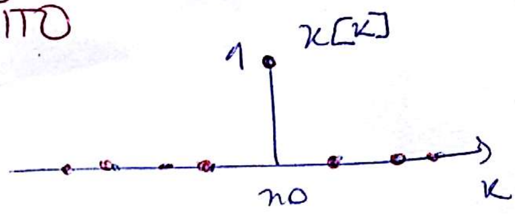
GABARITO

1)

a) $x[k] = \delta[k-n_0]$



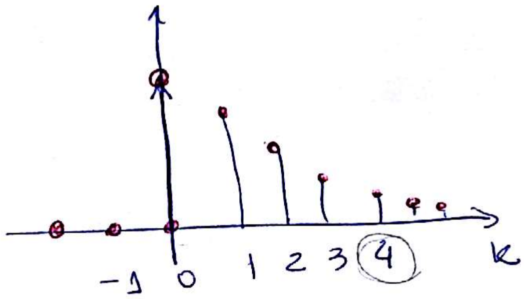
$$X(\Omega)=\sum_{k=-\infty}^{\infty} x[k]\,e^{-jk\Omega} = \sum_{k=-\infty}^{\infty}\delta[k-n_0]\,e^{-jk\Omega} = e^{-j\Omega n_0}$$

$$\boxed{\delta[k-n_0] \Longleftrightarrow e^{-j\Omega n_0}}$$

Par DTFT

b) $x[k]=\alpha^k \mu[k]$ $\quad |\alpha|<1$

(FILTRO)



$$X(\Omega)=\sum_{k=-\infty}^{\infty} x[k]\,e^{-jk\Omega} = \sum_{k=-\infty}^{\infty}\alpha^k \mu[k]\,e^{-j\Omega k}$$

$$= \sum_{k=0}^{\infty}\alpha^k e^{-j\Omega k} = \sum_{k=0}^{\infty}\left(\alpha e^{-j\Omega}\right)^k$$

Sabe-se que (fórmula para soma geométrica) $\boxed{\sum_{k=M}^{N}\beta^k = \frac{\beta^{N+1}-\beta^M}{\beta-1}}$

$$\therefore\ X(\Omega) = \frac{1}{1-\alpha e^{-j\Omega}}$$

Vamos analisar amplitude para se ter uma ideia do comportamento do filtro.

Amplitude $|X(\Omega)|$

Fase $\angle X(\omega)$

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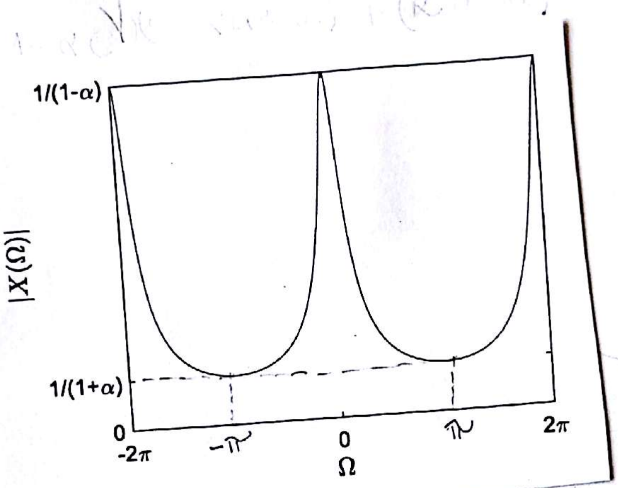
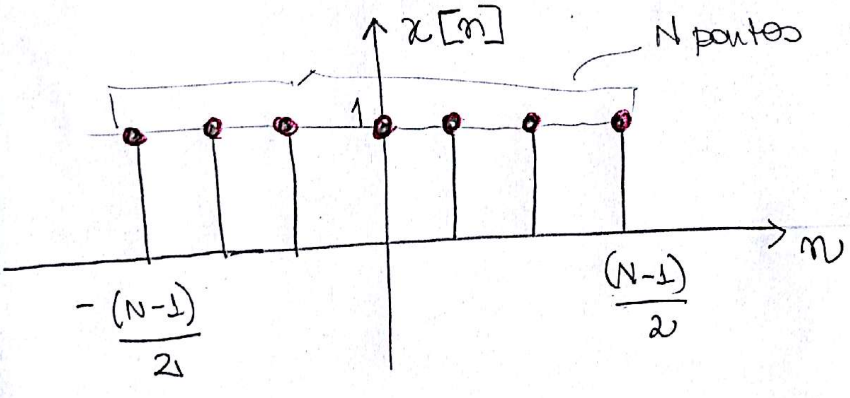
$$|X(\Omega)| = \left|\frac{1}{1-\alpha e^{-j\Omega}}\right| = \frac{1}{\left|\underbrace{(1-\alpha\cos\Omega)}_{\text{parte real}}+j\underbrace{\alpha \operatorname{sen}\Omega}_{\text{parte imaginária}}\right|}$$

