We have invited Mariana Bosch and Josep Gascón, the organizers of this Conference, to write a text for the *ICMI Bulletin* where they would present their personal view on this theory, on its development and on its outcomes. We asked them to keep in mind that this text had to be accessible to all those interested in mathematics education, whatever their cultural context was, and we hope that the reader will appreciate the substantial efforts the two authors have made in order to explain in a few pages the essence of a genuine approach in mathematics education that has been developing for more than two decades. We also hope that this text will be the first in a long series, and that through these ICMI will offer a useful contribution, even if modest, to the increase in mutual understanding that the field of mathematics education requires so much.

We are grateful to Carmen Batanero, our colleague of the ICMI Executive Committee, for calling our attention to the Baeza conference and for her support in bringing this project to completion.

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Twenty-Five Years of the Didactic Transposition

Marianna Bosch and Josep Gascón

In novels, the most important is to solve the point of view:
who contemplates reality and who offers this contemplation to others.
Manuel Vázquez Montalbán (2003)

In October 2005 a group of researchers, either working on the Anthropological Theory of the Didactic or interested in its developments, met in a conference in Baeza, a charming medieval town in the south of Spain. They shared and took stock of their researches carried out over the last 25 years, *since the term 'didactic transposition' was introduced into the newborn paradigm in mathematics education called 'didactics of mathematics'. We now have enough perspective to present what we understand as the three main contributions of the theory of didactic transposition in the progress of mathematics education as a field of research.*

Contents

1. The diffusion of the theory of 'didactic transposition'

The time of ideas often passes much slower than the time of people. Just over 25 years ago, at the 'First Summer School in Didactics of Mathematics' in Chamrousse (France, 7-19 July, 1980), Yves Chevallard gave his first course on the didactic transposition. We were at the early stages of a new paradigm set up by Guy Brousseau in the 70s with his Theory of Didactic Situations (Brousseau 1997a).

The notion of didactic transposition was rapidly integrated into the set of notions that gave this paradigm the beginning of an existence: didactic system, didactic and a-didactic situations, didactic contract, conceptual scheme, tool/object dialectic, didactic engineering, etc. Along with the notion of didactic transposition, other new terms appeared naming — and thus bringing into existence — new 'cuttings' of the social reality which didactics of mathematics planned to study: the bodies of knowledge, the 'noosphere' (or sphere of those who think about education), proto and para-mathematical knowledge and, methodologically, the 'illusion of transparency' of educational reality researchers should overcome by means of an enduring 'epistemological surveillance'.

With time, what was beginning to be called the 'theory of didactic transposition' started to spread in a very varied way, depending on the countries, the linguistic communities and the scientific or cultural affinities of the groups of researchers. The first edition of *La transposition didactique. Du savoir savant au savoir enseigné* (Chevallard 1985a) had its effect in the French-speaking community. It was followed by an important number of researches in the didactics of mathematics and experimental sciences that seemed to open a new domain of study, at least in the French-speaking community. Gilbert Arsac accurately depicts this evolution of the didactic transposition from its starting point till the 90s (Arsac 1992).

In the Spanish-speaking community, soon a 'grey' translation of the book done by Dilma Fregona appeared. Some years later, the Argentinean publishing house Aique took out a second translation that ended up spreading this theory widely, even outside the field of mathematics education: language, experimental sciences, philosophy, physical education, technology, social science, music and even chess! The diffusion in the international English-speaking community has been much slower, despite the fact that renowned investigators like Jeremy Kilpatrick soon knew how to put the new approach into practice (as, for instance, in Wan Kang's doctoral dissertation: Kang 1990, Kang and Kilpatrick 1992). Very few followed his footsteps. A fast check in Google shows that the French expression 'transposition didactique' has over 27 000 entries, the Spanish 'transposición didáctica' has more than 11 000 but there are less than 500 in English (including both translations: 'didactic transposition' and 'didactical transposition').

What does the didactic transposition consist of and what new elements does it provide for the research in mathematics education? Most of all, it formulates the need to consider that what is being taught at school ('contents' or 'knowledge') is, in a certain way, an exogenous production, something generated outside school that is moved — 'transposed' — to school out of a social need of education and diffusion. For this purpose, it needs to go through a series of adapting transformations to be able to 'live' in the new environment that school offers. For certain knowledge to be taught at school *transpositive work* needs to be carried out so that something that was not made for school changes into something that may be reconstructed inside school.

