EXERCÍCIOS (ELEMENTOS CARACTERÍSTICOS NA SEÇÃO) (Exercício item 6 - pg 1)

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4.1.1 (item d)

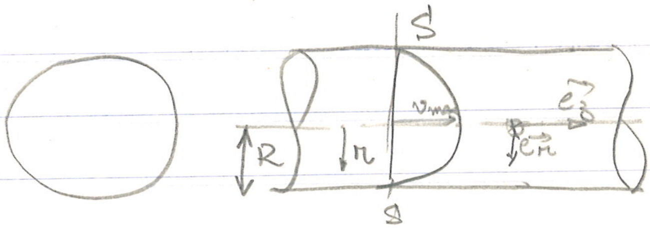
Parte Mov. LAMINAR EM TUBO CILÍNDRICO

Determinar

a) Vazão em Volume
b) Velocidade média
c) Coef. da En. Cinética
d) " " Q. Movimento
e) Fluxo de Energia Cinética
f) " da Quantidade de Movimento

Coord. cilíndrica

Tubo cilíndrico



$$v = V_{max}\left(1 - \frac{r^2}{R^2}\right)\vec{e_z}$$

a) $Q = \int_S \vec{v} \times \vec{n}\, dS$

No caso $\vec{e_n} = \vec{e_z}$

$$S = \pi r^2 \qquad dS = 2\pi r\, dr$$

$$Q = \int_0^R v_{max}\left(1 - \frac{r^2}{R^2}\right) 2\pi r\, dr = 2\pi v_{max}\int_0^R \left(1 - \frac{r^2}{R^2}\right) r\, dr$$

$$Q = 2\pi v_{max}\left[\int_0^R r\, dr - \int_0^R \frac{r^3}{R^2}\, dr\right] = 2\pi v_{max}\left[\frac{r^2}{2}\Big|_0^R - \frac{r^4}{4R^2}\Big|_0^R\right]$$

$$Q = 2\pi v_{max}\left[\frac{2R^2}{2\cdot 2} - \frac{R^2}{4}\right] = \frac{\cancel{2}\pi v_{max} R^2}{\cancel{4}_2} = \boxed{\frac{\pi v_{max} R^2}{2}}$$

Assim $\boxed{Q = \pi R^2 \frac{v_{max}}{2}}$ ou $\boxed{Q = \frac{v_{max}}{2} \cdot S}$

//

b) Velocidade Média

$$V = \frac{1}{S}\int_S \vec{v} \times \vec{n}\, dS \qquad \text{ou} \qquad V = \frac{Q}{S}$$

$$V = \frac{1}{S} \cdot \frac{\pi R^2 v_{max}}{2} \qquad \text{ou} \qquad V = \frac{1}{\cancel{S}} \cdot \cancel{S} \cdot \frac{v_{max}}{2}$$

Assim: $\boxed{V = \frac{v_{max}}{2}}$

c) $\alpha = \frac{C}{C_0}$

$$C = \frac{1}{2}\int_S v^2 \rho \vec{v} \times \vec{n}\, dS$$

$$C = \frac{1}{2}\int_S \rho v^3\, dS = \frac{\rho}{2}\int_0^R \left[v_{max}\left(1 - \frac{r^2}{R^2}\right)\right]^3 2\pi r\, dr$$

$$C_0 = \frac{1}{2}\rho V^2 Q$$ (onde $\rho = \rho_m \rightarrow$ massa específica média na seção)

$$C_0 = \frac{1}{2}\rho V^3 S = \frac{1}{2}\rho V^3 \pi R^2$$

Assim

$$\alpha = \frac{\rho/2}{\rho/2\ \pi R^2}\int_0^R \left(\frac{\cancel{v_{max}}\left(1 - \frac{r^2}{R^2}\right)}{\cancel{v_{max}}/2}\right)^3 2\pi r\, dr$$

$$\alpha = \frac{2\cancel{\pi}}{\cancel{\pi} R^2} \cdot 2^3 \left[\int_0^R \left(1 - 3\left(\frac{r^2}{R^2}\right) + 3\left(\frac{r^4}{R^4}\right) - \left(\frac{r^6}{R^6}\right)\right) r\, dr\right]$$