$$\therefore |X(\Omega)| = \frac{1}{\sqrt{(1-\alpha\cos\Omega)^2+(\alpha \operatorname{sen}\Omega)^2}}$$



$\therefore$ é um filtro passa baixa

c) Pulso no domínio do tempo



$$x[n] = \begin{cases} 1 & -\frac{(N-1)}{2} < n \leq \frac{(N-1)}{2} \\ 0 & \text{c.c.} \end{cases}$$

$$X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\Omega n} = \sum_{n=-M}^{M} e^{-j\Omega n} = e^{j\Omega M} \sum_{n=0}^{2M} e^{-j\Omega n}$$

onde $M = \frac{N-1}{2}$

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Aplicando soma finita,

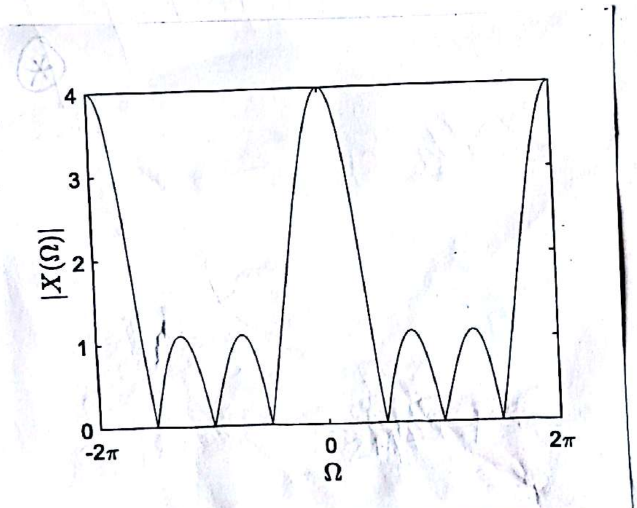
$$X(\Omega) = e^{j\Omega M}\left[\frac{1-e^{-j\Omega(2M+1)}}{1-e^{-j\Omega}}\right]$$

$$X(\Omega) = \underbrace{e^{j\Omega M}\,\frac{e^{-j\Omega\frac{(2M+1)}{2}}}{e^{-j\Omega/2}}}_{=1}\left[\frac{e^{j\Omega\frac{(2M+1)}{2}} - e^{-j\Omega\frac{(2M+1)}{2}}}{e^{j\Omega/2} - e^{-j\Omega/2}}\right]$$

$$X(\Omega) = \frac{\operatorname{sen}\left[(2M+1)\,\Omega/2\right]}{\operatorname{sen}\left[\Omega/2\right]}$$

$$2M+1 = 2\left(\frac{N-1}{2}\right)+1 = N-1+1 = N$$

$$\therefore\ X(\Omega) = \frac{\operatorname{sen}\left[N\,\Omega/2\right]}{\operatorname{sen}\left[\Omega/2\right]}$$



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2) Do item (b) do ex. 1, para $\alpha = 0{,}25$

$$h[n] = \left(\frac{1}{4}\right)^n u[n]$$

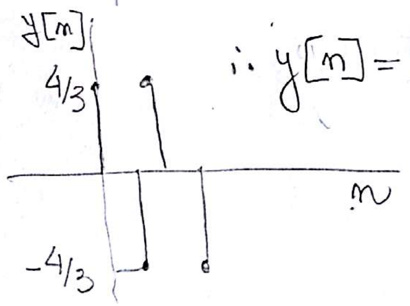
Sabe-se que a resposta $y[n]$ no domínio do tempo é dada pela convolução entre $h[n]$ e $x[n]$,

