# Didactic Transposition in Mathematics Education

Encyclopedia of Mathematics Education · Article ID: 313227 · Chapter ID: 48

## Keywords

Anthropological theory of the didactic, scholar knowledge, knowledge to be taught, institutional transposition, *noosphere*, ecology of knowledge, reference epistemological models.

## Definition

The process of didactic transposition refers to the transformations an object or a body of knowledge undergoes from the moment it is produced, put into use, selected and designed to be taught, until it is actually taught in a given educational institution. The notion was introduced in the field of didactics of mathematics by Yves Chevallard (1985). It highlights the fact that what is taught at school is originated in other institutions, constructed in concrete practices and organized in particular sets of objects. In the case of mathematics or any other subject, the *taught knowledge*, the concrete practices and bodies of knowledge proposed to be learned at school, originates from what is called the *scholarly knowledge*, generally produced at universities and other scholarly institutions, also integrating elements taken from a variety of related social practices. When one wishes to "transpose" a body of knowledge from its original habitat to school, specific work should be carried out to rebuild an appropriate environment with activities aimed at making this knowledge "teachable", meaningful and useful.

Different actors participate in this *transpositive work* (see Figure 1): producers of knowledge, teachers, curriculum designers, etc. They belong to what is called the *noosphere*, the sphere of those who "think" about teaching, an intermediary between the teaching system and society. Its main role is to negotiate and cope with the demands made by society on the teaching system, while preserving the illusion of "authenticity" of the knowledge taught at school, thus possibly denying the existence of the process of didactic transposition itself. It must appear that *taught knowledge* is not an invention of school. Although it cannot be a reproduction of *scholarly knowledge*, it should look like preserving its main elements. For instance, the body of knowledge taught at school under the label of "geometry" (or "mechanics", "music", etc.) has to appear as genuine. It is thus important to understand the choices made in the designation of the *knowledge to be taught* and the construction of the *taught knowledge*, to analyze what is transposed and why, what mechanisms explain its final organization and to understand what aspects are omitted and will therefore not be diffused.

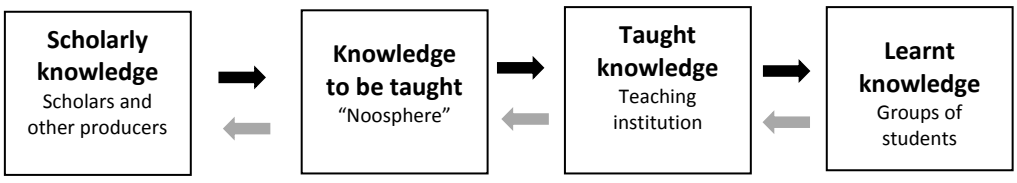


Figure 1. Diagram of the process of didactic transposition

## Scope

Besides mathematics, research on didactic transposition processes has been carried out in many other educational fields, such as the natural sciences,

philosophy, music, language, technology, physical education, etc. These investigations have spread faster in the French and Spanish speaking communities (Arsac 1992, Arsac et al. 1994, Bosch and Gascon 2006) than in the English speaking ones, although some prominent figures soon contributed to develop the first transpositive analyses (Kang and Kilpatrick 1992). The notion of didactic transposition has been generalized to *institutional transposition* (Chevallard 1992, Artaud 1995) when knowledge is transposed from one social institution to another. Because of social needs, bodies of knowledge originated and developed in different "places" or institutions of society need to "live" in other institutions where they should be transposed. They have to be transformed, deconstructed and reconstructed in order to adapt to their new institutional setting. For instance, the mathematical objects used by economists, geographers or musicians need to be integrated in other practices commonly ignored by the mathematicians who produced them. It is clear from the history of science that institutional transpositions—including didactic ones—do not necessarily produce degraded versions of the initial bodies of knowledge. Sometimes the transpositive work *improves* the organization of knowledge and makes it more understandable, structured and accurate, to the point that the knowledge originally transposed is itself bettered. The organization of knowledge in fields and disciplines as it exists today is the fruit of complex and changing historical interactional processes of institutional and didactic transpositions that are not well known yet.

## An emancipatory tool

In a field of research, new notions are not only introduced to describe reality but to provide new ways of questioning and new possibilities to modify it. The notion of didactic transposition is conceived, first of all, as an analytical instrument to avoid the "illusion of transparency" concerning educational phenomena and, more particularly, the nature of the knowledge involved, that is, to emancipate research from the viewpoint of the scholarly and the teaching institutions about the knowledge involved in educational processes.

Any *taught* field or discipline is the product of an intricate process the singularity of which should never be underrated. As a consequence, one should not take for granted the current, observable organization of a field or discipline taught at school, as if it were the only possible one. Instead one should see it against the (fuzzy) set of organizations that *could* have existed, some of which may someday turn into reality. Considering the "scholarly knowledge" as part of the object of study of research in didactics is part of this emancipatory movement of detachment. Although school teaching has to be legitimized by external entities that guarantee the pertinence and epistemological relevance of the knowledge taught (in a complex process of negotiations which includes crises and disagreements), researchers do not have to consider these institutional perspectives as the true or correct viewpoints, nor as the wrong ones; they just need to know them and integrate them in the analysis of educational phenomena.