The process of didactic transposition starts far away from school, in the choice of the bodies of knowledge that have to be transmitted. Then follows a clearly creative type of work — not a mere "transference", adaptation or simplification —, namely a process of de-construction and rebuilding of the different elements of the knowledge, with the aim of making it 'teachable' while keeping its power and functional character. The transpositive work is done by a plurality of agents (the 'noosphere'), including politicians, mathematicians ('scholars') and members of the teaching system (teachers in particular), and under historical and institutional conditions that are not always easy to discern. It makes teaching possible but it also imposes a lot of limitations on what can be and what cannot be done at school. It may happen that, after the transposition process, school loses the rationale of the knowledge that is to be taught, that is, the questions that motivated the creation of this knowledge: Why are triangles so important? What were limits of functions made for? Why do we need polynomials? In this case, we obtain what Chevallard (2004) called a 'monumentalistic' education, in which students are invited to contemplate bodies of knowledge the rationale of which have perished in time.

25 years ago research in mathematics education was very much influenced by the psychological aspects of learning. Making the existence of transpositive processes clear meant opening the field of study beyond the mathematical activities carried out by students and beyond the work done by teachers in the classroom. Taking didactic transposition into consideration also meant questioning the concrete way in which this process was carried out, the kind of constraints that limit it, the mechanisms that explain why a certain transposition is being done and not another. In short, considering the restrictions bearing on educational institutions contributes to explain, in a more comprehensive way, what teachers and students do when they teach, study and learn mathematics. In this sense, the theory of didactic transposition contributed to widen the object of study of research in mathematics education, bringing into existence a dimension of educational reality that had remained unnamed and, thus, unconsidered till then.

However, it was more than that. As we will see, with this broadening of the considered empirical reality, a new way of formulating and approaching mathematics educational problems appeared: what was called the ‘anthropological approach’ or ‘Anthropological Theory of the Didactic’ (Chevallard 1992, 1999, Chevallard, Bosch, Gascón 1997). It is thus not surprising that the notion of didactic transposition, seen now as the germ of this new approach, has spread at a different pace and in different ways through the work of mathematics education researchers, following transpositive phenomena that this time affects the didactics of mathematics itself as a discipline. What, for some of us, appeared as a ‘classic term’ in mathematics education, something that has always been here, can be even now a notion to discover for some other members of the community.

2. The didactic transposition within the field of the ‘didactics of mathematics’

The meaning and relevance of the theory of didactic transposition cannot be understood unless we start from the original project giving it sense: the new programme of research in mathematics education started by Guy Brousseau with the Theory of Didactic Situations (TDS). Actually, the TDS caused a radical change in our way of studying problems related to the teaching and learning of mathematics. This revolution has led to a change, not only in the notions used to study learning and teaching processes, but also in the particular way of questioning educational reality. The TDS has changed the problems, the used models and the system that is to be studied through a methodology that starts questioning mathematical knowledge as it is implicitly assumed in educational institutions: what is geometry, what is statistics, what are decimal numbers, what is counting, what is algebra, etc. It then proposes *specific epistemological models* of mathematical knowledge — the *situations* or ‘games against a *milieu’* — that are used at the same time as ways of designating mathematical notions as well as ways of constructing them in the classroom. In Brousseau's own words:

> A situation is the set of circumstances in which a person finds him/herself and the relations which unite him/her to a milieu. Taking as the object of study the circumstances that preside over the diffusion and learning of bodies of knowledge thus leads to study these situations. (Brousseau 1997b, p. 2, our translation.)
>
> *Situations* are minimum models that ‘explain’ how such knowledge intervenes in the specific relations a subject establishes with a milieu to exert a determined influence in that milieu. (Brousseau 2000, p. 2, our translation.)
>
> In didactics of mathematics, these ‘models’ are essentially used as research instruments, as means to prove the consistence of the analysis and explanation of didactic phenomena. Even when used to build up didactic engineering, they have never been presented as ‘examples’ to reproduce, not even as principles to be directly used to guide teachers’ decisions and to train future teachers. On the contrary, [they show] the complexity of the system society/teacher/pupil and the dangers of improvising extrapolations that ignore the validity field of these models and the misuse of metaphors. (Brousseau 2005, p. 56, our translation)