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\, x[n-k]$$ (fórmula discreta da convolução)

a) $$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u[k]\, e^{j\pi(n-k)}$$

$$= \sum_{k=0}^{\infty} \left(\frac{1}{4}\right)^k e^{j\pi n} e^{-j\pi k} = e^{j\pi n} \sum_{k=0}^{\infty} \left(\frac{1}{4} e^{-j\pi}\right)^k$$

Para resolver o problema vamos aplicar a fórmula para somatória finita já apresentada.



$$\therefore\ y[n] = e^{j\pi n}\left(\frac{1}{1 - \frac{1}{4}\underbrace{e^{-j\pi}}_{-1}}\right) = \frac{4}{3} e^{j\pi n} = \frac{4}{3}(-1)^n$$

b) $$y[n] = \sum_{k=-\infty}^{\infty} \left(\frac{1}{4}\right)^k u[k]\, e^{j\frac{\pi}{4}(n-k)}$$

Similarmente ao caso anterior, chega-se a:

$$\boxed{y[n] = \frac{e^{j\frac{\pi}{4}n}}{1 - \frac{1}{4} e^{-j\pi/4}}}$$

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3)

a) $x[k] = \frac{1}{N} \sum_{n=0}^{N-1} X[n] e^{j\frac{2\pi}{N}kn} = \frac{1}{4} \sum_{n=0}^{3} X[n] e^{j\frac{\pi}{2}kn}$

$$x[k] = \frac{1}{4}\left[5 + (3-j2)e^{j\frac{\pi}{2}k} - 3e^{j\pi k} + (3+j2)e^{j\frac{3\pi}{2}k}\right] \quad k=0,\dots 3$$

Substituindo os valores de k,

$$x[0] = \frac{1}{4}\left[5 + (3-\cancel{j2}) - \cancel{3} + (\cancel{3}+\cancel{j2})\right] = \frac{8}{4} = 2$$

$$x[1] = \frac{1}{4}\left[5 + (3-j2)e^{j\frac{\pi}{2}} - 3e^{j\pi} + (3+j2)e^{j3\pi/2}\right]$$

$$= \frac{1}{4}\left[5 + (3-j2)(j) - 3(-1) + (3+j2)(-j)\right] = 3$$

$$x[2] = \frac{1}{4}\left[5 + (3-j2)e^{j\pi} - 3e^{j2\pi} + (3+j2)e^{j3\pi}\right]$$

$$= \frac{1}{4}\left[5 + (3-j2)(-1) - 3(1) + (3+j2)(-1)\right] = -1$$

$$x[3] = \frac{1}{4}\left[5 + (3-j2)e^{j\frac{3\pi}{2}} - 3e^{j3\pi} + (3+j2)e^{j\frac{9\pi}{2}}\right]$$

$$= \frac{1}{4}\left[5 + (3-j2)(-j) - 3(-1) + (3+j2)(j)\right] = 1$$

$$x = [2, 3, -1, 1]$$

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(b) $X[n] = \sum_{K=0}^{N-1} x[K] e^{-j\frac{2\pi}{N}kn} = \sum_{n=0}^{3} X[k]\, e^{-j\frac{\pi}{2}kn}$

$\therefore X[n] = 2 + 3e^{-j\frac{\pi n}{2}} - e^{-j\pi n} + e^{-j\frac{3\pi}{2}n}, \quad n = 0,1,2,3$

Substituindo os valores,

$X[0] = 2+3-1+1 = 5$ ——— $|X[0]| = 5$; $\phi = \text{atan}\,\frac{0}{5} \Rightarrow \phi = 0$

$X[1] = 2+3e^{-j\pi/2} - e^{-j\pi} + e^{-j3\pi/2} =$ ——— $|X[1]| = \sqrt{3^2+(-2)^2}$

