Pergamon

Engineering Fracture Mechanics Vol. 54, No. 2, pp. 263–300, 1996
0013-7944(95)00178-6

0013-7944/96 $15.00 + 0.00

A HISTORY OF FATIGUE†

WALTER SCHÜTZ
IABG, D-85521 Ottobrunn, Germany

Abstract—The history of fatigue from 1838 to the present is described in detail, with special emphasis on the German contribution in the time period of 1920–1945. A number of distinguished scientists and engineers, and their contributions to the further development of fatigue knowledge are specifically mentioned. Copyright © 1996 Elsevier Science Ltd.

1. INTRODUCTION

MANY BOOKS and papers about fatigue start with a more or less detailed account of the historical development of this branch of technology; they are, however, mostly limited to the description of results. With the present paper, the author strives for an evaluation of the importance of scientists and engineers and their work for the further development of fatigue technology and knowledge.

For this evalution, two criteria were established:

- Were the results of the work useful for the following generations or not? A positive example would be, for example, the Palmgren–Miner rule, still being employed the world over, 50 respectively 71 yrs after its publication. A negative example would be the "Damage Line" of French, which has only caused confusion, or the "over-" and "understressing" works of Kommers, which uselessly haunted people's minds for decades.
- Does the work in question only contain results or did the researcher also draw conclusions? A positive example would be Wöhler's allowable stresses for railway axles in the finite life region.

Since the author obviously knows the important German fatigue efforts better than those of foreign engineers, this paper possibly has an entirely undesired nationalistic German touch. On the other hand, practically all Anglo–American historical descriptions of fatigue give a biased account as well, because they do not mention the decisive German contribution in the period of 1925–1945. In that period, however, the foundations were laid for what we know today in fatigue and fracture mechanics. This will be discussed in detail in Section 6.

Out of the large number of engineers and scientists who have worked on fatigue problems, because of limited space only three are described in detail, namely Wöhler, Thum and Gassner, who all three fulfill the criteria mentioned above. Of course, many other names are mentioned, albeit briefly. Some border areas of fatigue, for example non-destructive inspection, are not discussed. Others, such as metallurgy or the development of fatigue testing machines, will be mentioned only briefly and where absolutely necessary.

Some earlier papers on the history of fatigue [1–3] have made the author's work much easier.

When studying the old works some interesting points attract attention:

- Our predecessors were, in some respects, very modern. Wöhler, for example, as early as 1860 suggested design for finite fatigue life [4]. In other respects, however, their opinions were astonishingly primitive and erroneous.
- Knowledge about certain methods was highly developed in one location, while a few kilometers away it was nonexistent, for example on shot peening.
- Decades after final clarification of certain problems, they continue being discussed over and over again in the literature and, what is more, they still haunt people's minds, for example the influence of testing frequency on fatigue life.

---

†The original German paper, published in *Materialwissenschaft und Werkstofftechnik* **24,** 203–232 (1993), was dedicated to Prof. J. Schijve on the occasion of his 65th birthday. The present English version is slightly modified and modernized, and was translated by Dr J. Lincoln, WPAFB.

- It takes a long time, sometimes decades, until parameters known for ages are treated scientifically. Further decades pass before this scientific knowledge is applied to engineering practice. One example is the scatter of fatigue life, implicitly taken into account in 1860 by Wöhler [4], first specifically mentioned in 1927 as "scatter", scientifically treated from 1945 onwards, but still not considered even today in some branches of industry.
- Many well-known old papers treat additional important problems besides the subjects for which they are still remembered today. For example, Wöhler measured the service stresses on railway car axles before his well-known fatigue experiments, and Palmgren in his paper of 1924 stated not only the linear damage accumulation but also presented a four-parameter equation for the entire *SN* curve and already specified a B10-life.
- Some well-known fatigue scientists and engineers wrote only a few papers on the subject, others were extremely productive for decades; Miner only authored five papers, Palmgren only three, but Thum and coworkers no less than 574!
- Fatigue failures of machines, plants and vehicles in service always led to significant efforts and advancements of the state of the art. This was as true for the work of Albert in the 1830s — in the mines of the Oberharz the conveyor chains failed — as for the work of Wöhler and his predecessors in England — when the axles of railway cars and locomotives broke. This is also valid for the German efforts of the 1930s — here the cause was the fatal fatigue failure of a Dornier "Merkur" wing strut. The work of Thum was prompted by many fatigue failures of machine and vehicle components in service. After WW II, a number of aircraft crashes, notably of two "Comets", resulted in more aircraft fatigue problems and in 1969 the fatigue fracture of an F-111 wing led to a complete change of the structural specifications of the US Air Force, combined with an immense fracture mechanics programme that is continuing to this day. In more recent times (1988) the near-fatal accident to an Aloha Airlines Boeing 737 was the cause of renewed efforts and investigation activities into the structural integrity of old and poorly maintained aircraft.
- In every time period there are one or more unrealistic ideas and solutions which at the time are followed enthusiastically by some distinguished scientists and engineers; with the benefit of hindsight, however, their delusions appear incredible!

2. 1837–1858. THE TIME BEFORE WÖHLER

The history of fatigue begins with Albert [5], who was a Royal Hannoverian "Oberbergrat" (civil servant for mines). In 1837 he published in Clausthal the first fatigue-test results known. For this purpose he constructed a test machine for the conveyor chains which had failed in service in the Clausthal mines. As early as that, he therefore tested actual components, not just the material! Since chains at the time could only be replaced by hemp rope which had to be imported at great cost, Albert invented the wire rope — surely more important than those first fatigue tests.

In 1842, Rankine [6], better known from thermodynamics by the "Rankine process", discussed the fatigue strength of railway axles. He suggested that these axles be forged with a hub of enlarged diameter and large radii, so that the grain flow would not be cut more than necessary by machining. York [7] conducted experiments with railway axles.

In 1853 the Frenchman Morin in his book *Resistance des Matèriaux* [8] discussed reports of two engineers responsible for horse-drawn mail coaches. The replacement of the axles of the coaches was prescribed after 60 000 km, an early example of the "safe life" design approach. The axles of other mail coaches were to be inspected thoroughly after 70 000 km, cracks to be repaired by "fire-welding". It was noted that those cracks mainly occurred at section changes.

The term "fatigue" was mentioned for the first time by the Englishman Braithwaite in 1854 [9], contrary to a widespread belief which ascribes it to Morin. Braithwaite, however, says [9] that a Mr Field coined the term. In his paper Braithwaite describes many service fatigue failures of brewery equipment, water pumps, propeller shafts, crankshafts, railway axles, levers, cranes, etc. Allowable stresses for fatigue-loaded components are also discussed.

In this period many disastrous railroad accidents due to fatigue occurred; for example, on 5th October 1842, a locomotive axle broke at Versailles, claiming the lives of 60 people [10], about the same number as were lost in the two "Comet" crashes of 1954. In the history of the "Institution of Mechanical Engineers" in London of 1854 it is mentioned that a member had seen a collection of hundreds, if not thousands of failed railway axles. As late as 1887 English newspapers reported the "most serious railway accident of the week", and in many cases these were due to fatigue failures of axles, couplings and rails, and claimed many lives. More references on this period can be found in refs [11–16].

3. 1858–1870: WÖHLER

Wöhler, Royal "Obermaschinenmeister" of the "Niederschlesisch–Mährische" Railways in Frankfurt an der Oder, measured the service loads of railway axles with self-developed deflection gages as early as 1858 [17] and 1860 [4]. To the author's knowledge, this has not been noted before by the many authors describing Wöhler's work.

Specifically, this was accomplished for a number of four-wheeled and six-wheeled freight and passenger cars on trips between Breslau and Berlin as well as Frankfurt an der Oder and Berlin. The measurements were carried out for 22 000 km. The deflection of the axle was scratched on a zinc plate by a scriber by means of a compound lever system. Only the largest deflection per trip was measured. According to Wöhler: "In order to know the force necessary for a certain deflection, the axle was bent by a dynamometer, which was fastened to the rims of the wheels". This means in our words that Wöhler even then calibrated the forces acting on the axles. In the corresponding figures of his paper those forces are given. Wöhler then discusses the largest axle deflection per trip and the corresponding service load, and calculates the bending and torsional stresses of the axle. He then compares the measured bending forces with those caused by the static axle load and arrives at a factor of 1.33; that is, in our present-day terminology, he determined an impact factor of 1.33. Wöhler then draws the following conclusions from his measurements: "The number of such cycles per trip is considerably smaller than the number of miles the axle travels during its life. Therefore, the safety requirements are met if the axles can withstand the maximum stresses measured as many times as its expected life in miles. If we estimate the durability of the axles to be 200 000 miles with respect to wear of the journal bearings, it is therefore only necessary that it withstands 200 000 bending cycles of the magnitude measured without failure". Thus Wöhler implicitly suggested design for finite fatigue life, taking into consideration even the scatter of fatigue life, or in other words, the probability of failure. Since no fatigue-test data were available to him at that early date, he estimates them and arrives at an allowable axle load for 200 000 cycles of 136 "Zentner" (*ca* 6800 kg).

Beginning in 1860 [4, 18–21], Wöhler published the results of fatigue tests with railway axles. Since the rotating-bending test machine he designed and built ran at a very low frequency, he designed new machines for carrying out axial-bending and torsion tests on different notched and unnotched specimens. In 1870 [20, 21, 21a] he presented a final report containing the following conclusions, often called "Wöhler's laws": "Material can be induced to fail by many repetitions of stresses, all of which are lower than the static strength. The stress amplitudes are decisive for the destruction of the cohesion of the material. The maximum stress is of influence only in so far as the higher it is, the lower are the stress amplitudes which lead to failure". Wöhler therefore stated the stress amplitudes to be the most important parameter for fatigue life, but a tensile mean stress also to have a detrimental influence.

From his quantitative results he draws the following conclusions about this mean stress influence: "Components loaded in tension and compression like connecting rods, wheels, balances, etc. must be stronger by a factor of 9:5 than components loaded only in tension, like bridge members or roof beams. The springs of railway cars are loaded by small amplitudes, but high maximum stresses. Therefore the allowable maximum stresses can be much higher than usual. And indeed this is the case, with the spring steel of a loaded car reaching a stress of 180 hundredweight per square inch. Quite often this value even reaches 900 or 1000 hundredweight per square inch and with good steel that is safe. This is important for the smooth ride of railway cars".

After a discussion of why a safety factor is necessary, Wöhler comes back once more to finite life design: "It must be taken into consideration whether unlimited or limited life is required for the component. It follows that different components need different safety factors. In any case two such factors are necessary, one for the relation between the maximum stress in service and static strength, and the other for the allowable stress amplitude."

Wöhler then suggests a safety factor of two for static strength and an additional one of two for fatigue strength. In his opinion this is adequate for all circumstances. These factors, however, are only valid for unnotched sections, because "the strength of joints in the form of riveted joints, keyed joints and such kind, and different shapes require special tests. The results of the tests with sharply notched specimens have proved the necessity of such special tests". Thus Wöhler correctly does not present the additional safety factors for these joints, but requires special tests.

The safety factors given above are only valid for infinite design life, because Wöhler continues: "For components with finite fatigue life other considerations apply: if, for example, it is known that the maximum bending stresses on a railway car axle occur when traveling over switches, and if the number of such switches during the life of the axles is known, it is in accord with the requirements of safety that the allowable stresses in the axle are those which lead to failure after many millions of cycles, that is for iron 160 hundredweight per square inch and for cast steel up to 220 hundredweight per square inch".

To Dr Fischer, Thyssen-Kassel Research Centre, the author is obliged for the relevant information that even today the allowable stresses for railway car axles are roughly fixed by the number of switches traversed.

In another paper of 1870 [21] which to the author's knowledge has never been cited before, Wöhler describes the dimensioning, design and material selection for railway car axles. According to Wöhler "the axles were sized so that they should never fail in service according to experience. It was therefore all the more embarrassing when they failed in large numbers. The cause of these failures was not immediately apparent nor were means of preventing them available. Crystallization due to vibrations, by earth magnetism and other dark notions were resorted to before finally it was decided to believe that the axles had failed because they were too weak. When this was finally realized, the loads due to which the axles broke were soon found".

Wöhler then describes the forces acting on the axle in service, for example the static load, lateral loads due to cornering, wind pressure, etc. He calculates the service stresses via the measured loads and the axle diameter. By comparing these stresses with the result of his fatigue tests he concludes that axles are completely safe. Furthermore he describes the allowable axle loads according to the "Technical Regulations of the German Railways", which depend on the material, diameter, etc., and which also contain rules about the size of the radii between the axle and journal diameter. The "metallurgical size effect" was already taken into account at that time, i.e. the allowable stresses for thinner axles were higher than those for thicker axles, "because it was assumed that smaller dimensions allow the material to be worked better and therefore would result in higher fatigue strength".

Even crack propagation is already mentioned: "Those seams which are not entirely superficial, especially those which are radial, propagate in service. This property of crack propagation in service is more pronounced the more carbon the material contains. Most striking in this respect is cast steel. The author has observed several times that fine, hardly visible longitudinal cracks in cast steel axles which only appeared to be forged-in seams, after several years in service, propagated into the axle for 20 mm and more, and made its replacement necessary".

In summary one can only admire the work of Wöhler in its entirety, encompassing the measurement of service loads, the calculation of the corresponding service stresses, the design for finite life including scatter (probability of survival) up to the observation of crack propagation and the quantitative suggestions for the decrease of the notch effect. The next known mention of design for finite life occurred about 75 yrs later (Almen and Boegehold [22] for differential gears, and Thum and Bautz [23] in a more general sense).

The next service measurements known to the author date from the 1920s and 1930s, the first measured load spectra by Batson and Bradley [24], and Kloth and Stroppel [25, 26].

Wöhler incidentally represented his test results in the form of tables. Only his successor Spangenberg [27–29] as director of the "Mechanisch-Technische-Versuchsanstalt" in Berlin plotted them as curves, although in the unusual form of linear abscissa and ordinate. The *SN* curves were called "Wöhler curves" since 1936 [26].

Not until 1910 did the American Basquin [30] represent the finite life region of the "Wöhler curve" in the form "log σ_a on the ordinate, log N on the abscissa" and describe it by the simple formula

$$\sigma_a = CR^n,$$

which is still used even today.

In a large table Basquin gives some numerical values for C and n, based for the most part on Wöhler's tests 50–60 yrs before! We can deduce from this that Wöhler was well known to the research scientists of the following generations and that not very many new data had been obtained in the meantime.

In 1867 the English technical journal *Engineering* commented on Wöhler's exhibits at the Paris World Trade Fair: "Long after the many exhibits which have won medals at this fair will have been forgotten, the fundamental work of Wöhler will be remembered" [31]. The Englishman Gough honoured Wöhler in 1924 in his book, the first on fatigue [32], in the following way: "Wöhler's work will survive the time as a monument to his genius as an engineer and scientist". The Americans Moore and Kommers dedicated two pages of the introduction to their book [33] to Wöhler's work and life.

In the author's opinion, Wöhler differs from all of his predecessors — and most of his successors, some of them to this day — in that he always had in mind the basic problems the engineer must solve when designing for fatigue:

- the service loads and stresses, and
- the endurable and, derived therefrom, the allowable stresses must be known.

On the other hand Wöhler's description of the influence of mean stresses and notches on fatigue life does not appear so important to the author. The influence of notches was known before him, albeit in a qualitative way. Wöhler just quantified it.

After Wöhler left the Prussian Civil Service, the Ministry for Trade and Business ordered the continuation of his experiments in the "Gewerbeakademie" in Berlin. In 1876 Wöhler [34] proposed official requirements for the tensile and yield strengths of steels and irons to the technical committee of the German Railways. In 1881 the steel makers of Germany [35] and Austria protested violently against his proposal — fortunately to no avail.

For setting up such quantitative requirements Wöhler suggested establishing official testing laboratories. According to Martens this was the founding initiative for the "Royal-Prussian Testing Laboratories" which, after many changes of name, are known today as the "Bundesanstalt für Materialforschung und -prüfung" in Berlin.

4. 1870–1905

The next name to be mentioned would be Bauschinger, Professor of Mechanics at the Munich Polytechnical School, which now is the Technical University of Munich. The Bauschinger effect, in his words [36–38] "the change of the elastic limit by often repeated stress cycles", is the basis for the hypotheses of Manson and Coffin which originated in the 1950s and which are still being utilized today in the LCF field and for fatigue-life prediction according to the local concept.

Kirsch [39] in 1898 was the first to calculate the stress concentration factor of 3.0 for a cylindrical hole in an infinite plate. The Englishmen Ewing and Humfrey [40] in 1903 observed so-called slip bands on the surface of rotating-bending specimens. This probably was the first metallurgical description of the fatigue process. Since that time a huge number of metallurgical papers on fatigue has been published. The author considers as important the dislocation theory of Polanyi [41] of 1934, which Orowan [42] in 1939 applied to fatigue, and which was the basis of many metallurgical papers on the fatigue behavior of metals.

Orowan suggested a process of cyclical strain hardening, which after depletion of the local ductility leads to cracking. The Orowan model therefore is a discontinuous process. In recent times, the observation of cracks in the electron microscope has shown that fatigue is a continuous process.

5. 1905–1925

The years between 1905 and 1925, in the opinion of the author, were the domain of the British and Americans. Names like Smith [43–46], Haigh [47–51], Gough [32, 52–56], Griffith [50, 58], Inglis [59], Kommers [33, 60–68], Moore [66–73], etc., appear in the literature.

The first full-scale fatigue test with a large aircraft component was carried out at the Royal Aircraft Establishment in the U.K. [74, 75]. In the literature the notch effect on actual components — as apart from that on specimens — is quoted. The term "notch effect" ("Kerbwirkung" in German) was probably coined by the German Heyn in 1914 [76], but implicitly it was already discussed by Rankine [6] in 1842 and by all his successors including Wöhler. The first experiments to improve the fatigue strength of components probably were carried out in the U.K. during the first World War [77]. Further important references of this period are found in refs [78–94].

