$, etc.?

Station 5 has the same equipment as station 4. If you're assigned to station 5 first, you should actually do activity 4 first, because it's easier.

station 5

Use the transformer to increase and decrease the voltage across the bulb. This allows you to control the filament's temperature. Sketch graphs of intensity as a function of wavelength for various temperatures. The inability of the wave model of light to explain the mathematical shapes of these curves was historically one of the reasons for creating a new model, in which light is both a particle and a wave.

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Appendix 3: Hints and Solutions

Answers to Self-Checks

Answers to Self-Checks for Chapter 1

Page 23, self-check A: Only 1 is correct. If you draw the normal that bisects the solid ray, it also bisects the dashed ray.

Answers to Self-Checks for Chapter 2

Page 32, self-check A: You should have found from your ray diagram that an image is still formed, and it has simply moved down the same distance as the real face. However, this new image would only be visible from high up, and the person can no longer see his own image.

Page 35, self-check B: Increasing the distance from the face to the mirror has decreased the distance from the image to the mirror. This is the opposite of what happened with the virtual image.

Answers to Self-Checks for Chapter 2

Page 48, self-check A: At the top of the graph, d_i approaches infinity when d_o approaches f . Interpretation: the rays just barely converge to the right of the mirror.

On the far right, d_i approaches f as d_o approaches infinity; this is the definition of the focal length.

At the bottom, d_i approaches negative infinity when d_o approaches f from the other side. Interpretation: the rays don't quite converge on the right side of the mirror, so they appear to have come from a virtual image point very far to the left of the mirror.

Answers to Self-Checks for Chapter 4

Page 63, self-check A: (1) If n_1 and n_2 are equal, Snell's law becomes $\sin \theta_1 = \sin \theta_2$, which implies $\theta_1 = \theta_2$, since both angles are between 0 and 90° . The graph would be a straight line along the diagonal of the graph. (2) The graph is farthest from the diagonal when the angles are large, i.e., when the ray strikes the interface at a grazing angle.

Page 68, self-check B: (1) In 1, the rays cross the image, so it's real. In 2, the rays only appear to have come from the image point, so the image is virtual. (2) A ray is always closer to the normal in the medium with the higher index of refraction. The first left turn makes the ray closer to the normal, which is what should happen in glass. The second left turn makes the ray farther from the normal, and that's what should happen in air. (3) Take the topmost ray as an example. It will still take two right turns, but since it's entering the lens at a steeper angle, it will also leave at a steeper angle. Tracing backward to the image, the steeper lines will meet closer to the lens.

Answers to Self-Checks for Chapter 5

Page 81, self-check A: It would have to have a wavelength on the order of centimeters or

meters, the same distance scale as that of your body. These would be microwaves or radio waves. (This effect can easily be noticed when a person affects a TV's reception by standing near the antenna.) None of this contradicts the correspondence principle, which only states that the wave model must agree with the ray model when the ray model is applicable. The ray model is not applicable here because λ/d is on the order of 1.

Page 83, self-check B: At this point, both waves would have traveled nine and a half wavelengths. They would both be at a negative extreme, so there would be constructive interference.

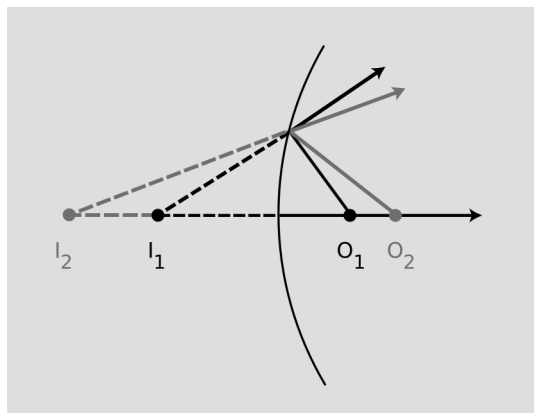
Page 87, self-check C: Judging by the distance from one bright wave crest to the next, the wavelength appears to be about $2/3$ or $3/4$ as great as the width of the slit.

Page 88, self-check D: Since the wavelengths of radio waves are thousands of times longer, diffraction causes the resolution of a radio telescope to be thousands of times worse, all other things being equal. (To compensate for the wavelength, it's desirable to make the telescope very large, as in figure y on page 88.)

Solutions to Selected Homework Problems

Solutions for Chapter 3

Page 56, problem 2: See the ray diagram below. Decreasing θ_o decreases θ_i , so the equation $\theta_f = \pm\theta_i + \pm\theta_o$ must have opposite signs on the right. Since θ_o is bigger than θ_i , the only way to get a positive θ_f is if the signs are $\theta_f = -\theta_i + \theta_o$. This gives $1/f = -1/d_i + 1/d_o$.



Page 57, problem 10: (a) The object distance is less than the focal length, so the image is virtual: because the object is so close, the cone of rays is diverging too strongly for the mirror to bring it back to a focus. (b) At an object distance of 30 cm, it's clearly going to be real. With the object distance of 20 cm, we're right at the crossing-point between real and virtual. For this object position, the reflected rays will be parallel. We could consider this to be an image at infinity. (c),(d) A diverging mirror can only make virtual images.

Solutions for Chapter 4

Page 74, problem 13: Since d_o is much greater than d_i , the lens-film distance d_i is essentially the same as f . (a) Splitting the triangle inside the camera into two right triangles, straightforward trigonometry gives

$$\theta = 2 \tan^{-1} \frac{w}{2f}$$

for the field of view. This comes out to be 39° and 64° for the two lenses. (b) For small angles, the tangent is approximately the same as the angle itself, provided we measure everything in radians. The equation above then simplifies to

$$\theta = \frac{w}{f}$$

The results for the two lenses are $.70 \text{ rad} = 40^\circ$, and $1.25 \text{ rad} = 72^\circ$. This is a decent approximation.