It was Brousseau who first postulated the existence of *didactic phenomena* which appear as unintentional regularities in the processes of generation and diffusion of mathematics in social institutions and are irreducible to the corresponding cognitive, sociologist or linguistic ones. It also supposes that ‘teaching’ and ‘learning’ mathematics are not primary objects of research any more and become secondary ones (which does not mean less important!) because they can be defined in the primitive terms of the considered epistemological model. The necessity to bring into question spontaneous epistemological models, commonly accepted, to elaborate our own models for research and improvement of teaching and learning call for the emergence of what we have named the 'epistemological programme' (Gascón 2003).[1]

By stressing the necessity of carrying out an 'epistemological enquiry', the theory of didactic transposition clarified the project of the TDS, contributing to the shattering of the illusion of a unique mathematical knowledge already defined and for which the best teaching method was to be found. Bodies of knowledge are constructed outside school as the answer to some particular needs and formulated according to some very specific conditions. There exists a process, a social construction with multiple actors and different temporalities, through which some of these bodies of knowledge have to be selected, delimited, reorganised and, thus, redefined until reaching the classroom. The study of this process is an important step towards understanding what is being done in the classroom, even if the teaching act itself has to deny the existence of this process (that is, the reality of all these redefinitions) and maintain the illusion of the uniqueness of the knowledge that legitimizes its teaching.

> To the didactician, [the concept of didactic transposition] is a tool that allows to stand back, question the evidence, erode simple ideas, get rid of the tricky familiarity on the object of study. It is one of the instruments of the *rupture* didactics should make to establish itself as a proper field; it is the reason why the 'entrance through the knowledge' into didactic problems passes from power to action: because the 'knowledge' becomes *problematic* through it and, from now on, can figure as a term in the formulation of problems (either new or reformulated ones), and in their solution. (Chevallard 1985a, p. 15, our translation.)

The audacity of the project of a science of didactics put forward by the TDS was in this way reinforced at the time that its empirical unit of analysis started to extend considerably. Since mathematics is knowledge brought about, taught, learnt, practised and diffused in social institutions, to be able to understand which mathematics is done at school it is necessary to know the mathematics that motivates and justifies its teaching as well as how this mathematics is being interpreted in the different teaching institutions.

3. First contribution: the enlargement of the empirical unit of analysis

One of the first contributions of the theory of didactic transposition was to make clear that it is not possible to interpret school mathematics properly without taking into account the phenomena related to the school reconstruction of mathematics, whose origin has to be found in the institutions that produce mathematical knowledge. A distinction is established among: the 'original' or 'scholarly' mathematical knowledge as it is produced by mathematicians or other producers; the mathematical

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[1] This expression derives from the fact that Brousseau initially designated as 'experimental epistemology' what was then baptised as 'didactics of mathematics'. It stresses the importance of the knowledge questioning and avoids other more exclusive designations as referring to geography or language (neither is all research of the epistemological programme French, nor does all the French didactics take part in it).

knowledge 'to be taught' as it is officially designed by curricula; the mathematical knowledge as it is actually taught by teachers in their classrooms and the mathematical knowledge as it is actually learnt by students and that can be considered at the same time the end of the didactic process and also the starting point of new ones. Figure 1 illustrates the steps of the didactic transposition process.

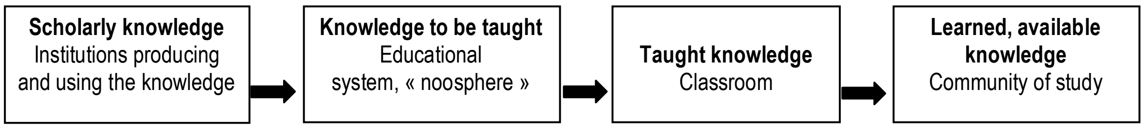

Fig. 1. The didactic transposition process

Didactic transposition processes underline the *institutional relativity of knowledge* and situate didactic problems on an institutional level, beyond individual characteristics of the subjects of the considered institutions. Its main consequence is that the *minimum* unity of analysis of any didactic problem must contain all steps of the process of didactic transposition. It is essential to take as an empirical basis data coming from each and every one of these institutions.

The first step corresponds to the study of the *formation* of the 'teaching text' pointing out the 'knowledge to be taught' through the productions of the noosphere (official programmes, textbooks, recommendations to teachers, didactic materials, etc.) and highlights the conditions and constraints under which the 'knowledge to be taught' is constituted and evolves (or remains fixed) in time. Thus, the analysis of the didactic transposition of a teaching domain (which includes the delimitation and designation of the domain itself) cannot be reduced to the reviewing of mathematics textbooks, even if they constitute a privileged empirical material for researchers. What is important is the kind of questions that are asked (why teaching this? why in this organisation? where does it come from?) and the kind of phenomena textbooks show (or hide).