In some cases, the "scholarly legitimation" of school knowledge can be questioned by the noosphere, on behalf of its cultural relevance: "Is this the geometry citizens need?" Such a conflict situation can change significantly the conditions of teaching and learning, by allowing a self-referential, epistemologically confined teaching. Moreover, there are certain teaching processes in which the scholarly body of knowledge is created afterwards, because of the need to teach a given content that has to be organized, labeled and recognized as something relevant(an illustrative example is the case of accounting and its corresponding body of knowledge, accountancy). It is also possible that something that is not even commonly recognized as a proper body of knowledge may appear as "scholarly knowledge" for the role it assumes in a given educational process. For instance, in the teaching of sports, the scholarly knowledge, albeit not academically tailored, includes that of high level sport

players, even if they are a far cry from what we normally consider "scholars" to be!

## Enlargement of the object of study

The second consequence of the detachment process introduced by the notion of didactic transposition is the evolution of the object of study of didactics as a research discipline. Besides studying students' learning processes and how to improve them through new teaching strategies, the notion of didactic transposition points at the object of the learning and teaching itself, the "subject matter", as well as its possible different ways of living—its diverse *ecologies*—in the institutions involved in the transposition process.

Let us take an example on negative numbers. Regarding the transpositive process, the first issue is to consider what the *taught knowledge* is made of (what concrete activities that are proposed to the students, their organization, the domain or block of contents they belong to, etc.) and how official guidelines and *noospherian* discourses present and justify these choices (the *knowledge to be taught*). Today, at most schools, negative numbers are officially related to the measure of quantities with opposite directions and introduced in the context of real-life situations. Where does this school organization come from? It results from different scholar ("New Mathematics") or social ("back-to-basics") pressures, canalized by the noosphere, that cannot be presented here but that delimit the kind of mathematical practices our students learn (or fail to learn) about this body of knowledge. If we look at scholarly knowledge, the environment is different: negative numbers are defined as an extension of the set of natural numbers **N** and form the ring of integers **Z**, without any specific discussion(http://www.encyclopediaofmath.org/index.php/Integer). This has not always been the case: it is very well known that until the mid-19$^{th}$ century, the possibility of "quantities less than zero" was still denied by many scholars. Their final acceptation was strongly related to the needs of algebraic work, which explains why, for a long time, integers were called "algebraic numbers". It also explains why the introduction of negative numbers was considered one of the main differences between Arithmetic and Algebra. This relationship to elementary algebraic work has now completely disappeared from the scholar's and school's conception of negative numbers, despite the fact that some practices of calculation—for instance those involving the product of integers—acquire their full sense when interpreted in this context.

Various other analyses have brought similar results regarding how the transposition process has affect other different mathematical contents (school algebra, linear algebra, limits of functions, proportionality, geometry, irrational numbers, functions, arithmetic, statistics, proof, modeling, etc.): more generally speaking, there is no such thing as an eternal, context-free notion or technique, the matter taught being always shaped by institutional forces that may vary from place to place and time to time. These investigations underline the institutional relativity of knowledge and show to what extend most of the phenomena related to the teaching and learning of mathematics are strongly affected by constraints coming from the different steps of the didactic transposition process. Consequently, the empirical unit of analysis of research in didactics becomes clearly enlarged, far beyond the relationships between teachers and students and their individual characteristics.

## The need for researchers' own epistemological models

Taking into consideration *transpositive phenomena* means moving away from the classroom and being provided with notions and elements to describe the bodies of knowledge and practices involved in the different institutions at different moments of time. To do so, the epistemological emancipation from scholarly and school institutions requires researchers to create their own perspective on the

different kinds of knowledge intervening in the didactic transposition process, including their own way of describing knowledge and cognitive practices, their own *epistemology*. In a sense, there is no privileged reference system from which to observe the phenomena occurring in the different institutions involved in the teaching process: the scholarly one, the *noosphere*, the school, the classroom. Researchers should build their own *reference epistemological models* (Barbé et al. 2005) concerning the bodies of knowledge involved in the reality they wish to approach (see Figure 2). The term "model" is used to emphasize the fact that any perspective provided by researchers (what mathematics is, what algebra is, what measuring is, what negative numbers are, etc.) always constitutes a methodological proposal for the analysis; as such, it should constantly be questioned and submitted to empirical confrontation.