6. 1920–1945, WITH SPECIAL EMPHASIS ON THE GERMAN CONTRIBUTION

In this period of time the foundations were laid for almost all the fatigue knowledge we enjoy today. The following topics originated or were investigated:

- the "Gestaltfestigkeit" by Thum and Bautz [95, 96];
- the fatigue strength under variable amplitudes ("Betriebsfestigkeit") by Gassner [97];
- the measurement of fatigue loads and stresses, on many components, for example by Lehr [98];
- the measurement of load spectra on automobile springs by Batson and Bradley [24], on agricultural machinery by Kloth and Stroppel [25, 26], on aircraft from 1932 onwards by Kaul, Filzek, Freise, etc. of the DVL [99–106], and from 1931 by the NACA [107–110];
- the realization that higher-strength materials do not result in higher fatigue strengths of components because they always contain notches, by Gassner for aircraft aluminum alloys [111] and by Graf [112] for welded and riveted joints of structural steels;
- the mechanical methods to improve fatigue strength by inducing compressive residual stresses, like cold-rolling, shot-peening and coining by Föppl [113–115], and Thum and coworkers [116–118];
- the first books on fatigue in 1924 by Gough in the U.K. [32], 1927 by Moore and Kommers [33] in the USA, 1929 by Graf in Stuttgart [119], 1929 by Föppl in Braunschweig [120], 1934 by Herold in Vienna [121], 1937 by Cazaud in France [122], 1937 by Serensen in Russia [123], 1941 by the Battelle Institute in the USA [124]. (Herold's book in 1944 was translated into English; J. Edwards, Ann Arbor, Mich.);
- the damage accumulation hypotheses for fatigue-life prediction under variable amplitudes in 1924 by the Swede Palmgren [123], 1937 by the American Langer [126], 1938 by the Russian Serensen [127] and 1945 by the American Miner [128];
- inductive strain gages [130], for example by Lehr [98], and as a direct counting device giving the spectrum of level crossings by Svenson [129];
- the electrical wire strain gage in 1939 by the Americans Ruge and de Forest from MIT [135], and independently by Simmons and Clark from Caltech [131];
- the statistical scatter of the static strength of materials in 1939 by Weibull [132];
- fatigue tests with large numbers of specimens and their statistical evaluation by Müller–Stock in Braunschweig [133, 134];
- the first crack-propagation tests in 1936 by the American de Forest [135];
- the foundation of fracture mechanics in 1920 by Griffith [57, 58];
- seminars and meetings about fatigue [136–151] by Thum, the VDI, the Lilienthal-Gesellschaft, the VDEh, the DGM in Germany, the Institution of Mechanical Engineers in the U.K., etc.

As the above description shows, this time period was the domain of the Germans — at least in the opinion of the author. The proof of this may be seen in Fig. 1 showing the number of publications in the area of fatigue between 1925 and 1945, subdivided by the countries U.S.A., U.K. and Germany. Other countries did not play a role, at least numerically. The number of papers per year certainly is not positive proof of the importance of the work carried out in the individual countries. However, looking at the papers in more detail, one finds that the essential ones in the area of Gestaltfestigkeit and the application to industrial problems were almost exclusively

German. As further proof of the author's opinion, an English book of 1935 [151] may be mentioned which contains the most important papers from 1927 to 1935 — at least this is claimed in its introduction: about 50% of all references are German. Finally, as a third proof Spaulding of Lockheed stated in 1957 [152] that for lack of design data of their own in the 1940s the load-spectrum measurements of Freise [103] were used in the U.S.A.

A good example for the state of the art in Germany at that time is the paper "Formgebung und Werkstoffausnutzung" by Lehr [98], MAN Augsburg, who in 1941 described static and dynamic strain measurements on connecting rods of an automobile engine up to 2500 rpm utilizing induction strain gages. He also describes fatigue tests with welded and riveted joints for steel bridges, the optimum shape of a lug, the influence of tensile strength on the fatigue stress-concentration factor K_f of shafts under bending and torsion, the size effect, the allowable stresses for hubs, the optimum shape for hollow crankshafts (this is very modern today!) and the load spectra of various aircraft. Employing just these data and the references given in ref. [98], even today a quite modern seminar could be given to designers!

On the other hand, Lehr stated his astonishment that by "blowing steel shot of a certain size at torsion bars" (i.e. shot-peening) the fatigue life could be improved. In the same year, Walz of the Mauserwerke in Oberndorf, only 100 miles from Augsburg, wrote a brochure "Federfragen" [153], in which the influence of shot-peening time, shot size, type of peening machine, etc., on the fatigue life of springs, even of case-hardened ones, was discussed at a very modern level; while Opel, probably based on General Motors experience, shot-peened their valve springs from 1935 onwards.

In 1920 Griffith of the Royal Aircraft Establishment, U.K., developed the basis of fracture mechanics [57]. Griffith later became chief engineer of Rolls Royce aircraft engines and also distinguished himself in the development of the gas turbine. In ref. [57] he showed by tests on the

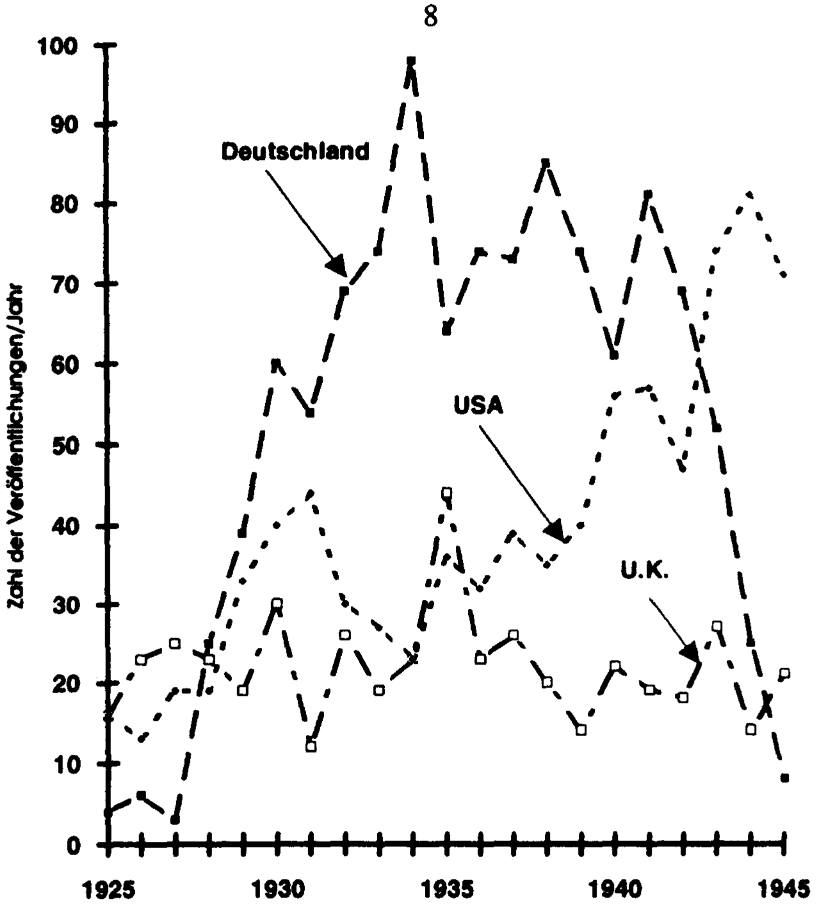

Fig. 1.

brittle material glass that small cracklike scratches considerably reduced the breaking strength and that the crack size also had an influence. He also described this process by formulae.

The 1924 book of Gough [32] contains the first account of the influence of surface roughness on the fatigue limit and also the stress-concentration factors of V-shaped notches on the basis of photoelastic measurements of Coker. By comparing the result of fatigue tests with Coker's data, Gough concludes that the stress-concentration factor is not fully effective, i.e. $K_t > K_f$. The important conclusion for design, however, apparently was not drawn by Gough, but was left to Thum some years later.

The Englishman Haigh, well known for his other fatigue work, in 1917 first mentioned corrosion fatigue [51], while in 1929 the American McAdam [154] carried out many corrosion-fatigue tests which are still referenced today by some authors, probably in ignorance of the huge mass of corrosion-fatigue data obtained in the ECSC offshore steels programme [155, 156].

All fatigue data show scatter; this was explicitly mentioned for the first time by Moore and Kommers [33] in 1927; the obvious conclusions for design purposes, however, were not drawn until much later. In the same year Hort [157] wrote for example at a fatigue meeting of the German Metallurgical Society: "If the service loads are known, the fatigue strength only has to be higher by a very small amount". Müller–Stock [133] in 1937 evaluated his almost 300 fatigue tests statistically, employing the Gaussian Normal Distribution and concluding that it did not fit his data. In a later paper [134] he utilized the log–normal distribution — probably for the first time — which fitted the number of cycles for failure very well.

In 1924 the Swede Palmgren authored his famous paper [125]. Besides the Palmgren–Miner rule — as it is called today — it contains a four-parameter equation extending from the tensile strength to the fatigue limit for the *SN* curve, which the Swede Weibull also used much later. Palmgren also specified the B10-life as a design criterion, to the author's knowledge the first numerical description of a probability of survival for fatigue-loaded components — albeit only for ball bearings.

In 1937 the American Langer [126] stated the identical damage-accumulation hypothesis — probably in ignorance of Palmgren's work. Langer's paper found widespread interest, as can be deduced from Kommers, Thum and Bautz's discussions. Langer already separated the fatigue life into the crack initiation and crack propagation phases and suggested a damage sum of 1.0 for each phase. He also wrote that for the application of his hypothesis crack-propagation *SN* curves were necessary.

The damage-accumulation hypothesis

$$\sum_{N_i}^{n_i} = 1.0$$

is again presented in 1945 by Miner [128]. Contrary to his predecessors, he also stated many restrictions which would have made the application of the hypothesis impossible in practice, had later users, including himself [158], taken them into consideration. On the other hand, Miner was the first to check his hypothesis by fatigue tests — if unsuitable ones.

The electrical resistance strain gage was invented concurrently in 1939 by the Americans Ruge and de Forrest at the MIT [135], and by Simmons and Clark at Caltech [131]. Although it was at first considered useless by MIT "experts", it was already utilized to a great extent in the U.S.A. during WW II [159–163].

The comprehensive German measurements of aircraft spectra discussed below were not hindered much by not having this type of strain gage available. The DVL (Deutsche Versuchsanstalt für Luftfahrt) developed an induction strain gage weighing only five grams [164], the output of which was registered on the DVL glass scratch gage [164] which already had a mechanical feed, in contrast to the concurrent NACA VG recorder. On stationary structures, Lehr described in ref. [165] that he carried out 500–1000 strain measurements on a large component employing his inductive strain gage.

A lot of unnecessary confusion was caused by the "under-" and "overstressing" effects maintained by Kommers and others [166–168] — from 1930 until long after WW II. Had they existed, they would have been of no practical significance. However, a statistical [169, 170] check of the "coaxing" due to understressing proved they were nonexistent. Kommers had overlooked the relatively large scatter of the fatigue limit; that is, the alleged effects had occurred by accidentally using specimens with a high fatigue limit for the understressing tests. The following example, however, shows the influence of Kommers' claims: in a German paper of that time, in all seriousness the importance of cautiously breaking-in the crankshaft of a new automobile engine is stressed in order that its fatigue limit be thus increased by "understressing"!

Similar objections can be raised against the "damage line" of French [171], for which even Gassner and Bautz fell. As the tests of Müller–Stock [133, 134] proved as early as 1938, there is no unique damage line. Even a few cycles in the finite-life region above the fatigue limit do damage; by increasing the number of these cycles, the corresponding Wöhler curve is shifted more and more to the left and down. The author confirmed this by hundreds of similar fatigue tests in the 1960s [172].

The German dominance claimed by the author in the time period between 1920 and 1945 can be attributed primarily, but not exclusively, to Thum, Föppl, Graf, and Gassner and his colleagues at the DVL in Berlin and Göttingen.

Graf at the Technical University of Stuttgart mainly investigated the fatigue behavior of riveted and welded joints in structural steels [112, 119, 173–193]. He showed that it does not depend on the static strength of the steels employed — a knowledge which is new even today to some so-called experts.

Föppl and coworkers at the Technical University of Braunschweig [114, 115, 120, 194–205] primarily dedicated themselves to the mechanical procedures to improve the fatigue behavior, i.e. cold-rolling, shot-peening and coining. Föppl obtained a German patent [206] for cold-rolling in 1929 and two US patents in 1934 [207]. The first patent for shot-peening was granted to the Röchling Steel Works in 1929 [208], although the US automobile industry had used shot-peening for the increase of the fatigue strength before that time. Nevertheless the report B-84 of the Materials Committee of the Verein deutscher Eisenhüttenleute (VDEh) of 17 July 1924 [208] mentions this possibility of improving the fatigue strength of automobile springs. Föppl himself in 1942 lamented [209] that the industrial application of shot-peening and cold-rolling had probably progressed further in the U.S.A. than in Germany.

Thum held the first chair for Metallurgy in Germany from 1927 onwards at the Technical University of Darmstadt. He founded the doctrine of "Gestaltfestigkeit", which maintains that for a high fatigue strength the shape of the components, as developed by the designer, is much more important than the material itself. The term "Gestaltfestigkeit" was not coined by him, but — according to Dr Sigwart [210] — by one Mr von Moellendorf. According to Thum, Gestaltfestigkeit "is a strength value which depends on the magnitude and the type of loading as well as on the material, and especially on the component's shape".

The fatigue stress concentration factor K_f (in German called β_k) was also created by Thum. He correctly called it a "crutch with which one can limp from the unnotched specimen to the notched component". This means he already knew that K_f is not a characteristic material value — much less a characteristic component value — and that it has to be determined anew in every case by exactly those fatigue experiments which it is supposed to make unnecessary.

The importance of Thum's work is that he never just carried out ad-hoc tests, but that he collected the knowledge scattered all over the literature, closed gaps in this knowledge purposefully by dissertations and adapted them in a form useful to the normal engineer and designer. In the opinion of the author, this is far more valuable than some new scientific findings — at least in the field of fatigue.

Thum, therefore, was of enormous importance for the engineering application of the available knowledge of the time. The 524 (!) papers of Thum and his coworkers from 1922 to 1956 are testimony of his efforts. Below is a small selection of the topics treated:

- stress-concentration factors [211–215]
- quick methods of determining the fatigue limit [216]
- axles [217]
- influence of machining marks on the fatigue limit [218]
- influence of the testing frequency on fatigue limit, also under corrosion [219, 220]

- flame-hardening [221]
- laws and rules of fatigue fracture [222]
- the fatigue limit at low temperatures [223]
- the influence of prestressing on the fatigue limit [224]
- Goodman diagrams for cast iron [225], magnesium alloys [226] and bolts [227]
- bolts with shanks of reduced diameter [214]
- residual static stresses and their influence on fatigue strength [228–230]
- nitriding [231]
- case-hardening [232]
- fatigue strength of automobile springs [231, 233]
- size effect [234]
- cast crankshafts [235]
- drive shafts [236]
- welded joints [237, 239], riveted joints [238]
- splined shafts [218]
- corrosion fatigue [219, 230]
- shot-peening [240, 241]
- fatigue strength of hub joints [215]
- fretting corrosion [242]
- malleable iron [223]
- gears [243]
- fatigue strength in the finite life region [23, 244]
- allowable fatigue stresses [245].

Further papers by Thum and coworkers are refs [246–254], three important German papers of this time are refs [255–257].

Thus, Thum in an ideal manner fulfills the criteria set up by the author in the introduction:

- he not only carried out tests, but he drew useful conclusions from them, shown by most of the papers mentioned above;
- his work was made use of by his contemporaries and successors. This was helped by many of his assistants rising to important positions in industry, where they disseminated the ideas of "Gestaltfestigkeit". In the 1950s and 1960s there was hardly any large German firm in which a former coworker of Thum was not at work. Representative of a large number of such engineers are Dr Sigwart at Daimler–Benz, Dr Petersen at the Metallgesellschaft and Dr Erker at the MAN.

As many of Thum's papers show, he brought component fatigue testing back into favor — as against unnotched specimen testing. Even today, component testing is the only way to ensure sufficient fatigue strength in service. Thum wisely realized that it is difficult, if not impossible, to transfer the fatigue behavior of specimens to that of an actual component — which is true even today. As an example of such component testing, the paper by Bandow [235] may be cited, in which the fatigue optimization of a hollow cast crankshaft is described in many individual steps, something which is praised today as a modern achievement!

Almen and Boegehold in 1935 [22] discussed finite fatigue life as a design goal (for transmission gears of cars); Thum and Bautz in 1937 [23] and Bautz in 1942 [244] also treated this topic, but in a much more general form.

On Thum's advice his coworkers Bautz and Bergmann in 1938 founded the "Bautz–Bergmann–Werkstoffberatungsgesellschaft", which later became one of the founding institutes of the "Laboratorium für Betriebsfestigkeit" (LBF).

According to the VDI [244], an "Atlas der Gestaltfestigkeit" was started in 1941 in close cooperation with Thum, but unfortunately it was not published. It was to contain fatigue tests on components and models of components, including materials and machining details, and was to serve as a starting point for the calculation and design of fatigue-loaded components. Such a book would be a valuable mine-source for every fatigue expert even today, more valuable than many new books on fatigue.

Since the sizing and design of a component for fatigue is not possible if the service loads and stresses are not known — Wöhler had realized that much before, by about 1930, the measurement of service stresses set in. The lack of simple strain gages was not a serious handicap — as mentioned above, the electrical resistance strain gage was invented only in 1939.

In 1933 Teichmann and Michael [258] stated: "In aircraft service, stress amplitudes are not of identical magnitude as in a laboratory test, but large and small amplitudes occur with different frequencies". Küssner at the DVL Göttingen expressed similar ideas even in 1931 [259, 260]. It is therefore surprising that the first measurements of such load spectra did not occur in a hi-tech area but in the automobile and agricultural machinery sector. In 1929 Batson and Bradley [24] showed a load spectrum for an automobile spring, while Kloth and Stroppel [25, 26] measured load spectra for agricultural machines in 1932 and 1936; the author does not know whether these English and German measurements had any consequences, i.e. were used for design. It was a different story, however, with the aircraft measurements of the DVL. Load spectra were measured from 1932 onwards and published from 1938 onwards [99–106]. Kaul [99] in 1938 wrote: "As a measure of the loading of the wing, the acceleration at the centre of gravity, the wing deflection or the strains of highly loaded wing components can be chosen". All three types of measurements were utilized, and Kaul mentioned in ref. [99] dynamic overswing, suggested the level-crossing counting method as well as a standard load spectrum and also stated the basic idea of the variable-amplitude test. This paper [99], like many other German publications of this time [100–106, 260, 261] is distinguished by an extremely high standard, a standard which was reached internationally only 15–20 yrs later — at least in the author's opinion.

Freise [103] utilized the DVL glass-scratch strain gage, described in 1932 [164], to measure the strains in the wing spars of two aircraft types of Lufthansa for about 60 flying hrs. Kaul's measurements of 1938 [99] contained the c.g. acceleration spectra for six other aircraft over about 700 hrs. In 1941 combat load spectra for 300 flying hrs were measured [106], among them spectra for the famous–infamous dive bomber Ju-87 or the Ju-88 in air raids on Malta. In ref. [102] a further 500 hrs on modern civil aircraft are presented. Gassner and Teichmann at the beginning of 1945 describe further combat load spectra for another 1000 flying hrs.