$= 2 + 3(-j) - (-1) + (j) = 3-2j$

$X[2] = 2+3e^{-j\pi} - e^{-j2\pi} + e^{-j3\pi}$ ———

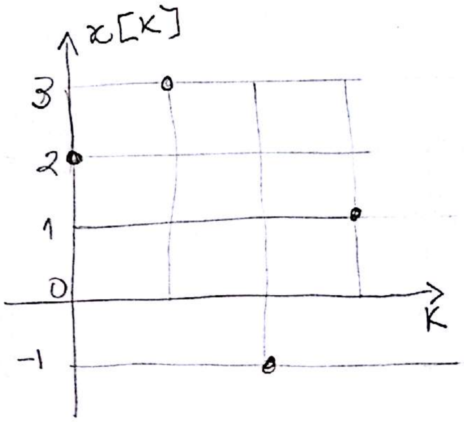
$= 2+3(-1) - (1) + (-1) = -3$

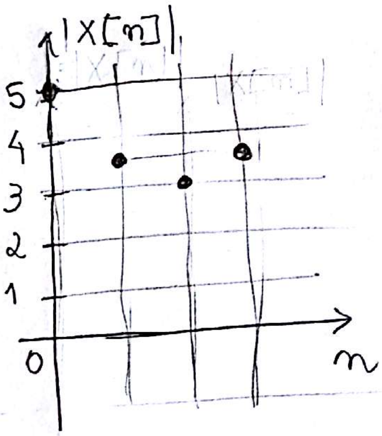
$X[3] = 2+3e^{-j\frac{3\pi}{2}} - e^{-j3\pi} + e^{-j\frac{9\pi}{2}}$ ———

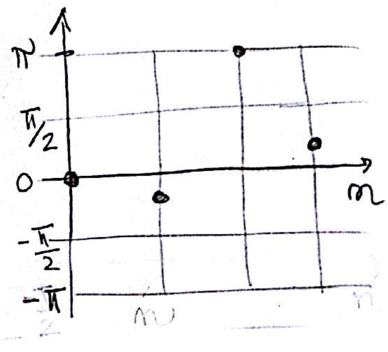
$= 2+3(j) - (-1) + (-j) = 3+2j$

Veja que as sequências $x[k]$ e $X[k]$ são pares da TDF, aperiódicos de comprimento $N=4$.

(c)







$|X[0]| = 5$

$|X[1]| = \sqrt{3^2+(-2)^2} \cong 3{,}6$

$|X[2]| = 3$

$|X[3]| \cong 3{,}6$

$\phi_0 = \text{atan}\,\frac{0}{5}$

$\phi_1 = \text{atan}\,\frac{-2}{3}$

$\phi_2 = \text{atan}\,\frac{0}{-3}$

$\phi_2 = \text{atan}\,\frac{2}{3}$

$rad = deg \times \frac{pi}{180}$

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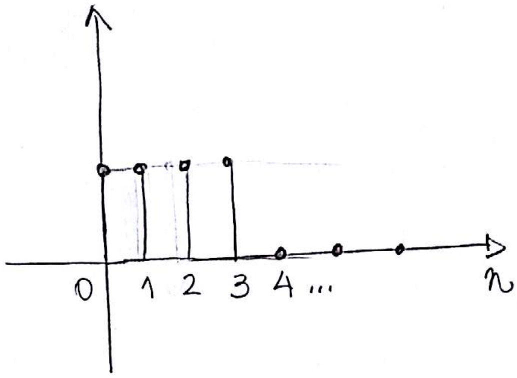
4)



DFT

$$X[k] = \sum_{n=0}^{3} 1\, e^{-j\frac{2\pi}{4}kn}$$

$$X[k] = 1 + e^{-j\pi/2\, k} + e^{-j\pi k} + e^{-j\frac{3\pi}{2}k}$$

$$(1)\begin{cases} X[0] = 1 \\ X[1] = 1 + e^{-j\pi/2} + e^{-j\pi} + e^{-j\frac{3}{2}\pi} = \emptyset \\ X[2] = 1 + e^{-j\pi} + e^{-j2\pi} + e^{-j3\pi} = \emptyset \\ X[3] = 1 + e^{-j\frac{3}{2}\pi} + e^{-j3\pi} + e^{-j\frac{9}{2}\pi} = \emptyset \end{cases}$$

$$\therefore\ X[k] = \begin{cases} 1 & \text{se } k=0 \\ 0 & \text{c.c. } (k=1,2,3) \end{cases}$$

