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Prova 2 - Mecânica para Geociências (Gabarito)

1) (a) $T = T_{CM} + T_{Rot}$

$$\to T = \frac{mv^2}{2} + \frac{I\omega^2}{2}$$

Cilindro rola <u>sem deslizar</u> $\Rightarrow v = \omega R \Rightarrow \omega = \frac{v}{R}$

$$\therefore T = \frac{mv^2}{2} + \frac{Iv^2}{2R^2} \to \boxed{T = \frac{v^2}{2}\left(m + \frac{I}{R^2}\right)}$$

(b) $E_i = T = \frac{v^2}{2}\left(m + \frac{I}{R^2}\right) \to$ Energia inicial

Rolamento <u>sem deslizar</u> $\Rightarrow$ Conservação de energia mecânica

Toda energia cinética é convertida em energia potencial.

$E_f = mg\Delta y$; $\Delta y \to$ variação de altura do CM.

$$E_f = E_i \to mg\Delta y = \frac{v^2}{2}\left(m + \frac{I}{R^2}\right)$$

$\longrightarrow$

②

$$\rightarrow \Delta y = \frac{v^2}{2mg}\left(m + \frac{I}{R^2}\right) = \frac{v^2}{2g}\left(1 + \frac{I}{mR^2}\right)$$

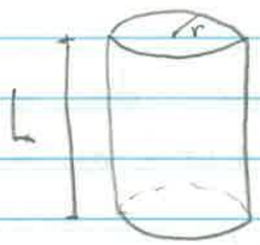
$$h = \Delta y + R \rightarrow h = R + \frac{v^2}{2g}\left(1 + \frac{I}{mR^2}\right)$$

(c) $h \propto I \rightarrow$ quanto maior I, maior será h

$$I = \int r^2\, dm \longrightarrow dm = 2\pi L\, \sigma(r)\, r\, dr$$

$L \rightarrow$ altura do cilindro

$\sigma(r) \rightarrow$ densidade (volumétrica) de massa do cilindro



$$\rightarrow I = 2\pi L \int_0^R \sigma(r)\, r^3\, dr$$

$$m = 2\pi L \int_0^R \sigma(r)\, r\, dr$$

Perfil 1: $\sigma_1(r) = K_1 r^\alpha$; K_1 constante positiva, $\alpha > 0$

$$\rightarrow I_1 = 2\pi L \int_0^R K_1 r^{(3+\alpha)}\, dr = 2\pi L K_1 \frac{R^{4+\alpha}}{4+\alpha}$$

$\longrightarrow$

$$m = 2\pi L \int_0^R k_1 r^{\alpha+1} dr = 2\pi L k_1 \frac{R^{\alpha+2}}{\alpha+2}$$

$$\rightarrow I_1 = \frac{m(\alpha+2)R^2}{\alpha+4} \rightarrow I_1 = \left(\frac{\alpha+2}{\alpha+4}\right) m R^2$$

Perfil 2: $\sigma_2(r) = k_2 r^{-\alpha}$; $\alpha > 1$; $\alpha \neq 4$; $\alpha \neq 2$

$$m = 2\pi L \int_0^R k_2 r^{1-\alpha} dr = 2\pi L k_2 \frac{R^{2-\alpha}}{2-\alpha}$$

$$I_2 = 2\pi L \int_0^R k_2 r^{3-\alpha} dr = 2\pi L k_2 \frac{R^{4-\alpha}}{4-\alpha}$$

$$\rightarrow I_2 = \left(\frac{2-\alpha}{4-\alpha}\right) m R^2$$

Perfil 3: $\sigma(r) =$ constante

$$I_3 = \frac{1}{2} m R^2$$

Perfil 4: $\sigma(r) = \begin{cases} 0, & \text{se } r < r_0 \\ k_4 r^{\alpha}, & \text{se } r \geq r_0 \end{cases}$; $\alpha > 1$

$$m = 2\pi L \int_{r_0}^R k_4 r^{\alpha+1} dr = 2\pi L k_4 \frac{1}{\alpha+2}\left(R^{\alpha+2} - r_0^{\alpha+2}\right)$$