These spectra were the basis for the variable-amplitude fatigue tests (Betriebsfestigkeitsversuche in German) of Gassner, who in 1939 described his fundamental ideas in the paper "Fatigue Tests on Aircraft Structures" [97]. Already in 1927 Brenner [263, 264] and in 1931 Hertel [265] had published constant-amplitude fatigue tests on large aircraft components, Hertel for example on wing spars of steel, aluminum and wood. The reason for these efforts was the crash of a Dornier "Merkur" on 23 September 1927 through fatigue failure of a wing spar attachment near Schleiz in Thuringia, killing the six people aboard [74, 266]. Shortly afterwards a Handley-Page W-10 crashed into the Channel, caused by the fatigue failure of a connecting rod bolt, and on 27 July 1934 a Curtis "Condor" accident killed 11 people by fatigue failure of a wing strut, again in Germany (near Tuttlingen in Swabia) [266].

Gassner in his Ph.D. Thesis of 13 October 1941 [267] describes his variable-amplitude fatigue test as follows: "The main idea is to apply stress cycles of various amplitudes in steps simulating the mixture of high and low loads in service". To call Gassner the "father" of the variable-amplitude test only is a massive depreciation of his merits. Much more complex variable-amplitude tests, for example, were performed by Lockheed [268] from 1943 onwards. But these were purely ad-hoc tests, fitted to the actual, individual problem, and therefore they allowed no generally valid conclusions. Gassner established the topic of "Betriebsfestigkeit" (operational fatigue strength) which can be described as follows: dimensioning (sizing) of a component for finite, but sufficient fatigue life under variable loads. This is accomplished by

- measuring the service stresses in the form of a stress spectrum employing the correct counting method, also counting the number of cycles per flying hr, km, etc.;
- determining the corresponding forces or moments for obtaining generally valid load assumptions for similar components;
- extrapolating the always too short measured spectrum to the one to be expected in service;
- selecting a suitable return period containing rare high stress cycles in order to include their nonlinear damaging effects;

- if at all possible, standardizing the shape of the spectrum, for example, specific spectra for civil aircraft, military aircraft and automobile components, respectively;
- simulating the service spectrum by a blocked variable-amplitude test (programme test) and — after this is possible with suitable test machines — by a random fatigue test with the component;
- plotting the results in the form of "Gassner"-curves with the maximum amplitude of the spectrum on the ordinate and the total number of cycles on the abscissa, both to a logarithmic scale;
- considering the scatter of fatigue lives by a safety factor, calculated on a statistical basis, to obtain the necessary probability of survival. Its numerical value depends on the component in question, i.e. an automobile component has to have a high, a ship component a low P_s.

The result is the "allowable" fatigue life in flying hrs, km, etc.

This short description pictures the state of the art of "Betriebsfestigkeit" in about 1960 (without the random sequence). But as early as 1939 [92, 269], 1941 [260] and 1945 [262] most of the above-mentioned steps can be found in Gassner's papers.

For the blocked variable-amplitude tests, new test machines were necessary. Gassner developed and patented these together with Erlinger at Schenck [270].

The German aircraft industry enthusiastically welcomed Gassner's variable-amplitude test [271], but there was also considerable criticism [272, 273]. Degenhardt at Junkers described [271] v.a. tests with the main spar of the Ju-88, including 57 (!) tests with various designs of the tension chord joint of the Ju-88, a large component. After a v.a. test with a sheet-metal wing spar of a transport aircraft, a residual static strength test was carried out in the cracked state that yielded a "just sufficient safety". Gassner and Teichmann stated in January 1945: "The v.a. test has found widespread acceptance with the manufacturers".

Teichmann [274, 275] and Gassner [267] discussed as early as 1939 and 1941 that the one-parametric level-crossing counting method — then customary — did not represent the actual fatigue process. Consequently they required a two-parametric counting method and made some suggestions towards a solution. However, it took a further 30 yrs until the Japanese Matsuishi and Endo in 1969 published the "rainflow" counting procedure [276] which met all the old requirements. It is less known, however, that at about the same time the Dutchman DeJonge at the NLR [277] also published the rainflow procedure; he called it "range-pair-range".

Besides "Betriebsfestigkeit", Gassner undertook many other investigations [278–283]. For example, he determined the fatigue properties of the then new ultra-high-strength aluminum alloy of the AlZnMg-type (7000-type). The result: they do not differ from those of the Dural Type AlCuMg (2000-type), which by the way had been developed by Wilms [284] in 1911 for the Zeppelin. This result would not be remarkable by itself — many of Gassner's contemporaries would also have discovered it — were it not for the conclusions Gassner drew: if the higher static strength of the new alloy is taken advantage of, a shorter fatigue life will result, because it only makes sense to use the higher-strength and more expensive material if higher allowable stresses are employed, and that results in a shorter fatigue life. By such conclusions Gassner — like Wöhler and Thum — distinguished himself from his contemporaries; therefore, in the opinion of the author, Gassner, like Thum and Wöhler, was a great engineer.

From 1931 onwards also the NACA, predecessor of NASA, measured gust loads on aircraft [107–110], using the NACA VG-recorder. As it had no feed, however, load spectra could not be determined. Pugsley in ref. [285] mentioned that during WW II many load or strain measurements were carried out on English aircraft.

Fatal crashes due to fatigue failures are known of in no less than 20 Vickers "Wellington" bombers [74, 266], while Gassner describes in ref. [286] the wing fatigue failure of a Junkers Ju-52 in South America, and according to Seelhorst [287] 10 Me-110 were lost due to fatigue failures of ribs of the horizontal tail, which caused flutter. (As late as 1959 two Lockheed "Electras" crashed after the fatigue failure of an engine mount, which resulted in wing flutter.)

Further well-known Germans in this period were von Houdremont [289–291], Pomp [292–298] and Hempel [299–304] of the Kaiser Wilhelm Institute, Mailänder of the Krupp Research Institute [305–312], Bollenrath [313–318] and Brenner at the DVL [263, 264, 319, 320], Siebel [165, 256, 321–324] at the Technical University of Stuttgart, Klöppel [140, 325, 326] at the

Technical University Darmstadt, Gürtler [327–330] and the previously mentioned Lehr [98, 331–336] in Stuttgart and later at the MAN in Augsburg [98, 331–336]. Lehr especially also had great merit in the dissemination of fatigue knowledge to industry.

Neuber in 1937 published the first comprehensive book [337] ("Kerbspannungslehre") on the theoretical calculation of stress concentration factors K_t and even fatigue stress concentration factors K_f. This book was translated into English in 1946 at the suggestion of Kuhn of NACA [338], and it was the basis for a large research programme of the NACA which will be mentioned in the time period 1945–1960. Bürnheim in his Ph.D. Thesis [339] of 1944 systematically investigated the fatigue strength of hundreds of riveted aluminum joints. This work was used as a reference for several decades.

In other countries the following engineers and scientists must be mentioned — in addition to those already quoted: Peterson at Westinghouse [340–348], whose later book *Stress Concentration Design Factors* became world famous; Horger at Timken [349–356], who investigated the size effect, the testing of large components and cold-rolling; Ròs at the Swiss "Eidgenössische Materialprüfanstalt" [357–360]; Templin at Alcoa [361–363]; and Zimmerli [364], who wrote many papers on shot peening.

7. THE PERIOD OF 1945–1960

This was the time period in which the harvest of the years 1920–1945 was brought in. In all industrial countries fatigue was investigated. The number of papers increased so considerably that only a small percentage can be mentioned here; fatigue meetings and books on fatigue however are all cited — as far as is known to the author. These efforts were furthered by failures on all types of fatigue-loaded structures and vehicles. Besides the well-known crashes of the "Comet", for example, there were a great number of fatigue failures of truck or automobile components. In the aircraft field, however, the fatigue problem was particularly acute, because many new English civil aircraft with single-spar wings were based on military aircraft of WW II [288], but required a much longer fatigue life of about 30 000 hrs as compared to 5000 flying hrs for a bomber.

The de Havilland "Comet", designed in about 1948, the first commercial jet aircraft of the Western world, had an operating altitude about twice that of contemporary propeller-driven aircraft. Therefore the pressurized fuselage had to support higher stresses. In 1954 two "Comets" crashed, one near Elba, one near Naples, by failure of the fuselage at a window cutout [365]. In a large research and test programme [366], the cause was clarified according to the level of knowledge of the day: the full-scale fatigue test had been carried out with a fuselage which had before been pressurized to twice the maximum pressure differential in service. This was done to save a static test aircraft. In the window corners beneficial residual compressive stresses were thereby induced, which obviously were not present in the accident aircraft. Besides, an ultra-high-strength aluminum alloy of the 7000 series had been used, the unfavorable fatigue behavior of which Gassner had already described in 1941. As late as 1987 Swift of the FAA [367] showed, however, that in reality a design fault had been the cause of the accidents: the fuselage frames of the "Comet" were in one piece, whereas in more modern aircraft types they are built up of two independent frames. Therefore, a fast fracture could not been contained in the "Comet" design.

Because of the "Comet" accidents, complex flight-by-flight tests with the complete aircraft structure, so-called full-scale fatigue tests, became the rule, the pressurized aircraft fuselage in a water tank (later air was used for cyclic pressurization), the wings loaded by servohydraulic cylinders with the ground-to-air cycle and 10–40 gusts of different magnitude per flight. Earlier aircraft had been tested in a much more simple way and often in parts, the wings for example only by the ground-to-air cycle and 10 gusts of equal magnitude per flight.

C.g. acceleration or stress spectra of commercial and military aircraft were measured over hundreds of thousands of flying hours, primarily in the U.K. [368] and the US [369–372] by NACA; for the Vickers "Viscount" alone Williams in ref. [388] presented stress spectra for 150 aircraft with eight different airlines over 300 000 flying hours. Special online counting devices were invented in the U.K.; for example, the Vickers "Strain-Range Counter" or the "RAE Fatiguemeter", employing statistical methods thought to be superior to the level-crossing method. As a consequence of the "Comet" accidents, the Royal Aircraft Establishment (RAE) quickly became a center of aircraft fatigue work.

Full-scale fatigue tests with more than one aircraft of one type, mostly of vintage WW II military ones, were performed in the U.K. [373, 374] and the US [375, 376], in order to gather general experience on the fatigue behavior of actual aircraft structures as opposed to small specimen data.

In addition, countries which had not come to the fore earlier supplied important contributions to the fatigue of aircraft structures, for example Australia: beginning in 1948 and until about 1970, the Aeronautical Research Laboratories (ARL) carried out full-scale fatigue tests with 180 wings of the P-51 "Mustang", left over from WW II [377, 379]. Topics investigated were the behavior at constant and variable amplitudes, a complete Goodman diagram for wings of 2024-T3 was established, the accuracy of Miner's Rule for the wing structure was investigated and a highly sophisticated statistical treatment of the scatter observed was developed, altogether a very meritorious research programme. The fatigue behavior of other military aircraft of WW II vintage was also investigated [380, 381].

Besides the "Comet", several other aircraft types suffered fatal crashes due to fatigue, for example the Lockheed "Electra" previously mentioned. Williams in 1965 [288] gave a comprehensive summary; one cause for accidents and incidents cited again and again in ref. [288] was: the high-strength aluminum alloys of the 7000 series have unfavorable fatigue properties (see Gassner in 1941!).

Beginning in about 1955 a discussion set in about "fail safe" and "safe life". Safe life means that the aircraft component in question has to be scrapped on reaching the end of its previously determined life; fail safe means that the failure of a primary member by fatigue or otherwise must not endanger flight safety. As early as 1919 (!) the Englishmen Pippard and Pritchard [382] had discussed this requirement and to a certain extent post-WW II piston-engined aircraft like the DC6, DC7 or Lockheed "Super-Constellation" were fail safe with regard to damage from without or within, as for example Spaulding showed [383]. Fail safety as a design requirement was, however, probably first employed with the Lockheed "Electra", and the first commercial jet aircraft B-707 and DC8. The large fail-safe test programme with the "Electra" fuselage [384], which withstood the sudden cutting of a fuselage frame at maximum differential pressure without failure, did not however prevent two fatal crashes of this aircraft due to fatigue fractures of engine mounts, since these were not fail safe. The required calculation procedures were also developed, still without the use of fracture mechanics. Generally, the Europeans (see Williams [288]) were more sceptical about fail safety than the Americans; Spaulding [383] goes so far as to claim that with a fail-safe design, full-scale fatigue tests à la "Comet" would not be necessary any more.

As experience has shown in the meantime, fail-safe construction is absolutely necessary, but it has not prevented several fatal crashes by structural failures due to fatigue. Certainly the fuselages of large jet aircraft have withstood hand grenade explosions from terrorists without endangering flight safety. However, up to most recent times the fact was not considered that at the fatigue failure of one element, most other, similarly stressed, elements would also suffer fatigue cracks. Thus, for example, many rivet holes in longitudinal fuselage lap splices may have cracks after a long service life — the infamous "Multiple Site Damage", which is a severe problem today. Fail-safe construction at any rate requires very thorough maintenance and inspection procedures, which are not performed by all airlines over the world.

The United States Air Force (USAF) in particular suffered many accidents due to fatigue. On 13 March 1958, two Boeing B-47 nuclear bombers crashed due to fatigue failure of the wing [385]; within two months two more aircraft also failed in the same manner. This was very dangerous for the national safety of the US, indeed of the Western world, because at the time the B-47 were the only aircraft which could reach the USSR, and the whole fleet had to be grounded until the critical components had been replaced. But most other military aircraft types of the USAF also suffered one or two fatal fatigue accidents. In some part, this was due to the extreme variations in the load spectra of individual aircraft of the same type [368]. Contrary to European Air Forces, the USAF always reported on these accidents quite openly, for example at a special conference in 1959 [387–400]. One paper at this conference read: "The aircraft fatigue problem: barely under control" [390].

The leading position of Germany in the fatigue field was obviously a thing of the past — with the exception of the automobile sector: the latter is due to Gassner's having to find a new field of work, since after WW II building aircraft in Germany was forbidden. In 1946 Gassner founded the "Physikalisch-Technisches Laboratorium" at Kempten, Bavaria, together with Svenson. In 1948 they moved to Darmstadt (at the suggestion of Opel) and merged with the previously mentioned "Bautz-Bergmann GmbH" to form the Laboratorium für Betriebsfestigkeit (LBF). Within a few years Gassner had convinced all German automobile and truck manufacturers of the benefits of his approach. His argument was weight- and therefore cost-saving, but with a sufficient fatigue life — through service-load measurement and v.a. testing. With the exception of NSU, who left all fatigue work to the LBF, all automobile manufacturers built up large fatigue laboratories and by about 1959 had become practically independent of the LBF. The many and relatively small German truck manufacturers of that time, however, remained LBF customers for much longer. The large body of experience collected by the German automobile makers since about 1950 to this day ensures them a significant lead in the fatigue field over their competitors in Europe and especially in the US — at least in the opinion of the author.

Gassner in 1954 described the state-of-the-art of fatigue testing and measurements in the automotive field [401], and in 1956 [286, 402] in the aircraft field. Svenson in 1952 reported on the measurement of stress and load spectra [403]. The LBF in 1960 had about 40 employees and had carried out about 1100 fatigue research and testing contracts from all fields of technology (aircraft, ships, automobiles, bridges, railways, etc.).

A multi-year test programme of the LBF over hundreds of thousands of kilometers deserves special mention: one rear swing axle of a Volkswagen Beetle operated a fatigue test machine with 10 notched specimens, which through interposed torsion springs were loaded at different stress levels. This therefore was an ideal variable-amplitude test which in no way differed from the actual service loading, including rest periods, corrosion, etc. The corresponding load spectra were counted simultaneously according to the level-crossing method. From this, eight-step blocked-programme tests were deduced and were run in an identical test machine in the laboratory. The purpose was to see if the fatigue life in actual service was equivalent to that in the laboratory under the blocked-programme test. The results will be discussed in Chapter 8.

Quite suddenly the up to now just marginally if at all, mentioned scatter of the number of cycles to failure and of the fatigue limit, respectively, was treated with the help of mathematical statistics. Weibull in Sweden extended his theory of static strength of 1939 [132] to fatigue life [404, 405] and carried out thousands of tests with bolts [406] and aluminum specimens [407] to prove his distribution and to obtain numerical data on the standard deviation of the number of cycles to failure.

Freudenthal and Gumbel [408] at the Columbia University on the other hand employed an extreme value distribution; Rossow at the Technical University of Berlin developed the arc sin $\sqrt{\pi}$-distribution [409], which like the three-parameter Weibull distribution had a lower limit > 0. It remains controversial to this day whether the "correct" distribution of the number of cycles to failure has such a lower limit > 0 or not.

All these distributions were mainly utilized to statistically evaluate the results of fatigue tests of 7–30 specimens, in order to obtain the mean and standard deviations, and to extrapolate only insignificantly. In this region all reasonable distributions yield very similar results. The main problem, namely the extrapolation to the high probabilities of survival required for actual, massproduced components (i.e. $> 10^5$ for automobile components, $> 10^6$ for helicopter components), cannot be solved correctly to this day with the help of mathematical statistics — at least in the opinion of the author. One has to rely on "safety factors" based on experience.

The Americans Ransom and Mehl [169], and Dieter *et al.* [170] proved by fatigue tests on a statistical basis that effects which had been claimed for decades, like coaxing by understressing, did not exist. They also determined the fatigue limit by the staircase method which had originally been developed by biologists.

Erker of the MAN, a coworker of Thum in 1958 [410], probably was the first to discuss the probability of survival with regard to the scatter of loads and the number of cycles to failure.

From about 1950 onwards, doubts about the validity of Miner's rule began to appear in the literature. Damage sums between 0.1 and 10 were found in fatigue tests which, however, were entirely unsuitable for this purpose. The unfortunate role of the "understressing" tests of the years 1930 to 1943 was now taken over by "overload" tests, i.e. by constant-amplitude tests with a few interspersed overloads. Since such stress-time histories do not occur in actual service, the proof that Miner's rule would work — or not — was entirely useless. A genuine check of Miner's rule was only possible at the end of the 1950s, as only then suitable test machines had been developed.

The Australians Head and Hooke [411] were the first who used the "white noise" of a thyratron to run a random fatigue test machine. The random fatigue machine described by Freudenthal [412, 413], however, produced anything but a random sequence because of its much too short return period of *ca* 1000 cycles. This resulted in the truncation of all cycles with a probability of occurrence of $< 10^{-3}$, so that all higher stress amplitudes were cut off. In addition, a blocked (programme) sequence was used, albeit with very short blocks. The results of Head and Hooke are also open to doubt, because inertia forces were apparently neglected and thus the command values were different from the achieved ones.

The extreme inaccuracies of Miner's rule mentioned above naturally caused a large number of "improvements", most of which — fortunately — are forgotten today. Efforts to give these improvements a scientific air by metallurgical foundations [414] appear particularly doubtful today. The hypothesis of Corten and Dolan [415] was a "self-fulfilling prophecy", because exactly those tests were required which the hypothesis was supposed to avoid, as the author showed in 1972 [416]. This, however, did not prevent its being employed in certain fatigue standards of East Germany. Corten–Dolan "proved" their hypothesis with the help of two-step tests on bending specimens of thin wire.