The term 'scholar' was used — in quite an ironical way — to characterise knowledge that guarantees and legitimates the teaching process. Quoting Kang and Kilpatrick (*op. cit*., p. 2): 'A scholarly body of knowledge is nothing other than knowledge used both to produce new knowledge and to organize the knowledge newly produced into a coherent theoretical assemblage.' The difficult reception of the expression 'scholarly knowledge' testifies the difficulty of considering it at the same level as the knowledge to be taught (the one proposed by standards and official programmes) or the knowledge as it is actually taught at school. What bodies of knowledge are chosen? How are they named? Why these ones and why with this kind of organisations? What are the reasons to these choices? Etc. It is no longer enough to study the 'knowledge to be taught' and the 'taught knowledge'. It is also necessary to analyse — examine minutely and break down — the spontaneous models of the 'scholarly knowledge' that are taken for granted in educational institutions. For this reason, 'scholarly knowledge' cannot appear in any case as the 'reference knowledge' (as called by Astolfi & Develay 1989): it certainly is the reference point of educational institutions, but not of researchers who consider these institutions as an object of study.

We are not commenting here on the other steps of the transposition process that delimit the degree of freedom left to teachers and students when carrying out their work in the classroom. They have been more studied as they correspond to the place where the teaching process is usually located. What we want to stress is the following: considering the transposition process as a new object of study allows researchers in mathematics education to free themselves from spontaneous epistemological models that are implicitly imposed by the educational institutions to which we belong. When looking at this new empirical object that includes all steps from scholarly mathematics to taught and learnt mathematics, we need to elaborate our own 'reference' model of the corresponding body of mathematical knowledge. Figure 1 can thus be completed by what we call the 'reference epistemological knowledge' (Bosch and Gascón 2005) that constitutes the basic theoretical model for the researcher and can be elaborated from the empirical data of the three corresponding institutions: the mathematical community, the educational system and the classroom (see figure 2).

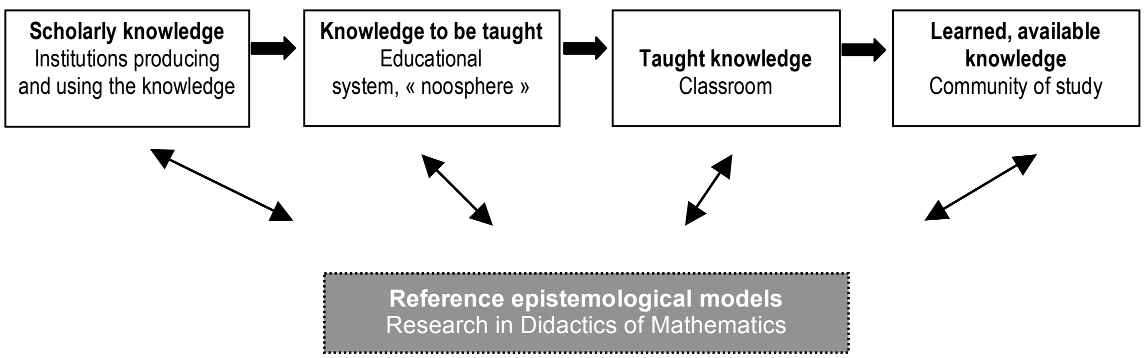

Fig. 2. The 'external' position of researchers

Research in didactics needs to elaborate its own models of reference to be able to avoid being subject to the different institutions observed, especially the 'dominant' ones. There is no privileged reference system for the analysis of the different bodies of knowledge of each step of the didactic transposition process. A reference model needs to be continuously developed by the research community and submitted to the proof of the facts. This is the sense we can attribute to the 'epistemological analysis' in didactics:

> Since 'scholarly knowledge' has been assigned its right place in the process of transposition, far from replacing epistemological analysis, in the strict sense, by the analysis of the didactic transposition, it turns out that it is indeed the concept of didactic transposition which allows the linking of the epistemological analysis to the didactic analysis, and from then on the guide of proper use of epistemology in didactics. (Chevallard 1985a, p. 20, our translation.)