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$$I_4 = 2\pi L \int_{r_0}^{R} K_4 \, r^{3+\alpha} \, dr = 2\pi L K_4 \cdot \frac{1}{4+\alpha} \left[R^{4+\alpha} - r_0^{4+\alpha} \right]$$

$$\rightarrow I_4 = \left(\frac{\alpha+2}{\alpha+4}\right) \left[\frac{1}{1 - \left(\frac{r_0}{R}\right)^{2+\alpha}} - \frac{1}{\left(\frac{R}{r_0}\right)^{4+\alpha} - \left(\frac{R}{r_0}\right)^{2}} \right] mR^2$$

Note que $\lim_{r_0 \to 0} I_4 = I_1$

$$I_4 > I_1 > I_3 > I_2$$

$\therefore$ Perfil 1 atinge altura maior e Perfil 2 atinge altura menor.

2) (a) Velocidade da massa antes do fio se romper: $v_A = \omega d$

Após o rompimento do fio a massa tem a mesma velocidade (em módulo) que possuía antes do rompimento do fio!

$\therefore v = \omega d$

(b) $p_c = 2 m v_c \longrightarrow$ momento do conjunto de 2 massas

$p_1 = m v \longrightarrow$ momento da massa que se soltou

$$p = 2 m v_c + m v$$

Antes do rompimento $\longrightarrow p = 0$

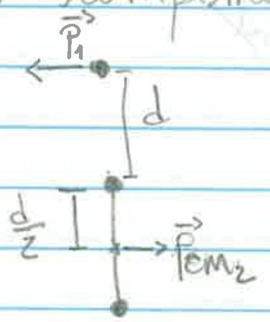
$$\Rightarrow 2m v_c = - m v \rightarrow v_c = -\frac{v}{2}$$

$$\therefore v_c = -\frac{\omega d}{2}$$

(c) Antes do rompimento

$$L = I\omega \; ; \; I = 2md^2 \Rightarrow \underline{L = 2md^2\omega}$$

Após o rompimento



$$\vec{p}_1 = m\omega d\,\hat{e}_x \; ; \; \vec{r}_1 = d\,\hat{e}_y$$

$$\vec{p}_{CM_2} = -\frac{m\omega d}{2} \; ; \; \vec{r}_{CM_2} = -\frac{d}{2}\,\hat{e}_y$$

$$\left.\begin{aligned} \vec{L}_1 &= \vec{p}_1 \times \vec{r}_1 = m\omega d^2\,\hat{e}_z \\ \vec{L}_{c_{ext}} &= \vec{p}_{CM_2} \times \vec{r}_2 = \frac{m\omega d^2}{4}\,\hat{e}_z \\ \vec{L}_{c_{int}} &= I_c \omega_c\,\hat{e}_z \end{aligned}\right\} L_c = L_{c_{ext}} + L_{c_{int}} \longrightarrow$$

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$L = L_1 + L_c$; $I_c = \frac{md^2}{2}$

$\Rightarrow L = m\omega d^2 + \frac{m\omega d^2}{4} + \frac{m\omega_o d^2}{2}$

Conservação do momento angular

$md^2\omega + \frac{md^2}{4}\omega + \frac{md^2}{2}\omega_c = 2md^2\omega$

$\rightarrow \frac{md^2}{2}\omega_c = \frac{md^2\omega}{2}$

$\Rightarrow \omega_c = \omega$

d) Energia cinética inicial:

$T_i = \frac{I\omega^2}{2}$; $I = 2md^2$

$\rightarrow T_i = md^2\omega^2$

Energia Cinética final

$T_f = T_1 + T_c$

$T_1 = \frac{mv^2}{2} = \frac{md^2\omega^2}{2}$

$\longrightarrow$

⑦

$$T_c = \frac{2mv_c^2}{2} + \frac{I_c\,\omega_c^2}{2} = \frac{md^2\omega^2}{4} + \frac{md^3\omega}{4}$$