The comprehensive check of Miner's rule of the author in ref. [416], which is still being cited today, is also open to argument, because most of the tests used were Gassner eight-step blocked-programme tests, and not random tests.

Only the servohydraulic fatigue test machines, first developed by Lockheed for a huge research programme of the USAF [417, 418] toward the end of the 1950s, permitted arbitrary stress–time histories to be applied to specimens and components at high frequencies. Only from this time onward realistic checks of Miner's rule were therefore possible; the results will be reported in the next chapter.

In 1959 Kowalewski of the DVL in his Ph.D. Thesis [419] used a similar machine to that of Head and Hooke, however without its inaccuracy. The object of his work was the comparison of the blocked-programme test of Gassner with a random test of identical spectrum, i.e. only the sequence of stress cycles was different. Result: the fatigue life of the random sequence was lower, i.e. the blocked-programme test was conservative. This (unfortunately) was not generally the case — as described in the following chapter.

Toward the end of the period described here, a large number of crack-propagation tests were carried out and crack-propagation hypotheses were developed — still without employing fracture mechanics — by Weibull [420], Frost of the NEL in the U.K. [421–423], as well as by McEvily and Illg [424], and Hudson and Hardrath [425] of NACA.

Kuhn of the NACA tried for about 15 yrs [426–428] to transform the calculation of the fatigue-stress concentration factor K_f into engineering practice with the help of Neuber's material "constant" ρ. However, the results were not accurate enough and too large an experimental effort was still necessary, as the author found out during work for his Ph.D. Thesis.

Based on Bauschinger's [36–38] ideas, Manson [429] of the NACA and Coffin [430, 431] of GE in 1954 described the behavior of metallic materials under cyclic inelastic strain amplitudes by a four-parameter formula. They thus created the new field of activity called LCF (low-cycle fatigue), which is employed for components loaded in service by relatively few cycles at elevated temperatures, for example discs of gas or steam turbines, pressure vessels, and so on. The aircraft turbine especially makes high demands on the disc: low weight, high fuel efficiency (i.e. high temperature), structural integrity and a long fatigue life ($>$ 20 000 flight hrs). Therefore the LCF field has had a strong growth since 1954.

In 1958 two large steam turbine rotors burst during test runs, as well as several "Polaris" rockets during pressure tests. In all cases the cause was crack-like material defects, which in the Polaris rocket were very small because an ultra-high-strength steel was used. Again, the national security of the U.S.A. and of the Western world was threatened. In the same year Irwin [432] of the US Navy had taken up the old ideas of Griffith [57] and had realized that the stress-intensity factor

$$K = S \cdot \sqrt{\pi \cdot a}$$

was the determining factor for static strength in the cracked state. If K reaches a certain critical number depending on the "fracture toughness" of the material in question, instant fast fracture occurs: linear elastic fracture mechanics (LEFM) was born. Its unbelievably rapid development for the case of static fracture will not be described here; however, the next chapter will deal with its employment for the description and calculation of crack propagation under cyclic loads.

In 1951, ICAF (International Committee on Aeronautical Fatigue) was founded at the suggestion of Dr Plantema of the NLL (later NLR) of the Netherlands. It brought together practically all aircraft fatigue experts from all Western countries every two yrs at conferences and symposia [433, 434]. At first, the delegates of the member countries only reported on the fatigue activities of their respective countries (ICAF Conferences). The founding countries were the Netherlands, the U.K., Sweden, Switzerland and Belgium; later the Federal Republic of Germany, the U.S.A., Australia, France and Italy joined in and, in the 1970s, Israel and Japan. As from 1959 [435], in addition a symposium with special lectures took place every two yrs. The ICAF Conferences in particular, even those of 30 yrs ago, are still a source of information for the fatigue expert, since there is hardly a fatigue problem which has not been discussed. For the author, the ICAF Symposia and Conferences represent the ideal of meetings as they should be, unparalleled as to the quality and the topics of their lectures, the level of the discussions and the frankness about aircraft fatigue problems.

The Dutch National Aircraft Laboratory NLL (NLR today) also distinguished itself in other respects. From about 1957 onwards, Schijve [436, 437] contributed important new ideas to the field of aircraft fatigue (see Section 8). The development of the rainflow counting method by DeJonge, concurrently with the Japanese Matsuishi and Endo in 1969, has already been mentioned.

In Sweden the FFA (Flytekniska–Försöks–Anstalten) has attained similar importance as the NLR in the Netherlands. Besides Weibull [404–407], the names of Wallgren [438, 439] and Lundberg [440] must be mentioned. Gassner's work was highly esteemed in Sweden already soon after the war, see for example ref. [152]. The car and truck maker Volvo and the truck manufacturer Scania also became followers of the idea of "Betriebsfestigkeit".

Many scientific and engineering societies organized fatigue meetings in the years 1945–1960, in part already on specific problems; examples are the ASTM on planning and performing fatigue tests [441], application of statistics [442], large fatigue test machines [443], fatigue of aircraft structures [444], fretting corrosion [445], low-cycle fatigue [446], etc.

General fatigue meetings took place in 1946 in Melbourne [447], in 1952 at the MIT [448], in 1955 in Stockholm [449], in 1956 at Columbia University [450]. In 1956 a meeting, still famous today, was held by the Institution of Mechanical Engineers together with the American Society of Mechanical Engineers [451], during which most of the well-known fatigue experts of the time gave lectures. In 1954 [452] a Russian meeting at a high level took place, on methods to improve fatigue strength by mechanical, thermal and thermochemical means, i.e. a special topic similar to that of the ASM meetings in Cleveland [453] in 1946 and Chicago [454] in 1950.

Besides the proceedings of the above meetings, a number of general books on fatigue were published: Cazaud's book saw several new editions [455–457] as well as an English translation [452]. The Battelle book of 1941 [124] was also reprinted [458]. Russian books were translated into German [459, 460] and English [461, 462]. The book of Hänchen, originally published in 1950 [463], saw several new editions; the one by Tauscher in East Germany by 1960 had reached its sixth edition [464]. Both books were simple "recipe" books, all the more important for engineering applications, similar to that of Lipson, Noll and Clock of the Chrysler Corporation [465].

Which fatigue expert has not used Peterson's book [340], a compilation of stress-concentration factors with a short introduction to design for fatigue? Locati of Fiat also published a book [466].

Two German books of the time deserve a special mention, although they are not well known:

- *Gestaltfestigkeit — Versuche mit Schwingern* by Berg [467], a disciple of Thum. Berg describes many fatigue tests on components, design or manufacturing changes, etc. The book contains many problems and their solution by simple means, as found during WW II at the "Deutsche Werke" in Kiel.

- *Atlas der Spannungsfelder in Technischen Bauteilen* by Kloth [468], who in 1932 had measured the first load spectra in Germany [25]. This book shows hundreds of excellent brittle-lacquer photographs of joints in frames and of other components of agricultural machinery; included were improvements by design changes.

Wiegand and Haas in ref. [469] summarized their experience on the fatigue strength of bolted joints. This book also saw several greatly improved editions, partly with new authors.

Pope was the first Englishman [470] to publish a book on fatigue after WW II.

Grover, Gordon and Jackson wrote a general book on fatigue [471], which in addition contained hundreds of *SN* curves in the form of tables.

In the book of Sines and Waisman [472], many well-known Americans of the time had their say, among them Miner, who still claimed that his rule would deliver conservative results if the "lower scatter bound" of the *SN* curves was used as a basis for the computation.

8. 1960–1994

From 1960 onwards the number of fatigue experts and therefore of publications increased still further. This must also be attributed to the rapid development of fracture mechanics, i.e. of fatigue-crack propagation. Therefore in this chapter only a few persons will be mentioned who have made special contributions to the state of the art or whose work has resulted in completely new points of view.

Schijve, already mentioned in the preceding section, first at the National Aerospace Labaratory (NLR) of the Netherlands, from 1972 onwards as Professor at the Technical University at Delft, has contributed greatly to the knowledge of fatigue of aircraft structures, for example fatigue-crack propagation under variable amplitudes in materials [473] and in actual structures [474]. By his investigations on the retarding effect of rare high amplitudes [475, 476] he has influenced most of the full-scale fatigue tests on commercial aircraft, for example all of the Airbuses from the A-300 to A-310, 320, 330 and 340, the Boeing 757 and 767. In his "classical" paper [476] he describes the seven counting methods then known, with special regard to fatigue-life prediction and variable-amplitude tests. He concludes that the range-pair procedure of the "Vickers" strain-range counter best describes the fatigue process in a random stress-time history. The range-pair-range [277] or "rainflow" method already mentioned was a further development, according to Schijve.

The development of "Arall" [477, 478] and "Glare" [479] (aramid aluminum laminate and glass fiber reinforced aluminum, respectively) deserves a special mention. Both materials consist of bonded layers of aluminum and the respective fiber material. In all investigations they have shown extremely favorable crack-propagation characteristics, orders of magnitude better than monolithic aluminum specimens and components [480]. At the same time, they avoid many drawbacks of CFRP; they can, for example, be machined like aluminum on the same production equipment. "Arall" and "Glare" are already used experimentally on aircraft [480].

Branger of Switzerland [481] also must be mentioned with regard to fatigue in aircraft structures. Against severe opposition and with great obstinacy he succeeded in carrying out a very complex full-scale fatigue test on the Swiss Airforce de Havilland "Venom" in the early 1960s. Hundreds of different flights, as measured in the special missions of mountainous Switzerland, were applied to the structure. The result was that the safe fatigue life of this aircraft was five times longer than originally foreseen by the manufacturer. The cost of these complex full-scale fatigue tests of many millions of Swiss francs must surely have repaid itself! This programme ran for over a decade and influenced full-scale fatigue tests on military aircraft all over the Western world, among them also the IABG tests on German military aircraft.

Paris of Lehigh University, in his Ph.D. Thesis of 1962 [482] and in a previous paper [483], established that fatigue-crack propagation could be described by the following equation — soon erroneously called a "law":

$$\frac{da}{dn} = C \cdot \Delta K^n.$$

This equation soon set out on a veritable triumphant advance around the world. It is almost exclusively used even today, although it contains neither the influence of mean stress on crack propagation, nor the static fracture on reaching the fracture toughness K_{IC}, nor the "fracture mechanics fatigue limit" ΔK_0, that is the stress-intensity range below which no fatigue-crack propagation occurs. The Paris equation therefore can be compared to an *SN* curve from a stress amplitude of infinity to one of zero, showing no influence of mean stress nor a fatigue limit. The complex process of crack propagation is undoubtedly described much too simply by this equation; this fact however did not prevent its — partly undiscriminating — use all over the world to this very day. Paris also maintained that the slope of his curve was $n = 4.0$ for all metallic materials. This could be compared to all *SN* curves of metallic materials having the same slope — and for all mean stresses as well.

In 1963 Paris extended his equation to variable amplitudes by inserting a "characteristic" stress "h" instead of the $\Delta\sigma$ in the formula for ΔK [484]. This value h was said to depend on the shape of the stress spectrum only. Paris further stated that the crack-propagation curve for constant and variable amplitudes would coincide if both were plotted against this h, this being somewhat similar to the root mean square of the stress amplitudes. Intentionally or unintentionally, Paris thus claimed to have solved the problem of damage accumulation — if only for fatigue-crack propagation. Paris' claims were soon proved incorrect. Numerical values for n ranged from 2.0 to 20 in experiments and the problem of damage accumulation, i.e. of the prediction of crack propagation under variable-stress amplitudes, has not been conclusively solved even today (see below).

These critical remarks are in no way meant to lessen Paris' merits. It was an enormous step forward to be able to calculate the propagation of cracks of arbitrary shape under arbitrary stress types (bending, axial, etc.) via ΔK, thereby considerably reducing the experimental effort. One simply had to determine one crack-propagation curve — a number of curves for different mean stresses — for one specimen and crack shape to find the constants C and n. With these, crack propagation for all other cases could be calculated.

The fundamental contribution to an improved calculation of crack propagation under service-like variable amplitudes was supplied by the German Elber. In his Ph.D. Thesis at the University of New South Wales [485] in 1968 he found out that after a high tensile load the crack closes before the load is reduced to zero. In contrast to many earlier hypotheses, which assumed a decrease of the mean stress after a high tensile load, Elber demonstrated experimentally that the amplitude decreased after such a high load; all promising crack-growth hypotheses since about 1975 are based on Elber's "crack closure".

In the time period in question, as an Institution surely the USAF (US Air Force) must be praised. In 1974 it introduced new structural specifications [486], the famous "Damage Tolerance Requirements", in which crack-like defects are assumed to be present from manufacture onwards in all critical points of the structure. These defects can be caused by machining processes during manufacture, or they can be caused by service loads. The aircraft manufacturer has to prove "by test and calculation" that in the cracked condition sufficient life (durability) and static strength ("damage tolerance") are available. Lincoln [386] published an outstanding survey of the historical development of these D.T. specifications.

The cause and motivation for this change in the structural specifications was that the USAF even after 1960 did not succeed in obtaining a sufficient durability and structural integrity of its aircraft. Fatal fatigue accidents occurred all too often [487]. The direct reason for the introduction of the new structural specifications was the crash of an F-111 after only 100 flight hrs owing to a wing failure due to a crack-like defect, which had not been detected during the prescribed inspections. The failed wing box consisted of the ultra-high-strength steel D6AC. In consequence of this accident, a huge theoretical and experimental research programme was started in which almost all US fracture mechanics experts had a part. One finding, for example, was that the fracture toughness K_{IC} of the D6AC steel was extremely sensitive to minute modifications of the heat treatment.

In this connection, the USAF managed to attract a large number of outstanding fracture mechanics and crack propagation experts. The names of Tiffany, Mar, Lincoln and Wood may be mentioned here as representative for many others. This certainly has facilitated the enforcement of the abovementioned specifications with the aircraft manufacturers, in contrast, for example, to the European Airforces — and especially the German one — which do not have experts of that level in their employment. This fact alone — in addition to the usually much smaller number of aircraft of one type in Europe, makes the relationship between the buyer and supplier much more difficult in Europe.

The D.T. specifications of the USAF are still being criticized today in Europe. In individual problems and cases this may be justified; the specifications are also continuously being developed further [386]. In summary, however, the decision of the USAF to completely change over to the new specifications can only be admired. After all, the success of the D.T. specifications speaks for itself: even with old USAF aircraft, which had not been designed and built to these damage tolerance requirements, there was only one fatal accident due to fatigue in 14 million flying hrs, in contrast to the many which would certainly have occurred without them. However, unforeseen and extremely expensive repair and maintenance efforts are still necessary, for example with the F-16.

The USAF is so convinced of its D.T. procedures that it has issued D.T. specifications for further components, for example for gas turbines [488]; others are worked on, for example for helicopters [489] and accessories, landing gears, hydraulic cylinders, etc.

Together with the D.T. specifications a large number of publications appeared, as well as the "Damage Tolerance Design Handbook" [490]; it contains a huge collection of toughness and crack-propagation data for practically all structural and engine materials as well as a volume "Guidelines" [491], in which the application of the specifications to practical cases is explained. These guidelines offer a mine of information on the actual utilization of fracture mechanics, and are worth much more than many theoretical books on fracture mechanics.

In December of each year, the USAF holds a "Structural Integrity Conference" [492, 492a, 492b]; the last one was held in December 1994, where the further development of the structural integrity specifications was discussed. Contrary to earlier expectations, a full-scale fatigue test with the complete structure of the aircraft appears to be still indispensable, combined with damage tolerance tests and calculations [492].

Damage tolerance requirements were also issued for commercial aircraft [493, 494]; they are, however, much less detailed; for that reason, they have been less successful than the USAF specifications — at least in the opinion of the author. In spite of alleged fail-safe construction, if before the D.T. requirements were issued, a Boeing 707 crashed in Lusaka, Africa, by fatigue failure of the empennage. Other fatal crashes include the DC10 accidents in Chicago by failure of an engine mount and on 10 September 1989 in Sioux City by the uncontained burst of an engine disc.

The "multiple site damage" (MSD) — also called "widespread fatigue damage" (WFD) — mentioned before has not been explicitly considered in the military and commercial D.T. specifications, although the original USAF damage tolerance specification [486] in a way addressed this possibility by requiring that each hole should have a small corner flaw at the time of manufacture. The 1978 revision of FAR 25-571 [494] also contained words about MSD, but was much less specific than MIL-A-83444. The authorities as well as the aircraft manufacturers and the airlines were alerted to MSD only after the nearly fatal accident to the Aloha Airlines Boeing 737 in 1988 [494]. The cause was corrosion and corrosion fatigue, to an unexpected extent, of the old ($> 90\,000$ flights), badly maintained Aloha Airlines aircraft which in addition were employed in a very corrosive environment; also the cold-bonding of the titanium crack stoppers had failed.

Due to this accident, huge investigation efforts, as well as many repair and maintenance programmes were undertaken, which for the Boeing 727 and 737 types alone cost more than a billion dollars. MSD also occurred in many other aircraft types, especially on longitudinal fuselage lap splices. Emmerson for example in ref. [495] mentions the BAC-111, the DC9, the Airbus A-300, the Boeing 747, the Fokker F-28 and several military aircraft. In the opinion of the author it is still unclear why MSD did not occur in the full-scale fatigue tests on most of the above aircraft, although these were usually conducted to more than twice the design life (exception: B-747 to one design life). Admittedly Williams as early as 1965 [288] stated MSD having occurred in certain full-scale fatigue tests. Anyway, "Aging Aircraft" is a very modern topic and several meetings have been held on this subject [495a, 495b, 495c, 495d].

A significant feature from 1960 onwards was the introduction of the servohydraulic fatigue test machine, which for the first time permitted the application of arbitrary stress-time histories at sufficiently high frequencies. Strictly speaking, only from that point in time was it possible to check Miner's rule and similar hypotheses — but also Gassner's blocked-programme test. The servohydraulic test machines at first proved to be extremely unreliable, mainly because of their punched-tape control systems. Only the introduction of digital computers in the 1970s [496] eliminated these deficiencies — but even today a high-level quality assurance programme is necessary [497] to assure the user that the machine actually performs as it should.

With a modern catchword: anything could be done, but — and this is also a modern phrase — is everything doable also useful? Therefore the standardization of stress-time histories for variable-amplitude fatigue tests, which Gassner already had required and carried out in the 1940s, again became significant: the standardized stress-time histories "Twist" for commercial aircraft, "Falstaff" for fighter aircraft, "Gauss" for general fatigue test purposes, "Helix–Felix" for helicopters, "Hot" and "Cold Turbistan" for gas turbine discs, "Wawesta" for steel mill drive systems, "Wash" for oil rigs in the ocean, "Wisper" for wind turbines and "Carlos" for automobiles evolved, practically all of them upon the initiative or at least with the cooperation of the author. A summary is given in ref. [498]. These standards are being employed all over the Western world; they are extremely useful, for example, in round-robin test programmes, which came into vogue in the 1970s [499]; fatigue problems of general interest can thus be jointly solved by many laboratories in many countries, lowering the cost per country and improving international cooperation.

