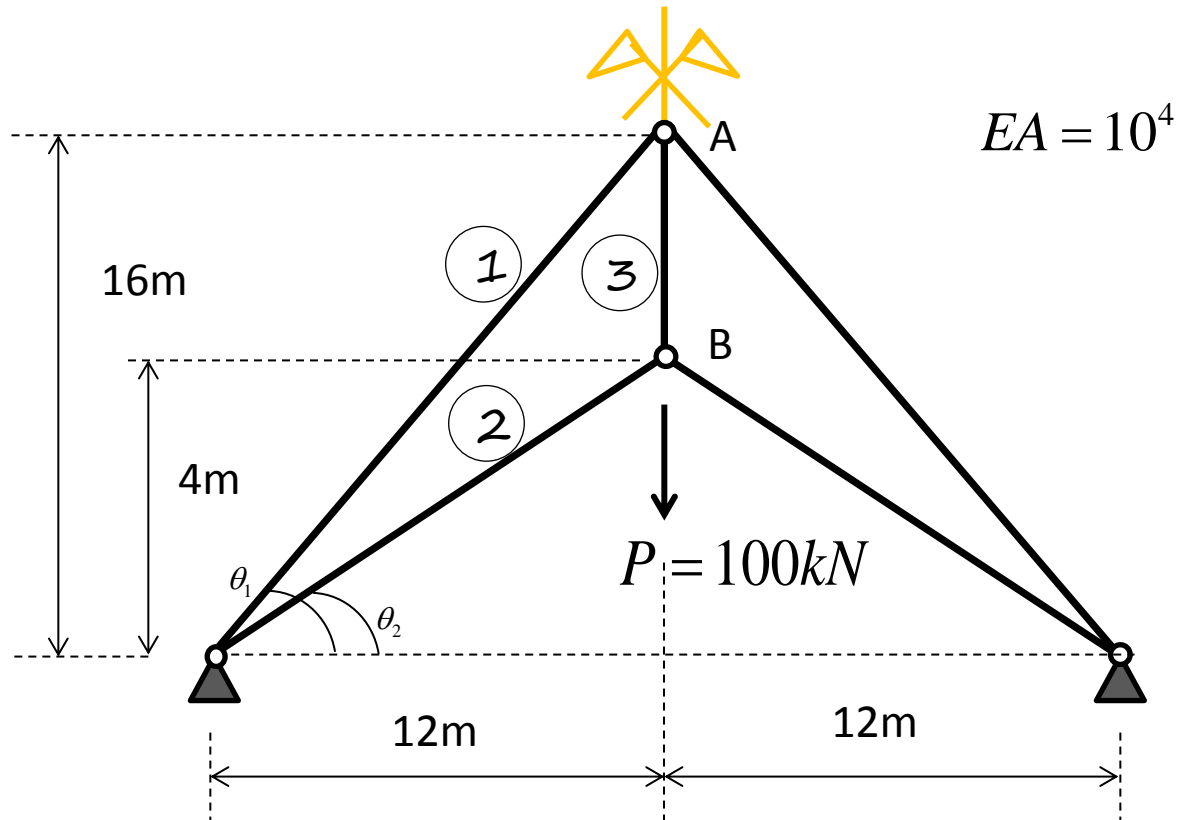


**Exercício:** Determinar os esforços nas barras da treliça:



$2n - b < r \therefore$  hiperestática

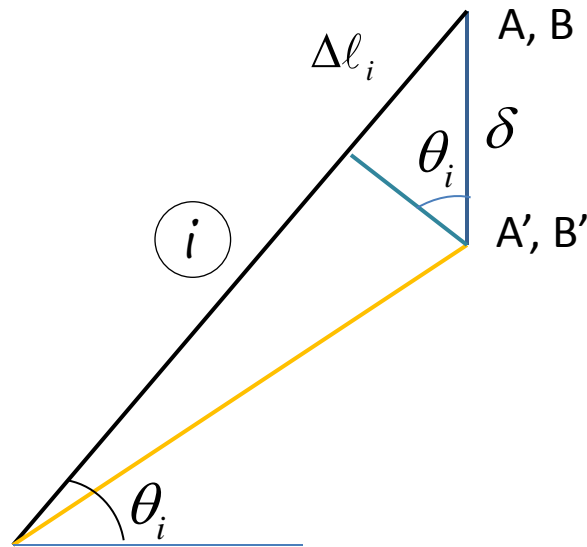
$= r \therefore$  isostática

$> r \therefore$  hipostática

$2 \cdot 4 - 5 = 3 < 4 \quad 1 \times$  hiper



# Compatibilidade de deformações



$$\Delta l_i \approx \delta_i \sin \theta_i \Rightarrow \Delta l_i = \frac{N_i l_i}{EA}$$