FÓRMULA P/ SOMA FINITA

$$\boxed{\sum_{k=M}^{N} \beta^k = \frac{\beta^{N+1} - \beta^M}{\beta - 1}}$$

DTFT

$$X(\Omega) = \sum_{n=0}^{3} 1_x e^{-j\Omega n} = \frac{1 - e^{-j4\Omega}}{1 - e^{-j\Omega}}$$

$$= \frac{e^{-j2\Omega}}{e^{-j\Omega/2}} \left(\frac{e^{j2\Omega} - e^{-j2\Omega}}{e^{j\frac{\Omega}{2}} - e^{-j\Omega/2}} \right)$$

$$\boxed{X(\Omega) = e^{-j\frac{3\Omega}{2}} \left[\frac{\operatorname{sen}(2\Omega)}{\operatorname{sen}(\Omega/2)} \right]}$$

$$X(\Omega)\Big|_{\Omega = \frac{2\pi}{4} = \frac{\pi}{2}}$$

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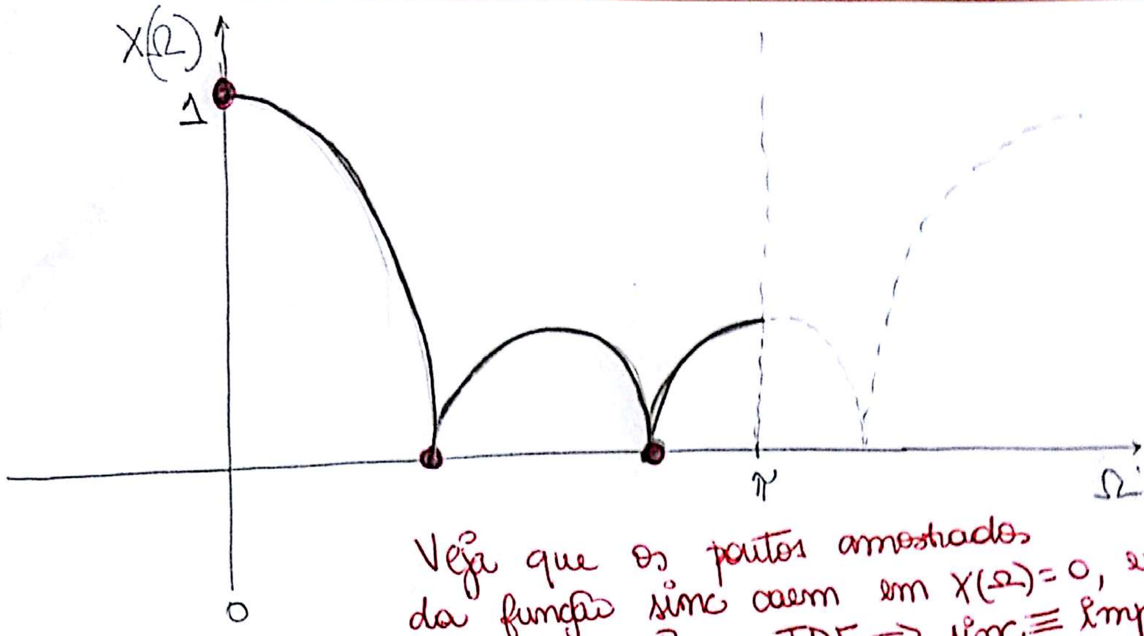

Veja que os pontos amostrados da função sinc caem em $X(\Omega)=0$, exceto para $\Omega=0$. Para TDF ⇒ sinc ≡ impulso.

OBS → Muitos autores preferem trabalhar com as equações (1) em forma de matriz.

Para isso, define-se: $W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$, de modo que

$$\boxed{X[n] = \sum_{k=0}^{N-1} x[k]\, W_N^{nk}}$$

A matriz W_N é definida como:

$$W_N^{nk} = e^{-j\frac{2\pi}{N}nk}$$

nessa linha e nessa coluna é 1, pois n ou k são zero

$$\therefore\ W_N = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & W_N^{11} & W_N^{12} & \cdots & W_N^{1(N-1)} \\ 1 & & \ddots & & \\ \vdots & & & & \\ 1 & & \cdots & & W_N^{(N-1)(N-1)} \end{bmatrix}$$

Dessa forma:

$$\begin{Bmatrix} X[0] \\ X[1] \\ X[2] \\ \vdots \\ X[N-1] \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & \cdots & 1 \\ 1 & & & & \\ 1 & & & \ddots & \\ \vdots & & & & \\ 1 & & \cdots & & W_N^{(N-1)(N-1)} \end{bmatrix} \begin{Bmatrix} x[0] \\ x[1] \\ x[2] \\ \vdots \\ x[N-1] \end{Bmatrix}$$

Aplicando, portanto, ao nosso exemplo:

W_4 é uma matriz 4×4,

$$W_4^{nK} = e^{-\pi/2 \, j nK}$$

$$W_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} \\ 1 & e^{-j\pi} & e^{-j2\pi} & e^{-j3\pi} \\ 1 & e^{-j\frac{3\pi}{2}} & e^{-j3\pi} & e^{-j\frac{9\pi}{2}} \end{bmatrix}$$

de modo que,

$$\begin{Bmatrix} X[0] \\ X[1] \\ X[2] \\ X[3] \end{Bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & e^{-j\frac{\pi}{2}} & e^{-j\pi} & e^{-j\frac{3\pi}{2}} \\ 1 & e^{-j\pi} & e^{-j2\pi} & e^{-j3\pi} \\ 1 & e^{-j\frac{3\pi}{2}} & e^{-j3\pi} & e^{-j\frac{9\pi}{2}} \end{bmatrix} \begin{Bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{Bmatrix}$$

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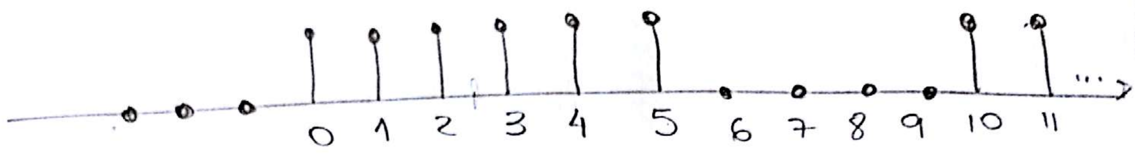
5) O caso mais geral é,

* Período N

* Largura do pulso: N_p deslocado para iniciar em $n=0$

Por exemplo,



Lembre-se que a DFT é a DTFT para um intervalo $0 \to N-1$. Implicitamente, a DFT considera que o sinal amostrado entre $0 \to N-1$ é periódico. (Vale um parêntese de que, por essa razão, a fórmula da DFT se assemelha à fórmula da Série de Fourier, → verifique as fórmulas nas notas de aula lembrando que a transformada de Fourier tem um fator de escala de $1/T$)

Para pulso ñ deslocado:

$$X[n] = \sum_{k=0}^{N-1} x[k]\, e^{-j\frac{2\pi}{N}kn} = \sum_{k=0}^{N_p-1} \underbrace{\left(e^{-j\frac{2\pi}{N}n}\right)}_{\beta}{}^{k}$$

$$\therefore X[n] = \frac{1-e^{-j\frac{2\pi}{N}nN_p}}{1-e^{-j\frac{2\pi}{N}n}} = \frac{\operatorname{sen}(n\pi N_p/N)}{\operatorname{sen}(\pi n/N)}$$

$\therefore$ Para pulso deslocado:

$$\boxed{X[n] = \frac{\operatorname{sen}(n\pi N_p/N)}{\operatorname{sen}(n\pi/N)}\, e^{-jn\frac{(N_p-1)}{\cancel{2}}\frac{\cancel{2}\pi}{N}}}$$

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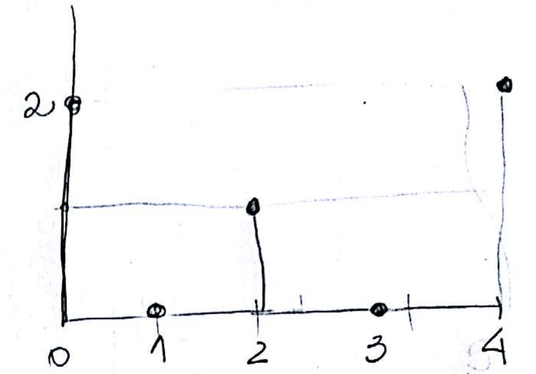
Portanto,

a) $N=6$

$N_p=3$

pulso ñ deslocado

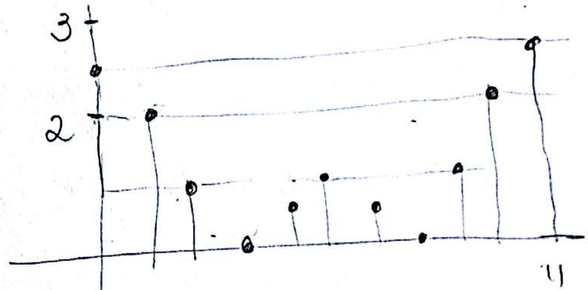
$$X[n] = \frac{sen(n\pi/2)}{sen(n\pi/6)}$$