(c) With the 28-mm lens, which is closer to the film, the entire field of view we had with the 50-mm lens is now confined to a small part of the film. Using our small-angle approximation $\theta = w/f$, the amount of light contained within the same angular width θ is now striking a piece of the film whose linear dimensions are smaller by the ratio $28/50$. Area depends on the square of the linear dimensions, so all other things being equal, the film would now be overexposed by a factor of $(50/28)^2 = 3.2$. To compensate, we need to shorten the exposure by a factor of 3.2.

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Index

- aberration, 52
- absorption, 15
- angular magnification, 36
- Bohr
 - Niels, 80
- brightness of light, 17
- Bush, George, 59
- color, 65
- concave
 - defined, 39
- converging, 33
- convex
 - defined, 39
- correspondence principle, 80
- diffraction
 - defined, 78
 - double-slit, 82
 - fringe, 79
 - scaling of, 79
 - single-slit, 87
- diffraction grating, 87
- diffuse reflection, 16
- diopter, 47
- diverging, 39
- double-slit diffraction, 82
- Empedocles of Acragas, 12
- endoscope, 66
- evolution, 59
- eye
 - evolution of, 59
 - human, 61
- Fermat's principle, 25
- flatworm, 60
- focal angle, 45
- focal length, 46
- focal point, 46
- fringe
 - diffraction, 79
- Galileo, 13
- Hertz, Heinrich
 - Heinrich, 82
- Huygens' principle, 81
- images
 - formed by curved mirrors, 33
 - formed by plane mirrors, 32
 - location of, 44
 - of images, 35
 - real, 34
 - virtual, 32
- incoherent light, 79
- index of refraction
 - defined, 62
 - related to speed of light, 63
- Io, 14
- Jupiter, 14
- lensmaker's equation, 70
- light
 - absorption of, 15
 - brightness of, 17
 - particle model of, 18
 - ray model of, 18
 - speed of, 13
 - wave model of, 18
- magnification
 - angular, 36
 - by a converging mirror, 34
 - negative, 55
- Maxwell, James Clerk, 82
- mirror
 - converging, 44
- mollusc, 60
- Moses, 59
- nautilus, 60
- Newton, Isaac, 36, 81
- particle model of light, 18, 81
- Porro prism, 76
- prism
 - Porro, 76

Pythagoras, 12

ray diagrams, 19

ray model of light, 18, 81

reflection

- diffuse, 16
- specular, 22

refraction

- and color, 65
- defined, 60

repetition of diffracting objects, 86

retina, 35

reversibility, 23

Roemer, 14

single-slit

- diffraction, 87

Snell's law, 62

- derivation of, 64
- mechanical model of, 64

Squid, 60

telescope, 88

time reversal, 23

total internal reflection, 66

vision, 12

wave model of light, 18, 82

Wigner, Eugene, 43

Young, Thomas, 82

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Useful Data

Metric Prefixes

M-	mega-	10^6
k-	kilo-	10^3
m-	milli-	10^{-3}
μ - (Greek mu)	micro-	10^{-6}
n-	nano-	10^{-9}
p-	pico-	10^{-12}
f-	femto-	10^{-15}

(Centi-, 10^{-2} , is used only in the centimeter.)

Conversions

Nonmetric units in terms of metric ones:

1 inch	= 25.4 mm (by definition)
1 pound-force	= 4.5 newtons of force
(1 kg) $\cdot g$	= 2.2 pounds-force
1 scientific calorie	= 4.18 J
1 kcal	= 4.18×10^3 J
1 gallon	= 3.78×10^3 cm ³
1 horsepower	= 746 W

When speaking of food energy, the word “Calorie” is used to mean 1 kcal, i.e., 1000 calories. In writing, the capital C may be used to indicate 1 Calorie=1000 calories.

Relationships among U.S. units:

1 foot (ft)	= 12 inches
1 yard (yd)	= 3 feet
1 mile (mi)	= 5280 feet

Notation and Units

quantity	unit	symbol
distance	meter, m	$x, \Delta x$
time	second, s	$t, \Delta t$
mass	kilogram, kg	m
density	kg/m ³	ρ
velocity	m/s	v
acceleration	m/s ²	a
force	N = kg \cdot m/s ²	F
pressure	Pa=1 N/m ²	P
energy	J = kg \cdot m ² /s ²	E
power	W = 1 J/s	P
momentum	kg \cdot m/s	p
period	s	T
wavelength	m	λ
frequency	s ⁻¹ or Hz	f
focal length	m	f
magnification	unitless	M
index of refraction	unitless	n

Some Indices of Refraction

substance	index of refraction
vacuum	1 by definition
air	1.0003
water	1.3
glass	1.5 to 1.9
diamond	2.4

Note that indices of refraction, except in vacuum, depend on wavelength. These values are about right for the middle of the visible spectrum (yellow).

Fundamental Constants

gravitational constant	$G = 6.67 \times 10^{-11}$ N \cdot m ² /kg ²
Coulomb constant	$k = 8.99 \times 10^9$ N \cdot m ² /C ²
quantum of charge	$e = 1.60 \times 10^{-19}$ C
speed of light	$c = 3.00 \times 10^8$ m/s

Subatomic Particles

particle	mass (kg)	radius (fm)
electron	9.109×10^{-31}	$\lesssim 0.01$
proton	1.673×10^{-27}	~ 1.1
neutron	1.675×10^{-27}	~ 1.1

The radii of protons and neutrons can only be given approximately, since they have fuzzy surfaces. For comparison, a typical atom is about a million fm in radius.