In the time period considered, the first really meaningful checks on Miner's rule were carried out, among them a large cooperative research programme of the German automobile makers, the German Ministry of Research and the LBF [500] and — in this case on automobile components proper — a similar programme jointly by the automobile makers and the IABG [501, 501a]. By far the largest effort in this respect, however, was spent by Kotte and Eulitz of the Technical University of Dresden [501], who collected hundreds of test programmes, i.e. many thousands of variable-amplitude and corresponding constant-amplitude tests and carried out new Miner calculations. One outstanding result of all these checks was that Miner's rule generally proved wildly unconservative; that is, the specimens — in the case of the IABG programme, the components — failed long before their predicted lives; factors of 10 and more on the unconservative side were not uncommon.

From about 1965 onwards, the Society of Automotive Engineers (SAE) in a large cooperative programme has tried to predict fatigue life using the so-called local-strain approach. Several US automobile makers and many universities were participants. Much too short stress–time histories were employed — probably because of the limited computer capacities of the time. Had it been proved that the fatigue life could be predicted with reasonable accuracy, this would not have been useful, because the rare high loads so typical of real service stress–time histories did not occur in the sequences chosen. However, as shown in the final report [503], the programme did not even succeed in correctly predicting fatigue life under these unrealistic sequences. In the opinion of the author, the programme therefore was a complete failure — despite innumerable claims to the contrary in the literature. This failure also did not prevent the local-strain approach from being employed in several countries — notably the US and U.K., less in Germany — although it has been shown in the meantime, for example by Eulitz *et al.* [501] in the large programme mentioned above — but also by many other checks [504] — to be even less accurate than Miner's rule. The extremely large but unaccountable scatter of the predictions is a further disadvantage of the local-strain approach.

The German efforts in the local-strain approach area, especially at the Technical University of Darmstadt, are ahead [504a, 504b, 504c, 504d] of the Anglo–Americans, but they are hardly even mentioned in foreign journals. The "Relative Miner Rule", first published by the author in 1972 [502], has also not fulfilled expectations. Furthermore, according to ref. [501] it can only be used in limited areas. The "Double Relative Miner Rule" developed in ref. [501] has also not succeeded in reliable and accurate enough fatigue life predictions.

The crack-propagation hypotheses based on Elber's crack closure as a rule have resulted in better predictions; however, in the opinion of the author none of them have been checked extensively enough to be really certain of their accuracy and reliability.

Even the time period of 1960 to today was not exempted from unnecessary, partly even absurd developments. An example is the so-called Fatigue Life Gage in which a permanent change of resistance of a wire by cyclic stresses was supposed to be a qualitative measure of damage, or the Dornier "Fatigue Measuring Gage" (FMG), where the change of reflectivity of a polished aluminum foil was said to fulfill the same purpose [502a]. A simple consideration — even by the inventors themselves — should have been sufficient to understand that a piece of wire or foil cannot be more intelligent than the many engineers and scientists who have struggled with damage accumulation since Palmgren — without, in the end, completely understanding and solving the problem. In addition, the inherent scatter of the change of reflectivity of the foil was also overlooked by the inventors; according to an IABG investigation this caused very different changes under identical stress–time histories.

A very unfortunate role is also still being played by trying to solve the problem of damage accumulation and fatigue-life prediction through the "equivalent" constant-stress amplitudes, equivalent that is to variable-stress amplitudes of equal number of cycles to failure. It is completely overlooked that the basic assumption here is a damage sum to failure of 1.0. Since this is generally not so (see above), any proof of the method having worked is only proof that purely by chance Miner's rule was correct in that particular case. In the author's Ph.D. Thesis of the 1960s [502] it was shown that there is no fixed relationship between constant and variable-stress amplitudes.

The "RMS" (Root Mean Square) approach belongs to the same category of trying to predict fatigue life under a stress spectum on the basis of constant-amplitude tests. Although it has been shown many times [502] what is basically wrong with this approach, this has not prevented its widespread employment. As with the "equivalent" stress–amplitude concept, a damage sum of 1.0 to failure is assumed and, in addition, a slope of the corresponding *SN* curve of 2.0. Since no *SN* curves of such steepness are to be found in the literature, occasional proof of the validity of the RMS approach only means that two principal errors, namely a damage sum of 1.0 and a *SN* curve with a slope of 2.0, have compensated each other in the individual case.

Finally the "PSD" (Power Spectral Density) method must be mentioned. Although Schijve and Weibull in ref. [476] have shown as early as 1961 that only with a Gaussian distribution the PSD contains the requisite parameters of fatigue life, this has not prevented its widespread employment with other stress spectra. A simple reflection would in the general case have shown the frequency information of the PSD to be unimportant for fatigue, while other, important information is missing in the PSD.

As soon as the servohydraulic machines were available, a check on Gassner's blocked-programme test became possible. Such checks took place at NASA [505], at the NLR in the Netherlands by Schijve [506, 507] and at the Lockheed Company with the huge investigation mentioned before [418]. The general result was that the blocked-programme test resulted in a longer life than the more realistic test with a random sequence (at equal spectrum shape), i.e. the blocked test was unconservative.

This, by the way, was also the result of the already mentioned comparative test programme in the Volkswagen Beetle, which ended in 1967; the results were only published in 1976 [508] and 1979 [509]. In spite of all these results — the most comprehensive ones having been obtained at the LBF itself — Gassner only became convinced by the much smaller test programme of Jacoby [510, 511] that his blocked-programme test had become outdated. One consequence of this was the huge cooperative investigation with the German automobile makers mentioned above.

On account of the increased demands on durability, of the importance of saving weight and — last but not least — of product liability, the German automobile makers from about 1975 onwards enlarged their already large fatigue laboratories even more. Their requirements on fatigue testing machines and test rigs also increased, so that since that time the most demanding requests on the capabilities of these machines no longer come from the aircraft field, but from the automobile makers. Examples of such complex and expensive test rigs are described in refs [512–516].

With the introduction of jet aircraft, sonic fatigue became a problem, i.e. resonance of the structure due to excitation by the gas turbine noise. Early jet aircraft with their noisy engines had severe difficulties [517]; however, the reduced noise of modern fan engines soon made this problem less critical.

In many Western countries committees were established which dedicated themselves exclusively to fatigue including crack propagation, for example in Germany the DVM Working Group "Betriebsfestigkeit", in the U.K. the Engineering Integrity Society (EIS) and in the US the Committees E-9 and E-24 of the ASTM.

The number of fatigue meetings increased, as well as the number of books on fatigue, of which only a few can be cited here [518–531]. Some of the books not mentioned here, in part also of German origin, were quite superfluous.

All books on the ICAF Conferences and Symposia (for example [532–534]), however, were outstanding; even quite old ones offer a veritable mine of information on all important military and commercial aircraft fatigue problems. The Structures and Materials Panel (SMP) of Agard also distinguished itself by many high-level meetings and reports on fatigue, and especially by several round-robin programmes.

One general result of these round-robin programmes was that nominally equal fatigue and crack-propagation tests showed an unexpectedly large scatter when carried out by different laboratories. This was especially so for LCF tests with their difficult control of loads and temperatures. The numbers of cycles to crack initiation in some of these programmes varied over several orders of magnitude, although the specimens came from one batch of material.

A number of books on fatigue was also published in Germany, in 1969 for example [526]. The *Leitfaden für eine Betriebsfestigkeitsrechnung* (manual for the calculation of fatigue life under variable-stress amplitudes) of the VBFEh [526] described in easily comprehensible form the many steps of such a calculation. In 1973 the quite good book *Schwingfestigkeit* [530] came out in East Germany. In spite of being cut off from Western sources, the German Democratic Republic was quite up to Western standards, as shown by many Ph.D. Theses [535, 536], or by the only fatigue journal in German language *IFL-Mitteilungen*, which unfortunately had to be discontinued after the German reunification.

The good and — for their time — modern fatigue Standards and Specifications (TGL) of East Germany deserve special mentioning, although their adoption and enforcement were certainly facilitated by the political system of this country. In contrast, the basic West German Standard on fatigue (DIN 50100) dates back to 1951! British Specifications have been brought out in large numbers and high quality, for example BS 5400 [537] for steel bridges or BS 8188 for aluminum constructions. The European Community also has its own standards, for example for welded joints the Eurocode 3 [538], which in the opinion of the author is an unpleasant compromise between the BS 5400 and Central European concepts.

Two German books with similar titles made their appearance within a brief time span: *Betriebsfestigkeit* [539] by Buxbaum and *Betriebsfestigkeit* [540] by Haibach, the latter one supplemented by a short version [541]; the first one is now available in its second, improved edition. Both books are concerned with Betriebsfestigkeit (operational fatigue strength) and therefore have no international competitors; for example both also deal with the measurement of variable loads in service and their evaluation.

Books containing fatigue data in the form of tables are appearing in ever larger numbers [542–547]. They are very useful — if only for the fatigue expert — because they save the time and therefore the cost of chasing after the many *SN* or crack-propagation curves scattered all over the vast literature. Their value for practical fatigue work is far higher than that of some other books on fatigue. However, they are no replacement for the experience of the fatigue expert, as some software houses seem to believe, which sell software packages containing unbelievable errors and misconceptions. For example, fatigue-life calculations based on Miner's rule with a damage sum of 1.0 are completely outdated in view of the modern checks on Miner's rule mentioned above. This is also true for the software packages employing the local-strain approach which, in the end, also uses Miner's rule.

The Damage Tolerance Handbook of the USAF mentioned before [490], and the Fatigue and Fracture Mechanics Data Sheets of the ESDU in Great Britain [548, 549] are also books of tables, but are of a somewhat higher category, because the raw data are first screened by a committee of experts before they are incorporated into the books; erroneous data are thus excluded. In the opinion of the author the concept of "Synthetic *SN* Curves" [550] must be rated still higher, because it describes the collected fatigue data by formulae, resulting in the best estimates for the mean values, for example of the *SN* curves of steels with a certain static strength. A further advantage is that *SN* curves for which no test data are available can be calculated by inter- and extrapolation.

Components or structures which had not been considered fatigue critical up to recent times, suddenly had fatigue problems, for example oil rigs in the North Sea. As a consequence, a large cooperative programme was started, the results of which were reported in two large meetings in Paris in 1980 [155] and in Delft in 1987 [156], a veritable mine of information on corrosion fatigue of steels.

All the same, many important problems of corrosion fatigue are far from being solved (see Section 9).

9. OUTLOOK

As mentioned several times above, many questions of fatigue and "Betriebsfestigkeit" (operational fatigue strength) have not been solved. Nevertheless the frequently expressed opinion "in no field of engineering has more money been spent with less effect than in fatigue" is nonsense. Several branches of industry, for example the automobile makers — at least in Germany — by a judicious mixture of engineering judgment, testing and calculation have succeeded in avoiding fatigue failure of their products in service, although many of the questions involved have not been solved in a scientific way.

Unsolved, as much as ever, is the prediction of fatigue life under variable amplitudes — despite innumerable claims to the contrary in the literature. Neither the Miner calculation in its many variations nor the local-strain approach attain a sufficient accuracy, as shown in refs [500, 501, 501a, 501b]. The prediction of crack propagation under variable amplitudes, on the other hand, is better, but has not been sufficiently checked.

The transferability to actual components of fatigue data obtained with specimens is completely unsolved [551]. This problem is — intentionally or unintentionally — passed over in silence. Yet the IABG fatigue tests mentioned above [501] have shown the damage sums to failure of actual components to be much lower than those of specimens — to mention just one problem of transferability. Some concepts — like the local-strain approach — blindly take transferability for granted — otherwise they cannot be employed.

Corrosion fatigue is another complex and unsolved problem. Although experimentally investigated since the 1920s, there is no explanation of many effects observed, for example why even a few minutes in the corrosive medium has a detrimental effect, which however hardly increases even after several days [552].

The combination of high temperature and fatigue (creep plus fatigue) is largely unsolved, although many hypotheses have been published, "proved" by a few tests of the respective author, and disproved by the next set of experiments.

Multiaxial fatigue stresses, especially if they are asynchronous, are already difficult under constant amplitudes, but can be considered to some extent as having been solved [553, 554]. Unsolved, on the other hand, is the problem when the amplitudes are variable, especially if there is no fixed correlation between the forces in the different directions. The calculation of fatigue life in such cases is then only possible on a probabilistic basis — even if all the other questions have been solved.

A probably unsolvable problem is the scientifically and practically correct calculation of fatigue life at very low probabilities of failure, because the type of distribution would have to be known.

Acknowledgements—This work could not have been written without the assistance of many persons. The author is especially grateful to Dr Fischer of Thyssen Research Centre of Kassel for several discussions and for literature, to Prof. Kloos of the Technical University of Darmstadt for many of Thum's and Föppl's publications, and an extended discussion, to Dr Döker of the DLR at Cologne–Porz for exclusive lists of DVL papers and to Prof. Buxbaum, LBF, for his permission to use the large LBF library. The author is also obliged to Dr Schmiedeck of the Institude of Railway Vehicles in Berlin for his reference to a hitherto unknown paper by Wöhler. Prof. Schijve, to whom the German version of this paper was dedicated, critically examined the original draft and made many suggestions, as did Prof. Zenner and Mr Naundorf of BMW.

Last but not least, the author owes a special debt of gratitude to Mrs Ingrid Bauer, the librarian of the LBF, who with the help of her phenomenal memory found many old papers and books. Finally, Mrs Brigitte Woitschach wrote the manuscript and with boundless patience inserted the many alterations, and Mrs Barbara Bonke, Florence, proof read and corrected the English manuscipt.

REFERENCES

[1] H. Zenner and D. Hanewinckel, *Schwingfestigkeit, eine Faksimilesammlung Historischer Arbeiten bis* 1950. TU Clausthal, Institut für Hüttenmaschinen und Maschinelle Anlagentechnik (1989).
[2] J. Y. Mann, *Bibliography on the Fatigue of Materials, Components and Structures*. 1838–1950, Vol. 1 (1970), 1951–1960, Vol. 2. Pergamon Press (1978).
[3] H. Lüling, Von den Anfängen der Materialermüdung bei Metallen. *Ferrum* **56** (1985).
[4] A. Wöhler, Versuche zur Ermittlung der auf die Eisenbahnwagenachsen einwirkenden Kräfte und die Widerstandsfähigkeit der Wagen-Achsen. *Zeitschrift für Bauwesen*. **X**, 583–616 (1860).
[5] W. A. J. Albert, Über Treibseile am Harz. Archiv für Mineralogie, Georgnosie. *Bergbau und Hüttenkunde* **10**, 215–234 (1837).
[6] W. J. M. Rankine, On the causes of the unexpected breakage of the journals of railway axles, and on the means of preventing such accidents by observing the law of continuity in their construction. *Institution of Civil Engineers, Minutes of Proceedings*, Vol. 2, pp. 105–108, London (1842).
[7] J. O. York, An account of a series of experiments on the comparative strength of solid and hollow axles. *Institution of Civil Engineers, Minutes of Proceedings*, Vol. 2, pp. 105–108, London (1842).
[8] A. Morin, Leçons de mécanique practique—résistance des matériaux. Paris, Librairie de L. Hachette et Cie, p. 456 (1853).
[9] F. Braithwaite, On the fatigue and consequent fracture of metals. *Institution of Civil Engineers, Minutes of Proceedings*, Vol. XIII, pp. 463–474, London (1854).
[10] R. A. Smith, The Versailles railway accident of 1842 and the first research into metal fatigue, in *Fatigue* '90, Vol. IV, EMAS, Birmingham (1990).
[11] P. Stopfl, Achsenbrüche an Lokomotiven, Tender und Wagen, ihre Erklärung und Beseitigung. *Org. Eisenbahnw*. **3**, 55–67 (1848).
[12] W. Fairbairn, Experiments to determine the effect of impact, vibratory action and long continued changes of load on wrought iron girders. *Phil. Trans. R. Soc.* **154**, 311–325 (1864).
[13] J. E. McConnell, On railway axles. *Proc. Inst. Mech. Engrs*, London (13–27 October 1849).
[14] J. E. McConnell, On the deterioration of railway axles. *Proc. Inst. Mech. Engrs*, pp. 5–19 (January 1850), pp. 3–14 (April 1850).
[15] Report of the commissioners appointed to inquire into the applications of iron to railway structures. William Clowes, London (1847).
[16] J. V. Poncelet, *Introduction à la Mécanique Industrielle, Physique ou Expérimentale*. Zweite Ausgabe. Paris, Imprimerie de Gauthier-villars (1839).
[17] A. Wöhler, Bericht über die Versuche, welche auf der königl. Niederschlesisch-märkischen eisenbahn mit Apparaten zum Messen der Biegung und Verdehung von Eisenbahnwagenachsen während der Fahrt angestellt wurden. *Zeitschrift für Bauwesen*, **VIII**, 641- 652 (1858).
[18] A. Wöhler, Über Versuche zur Ermittlung der Festigkeit von Achsen, welche in den Werkstätten der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a.d.O. angestellt sind. *Zeitschrift für Bauwesen*, **XIII**, 233–258 (1863).
[19] A. Wöhler, Resultate der in der Central-Werkstatt der Niederschlesisch-Märkischen Eisenbahn zu Frankfurt a.d.O. angestellten Versuche über die relative Festigkeit von Eisen, Stahl und Kupfer. *Zeitschrift für Bauwesen*, **XVI**, 67–84 (1866).
[20] A. Wöhler, Über die Festigkeits-Versuche mit Eisen und Stahl. *Zeitschrift für Bauwesen*, **XX**, 73–106 (1870).
[21] A. Wöhler, Über die Festigkeitsversuche mit Eisen und Stahl. Auf Anordnung des Ministers für Handel, Gewerbe u. öffentl. Arbeiten, Grafen Itzenblitz, angestellt. Berlin, Ernst und Korn (1870).
[21a] A. Wöhler, Achsen, deren Dimensionen, Form der Achsschenkel, Material, in *Handbuch für Spezielle Eisenbahn-Technik*. 2. Band. Der Eisenbahn-Wagenbau in seinem ganzen Umfange. E. Heusinger von Waldegg Hrsg. Verlag Wilhelm Engelmann, Leipzig (1870).
[22] J. O. Almen and A. C. Boegehold, Rear axle gears: factors which influence their life, in *Proc. American Society for Testing and Materials*, Vol. 35, pp. 99–146 (1935).
[23] A. Thum and W. Bautz, Zeitfestigkeit. *VDI-Zeitschrift*, **81**, 1407-1412 (1937).
[24] R. G. Batson and J. Bradley, Static and endurance tests of laminate springs, made of carbon and alloy steels. Department of Scientific and Industrial Research, London, Special Rep. No. 13. Researches on Springs No. 6 (1931).
[25] W. Kloth and Th. Stroppel, Der Energiefluss im Zapfwellenbinder. *Die Technik in der Landwirtschaft*. **13**, 49–91 (February 1932).
[26] W. Kloth and Th. Stroppel, Kräfte, Beanspruchungen und Sicherheiten in den Landmaschinen. *Z-VDI* **80**, 85–92 (1936).
[27] L. Spangenberg, Über das Verhalten der Metalle bei wiederholten Anstrengungen. *Z. Bauw.* **24**, 473–495 and **25**, 78–98 (1874).
[28] L. Spangenberg, *Über das Verhalten der Metalle bei wiederholten Anstrengungen*. Berlin, Ernst and Korn (1875).
[29] L. Spangenberg, Über Festigkeits-Versuche mit Eisen und Stahl. *Glasers Ann. Gew.* **5**, 6–15 (1879).
[30] O. H. Basquin, The exponential law of endurance tests. *Proc. Annual Meeting, American Society for Testing Materials*, Vol. 10, pp. 625–630 (1910).
[31] Wöhler's experiments on the strength of metals. *Engineering* **4**, 160–161 (1867).

