Econometria I - REC2301 Prof. Daniel Santos

Nome: \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_ # USP: \_\_\_\_\_\_\_\_\_\_

**ATENÇÃO**

**Só considerarei o que estiver escrito no espaço designado para a questão. Use o rascunho para organizar suas idéias.**

1. Considere o modelo:

$$y=a+bx+ε$$

Suponha que ε|X ~ N[0,σ2].

1. (1 ponto) Escreva a função de verossimilhança do modelo para uma amostra aleatória de tamanho N (obs: $f\_{ε|x}\left(t\right)=\frac{1}{\sqrt{2πσ^{2}}}e^{-\frac{1}{2}\left(\frac{t}{σ}\right)^{2}}$).

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1. (1 ponto) Calcule a fórmula do estimador de máxima verossimilhança de a, b e σ2.

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1. Considere o modelo y = a + bx + ε. Suponha que E[ε ] = 0 e ε ǁ X.
2. (0,5 ponto) Enuncie a Lei das Expectativas Iteradas

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1. (0,5 ponto) (ATENÇÃO: uma errada anula uma certa) Quais das propriedades abaixo tem o estimador de MQO neste caso?
2. $\hat{b}^{MQO}→b$ (c) $E\left(\hat{b}^{MQO}\right)=b$
3. $\hat{b}^{MQO}|X\~N\left(b,\frac{σ^{2}}{\sum\_{i=1}^{N}\left(x\_{i}-\overbar{x}\right)^{2}}\right)$ (d) $\hat{b}^{MQO}→N\left(b,\frac{σ^{2}}{NVar(x)}\right)$
4. Como sua resposta muda se adicionalmente supusermos a hipótese de homocedasticidade, isto é, Var(ε|X) = σ2?

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1. Escolha uma das propriedades marcadas acima (no item 2) e demonstre rigorosamente sua validade.

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1. Considere a seguinte base de dados:

|  |  |  |
| --- | --- | --- |
| Produtor | Sacas de Feijão | Trabalhadores |
| João | 2 | 2 |
| Mara | 4 | 2 |
| Carla | 4 | 10 |
| Jair | 6 | 10 |

Suponha que a função de produção de feijões seja do tipo F = a + bT + ε, onde F é o número de sacas de feijão, T é o número de trabalhadores, e ε é um fator de produção não-observável, onde E[ε | T] = 0. Para facilitar seu trabalho, foram calculados que, na amostra:

Cov(F,T) = 4

Var(F) = 2

Var(T) = 16

Calcule:

1. (0,5 ponto) Os estimadores de MQO de a e b:

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1. (0,5 ponto) Os valores de $\hat{ε}$ para João, Mara, Carla e Jair:

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1. (0,5 ponto) Os estimadores $\hat{σ}^{2}e S^{2}$ da variância de ε:

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1. (0,5 ponto) A variância de $\hat{b}^{MQO}$:

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1. (1 ponto) No espaço abaixo desenhe, em um gráfico FxT, (i) os pares (Fi,Ti) para os quatro indivíduos de sua amostra; (ii) a reta $\hat{F}=\hat{a}+\hat{b}T$
2. Suponha que **x** denote um vetor composto por duas variáveis aleatórias, x1 e x2, que conjuntamente se distribuem segundo uma Normal Multivariada do tipo:

$$\left(\genfrac{}{}{0pt}{}{x\_{1}}{x\_{1}}\right)\~N\left[\left(\genfrac{}{}{0pt}{}{1}{2}\right);\left(\begin{matrix}1&-1\\-1&25\end{matrix}\right)\right]$$

1. Qual a distribuição (marginal) de x1?

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1. Qual a distribuição de 3x1 – x2 + 8?

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1. Se $a=\left(\genfrac{}{}{0pt}{}{2}{3}\right) e B= \left(\begin{matrix}1&2\\3&4\end{matrix}\right)$, qual a distribuição de **y = a + Bx?**

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1. Seja **Σ** e **μ** a matriz de covariância e o vetor de esperanças de **x**, respectivamente**.** Qual a distribuição de w = (**x** – **μ**)’ **Σ-1**(**x** – **μ**)?

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