Whatever the considered didactic problem may be, its study requires adopting a particular 'point of view' about the involved mathematical practices. For instance, what are the 'limits of functions' taught at undergraduate level? Or what kind of 'proof' or 'problem solving' are we considering? Is it something existing in 'scholarly mathematics'? In what way? Does it exist as knowledge to be taught? Since when? In what terms? What kind of restrictions does it impose on the teachers' practice? On the students' practice? Etc. From this point of view, the TDS appears as a 'machine' producing reference epistemological models. Situations are classically considered as tools to implement mathematical knowledge in the classroom (didactic engineering) and to analyse phenomena related to learnt and taught knowledge. However, they have also shown their pertinence to describe scholarly knowledge and the evolution of knowledge to be taught, as for instance in Brousseau's preliminary study on the teaching of decimal numbers (Brousseau 1980).

Over the last 25 years, didactic transposition processes of some of the main specific areas of the 'mathematics to be taught' have been analysed. For instance, concerning secondary school teaching we can mention among others: elementary algebra (Chevallard 1985b, Kang 1990, Coulange 2001), proportionality and magnitudes (Bolea *et al.* 2001, Comin 2002, Hersant 2005), geometry (Tavignon 1991, Chevallard and Jullien 1991, Matheron 1993, Bolea 1995), 'idecimal' and irrational numbers (Assude 1992, Bronner 1997), functions and calculus (Artigue 1993, 1998, 2000; Ruiz Higueras 1994, 1998; Chauvat 1999; Amra 2004; Barbé *et al.* 2005), linear algebra (Ahmed and Arsac 1998, Dorier 2000, Gueudet 2000), arithmetic (Ravel 2002), demonstration (Arsac 1989, Cabassut 2004), modelling (García 2005), statistics (Wozniak 2005), mathematics and economics (Artaud 1993, 1995), mathematics and sciences (Arsac *et al.* 1994).

These researches show that most phenomena related to the teaching of mathematics have as a main component a particular phenomenon of didactic transposition. It is in this sense that we can say that *phenomena of didactic transposition are at the very core of any didactic problem*. At the same time, these phenomena cannot be separated from those related to the production, use and diffusion of mathematics. School mathematical activity is thus inseparably integrated into the much larger problem of institutional mathematical activities which leads to a more general definition of didactics of mathematics: the 'science of specific (imposed) diffusion conditions of the mathematical bodies of knowledge useful to people and human institutions' (Brousseau 1994). This definition enlarges the field of didactics beyond educational institutions to include all institutions in which any kind of mathematical activity takes place.

Guy Brousseau has always insisted on the importance for mathematics education to remain close to the mathematical community because of the central role of epistemological analysis in didactics. However, in order to progress in their widened field of investigation, researchers have to get rid of their positions of 'teachers' and 'mathematicians', guardians of orthodoxies. A new position is necessary. Resort to the 'anthropological didactics' (Chevallard 1992) will highlight the need to work on the construction of this position.

4. Second contribution: the description of mathematical and didactic activities

Between 1980 and 1995, the study of phenomena of didactic transposition was formulated in terms of *objects of knowledge* and *relations to objects* in the broader frame of the *institutional ecology of knowledge objects* (Chevallard 1992, Artaud 1993, 1995). The search for a more detailed tool to model mathematical practice, including its material dimension, and mathematical bodies of knowledge, as inseparable from this practice, gives rise to the notion of *mathematical praxeology* or *mathematical organisation* within the frame of the 'Anthropological Theory of the Didactic' (Chevallard 1999, 2002a, 2002b). This theory is based on the assertion that mathematical activity has to be interpreted as an ordinary human activity, along with other forms of activity, and thus proposes a general model of human activities (the *praxeologies*) that links and gives the same importance to their theoretical (knowledge) and practical (know-how) dimension. According to Chevallard (2006):

> *A* praxeology is, in some way, the basic unit into which one can analyse human action at large. […] What exactly is *a* praxeology? We can rely on etymology to guide us here — one can analyse any human doing into two main, interrelated components: *praxis*, i.e. the practical part, on the one hand, and *logos*, on the other hand. "*Logos*" is a Greek word which, from pre-Socratic times, has been used steadily to refer to human thinking and reasoning — particularly about the cosmos. […] [According to] one fundamental principle of ATD — the anthropological theory of the didactic —, no human action can exist without being, at least partially, "explained", made "intelligible", "justified", "accounted for", in whatever style of "reasoning" such an explanation or justification may be cast. *Praxis* thus entails *logos* which in turn backs up *praxis*. For *praxis* needs support just because, in the long run, no human doing goes unquestioned. Of course, a praxeology may be a *bad* one, with its "praxis" part being made of an inefficient technique — "technique" is here the official word for a "way of doing" —, and its "logos" component consisting almost entirely of sheer nonsense — at least from the praxeologist's point of view!