[32] H. J. Gough, *The Fatigue of Metals*. London, Scott, Greenwood (1924).

[33] H. F. Moore and J. B. Kommers, *The Fatigue of Metals*. New York, McGraw-Hill (1927).

[34] R. Blaum and A. Wöhler. *Beiträge zur Geschichte der Technik*. **8** (1918).

[35] Gutachten der zur Revision der Classifications-Bedingungen für Eisen und Stahl eingesetzten Commission, redigiert nach den Beschlüssen der General-Versammlung (des Vereins Deutscher Eisenhüttenleute). Vols 28 and 29 (May 1881).

[36] J. Bauschinger, Über die Veränderung der Elastizitätsgrenze und der Festigkeit des Eisens und Stahls durch Strecken und Quetschen, durch Erwärmen und Abkühlen und durch oftmals wiederholte Beanspruchung. *Mitt. Mech.-Tech. Lab. Münch.* **13,** 1–115 (1886).

[37] J. Bauschinger, Über die Veränderung der Elastizitätsgrenze und des Elastizitätsmoduls verschiedener Metalle. *Civilingenieur XXVII*, pp. 289–348, Felix, Leipzig (1881).

[38] J. Bauschinger, Über das Kristallinischwerden und die Festigkeitsverminderung des Eisens durch den Gebrauch. *Dinglers J.* **235,** 169–173 (1880).

[39] Kirsch, *ZVDI*. **16** (July 1898).

[40] J. A. Ewing and J. C. W. Humfrey, The fracture of metals under repeated alternations of stress. *Phil. Trans.* Royal Society, London, Vol. CC, pp. 241 – 250 (1903).

[41] Polanyi, *Zeitschrift für Physik* **89,** 660 (1934).

[42] E. Orowan, Theory of the fatigue of metals. *Proc. Royal Society*, *Ser. A*, Vol. 171, pp. 79–105 (1939).

[43] J. H. Smith, A fatigue testing machine. *Engineering* **88,** 105–107 (1908).

[44] J. H. Smith, A new testing machine for reversals of stress. *Engineering* **79,** 307–308 (1905).

[45] J. H. Smith, Some experiments on fatigue of metals. *J. Iron Steel Inst.* **82,** 246–318 (1910).

[46] J. H. Smith and G. A. Wedgwood, Stress–strain loops for steel in the cyclic state. *J. Iron Steel Inst.* **91,** 365–397 (1915).

[47] B. P. Haigh, A new machine for alternating load tests. *Engineering* **24,** 721–723 (1912).

[48] B. P. Haigh, Report on alternating stress tests of a sample of mild steel received from the British Association Stress Committee. 85, p. 163–170. Report Brit. Association (1915).

[49] B. P. Haigh, The endurance of metals under alternating stresses. *J. W. Scot. Iron St. Inst.* **23,** 17–56 (1915–1916).

[50] B. P. Haigh, The endurance of metals under alternating stresses. *Mech. World* **59,** 177–178 (1916).

[51] B. P. Haigh, Experiments on the fatigue of brasses. *J. Inst. Met.* **18,** 55–86 (1917), *Engineering* **104,** 315–319 (1917).

[52] H. J. Gough, Improvements in methods of fatigue testing. *Engineer* **132,** 159–162 (1921).

[53] H. J. Gough, Some experiments on the fatigue of materials under alternating torsion. *Pep. Memor. Aero. Res. Comm., Lond.* No. 743 (1921).

[54] H. J. Gough and D. Hanson, The behaviour of metals subjected to repeated stresses. *Proc. Roy. Soc.* (*A*) **104,** 538–565 (July 1923).

[55] H. J. Gough, The effect of keyways upon the strength and stiffness of shafts subjected to torsional stresses. *Rep. Memor. Aero. Res. Comm., Lond.* No. 864 (1925).

[56] R. V. Southwell and H. J. Gough, On the concentration of stress in the neighbourhood of a small spherical flaw; and on the propagation of fatigue fractures in statistically isotropic materials. *Phil. Mag.* **1,** 71–97 (1926). *Rep. Memor. Aero. Res. Comm., Lond.*, No. 1003 (1926).

[57] A. A. Griffith, The phenomena of rupture and flow in solids. *Phil. Trans. Roy. Soc.* **221A,** 163 (1920).

[58] A. A. Griffith, Stress concentrations in design and practice. Rep. Brit. Ass., pp. 316–324 (1921).

[59] C. E. Inglis, Stresses in a plate due to the presence of cracks and sharp corners. *Trans. Institution of Naval Architects* **55,** 219–242 (1913).

[60] J. B. Kommers, Repeated stress testing. I An investigation of a commercial endurance test. II A proposed quality factor. *Congr. Int. Ass. Test. Mater., Sixth Congr.*, New York (1912).

[61] J. B. Kommers, Essais de durée I recherche sur un essai commercial d'endurance. *Rev. Métall.* **10,** 79–87 (1913).

[62] J. B. Kommers, Essais aux efforts alternatifs II Propositiond'un facteur de qualité et d'une forme normale d'essai, avec des resultats pour divers matériaux. *Rev. Métall.* **10,** 88–92 (1913).

[63] J. B. Kommers, Repeated stress tests of steel. *Amer. Mach.* **42,** 551–553 (1915).

[64] J. B. Kommers, Broader use of Johnson's formula for repeated stress. *Engng News Rec.* **83,** 942–944 (1919).

[65] J. B. Kommers, Repeated stress safety factors quickly determined. *Engng News Rec.* **85,** 393–394 (1920).

[66] H. F. Moore, The fatigue of metals. *J. Engrs Cl. Phil.* **36,** 138–143 (1919).

[67] H. F. Moore, Fatigue of metals under repeated stress. *J.West. Soc. Engrs* **24,** 331–340 (1919).

[68] H. F. Moore and A. G. Gehring, Some fatigue tests of nickel steel and chrome–nickel steel. *Proc. Amer. Soc. Test. Mater.* **19,** 206–223 (1919).

[69] H. F. Moore and J. B. Kommers, Fatigue of metals under repeated stresses. *Iron Age* **105,** 1595–1598 (1920).

[70] H. F. Moore and J. B. Kommers, An investigation of the fatigue of metals. A report of the investigation conducted by the engineering experiment station, University of Illinois Bulletin No. 124 (October 1921).

[71] H. F. Moore and T. M. Jasper, Recent developments in fatigue of metals. *Iron Age* **110,** 779–784 (1922).

[72] H. F. Moore and W. J. Putman, Effect of cold working and of rest on resistance of steel to fatigue under reversed stress. *Trans. Amer. Inst. Min.* (*Metall.*) *Engrs* **62,** 397–419 (1918-1919).

[73] H. F. Moore and F. B. Seely, The failure of materials under repeated stress. *Proc. Amer. Soc. Test. Mater.* **15,** 437–466 (1915).

[74] J. Y. Mann, Aircraft fatigue — with particular emphasis on Australian operations and research. *Proc. 12th ICAF Symp. Toulouse*, Centre d'Essais Aéronautique (1983).

[75] W. D. Douglas, Methods employed at the Royal Aircraft Establishment for the experimental determination of the ultimate strength of aeroplane structures. *Advis. Comm. Aero. Rep. Memo.* No. 476 (June 1918).

[76] E. Heyn, Die kerbwirkung und ihre bedeutung für den konstrukteur. *ZVDI* **58,** 383–391 (1914).

[77] T. E. Stanton and R. G. C. Batson, The effect on the resistance to fatigue of crankshafts of a variation on the radius of curvature of fillets. Advisory Committee for Aeronautics (Great Britain), Internal Combustion Engine Sub-Committee Report 15, p. 6 (October 1917).

[78] G. A. Hankins, Alternating stress tests of aluminium alloys. *Advis. Comm. Aero. Light Alloys Sub-Committee* Report no. 12 (July 1917).

[79] E. Preuss, Zur Geschichte der Dauerversuche mit Metallen. ***Baumaterialienkunde* 11,** 245–249 (1906).

[80] E. Preuss, Ergebnisse neuerer Dauerversuche an Metallen. *Dinglers J.* **322,** 100–102 (1907).

[81] E. Preuss, Kerbwirkung bei Dauerschlagbeanspruchung. *Z. Ver. Dtsch. Ing.* **58,** 701–703 (1914).

[82] E. Preuss, Die Festigkeit von Schweisseisen gegenüber Stoßbeanspruchung. *Stahl und Eisen* **34,** 1207–1208 (1914).

[83] R. E. Stanton, Repeated impact testing machine at the National Physical Laboratory. *Engineering, Lond.* **82,** 33–34 (1906).

[84] R. E. Stanton, A factor in the design of machine details. *Engineering, Lond.* **83,** 505 (1907).

[85] T. E. Stanton, A new fatigue test for iron and steel. *J. Iron St. Inst.* **76,** 54–70 (1908).

[86] T. E. Stanton, Recent researches made at the National Physical Laboratory, Teddington, England, on the resistance of metals to alternating stresses. *Congr. Int. Ass. Test. Mater. Sixth Congr.*, New York, Paper VI (1912).

[87] T. E. Stanton, Récentes recherches faites au National Physical Laboratory de Teddington (Angleterre) sur la résistance des métaux aux efforts alternatifs. *Rev. Métall.* **10,** 53–57 (1913).

[88] T. E. Stanton and L. Bairstow, On the resistance of iron and steel to reversals of direct stress. *Min. Proc. Instn Civ. Engrs* **166,** 78–134 (1905–1906).

[89] T. E. Stanton and L. Bairstow, The resistance of materials to impact. *Proc. Inst. Mech. Engrs, Lond.* 889–919 (1908).

[90] T. E. Stanton and R. G. C. Batson, On the fatigue resistance of mild steel under various conditions of stress distribution. *Rep. Brit. Ass., Sect. G,* 288–291 (1916). *Engineering, Lond.* **102,** 269–270 (1916).

[91] T. E. Stanton and J. R. Pannell, Experiments on the strength and fatigue properties of welded joints in rion and steel. *Min. Proc. Inst. Civ. Engrs* **188,** 1–77 (1911–1912).

[92] T. E. Stanton and J. R. Pannell, Experiments on the strength and fatigue properties of welded joints in iron and steel. Collected Researches, National Physical Laboratory, Teddington, Vol. 9, pp. 191–217 (1913).

[93] Report on the fracture of a propeller shaft submitted by the Air Department of the Admiralty. *Advis. Comm. Aero. Rep. Memo.* No. 215 (July 1915).

[94] E. M. Eden, W. N. Rose and F. L. Cunningham, The endurance of metals — experiments on rotating beams at university college. *London. Proc. Inst. Mech. Engrs., Lond.*, pp. 839–974 (1911).

[95] A. Thum and W. Bautz, Die Gestaltfestigkeit. *Stahl und Eisen* **55,** 1025–1029 (1935).

[96] A. Thum, Die Entwicklung der Lehre von der Gestaltfestigkeit. *VDI-Zeitschrift* **88,** 609–615 (1944).

[97] E. Gaßner, Festigkeitsversuche mit wiederholter Beanspruchung im Flugzeugbau. *Deutsche Luftwacht, Ausg. Luftwissen* **6,** 61–64 (1939).

[98] E. Lehr, Formgebung und Werkstoffausnutzung. *Stahl und Eisen*, Zeitschrift für das deutsche Eisenhüttenwesen, 61. Jahrgang, Heft 43 (1941).

[99] H. W. Kaul, Statistische Erhebungen über Betriebsbeanspruchungen von Flugzeugflügeln. Jahrbuch 1938 der deutschen Luftfahrtforschung, Ergänzungsband, pp. 307–313 (1938).

[100] H. W. Kaul, Statistische Erhebungen über Betriebsbeanspruchungen von Flugzeugflügeln, in *Bericht*, Vol. 106, Lilienthal-Gesellschaft für Luftfahrtforschung, Stuttgart (1939).

[101] H. W. Kaul, Beanspruchung des Tragwerks und Höhenleitwerks beim abfangen und im kurvenflug. *Ringbuch der Luftfahrttechnik* (1940).

[102] H. W. Kaul and B. Filzek, Meßergebnisse über Betriebsbeanspruchungen von Flugzeugflügeln. *Luftwissen* 191, **8,** 20–24.

[103] H. Freise, Spitzenwerte und Häufigkeit von Böenbelastungen an Verkehrsflugzeugen. Jahrbuch 1938 der deutschen Luftfahrtforschung, pp. 289–302 (1938).

[104] H. Freise, Der DVL-Beschleunigungsmesser. *Luftwissen* 7, p. 30 (1940).

[105] H. Freise, Ritzgeräte zum Aufzeichnen schnell wechselnder Spannungen. *Drücke und Kräfte, Z-VDI* **82,** 457–461 (1938).

[106] B. Filzek, Frontmessungen, in *Bericht*, Vol. 152. Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[107] R. V. Rhode and E. E. Lundquist, Preliminary Study of Applied Load Factors in Bumpy Air. NACA Tech. Note 374 (1931).

[108] R. V. Rhode, Gust loads of airplanes. *SAE Trans.* **32,** 80–88 (1937).

[109] R. V. Rhode and P. Donely, Frequency of occurrence of atmospheric gusts and of related loads on airplane structures. NACA ARR L4121, WR-L-121 (1944).

[110] W. G. Walker, Summary of revised gust velocity data obtained from V.G. records taken on civil transport airplanes from 1933–1950. NACA Tech. Note 3041 (1953).

[111] E. Gaßner, Ergebnisse aus Betriebsfestigkeits-Versuchen mit Stahl- und Leichtmetallbauteilen, in *Bericht*, Vol. 152. Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[112] O. Graf, *Dauerfestigkeit von Stählen mit Walzhaut ohne und mit Bohrung, von Niet- und Schweißverbindungen*. VDI, Berlin (1931).

[113] O. Föppl, *Oberflächendrücken und Druckeigenspannungen*. Mitteilungen des Wöhler-Instituts Braunschweig, Vol. 33, Friedr. Vieweg and Sohn, Braunschweig (1938).

[114] O. Föppl, *Geschichtliche Entwicklung des Oberflächendrückens zum Zwecke der Steigerung der Dauerhaltbarkeit*. Mitteilungen des Wöhler-Instituts Braunschweig, Vol. 36, Friedr. Vieweg and Sohn, Braunschweig (1939).

[115] O. Föppl, *Die Zweckmäßigste Art der Durchführung des Oberflächendrückens*. Mitteilungen des Wöhler-Instituts Braunschweig, Vol. 38, Friedr. Vieweg and Sohn, Braunschweig (1941).

[116] A. Thum and W. Bautz, Ursachen der Steigerung der Dauerhaltbarkeit gedrückter Stäbe. *Forschung auf dem Gebiete des Ingenieurwesens* 6, **3** (1935).

[117] A. Thum and H. Oschatz, Möglichkeiten zur Steigerung der Dauerhaltbarkeit von Konstruktionsteilen. *Der Maschinenschaden* **2,** 17–24 (1933).

[118] A. Thum and H. Weiß, Versuche zur Steigerung der Verdrehdauerhaltbarkeit quergebohrter Wellen durch Kaltverformung. *ATZ* **24** (1938).

[119] O. Graf, *Die Dauerfestigkeit der Werkstoffe und Konstruktionselemente*. Julius Springer, Berlin (1929).

[120] O. Föppl, E. Becker and G. von Heydekampf, *Die Dauerprüfung der Werkstoffe hinsichtlich ihrer Schwingfestigkeit und Dämpfungsfähigkeit*. Julius Springer, Berlin (1929).

[121] W. Herold, *Die Wechselfestigkeit Metallischer Werkstoffe — Ihre Bestimmung und Anwendung*. Julius Springer, Wien (1934).

[122] R. Cazaud and L. Persoz, *La Fatigue des Métaux*. Dunod Press, Paris (1937).

[123] S. V. Serensen, *The Endurance of Metals and the Design of Machine Components*. M.L.Gos Izd-vo Teckhnich Liter. (1937).

[124] *Battelle Memorial Institute*, *Prevention of the Failure of Metals under Repeated Stresses*. John Wiley, New York (1941).

[125] A. Palmgren, Die Lebensdauer von Kugellagern. *VDI-Zeitschrift* **68**, 339–341 (1924).

[126] B. F. Langer, Fatigue failure from stress cycles of varying amplitude. *Trans. ASME J. appl. Mech.* **59**, A160–A162 (1937).

[127] S. V. Serensen, Theory of strength under variable loading. Akademici Nauk Ukrainskoc, SSR. Nukwoi pratsi Instytuta Budivel noi K. McKhani [Abstract in *Stahl und Eisen* **60**, 285 (1940)].

[128] M. A. Miner, Cumulative damage in fatigue. *Trans. ASME J. appl. Mech.* **12**, A159–A164 (1945).

[129] A. Thum, O. Svenson and H. Weiss, Neuzeitliche Dehnungsmeßgeräte. *Forschung auf dem Gebiet des Ingenieurwesens* **9**, 229–234 (1938).

[130] F. G. Tatnall, Tatnall on testing. *Auflage* **2**. American Society for Metals (1967).

[131] E. E. Simmons, U.S. Patent 320327 (1942).

[132] W. Weibull, *A Statistical Theory of the Strength of Materials*. Ingeniörs Vetenskaps Akademiens Handlingar No. 151 Generalstabens Litografisky Anstalts Förlag, Stockholm (1939).

[133] H. Müller-Stock, Der Einfluß dauernd und unterbrochen wirkender, schwingender Überbeanspruchung auf die Entwicklung des Dauerbruchs. Dissertation, TH Braunschweig (1937).