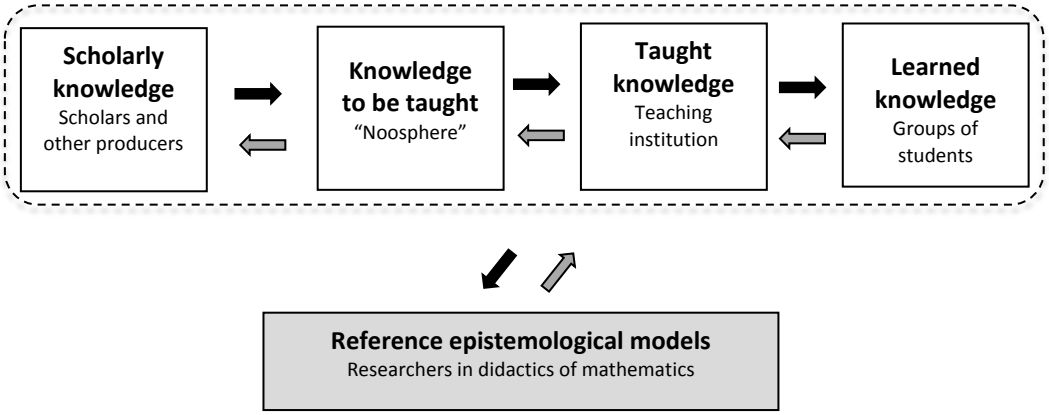


Figure. 2. The external position of researchers

## From didactic transposition to the anthropological approach

When knowledge is considered a changing reality embodied in human practices taking place in social institutions, one cannot think about teaching and learning in individualistic terms. The evolution of the research perspective towards a systematic epistemological analysis of knowledge activities explicitly appears at the foundation of the Anthropological theory of the didactic (Chevallard 1992 and 2007, Winslow 2011). It is approached through the study of the conditions enabling and the constraints hindering the production, development and diffusion of knowledge and, more generally, of any kind of human activity in social institutions.

## Cross references

Mathematics curriculum design and conception; Anthropological Approaches in Mathematics Education - French Perspectives; Didactic Situations in Mathematics Education; Didactic Engineering in Mathematics Education

## References

Arsac G (1992) The evolution of a theory in didactics: the example of didactic transposition. In: Douady R, Mercier A (eds) Research in Didactique of Mathematics. Selected Papers. La Pensée sauvage, Grenoble, France, pp 107-130.

Arsac G, Chevallard Y, Martinand J L, Tiberghien A, & Balacheff N (1994) La transposition didactique à l'épreuve. La Pensée sauvage, Grenoble, France.

Artaud M (1995) La mathématisation en économie comme problème didactique. La communauté des producteurs de sciences économiques : une communauté d'étude. In: Margolinas C (ed) Les débats de didactique des mathématiques. La Pensée sauvage, Grenoble, France, pp 113-129.

Barbé J, Bosch M, Espinoza L, Gascón J (2005) Didactic restrictions on the teacher's practice. The case of limits of functions in Spanish High Schools. Educational Studies in Mathematics 59: 235-268.

Bosch M, Gascón J (2006) Twenty-five years of the didactic transposition. ICMI Bulletin 58: 51-64.

Chevallard Y (1985) La Transposition Didactique. Du savoir savant au savoir enseigné, 2nd edn 1991, La Pensée sauvage, Grenoble, France. [Spanish translation: Chevallard Y (1997) La transposición didáctica. Del saber sabio al saber enseñado. AIQUE, Buenos Aires.]

Chevallard Y (1992) Fundamental concepts in didactics: Perspectives provided by an anthropological approach. In: Douady R, Mercier A (eds) Research in Didactique of Mathematics. Selected Papers. La Pensée sauvage, Grenoble, France, pp 131-167.

Chevallard Y (1989) On didactic transposition theory: some introductory notes. In: Proceedings of the International Symposium on Selected Domains of Research and Development in Mathematics Education, Bratislava, pp 51-62.http://yves.chevallard.free.fr/spip/spip/IMG/pdf/On_Didactic_Transposition_Theory.pdfAccessed 25 October 2012.

Chevallard Y (1992) A Theoretical Approach to Curricula. Journal für Mathematikdidaktik,13(2/3): 215-230.http://yves.chevallard.free.fr/spip/spip/IMG/pdf/A_Theoretical_Approach_to_Curricula.pdf.Accessed 25 October 2012.

Chevallard Y (2007) Readjusting Didactics to a Changing Epistemology, European Educational Research Journal, 6(2): 131-134.http://www.wwwords.co.uk/pdf/freetoview.asp?j=eerj&vol=6&issue=2&year=2007&article=4_Chevallard_EERJ_6_2_web . Accessed 25 October 2012.

Kang W, Kilpatrick J (1992) Didactic transposition in mathematics textbooks, For the Learning of Mathematics 12(1): 2-7.

Winslow C (2011) Anthropological theory of didactic phenomena: Some examples and principles of its use in the study of mathematics education. In Bosch M et al. (eds) Un panorama de la TAD. An overview of ATD. CRM Documents, vol 10, Centre de Recerca Matemàtica, Barcelona, Spain, pp 533-551. http://www.recercat.net/bitstream/handle/2072/200617/Documents10.pdf?sequence=1. Accessed 25 October 2012.