In order to have the most precise tools to analyse the institutional didactic processes, Chevallard (1999) classifies mathematical praxeologies into a sequence of increasing complexity. The "simpler" ones — *point*-mathematical organisations —, built up around a single kind of problem, can be linked according to their theoretical background to give rise to *local*, *regional* or *global* praxeologies that cover respectively a whole mathematical theme, an sector or a domain. The analysis of didactic transposition processes acquires a new functionality since the modelling in terms of praxeologies can be used to describe all steps of the didactic transposition process: from the 'official' scholarly bodies of knowledge that can be found in mathematical treatises or those more informal ones 'activated' by researchers in their daily work, till the contents explicitly taught in classrooms or the less explicit mathematical knowledge learnt by, and thus available for, a group of students. It particularly becomes a very useful tool to clarify the reference epistemological models that guide researchers pointing out the strict constraints 'praxeologies to be taught' may bear onto teachers' and students' practice.

Bolea, for instance, presents an original specific model of elementary algebra that allows describing didactic restrictions on the teaching of algebra as a modelling tool (Bolea *et al*. 2004). Garcia (2005) extends this model to reach the link between algebraic and functional modelling, showing the isolated character of proportionality in Spanish curricula (García & Ruiz 2006). A very simple model in terms of a double-sided praxeology can show the extremely narrow space in which a Spanish high school teacher when teaching limits of functions can move (Barbé *et al*. 2005). Other analyses of didactic transposition processes in terms of praxeologies can be found in recent doctoral dissertations, such as: Cabassut (2004) analysing what he calls a 'double transposition' on the teaching of proofs as a mathematical and a social knowledge; Hersant (2005) depicting the evolution of the teaching of proportionality in France and questioning it on the poor place devoted to the study of magnitudes; Ravel (2004) studying the difficult reintroduction of arithmetic at high school level in France; Amra (2004) on the teaching of functions; Rodríguez (2005) on the teaching of problem solving and metacognitive skills; and Wozniak (2005) on statistics.

We have said that mathematical knowledge can be described in terms of praxeologies. What about the teaching and learning of mathematics, that is, the didactic process itself? In fact, in the same way knowledge is never a definitive construction, *mathematical praxeologies* do not emerge suddenly and do not acquire a definite form. They are the result of complex and ongoing activities, with complex dynamics, that in their turn have to be modelled. There appear two aspects very close to the mathematical activity: the process of mathematical construction — the *process of study* — and the result of this construction — the *mathematical praxeologies*. Once again, this process of study, as a human activity, is to be modelled in terms of praxeologies, which are now called *didactic praxeologies* (Chevallard 1999). Thus the notion of *study* provides a unitary field to describe didactic praxeologies the object of which is the setting up of mathematical praxeologies in different institutions (production, diffusion, using and teaching institutions).

We are not developing this point here, but let us just say that a new conception of didactics of mathematics appears in which *didactics* is identified to anything that can be related to *study* and *aid to study*:

> Didactics of mathematics is the science of study and aid to study mathematics. Its aim is to describe and characterize the study processes (or didactic processes) in order to provide explanations and solid responses to the difficulties of people (students, teachers, parents, professionals, etc.) studying or helping others to study mathematics. (Chevallard, Bosch, Gascón 1997, p. 60, our translation).
>
> The concept of praxeology enables us to completely formulate the object of didactics: *didactics is devoted to study the conditions and restrictions under which praxeologies start living, migrating, changing, operating, dying, disappearing, reviving, etc., within human groups*. Naturally there is a considerable enlargement of the field of research of didactics: *the* didactics studies *the* didactic, whenever it can be found, whatever form it can take. Giving its proper object, didactics can hope to progressively escape from a status dominated by disciplinary fields established at school. (Chevallard 2005, our translation)

If we replace 'conditions and restrictions' by 'ecology', we can say, in a shorter way, that didactics is devoted to the study of the institutional *ecology of mathematical and didactic praxeologies*. As was showed some years ago by Artaud (1993, 1995), the didactic transposition appears as a particular form of institutional transposition, that is of social diffusion of mathematical knowledge.