[134] H. Müller-Stock, E. Gerold and E. H. Schulz, Der Einfluß einer Wechselvorbeanspruchung auf Biegezeit- und Biegewechselfestigkeit von Stahl st 37. *Archiv für Eisenhüttenwesen* **12**, 141 (1938).

[135] A. V. De Forest, The rate of growth of fatigue cracks. *Trans. ASME* **58**, A23–A25 (1936).

[136] Dauerbruchtagung des DGM. Fachheft der Zeitschrift für Metallkunde (1928).

[137] O. Graf, Dauerfestigkeit von Nietverbindungen. *Intern. Vereinigung für Brückenbau und Hochbau*. Beitrag zum Vorbericht für den 2. Kongreß, Berlin–München (1936).

[138] K. Heyer, Mehrstufenversuche an Konstruktionselementen, in *Bericht* **152**. Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[139] E. Hottenrott, Über Beanspruchungsmessungen an den Schrauben eines lösbaren Holmstoßes, in *Bericht* **152**. Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[140] K. Klöppel, Dauerfestigkeit im Stahlbrückenbau, in *Bericht* **106**. Stuttgart, Lilienthal-Gesellschaft für Luftfahrtforschung, Berlin (1939).

[141] 25-Jahrfeier der Staatlichen Materialprüfanstalt an der TH Darmstadt. Ansprachen und Festvorträge. Schriften der Hessischen Hochschulen, Jahrgang 1932, Vol. 4, Darmstadt (1932).

[142] Werkstofftechnisches Kolloquium der Staatlichen Materialprüfungsanstalt an der TH Darmstadt. Schriften der Hessischen Hochschulen, Jahrgang 1934, Vol. 3, Darmstadt (1934).

[143] VDI-Hauptversammlung Darmstadt 1936 und 80-Jahrfeier des Vereins deutscher Ingenieure. *Fachvorträge*. VDI, Berlin (1936).

[144] Ausschuß flugzeugkonstruktion, Bericht über die Tagung "Zeit- und Dauerfestigkeit im Flugzeugbau", in *Bericht* **106**. Lilienthal-Gesellschaft für Luftfahrtforschung (1939).

[145] F. Wunderlich, Festigkeit und Formgebung. *Auszug aus einem Lehrgang für Konstrukteure*, VDI, Württembergischer Bezirksverein, Gruppe Konstruktion, Stuttgart (1937).

[146] W. Kloth, Beanspruchungen, Werkstoffe und Gestaltung im Landmaschinenbau. 74. Hauptversammlung Darmstadt und Achtzigjahrfeier des VDI, VDI, Berlin (1936).

[147] A. Thum, Neuere Anschauungen in der Gestaltung. 74. Hauptversammlung Darmstadt und Achtzigjahrfeier des VDI, VDI, Berlin (1936).

[148] A. Thum, Geschmiedete und gegossene Kurbelwellen. 74. Hauptversammlung Darmstadt und Achtzigjahrfeier des VDI, VDI, Berlin (1936).

[149] F. Meyercordt, Beurteilung der Beanspruchungsverhältnisse aus dem Bruchaussehen. 74. Hauptversammlung Darmstadt und Achtzigjahrfeier des VDI, VDI, Berlin (1936).

[150] W. Bautz, Möglichkeiten zur Steigerung der Dauerhaltbarkeit von Konstruktionsteilen mit unvermeidbarer Kerbwirkung. 74. Hauptversammlung Darmstadt und Achtzigjahrfeier des VDI, VDI, Berlin (1936).

[151] R. A. MacGregor, W. S. Burn and F. Bacon, *The Relation of Fatigue to Modern Engine Design*. The North East Coast Institution of Engineers and Shipbuilders, Bolbec Hall, London (1935).

[152] W. Weibull, History of service simulated load spectrum fatigue testing. WADC-TR-57-267.

[153] K. Walz, *Federfragen Insbesondere aus dem Gebiet der Zeitfestigkeit bei Sinusförmiger und Schlagartiger Beanspruchung*. Mauser (1941).

[154] D. J. McAdam, Corrosion of metals under cyclic stress. *Proc. ASTM* **29**, 250–313 (1929).

[155] Steel in marine structures. *Int. Conf.*, Paris. Kommission der Europäischen Gemeinschaften (1980).

[156] Steel in marine structures. *Int. Offshore Conf.*, Delft. Kommission der Europäischen Gemeinschaften (1987).

[157] W. Hort, Dauerbruch als dynamische, schwingungstechnische Erscheinung. *Zeitschrift für Metallkunde* (1928).

[158] M. A. Miner, Estimation of fatigue life with particular emphasis on cumulative damage, in *Metal Fatigue* (Edited by G. Sines and J. L. Waisman), pp. 278–289. McGraw-Hill (1959).

[159] C. H. Gibbons, The use of the resistance wire strain gage in stress determination. *Proc. Soc. Exp. Stress Analysis* **I**, 41 (1943).

[160] H. J. Grover, The use of electric strain gages to measure repeated stress. *Proc. Soc. Exp. Stress Analysis* **I**, 110 (1943).

[161] M. Tucher, Vibratory stress measurement in aircraft engine parts. *Proc. SESA* **1**, 11–17 (1943).

[162] A. Goloff, Determination of operating loads and stresses in cranckshafts. *Proc. SESA* **2**, 149–149 (1944).

[163] G. M. Magee and R. Ferguson, Applications of electric measuring equipment in railroad research. *Proc. SESA* **II**, 18–33 (1944).

[164] F. Seewald, Messungen mit dem Glasritzdehnungsschreiber der Deutschen Versuchsanstalt für Luftfahrt. *Maschinenbau* **10**, 725–727 (1932).

[165] E. Siebel, *Handbuch der Werkstoffprüfung* 1. Band: Prüf- und Meßeinrichtungen. Springer, Berlin (1940).

[166] J. B. Kommers, Effect of overstressing and understressing in fatigue. *Proc. ASTM* **43**, 749 (1943).

[167] J. B. Kommers, The effect of understressing of cast iron and openheath iron. *Proc. ASTM* **30**, 368–383 (1930).

[168] J. B. Kommers, Overstressing and understressing in fatigue. *Engineering* **143,** 620–622 (1937) and 676–678 (1937).

[169] J. T. Ransom and R. F. Mehl, The statistical nature of the endurance limit. *Metals Trans.* **185,** 364–365 (1949).

[170] G. E. Dieter, G. T. Horne and R. F. Mehl, Statistical study of overstressing in steel. NACA TN 3211 (1954).

[171] H. J. French, Fatigue and the hardening of steels. *Trans. A.S.S.T.* 899–946 (1933).

[172] E. Gaßner and W. Schütz, Einfluß unterschiedlich hoher und häufiger Vorbelastungen auf die Schwingfestigkeit gebohrter Flachstäbe aus st 37. LBF-Bericht FB-58 (1966).

[173] O. Graf, Einige Bemerkungen über die Ermittlung der Dauerfestigkeit und der zulässigen Anstrengungen der Werkstoffe. Stahlbau, Beilage zu Bautechnik (1931) s. 285; Intern. Verb. F. Materialprüfung, Kongreß Zürich S. 328 (1931).

[174] O. Graf, Versuche mit Nietverbindungen bei oftmals wiederholter Belastung. VDI-Z. 76 S. 438 (1932).

[175] O. Graf. Dauerfestigkeit der Niet- und Schweißverbindungen. Abhandlungen d. Intern. Vereinigung für Brücken-Hochbau, 1. Band, Zürich S. 215 (1932).

[176] O. Graf, 200-t-Zug-Druck-Dauerversuch. VDI-Nachrichten No. 22 (1932).

[177] O. Graf, Dauerversuche mit Nietverbindugnen aus ST 37 zur Erkundung des Einflusses von s: t. Bauingenieur S. 389 (1932).

[178] O. Graf, Dauerversuche mit Schweißverbindungen. Bautechnik S. 395 u. 414 (1932).

[179] O. Graf, Aus Dauerversuchen mit Lichtbogenschweißungen. Ein Beitrag zur Frage der Bemessung und Anordnung der Kehlnähte. Messungen von Spannungen in Schweißverbindungen. Stahlbau S. 177 (1932).

[180] O. Graf, Die Dauerfestigkeit von genieteten und geschweißten Verbindungen aus Baustahl St. 52. Stahl und Eisen 53 S. 861 (1933).

[181] O. Graf, Dauerversuche mit Schweißverbindungen. Mitt. des Fachausschusses f. Schweißtechnik i. VDI Nr. 23, S. 3 (1933).

[182] O. Graf, Über Dauerversuche mit I-Trägern aus St 37. Stahlbau, Beilage zu Bautechnik H. 22, S. 169 (1934).

[183] O. Graf, *Dauerfestigkeit von Schweißverbindungen*. VDI-Z. 78 S. 1423–1427 (1934).

[184] O. Graf, On the fatigue strength of constructional members, and particularly of welded joints. *Symp. Welding of Iron and Steel*, pp. 19–22. Iron and Steel Institute (May 1935).

[185] O. Graf, Dauerversuche mit großen Schweißverbindungen bei oftmaligem Wechsel zwischen Zug-und Druckbelastung. *Stahlbau, Beilage zu Bautechnik* H. 21, S. 164–165 (1935).

[186] O. Graf, Versuche über den Einfluß der Zahl der minütlich auftretenden Lastwechsel auf die Ursprungszugfestigkeit von Nietverbindungen. Stahlbau H. 6, S. 48 (1936).

[187] O. Graf, Dauerbiegeversuche mit geschweißten Trägern I 30 aus St 37. Stahlbau H. 9, S. 71 (1936).

[188] O. Graf, Einfluß der Gestalt der Schweißverbindungen auf ihre Widerstandsfähigkeit. Intern. Vereinigung für Brückenbau und Hochbau. Beitrag zum Vorbericht für den 2. Kongreß, Berlin–München, 1–11 October 1936. Berlin: Ernst (1936).

[189] O. Graf, Festigkeit des Betons und des Eisenbetons bei dauernder und bei oftmals wiederholter Belastung. Intern. Vereinigung für Brückenbau und Hochbau. Beitrag zum Vorbericht für den 2. Kongreß, Berlin–München, 1–11 October 1936. Berlin: Ernst (1936).

[190] O. Graf, Versuche über die Längenänderungen und über die Tragfähigkeit von Nietverbindungen aus St 52 unter oft wiederkehrenden Wechseln zwischen Zug- und Druckbelastung. Stahlbau S. 185–188 (1936).

[191] O. Graf, Die Dauerbiegefestigkeit von geschweißten Schienen. *Industrieblatt* 37 S. 10 (1937).

[192] O. Graf, Versuche über das Verhalten von genieteten und geschweißten Stößen in Trägern I 30 aus St 37 bei oftmals wiederholter Belastung. Stahlbau S. 9–16 (1937).

[193] O. Graf und E. Brenner, Versuche zur Ermittlung der Widerstandsfähigkeit von Beton gegen oftmals wiederholte Druckbelastung. *Deutscher Ausschuß für Eisenbeton* H. 76, S. 1–13 (1934).

[194] O. Föppl, Dämpfungsfähigkeit der Werkstoffe, Oberflächendrücken, Resonanzschwingungsdämpfer für Kurbelwellen. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 30, Friedr. Vieweg, Braunschweig (1937).

[195] O. Föppl, Größtspannung und Tiefenwirkung beim Oberflächendrücken mittels Rolle. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 40, Friedr. Vieweg, Braunschweig (1948).

[196] O. Föppl, Die mit dem Oberflächendrücken verbundenen plastischen und elastischen Formänderungen Spannungskonzentration und Ermüdungsbrüche. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 42, Friedr. Vieweg, Braunschweig (1949).

[197] H. Isemer, Die Steigerung der Schwingungsfestigkeit von Gewinden durch Oberflächendrücken. Dissertation an der TH Braunschweig (1931).

[198] H. Koch, Die Biegewechselfestigkeit einer Keilverbindung (Paßfederanordnung) und die Erhöhung der Dauerhaltbarkeit durch das Oberflächendrücken. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 20, NEM-Verlag Berlin (1934).

[199] W. Meyer, Die Drehwechselfestigkeit genuteter Stäbe und die Erhöhung der Dauerhaltbarkeit durch Oberflächendrücken. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 18, NEM-Verlag Berlin (1934).

[200] E. Wedemeyer, Die Steigerung der Dauerhaltbarkeit von Schrauben durch Gewindedrücken. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 33, Friedr. Vieweg, Braunschweig (1938).

[201] H. Wiecker, Die Biegewechselfestigkeit genuteter Stäbe und die Erhöhung der Dauerhaltbarkeit durch das Oberflächendrücken. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 19, NEM-Verlag GmbH Berlin (1934).

[202] P. Behrens, Das Oberflächendrücken zur Erhöhung der Drehschwingungsfestigkeit. Veröffentlichungen des Wöhler-Instituts Braunschweig, Heft 6, NEM-Verlag Berlin (1930).

[203] P. Berg, Die Steigerung der Dauerhaltbarkeit von Keilverbindungen durch Oberflächendrücken. Mitteilungen des Wöhler-Instituts Braunschweig, Heft 26, NEM-Verlag Berlin (1935).

[204] H. Döring, Das Drücken der Oberfläche und der Einfluß von Querbohrungen auf die Biegungsschwingungsfestigkeit. Dissertation an der TH Braunschweig (1930).

[205] O. Föppl, Oberflächendrücken zum Zwecke der Steigerung der Dauerhaltbarkeit mit Hilfe des Stahlkugelgebläses. Mitteilungen des Wöhlerinstituts Braunschweig, Heft 36, Friedr. Vieweg, Braunschweig (1939).

[206] O. Föppl, DRP 521465 (1929).

[207] O. Föppl, US-Patente 1946340 und 194792 (February 1934).

[208] DRP 573630, Röchling-Stahlwerke, 1929 a Diskussion zum Artikel, H. Kändler, E. H. Schulz: Ein neuer Weg zur Verminderung der Dauerbruchgefahr. Bericht Nr. 48 des Werkstoffausschusses des VDEh (1924).
[209] O. Föppl, Das Oberflächendrücken als Mittel zur Steigerung der Dauerhaltbarkeit der im Kfz-Bau verwendeten Federn. *ATZ* **45**, Nr. 12, 321–325 (1942).
[210] H. Sigwart, Aus der Geschichte der Werkstoffprüfung und der Festigkeitslehre. Informationen der Zentralwerkstofftechnik, Daimler–Benz (1985).
[211] A. Thum and W. Bautz, Zur Frage der Formziffer. VDI-Zeitschrift Bd. 79 S. 1303–1306 (1935).
[212] A. Thum and O. Svenson, Die Verformungs- und Beanspruchungsverhältnisse von glatten und gekerbten Stäben, Scheiben und Platten in Abhängigkeit von deren Dicke und Belastungsart. Forschung auf dem Gebiete des Ingenieurwesens, Band 13, Nr. 1, VDI-Verlag Berlin (1942).
[213] A. Thum and O. Svenson, Beanspruchung bei mehrfacher Kerbwirkung. Entlastungs- und Überlastungskerben. Schweizer Archiv, 15. Jahrgang, Nr. 6 (1949).
[214] H. Wiegand, Über die Dauerfestigkeit von Schraubenwerkstoffen und Schraubenverbindungen. Dissertation an der TH Darmstadt (1933).
[215] F. Wunderlich, Die Dauerbiegefestigkeit von Konstruktionsteilen an Einspannungen, Nabensitzen und ähnlichen Kraftangriffsstellen. Dissertation an der TH Darmstadt (1933).
[216] A. Thum, Festigkeitsprüfung bei schwingender Beanspruchung. Siebel, Handbuch der Werkstoffprüfung, 2. Bd. S. 175/231, Verlag Springer Berlin (1939).
[217] A. Thum and F. Wunderlich, Zur Festigkeitsberechnung von Fahrzeugachsen. Z. VDI 78 S. 823/824 (1934).
[218] A. Thum and K. Federn, Spannungszustand und Bruchausbildung. Verlag Springer Berlin (1939).
[219] H. Ochs, Einfluß der Korrosion auf die Dauerfestigkeit. Schr. Hess. Hochsch. Heft 4, S. 55–59 (1932).
[220] A. Thum and H. Ochs, Korrosion und Dauerfestigkeit. Mitt. der Mat.-Prüfanst. Darmstadt Heft 9, VDI-Verlag Berlin (1937).
[221] A. Thum, Der Werkstoff in der konstruktiven Berechnung. Vortrag Eisenhüttentag 1938, Stahl u. Eisen 59 S. 252–263 (1939).
[222] A. Thum and E. Bruder, Flanschwellen-Dauerbrüche und ihre Ursachen. Deutsche Kraftfahrtforschung, Heft 41, VDI-Verlag Berlin (1940).
[223] W. Buchmann, Festigkeitseigenschaften von Magnesiumlegierungen. Beck Magensium S. 131–271, Verlag Springer Berlin (1939).
[224] A. Thum and W. Buchmann, Dauerfestigkeit und Konstruktion. Mitt. d. Mat.-Prüf.-Anst- Darmstadt (1932) Heft 1, VDI-Verlag Berlin (1932).
[225] F. Meyercordt, Über die Gestaltfestigkeit des Gußeisens und die innere Mechanik seiner Festigkeit. Diss. Darmstadt (1937).
[226] W. Buchmann, Dauerfestigkeitseigenschaften von Elektronlegierungen, insbesondere Kerbempfindlichkeit der Knetlegierungen. Jahrbuch der Deutschen Luftfahrtforschung, Teil I, Flugwerk. S. 524–528 (1938).
[227] H. Wiegand, Die Dauerfestigkeit der Schraube in Abhängigkeit von der Mutterform. Schr. Hess. Hochsch. Heft 2, S. 67–72 (1933).
[228] H. Oschatz, Gesetzmäßigkeiten des Dauerbruchs und Wege zur Steigerung der Dauerhaltbarkeit gekerbter Konstruktionen. Dissertation an der TH Darmstadt (1932).
[229] W. Bautz, Steigerung der Dauerhaltbarkeit von Formelementen durch Kaltverformung. Dissertation an der TH Darmstadt (1935).
[230] A. Thum and H. Ochs, Die Bekämpfung der Korrosionsermüdung durch Druckvorspannung. Z. VDI 76 S. 915–916 (1932).
[231] R. Zoege von Manteuffel, Einfluß der Oberflächenbeschaffenheit auf die Dauerhaltbarkeit von wärmebehandelten Federn. Deutsche Kraftfahrtforschung, Zwischenbericht Nr. 20 (1935).
[232] H. Wiegand, Werkstoff-, Gestaltungs- und Behandlungsfragen bei Stählen im Flugzeugbau. Luftwissen 6 S. 47–53 (1939).
[233] R. Zoege von Manteuffel, Einfluß der Wärmebehandlung auf die Dauerhaltbarkeit von Federn. Deutsche Kraftfahrtforschung, Zwischenbericht Nr. 10 (1937).
[234] A. Thum and H. Weiß, Dauerverdrehversuche an Kurbelwellenstählen. Zentr. f. wiss. Berichtswes. über Luftfahrtforschung F.B. Nr. 966 (1938).
[235] K. Bandow, Dauerhaltbarkeit von Stahl- und Gußkurbelwellen. Deutsche Kraftfahrtforschung Heft 14, VDI-Verlag Berlin (1938).
[236] A. Thum, Querbrüche an Kardanwellen. Deutsche Kraftfahrtforschung, Heft 6, VDI-Verlag Berlin (1938).
[237] A. Erker, Zeit- und Dauerfestigkeit von Schweißverbindungen, in Bericht 152 in Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).
[238] A. Erker, Beeinflußung der Dauerhaltbarkeit von Rahmenträgern durch Bohrungen, Nietungen und Schweißungen. Deutsche Kraftfahrtforschung, Heft 35, S. 1/12, VDI-Verlag Berlin (1939).
[239] A. Thum and Th. Lipp, Zur Frage der Dauerhaltbarkeit geschweißter und gegossener Konstruktionsteile. Diss. Darmstadt (1934), Gießerei (1934) S. 41–49, S. 64–71, S. 89–95, S. 131–141.
[240] A. Thum and R. Rüttger, Das Verhalten von Rohrverbindungen bei verschiedener Oberflächenbeschaffenheit und Formgebung. Mitt. Ver. Großkesselbes. Nr. 53, s. 127–133 (1935).
[241] H. Wiegand, Einflüsse der Oberflächenbehandlung auf die Festigkeitseigenschaften von Leichtmetallen. Metallwirtsch. 20, S. 165–168 (1941).
[242] F. Wunderlich, Die Reiboxydation, Reibung und Verschleiß. VDI-Verschleißtagung, S. 87–95 (1938).
[243] K. Richard and E. Baumeister, Prüfung von Getriebezahnrädern auf Abnutzung und Grüchenbildung (Pittings) bei verhältnismäßig hohen Öltemperaturen. Deutsche Kraftfahrtforschung, Zwischenbericht Nr. 40 (1938).
[244] W. Bautz, Kritik der Dauerfestigkeit als Bemessungsgrundlage. Forschung auf dem Gebiete des Ingenieurwesens, Band 12, Nr. 4, VDI (1941).
[245] A. Thum, Die werkstofftechnischen Grundlagen einer neuen Konstruktionslehre. Schr. Hess. Hochsch. Heft 4 S. 25–36 (1932).
[246] W. Bautz, Erscheinungsformen des Dauer- und Gewaltbruches. Sonderdruck des Vortrags, Betriebsleitertagung 1935, Berlin, Allianz und Stuttgarter Verein Vers.-A.-G. (1935).