b) $N=12$

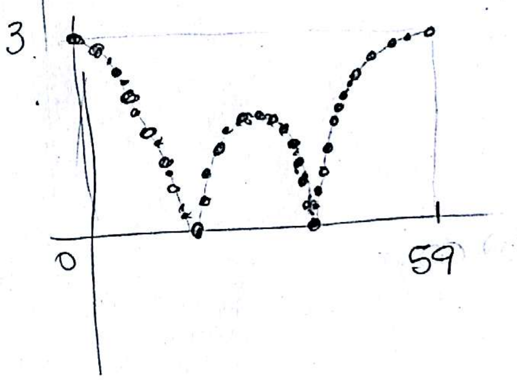
$N_p=3$

$$X[n] = \frac{sen(n\pi/4)}{sen(n\pi/12)}$$



c) $N=60$ $\qquad X[n] = \dfrac{sen(n\pi/20)}{sen(n\pi/60)}$

$N_p=3$



Para melhor visualizar o gráfico,

faça no Matlab,

```
N=   ;
Np=   ;
n=0:n-1;
X=sin(pi*Np/N.*n)./sin(pi*N.*n);
Stem(n, abs(X))
```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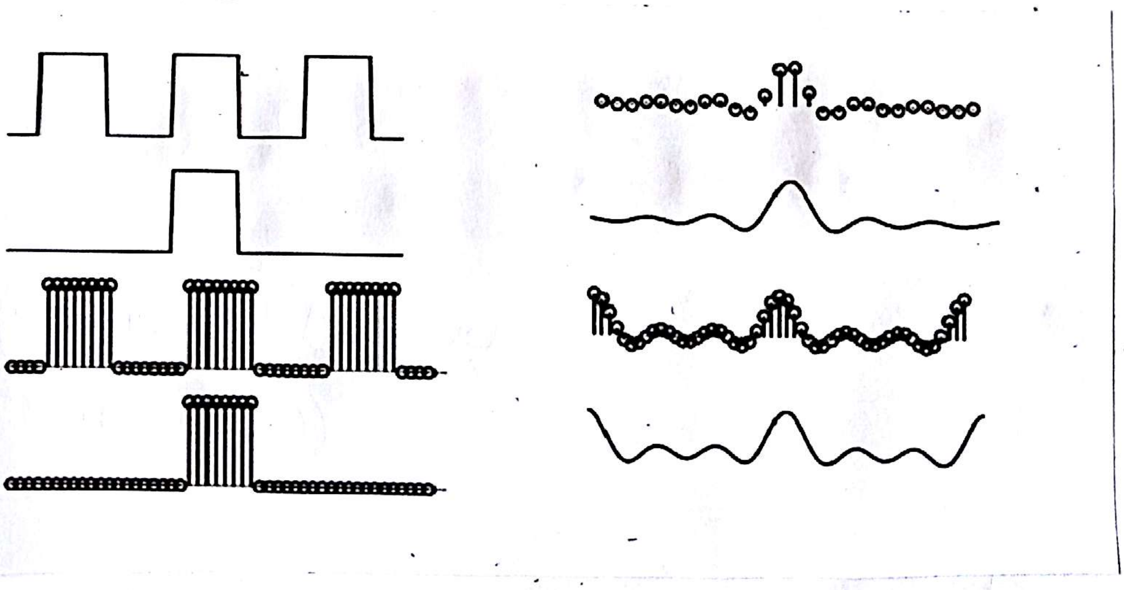
⑥

Tempo contínuo	Duração Infinita	Periódico	(I) ou (III)
Tempo contínuo	Duração Infinita	Não Periódico	(III)
Tempo contínuo	Duração Finita	Não Periódico	(III) ou (I)*
Tempo discreto	Duração Infinita	Periódico	(II) ou (IV)
Tempo discreto	Duração Infinita	Não Periódico	(IV)
Tempo discreto	Duração Finita	Não Periódico	(IV) ou (II)*

* ambos os sinais são não periódicos, porém de duração finita, então a Transf discreta de Fourier pode ser usada para expressar um sinal periódico formado por réplicas do sinal de duração finita.

⑦



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