5. Third contribution: constraints at different levels of determination

The study of the ecology of mathematical and didactic praxeologies states that, when the teacher and the students meet around a knowledge to be taught, what can happen is mainly determined by conditions and restrictions that cannot be reduced to those immediately identifiable in the classroom: teacher's and students' knowledge, didactic material available, software, temporal organisation, etc. Even if these conditions and restrictions play an important role, Chevallard recently proposed to consider a scale of 'levels of determination' (see figure 3) that may help researchers to identify conditions that go beyond the narrow space of the classroom and the subject that has to be studied in it (Chevallard 2002b, 2004).

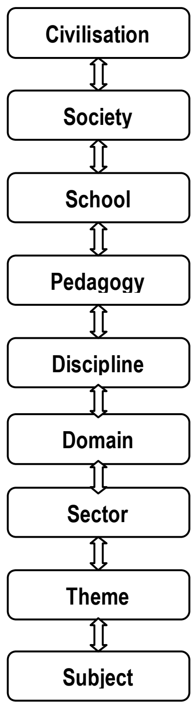

Figure 3. Scale of levels of determination

Why such a new broadening of the object of study with the corresponding complexity of the theoretical framework? The answer is always the same: to get free from the spontaneous conceptions of mathematical knowledge that researchers could assume without bringing them into question. 'Specific', 'local', 'regional' and 'global' praxeologies correspond to the low levels of the topic, the theme, the sector and the domain. Maybe due to the closeness to the 'teacher's problem' (the simplest formulation of which could be: 'given a mathematical content to be taught to a group of students, which is the best way to do it?'), researchers often took for granted the specific delimitation of contents that is given by scholarly or educational institutions. Why are mathematical contents divided in these or those particular blocks? Which are the criteria for this division and what kind of restrictions on the concrete activity of teachers and students does it cause?

Within the discipline of mathematics, the high value attributed to geometry to approach reasoning and proof, the lower consideration of elementary algebra and the difficulty in some European countries to introduce the teaching of statistics as a 'normal' block of contents, are phenomena that originate at higher levels of determination (school, society and civilisation). Not to mention the closing of mathematics in itself and the difficult relationships between mathematics and other disciplines. These are clearly strong constraints against, for instance, the learning of modelling, statistics or other practices that require the construction of mathematical praxeologies mixed up with non-mathematical objects (more developments can be found in Wozniak 2005).

The main problem is to know which kind of restrictions, coming from which level, are becoming crucial to the ecology of mathematical praxeologies. At the beginning of this paper we mentioned the process of monumentalization of mathematical knowledge. Nowadays it really appears as an essential transpositive phenomenon that goes beyond mathematical education and affects almost all kinds of praxeologies taught at school. The most recent works of Chevallard (2004, 2005, 2006) call for the construction of a new school epistemology that places the rationale of bodies of knowledge and their functionalities at the core of learning:

> [We need] an updating of mathematics to turn it, in a sense, into *responsible* mathematics. Mathematics clearly showing to new generations that school will not let them down but is, on the contrary, highly concerned about endowing them with the necessary tools to think about the world around them and step into it armed with knowledge and reason. (Chevallard 2004, our translation)

The scale of levels of determination clarifies a new opening in the field of study of didactic phenomena that was incipient in the first formulations of the theory. 25 years ago Chevallard's work impelled us to take into account constraints coming from the didactic transposition process and the concrete way this process organises mathematical contents at school: from the division into disciplines and blocks of contents, till the low-level concatenation of subjects. A further step seems necessary now, looking at constraints coming from the way Society, through School, organises the study of disciplines. It concerns, more generally, the status and functions our societies assign to disciplines and 'study activities'. It seems that this last development will allow us to reassess our common views about education and learning and establish the 'alternative epistemology' that Kang and Kilpatrick (1992, p. 2) were able to make out in the original transposition theory:

> We may need an alternative epistemology if we admit that most of the knowledge in school mathematics is a compound of knowledge that fits observations together with our values, instructional purposes, mathematical skills, and so on. […] Can we construct and epistemology that allows us to treat knowledge at least "as if" it existed independently outside of the knower without violating much of the constructivist position? One epistemological model that gives a positive answer can be found in the didactic transposition theory of Chevallard.

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