$$\alpha = \frac{16}{R^2}\left[\frac{R^2}{2} - \frac{3R^{\cancel{4}\,2}}{4\cancel{R^2}} + \frac{\cancel{3}R^{\cancel{6}\,2}}{\cancel{6}_2\cancel{R^4}} - \frac{R^{\cancel{8}\,2}}{8\cancel{R^6}}\right] = \frac{16}{R^2}\left[\frac{4R^2 - 6R^2 + 4R^2 - R^2}{8}\right]$$

$$\alpha = \frac{16}{\cancel{R^2}}\left[\frac{\cancel{R^2}}{8}\right] = 2$$

$$\boxed{\alpha = 2}$$

d) $\underline{\underline{\beta}} = \dfrac{|\vec{X}|}{|\vec{X}_0|}$

$$\vec{X}(S;t_0) = \int_S \rho \vec{v} \cdot \vec{v} \times \vec{n} \, dS = \rho \int_0^R v^2 dS = \rho \int_0^R \left(v_{max}^2 \left(1 - \frac{r^2}{R^2}\right)\right)^2 2\pi r \, dr$$

$$\vec{X}_0(S, t_0) = \rho_m \cdot V^2 \cdot S \cdot \vec{n} = \vec{e_z} = \rho V^2 \cdot \pi R^2 \cdot \vec{e_z} = \rho \frac{v_{max}^2}{4} \pi R^2 \cdot \vec{e_z}$$

<u>ASSIM</u>

$$\beta = \frac{\cancel{\rho} \int_0^R \left(\cancel{v_{max}^2} \left(1 - \frac{r^2}{R^2}\right)\right)^2 2\pi r \, dr}{\cancel{\rho} \frac{\cancel{v_{max}^2}}{4} \pi R^2} = \frac{8\cancel{\pi}}{\cancel{\pi} R^2} \int_0^R \left(1 - \frac{r^2}{R^2}\right)^2 r \, dr$$

$$\beta = \frac{8}{R^2} \int_0^R \left(1 - \frac{2r^2}{R^2} + \frac{r^4}{R^4}\right) r \, dr =$$

$$\beta = \frac{8}{R^2} \left[\frac{R^2}{2} - \frac{2R^{\cancel{4}\,2}}{4\cancel{R^2}} + \frac{R^{\cancel{6}\,2}}{6\cancel{R^4}} \right] = \frac{\cancel{8}\,2}{R^2} \left[\frac{\cancel{6R^2} - \cancel{6R^2} + 2R^2}{\cancel{12}\,3} \right]$$

$$\beta = \frac{4\cancel{R^2}}{3\cancel{R^2}} \Rightarrow \boxed{\beta = 4/3}$$ //

e) FLUXO (VAZÃO) DE ENERGIA CINÉTICA $(\mathbb{C})$ ou (ϕ_c)

$$\mathbb{C} \Rightarrow \quad \mathbb{C} = \alpha \cdot \mathbb{C}_0 = 2 * \frac{1}{2} \rho V^3 S$$

$$\mathbb{C} = \rho \left(\frac{v_{max}}{2}\right)^3 \cdot \pi R^2 = \frac{\rho v_{max}^3 \pi R^2}{8} = \boxed{\frac{\pi \rho R^2 v_{max}^3}{8}}$$ //

f) FLUXO DA QUANTIDADE DE MOVIMENTO ($\vec{\chi}$) (Ex(6) pag 4)

$$\vec{\chi} = \beta \rho V^2 S \vec{e_z}$$

$$|\vec{\chi}| = \frac{4}{3} \rho V^2 S = \frac{4}{3} \rho V^2 \pi R^2$$

$$\vec{\chi} = \frac{4}{3} \pi \rho R^2 \cdot \frac{v_{max}^2}{\cancel{4}} \vec{e_z}$$

$$\boxed{\vec{\chi} = \frac{\pi \rho R^2}{3} v_{max}^2 \vec{e_z}}$$

VERIFICANDO RELAÇÃO: $\alpha \cong 3\beta - 2$

$$2 = \frac{4 \cdot \cancel{3}}{\cancel{3}} - 2 \quad \therefore \quad 2 = 2 \quad \text{OK!}$$

PARA CASA:

Fazer Ex: 4.1.1 itens a), b), c), e), f)

Atenção p/ caso importante (trabalhoso)

item e) Tubo cilíndrico - Mov turbulento

$$\vec{v} = v_{max} \left(\frac{R-r}{R}\right)^{1/7} \vec{e_z}$$

Atenção p/ Resposta: $V = \frac{98}{120} v_{max}$

$\alpha = 1,06$ $\beta = 1,02$