$$\rightarrow T_c = \frac{md^2\omega}{2}$$

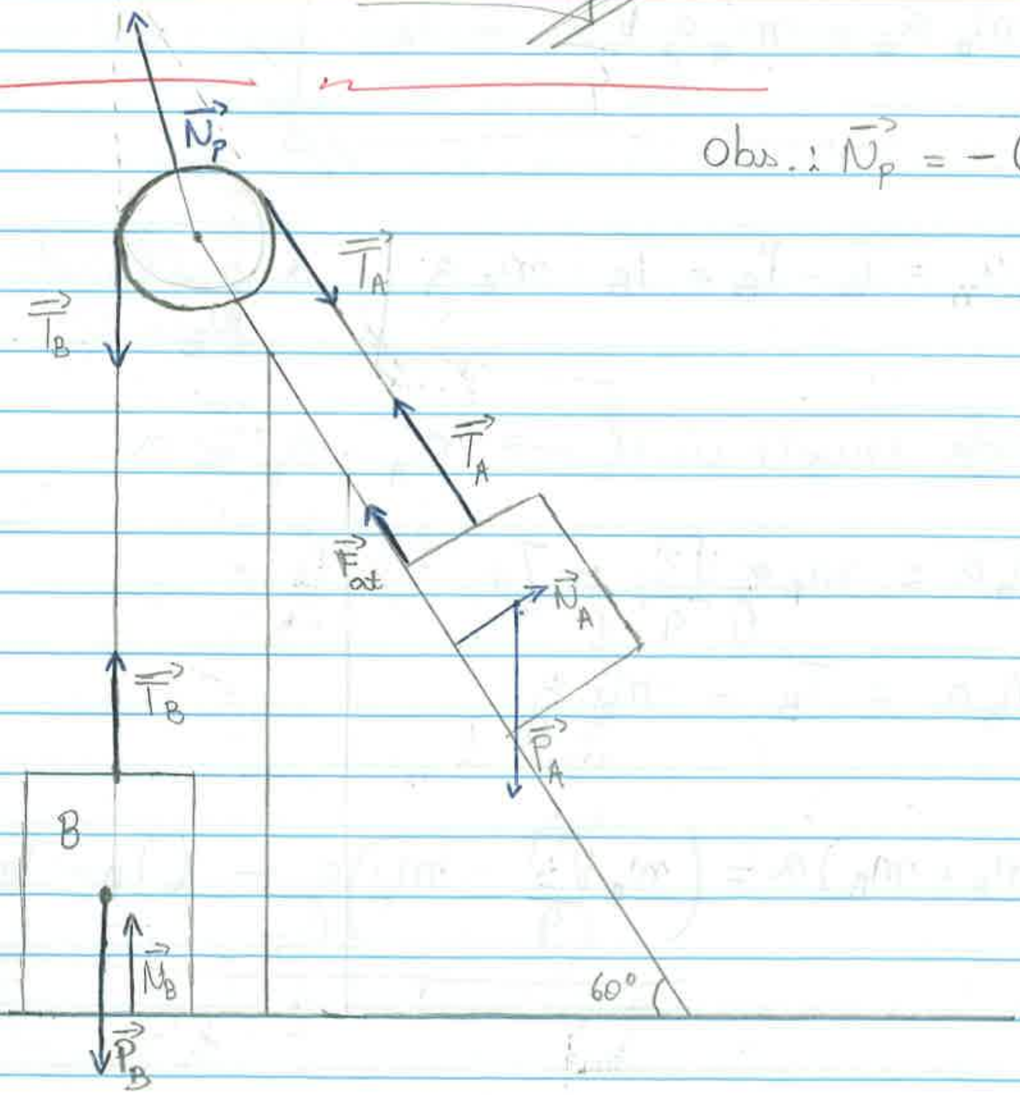
$$\Rightarrow \underline{T_f = md^2\omega}$$

$$\Delta T = T_f - T_i = md^2\omega - md^2\omega$$

$$\therefore \underline{\Delta T = 0}$$

3)

Obs.: $\vec{N_p} = -(\vec{T_A} + \vec{T_B})$



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(b) $m_A a_A = P_A \sin 60° - T_A - F_{at}$