$$\Delta l_1 \approx \delta_1 \sin \theta_1 \Rightarrow \Delta l_1 = \frac{N_1 l_1}{EA} = \delta_1 \sin \theta_1$$

$$N_1 = -\frac{EA}{l_1} \delta_1 \sin \theta_1$$

$$\Delta l_2 \approx \delta_2 \sin \theta_2 \Rightarrow \Delta l_2 = \frac{N_2 l_2}{EA} = \delta_2 \sin \theta_2$$

$$N_2 = -\frac{EA}{l_2} \delta_2 \sin \theta_2$$

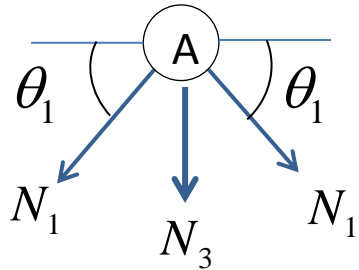
$$\Delta l_3 = \delta_2 - \delta_1$$

$$\frac{N_3 l_3}{EA} = \delta_2 - \delta_1$$

$$N_3 = \frac{EA}{l_3} (\delta_2 - \delta_1)$$

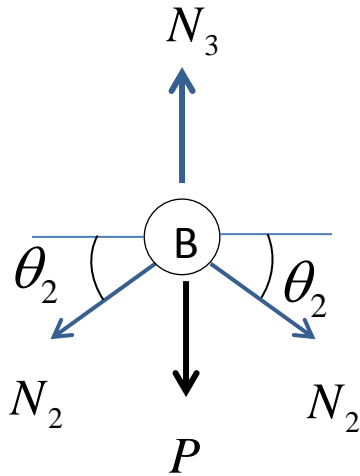


## Equilíbrio:



$$\sum F_Y = 0 \quad \therefore \quad 2N_1 \sin \theta_1 + N_3 = 0$$

$$N_3 = -2N_1 \sin \theta_1 \quad (1)$$



$$\sum F_Y = 0 \quad \therefore \quad 2N_2 \sin \theta_2 - N_3 = P \quad (2)$$

*Substituindo (1) em (2), temos:*

$$2N_2 \sin \theta_2 + 2N_1 \sin \theta_1 = P$$

$$2(N_2 \sin \theta_2 + N_1 \sin \theta_1) = P \quad (3)$$



$$l_1 = \sqrt{12^2 + 16^2} = 20m$$

$$l_2 = \sqrt{12^2 + 4^2} = \sqrt{160}m$$

$$l_3 = \sqrt{160}m = 12m$$

$$\sin\theta_1 = \frac{16}{20} = 0.8$$

$$\sin\theta_2 = \frac{4}{\sqrt{160}} = 0.316$$

$$N_1 = -\frac{10^4}{20} \delta_1 0.8 = -400\delta_1 \quad (4)$$

$$N_2 = -\frac{10^4}{\sqrt{160}} \delta_2 0.316 = -248.82\delta_2 \quad (5)$$

$$N_3 = \frac{10^4}{12} (\delta_2 - \delta_1) = 833.33(\delta_2 - \delta_1) \quad (6)$$



*Substituindo (4) e (5) em (3), temos:*

$$2(N_2 \sin \theta_2 + N_1 \sin \theta_1) = P$$

$$2(-248.82\delta_2 0.316 - 400\delta_1 0.8) = -100$$

$$(157.25\delta_2 + 640\delta_1) = 100 \quad (7)$$

*Substituindo (4) e (6) em (1), temos:*

$$833.33(\delta_2 - \delta_1) - 640\delta_1 = 0$$

$$\boxed{\delta_2 = 1.768\delta_1} \quad (8)$$

*Finalmente, substituindo (8) em (7), temos:*

$$(278\delta_1 + 640\delta_1) = 100$$

$$\boxed{\delta_1 = 0.1089m}$$

$$\boxed{\delta_2 = 0.1926m}$$

(9)



*Substituindo  $\delta_1$  e  $\delta_2$  em (4), (5) e (6) temos:*

$$N_1 = -400\delta_1 = -43.56\text{kN}$$

$$N_1 = -43.56\text{kN}$$

$$N_2 = -\frac{10^4}{\sqrt{160}}\delta_2 \cdot 0.316 = -48.12\text{kN}$$

$$N_2 = -48.12\text{kN}$$

$$N_3 = \frac{10^4}{12}(\delta_2 - \delta_1) = 69.75\text{kN}$$

$$N_3 = 69.75\text{kN}$$