$F_{at} = \mu N_A = \mu m_A g \cos 60° \rightarrow F_{at} = \frac{\sqrt{3}}{2} m_A g \cdot \frac{1}{2}$

$\rightarrow F_{at} = \frac{\sqrt{3}}{4} m_A g$

$\Rightarrow m_A a_A = m_A g \frac{\sqrt{3}}{2} - T_A - \frac{\sqrt{3}}{4} m_A g$

$\Rightarrow m_A a_A = m_A g \frac{\sqrt{3}}{4} - T_A$

$m_B a_B = T_B - P_B = T_B - m_B g$ admitindo $N_B = 0$

Corda inextensível $\rightarrow a_A = a_B = a$

$$\begin{cases} m_A a = m_A g \frac{\sqrt{3}}{4} - T_A \\ m_B a = T_B - m_B g \end{cases}$$

$\Rightarrow (m_A + m_B) a = \left(m_A \frac{\sqrt{3}}{4} - m_B\right) g - (T_A - T_B)$ (*)

• Torque produzido pelas Trações sobre a polia

$$\tau = (T_A - T_B) R \; ; \; \tau = \frac{dL}{dt} = \frac{d}{dt}(I\omega) = I \frac{d\omega}{dt}$$

$$a = R\frac{d\omega}{dt} \rightarrow \frac{d\omega}{dt} = \frac{a}{R} \rightarrow (T_A - T_B) R = I \frac{a}{R}$$

$$\rightarrow (T_A - T_B) = \frac{I a}{R^2}$$

$$I = \frac{MR^2}{2} \rightarrow (T_A - T_B) = \frac{Ma}{2} \qquad (**)$$

Substituindo (**) em (*)

$$(m_A + m_B) a = \left(m_A \frac{\sqrt{3}}{4} - m_B\right) g - \frac{Ma}{2}$$

$$\rightarrow \left(m_A + m_B + \frac{M}{2}\right) a = \left(\frac{\sqrt{3}}{4} m_A - m_B\right) g$$

$$\rightarrow a = \left(\frac{\sqrt{3}}{4} m_A - m_B\right) g \Big/ \left(m_A + m_B + \frac{M}{2}\right)$$

$a > 0 \rightarrow$ Sistema sai do repouso

$$\rightarrow \left(\frac{\sqrt{3}}{4} m_A - m_B\right) g \Big/ \left(m_A + m_B + \frac{M}{2}\right) > 0 \longrightarrow$$

spiral

$$\rightarrow \left(\frac{\sqrt{3}}{4} m_A - m_B\right) > 0 \Rightarrow m_A > m_B \cdot \frac{4}{\sqrt{3}}$$

$$\therefore \boxed{m_A > \frac{4\sqrt{3}}{3} m_B}$$

(c) Conforme desenvolvido no item (b)

$$\boxed{a = \left(\frac{\sqrt{3}}{4} m_A - m_B\right) g \Big/ \left(m_A + m_B + \frac{M}{2}\right)}$$

(d) $m_B a = T_B - m_B g$

$$\rightarrow T_B = m_B(a+g)$$

$$T_B = m_B \left\{ \frac{\left(\frac{\sqrt{3}}{4} m_A - m_B\right) g}{\left(m_A + m_B + \frac{M}{2}\right)} + g \right\}$$

$$\boxed{T_B = m_B g \left\{ 1 + \left[\left(\frac{\sqrt{3}}{4} m_A - m_B\right) \Big/ \left(m_A + m_B + \frac{M}{2}\right)\right] \right\}}$$

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Pelo item (b)

$$T_A - T_B = \frac{Ma}{2}$$

$$T_B = m_B(a+g) \longrightarrow T_A = \frac{Ma}{2} + m_B a + m_B g$$

$$T_A = \left(\frac{M}{2} + m_B\right) a + m_B g$$

$$\boxed{T_A = \left(\frac{M}{2} + m_B\right) \frac{\left(\frac{\sqrt{3}}{4} m_A - m_B\right) g}{\left(m_A + m_B + \frac{M}{2}\right)} + m_B g}$$