[247] W. Bautz, Der Stand der Modellversuche zur Ermittlung der Spannungsverteilung in drehbeanspruchten Querschnitten. Bericht über die Tagung des Fachausschusses für Maschinenelemente in Aachen 1935, VDI-Verlag Berlin, S. 23–28 (1936).

[248] W. Bautz, Untersuchungen über die Schadenslinie bei Leichtmetallen. Z. VDI 84, S. 918–919 (1940).

[249] A. Thum and O. Svenson, Die Verformungs- und Beanspruchungsverhältnisse an Bauelementen des Fahrzeugbaues. Deutsche Kraftfahrtforschung, Heft 71, VDI-Verlag Berlin (1942).

[250] A. Thum and O. Svenson, Kontakt-Dehnungsmesser zum Messen von Betriebsbeanspruchungen. VDI-Zeitschrift Bd. 88, Nr. 11/12, S. 153–154 (1944).

[251] A. Thum and O. Svenson, Dehnungsmessungen an einfachen Bauelementen. ATZ Jahrgang 46, Nr. 21/22 (1943).

[252] A. Thum, O. Svenson and H. Weiss, Neuzeitliche Dehnungsmeßgeräte. Forschung 9. Band, Heft 5, Sept./Okt. S. 229–234 (1938).

[253] H. Wiegand, Oberfläche und Dauerfestigkeit. Berlin-Spandau, BMW-Flugmotorenbau, 1940, auch im Verlag E. Müller, Berlin (1941).

[254] H. Oschatz, Eine Dauerprüfmaschine zur Bestimmung der Dauerhaltbarkeit von Proben und Formelementen. Metallwirtsch. 13, S. 443–448 (1934).

[255] K. Lürenbaum, Belastung und Tragfähigkeit von Flugmotoren-Kurbelwellen. Jb. Lilienthal-Gesellschaft, Luftfahrtf. 296–304 (1937).

[256] E. Siebel and G. Stähli, Prüfung von Kurbelwellen. Gießerei 28, S. 145–150 (1941).

[257] F. Körber, Über den Dauerbruch metallischer Werkstoffe. Schriften der deutschen Akademie der Luftfahrtforschung, Heft 15 (1939).

[258] A. Teichmann and F. Michael, Sicherheit und Gestaltung im Flugzeugbau. ATZU 1934, Heft 2, S. 28–36 (1934).

[259] H. G. Küssner, Beanspruchung von Flugzeugflügeln durch Böen. *Z. Flugtech.* **22,** 579–586 and 605–615 (1931).

[260] H. G. Küssner, Häufigkeitsbetrachtungen zur Ermittlung der erforderlichen Festigkeit von Flugzeugen. Luftfahrtforschung, Band 12, Nr. 2 (1935).

[261] H. W. Kaul, Die erforderliche Zeit- und Dauerfestigkeit von Flugzeugtragwerken. Jahrbuch 1938 der deutschen Luftfahrtforschung, S. 274–288 (1938).

[262] E. Gaßner and A. Teichmann, Ansatz und Durchführung von Betriebsfestigkeits-Versuchen. Gemeinschaftsbericht der DVL e.V., Berlin-Adlershof, und des Lehrstuhls für Flugzeugbau an der TH Aachen (1945).

[263] P. Brenner, Dynamische Festigkeit von Flugzeug-Konstruktionsteilen. *Luftfahrtforschung* **3,** Nr. 3, 17. 59–65 (1929).

[264] P. Brenner, Baustoffragen bei der Konstruktion von Flugzeugen. *Z. Flugtech.* **22,** Nr. 21, 637–648 (1931).

[265] H. Hertel, Dynamische Bruchversuche mit Flugzeugbauteilen. *Z. Flugtech.* **22,** Nr. 15, 465–474 (1931) und Nr. 16, 489–502 (1931).

[266] G. S. Campbell and R. T. C. Lahey, A survey of serious aircraft accidents involving fatigue fracture. Vol. 1, Fixed-Wing Aircraft, Aeronautical Note NAE AN-7, NRC No. 21276, Ottawa (1983).

[267] E. Gaßner, Auswirkung betriebsähnlicher Belastungsfolgen auf die Festigkeit von Flugzeugbauteilen. Kurzfassung der Dissertation gleichen Titels. Jahrbuch 1941 der deutschen Luftfahrtforschung, S. I472–I483 (1941).

[268] H. W. Foster, Fatigue testing of airframe structural components, in *Symp. on Large Fatigue Testing Machines and Their Results*. ASTM STP **216** (1958).

[269] E. Gaßner, Über bisherige Ergebnisse aus Festigkeitsversuchen im Sinne der Betriebsstatistik, in Bericht 106, Lilienthal-Gesellschaft für Luftfahrtforschung Berlin, Stuttgart (1939).

[270] DRP 759264, Erfinder Erlinger und Gaßner (1940).

[271] H. Degenhardt, Ergebnisse von Betriebsfestigkeits-Versuchen mit Flugzeugbauteilen, in Bericht 152 in Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[272] K. Heyer, Mehrstufenversuche mit Konstruktionselementen, in Bericht 152 in Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[273] G. Boelk, Über Streuungen bei Zeitfestigkeitsversuchen an Flugzeugbauteilen und ihre Auswirkung, in Bericht 152 in Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[274] A. Teichmann, Grundsätzliche Betrachtungen über Festigkeitsversuche im Sinne der Betriebsstatistik, in Bericht 106, Teil I in Stuttgart, Lilienthal-Gesellschaft für Luftfahrtforschung, Berlin (1939).

[275] A. Teichmann, Grundsätzliches zum Betriebsfestigkeitsversuch. Jahrbuch 1941 der deutschen Luftfahrtforschung, S. I467 - I471 (1941).

[276] M. Matsuishi and T. Endo, Fatigue of metals subjected to varying stress. Presented to Kyushu District Meeting, *Jap. Soc. Mech. Engng* (March 1968).

[277] J. B. DeJonge, Fatigue load monitoring of tactical aircraft. NLR Report TR 69 0330 (August 1969).

[278] E. Gaßner and A. Lüttner, Betriebsfestigkeitsversuche mit Widerstands-Abbrenn-Schweißverbindungen aus Chrom-Vanadium-Stahl. DVL-Bericht UM 1385 (1944).

[279] E. Gaßner, Zur Gestaltung eines dauerbruchsicheren Tragschrauben-Holmanschlusses. DVL-Bericht, VM 466 (1937).

[280] E. Gaßner and Pries: Zeit- und Dauerfestigkeit für stabartige Bauteile aus Cr-Mo-Stahl, Duralumin, Hydronalium und Elektron. Forschungsbericht Nr. 1294, DVL (1940).

[281] E. Gaßner and A. Teichmann, Zur Betriebsfestigkeit gekerbter Bauteile. *Technische Berichte* **11,** 459–460 (1944).

[282] E. Gaßner, Lastannahmen für doppelkielige Leitwerke. *Technische Berichte* **11,** ZWB (1944).

[283] E. Gaßner, Zur Aufstellung von Bemessungsregeln für Flugzeugbauteile. Jahrbuch 1944 der deutschen Luftfahrtforschung (1944).

[284] A. Wilm, *Metallurgie.* **8,** 225 (1911).

[285] A. G. Pugsley, Repeated loading on structures, in *The Failure of Metals by Fatigue*. Proc. of a Symposium, Melbourne, Melbourne University Press (December 1946).

[286] E. Gaßner, Performance fatigue testing with respect to aircraft design, in *Fatigue in Aircraft Stuctures*. Academic Press, New York (1956).

[287] H. Seelhorst, Einige Erfahrunge über das Auftreten von Ermüdungsschäden, in Bericht 152 in Jena, Lilienthalgesellschaft für Luftfahrtforschung, Berlin (1942).

[288] J. K. Williams, The airworthiness approach to structural fatigue, in *Fatigue Design Procedures*. Proc. of the 4th ICAF-Symposium (Edited by E. Gassner and W. Schütz), Munich. Pergamon (1965).

[289] E. v. Houdremont and R. Mailänder, Dauerbiegeversuche mit Stählen. *Stahl und Eisen* **49,** Nr. 23, 833–839 (1929).

[290] E. v. Houdremont and H. Bennek, Federstähle. *Stahl und Eisen* **52,** Nr. 27, 653–662 (1932).

[291] E. v. Houdremont, Einige Gesichtspunkte zur Entwicklung der Forschung auf dem Gebiet der Dauerfestigkeit. *Techn. Mitt. Krupp A,* **2,** S. 1–8 (1939).

[292] A. Pomp and M. Hempel, Über das Verhalten von Gußeisen und Temperguß unter wechselnder Beanspruchung. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **22,** Nr. 11, S. 169–201 (1940).

[293] A. Pomp and M. Hempel, Biegewechselversuche an Chrom-Molybdän-Vergütungs- und Einsatzstählen im Vergleich zu nickelhaltigen Stählen. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **22,** Nr. 10, S. 149–168 (1940).

[294] A. Pomp and M. Hempel, Das Verhalten von Gußeisen unter Zug-Druck-Wechselbeanspruchung. *Stahl u. Eisen* **57,** Nr. 40, S. 1125–1127 (1937).

[295] A. Pomp and C. A. Duckwitz, Dauerprüfungen unter wechselnden Zugbeanspruchungen an Stahldrähten. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **13,** Nr. 5, S. 79–91 (1931).

[296] A. Pomp and M. Hempel, Untersuchungen an Stahlstäben bei wechselnder Zugbeanspruchung. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **15,** Nr. 18, S. 247–254 (1933).

[297] A. Pomp and M. Hempel, Dauerfestigkeits-Schaubilder von Stählen bei verschiedenen Zugmittelspannungen unter Berücksichtigung der Prüfstabform. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **18,** Nr. 1, S. 1–14 (1936).

[298] A. Pomp and M. Hempel, Dauerfestigkeits-Schaubilder von gekerbten und kalt-verformten Stählen sowie von 1″- und 1 1/8″-Schrauben bei verschiedenen Zugmittelspannungen. *Mitt. K.-Wilh.-Inst. Eisenforsch.* **18,** Nr. 14, S. 205–215 (1936).

[299] M. Hempel, Gußeisen und Temperguß unter Wechselbeanspruchung. *Z. Ver. dtsch. Ing.* **85,** Nr. 12, S. 290–292 (1941).

[300] M. Hempel, Magnetpulverbild und Dauerhaltbarkeit von Schraubenfedern. *Arch. Eisenhüttenw.* **13,** Nr. 11, S. 479–487 (1940).

[301] M. Hempel, Das Verhalten einiger Werkstoffe bei dynamischer Biegungsbeanspruchung. *Forsch. Arb. IngWes.* **2,** Nr. 9, S. 327–334 (1931).

[302] M. Hempel and H. E. Tillmanns, Verhalten des Stahles bei höheren Temperaturen unter wechselnder Zugbeanspruchung. *Mitt. K.-Wilh.-Ing. Eisenforsch.* **18,** Nr. 12, S. 163–181 (1936).

[303] M. Hempel, Einfluß der Beanspruchungsart auf die Wechselfestigkeit von Stahlstäben mit Querbohrungen und Kerben. *Arch. Eisenhüttenw.* **12,** Nr. 9, S. 433–444 (1939).

[304] M. Hempel, Zur Frage des Dauerbruches: Magnetpulverbild und Dauerbruchanrisse. *Mitt. K.-Wilh.-Ing. Eisenforsch.* **21,** Nr. 9 (1939).

[305] R. Mailänder, Ermüdungserscheinungen und Dauerversuche. *Stahl und Eisen* **44,** Nr. 21, S. 585–589, Nr. 22, S. 624–629, Nr. 23, 657–661, No. 24, S. 684–691, Nr. 25, S. 719–725 (1924).

[306] R. Mailänder, Die Ermüdung der Metalle. *Stahl und Eisen* **45,** Nr. 41, S. 1713–1715 (1925).

[307] R. Mailänder and W. Bauersfeld. Einfluß der Probengröße und Probenform auf die Dreh-, Schwingungsfestigkeit von Stahl. *Tech. Mitt. Krupp* **2,** Nr. 5, S. 577–581 (1934).

[308] R. Mailänder and W. Ruttmann, Einfluß von Oberflächenbeschaffenheit, Randschichten und Korrosion auf die Dauerfestigkeit. *Maschinenbau* **14,** Nr. 3/4, S. 73–77 (1935).

[309] R. Mailänder, Ergebnisse von Dauerversuchen an Stählen. *Tech. Mitt. Krupp A.* **2,** S. 8–15 (1939).

[310] R. Mailänder, Über die Dauerfestigkeit von nitrierten Proben. *Z. Ver. dtsch. Ing.* **77,** Nr. 10, S. 271–274 (1933).

[311] R. Mailänder, Über die Dauerfestigkeit von Gußeisen, Temperguß und Stahlguß. *Tech. Mitt. Krupp* **4,** Nr. 3, S. 59–66 (1936).

[312] R. Mailänder, W. Szubinski and H. J. Wiester, Biegewechselversuche und metallographische Untersuchungen an geschweißten Dünnblechen aus Stählen höherer Festigkeit. *Tech. Mitt. Krupp A.* **3,** 199–221 (1940).

[313] F. Bollenrath and K. Bungardt, Untersuchungen über die Korrosionsermüdung von Aluminium- und Magnesium-Knetlegierungen. *Z. Metallk.* **30,** Nr. 10, S. 357–359 (1938).

[314] F. Bollenrath, Zeit- und Dauerfestigkeit der Werkstoffe. Gesamm. Vortr. Hauptversamml. Lilienthal-Ges. Luft Forsch., S. 147–157 (1938).

[315] F. Bollenrath and K. Bungardt, Dauerfestigkeit einiger Leichtmetallknetlegierungen bei verschiedenen Arten der Beanspruchung; Einfluß der Kaltverformung. Jb. 1939 dtsch. Luftfahrtf. S. I595–I599 (1939).

[316] H. Cornelius and F. Bollenrath, Verdreh-Dauerhaltbarkeit von einsatzgehärteten, hohlen Kurbelwellenzapfen mit Innenverstärkung an der Ölbohrung. *Z. Ver. dtsch. Ing.* **83,** Nr. 48, S. 1257–1258 (1939).

[317] H. Cornelius and F. Bollenrath, Verdrehwechselfestigkeit von Stahlwellen mit hoher Zugfestigkeit. *Z. Ver. dtsch. Ing.* **86,** Nr. 7/8, S. 103–108 (1942).

[318] H. Cornelius and F. Bollenrath, Gegossene Nocken-und Kurbelwellen. *Giesserei* **23,** Nr. 10, S. 229–236 (1936).

[319] P. Brenner, Problems involved in the choice and use of materials in airplane construction. *Tech. Memor. nat. Adv. Comm. Aero., Wash.,* No. 658, 25 (1932).

[320] P. Brenner and H. Kostron, Der Einfluß der Faserrichtung auf die Dauerfestigkeit einer Al–Cu–Mg Legierung. *Luftwissen* **5,** Nr. 1, S. 309–312 (1938).

[321] E. Siebel, Statische und dynamische Werkstoffprüfung. *Vortr. dtsch. Ges. Metallk.* 11–15 (1938).

[322] E. Siebel and K. Wellinger, Festigkeitsverhalten von Rohrschweissungen bei Dauerbelastung. *Z. Ver. dtsch. Ing.* **84,** Nr. 4, 57–59 (1940).

[323] E. Siebel, Das Verhalten der Werkstoffe bei schwingender Beanspruchung. *Metallwirtschaft* **20,** Nr. 17, 409–414 (1941).

[324] E. Siebel and G. Stähli, Versuche zum Nachweis von Schädigung und Verfestigung im Gebiet der Zeitfestigkeit. *Arch. Eisenhüttenw.* **15,** Nr. 11, 519–527 (1942).

[325] K. Klöppel, Gemeinschaftsversuche zur Bestimmung der Schwellzugfestigkeit voller, gelochter und genieteter Stäbe aus St37 und St52. *Stahlbau* **9,** Nr. 13/14, S. 97–112 (1936).

[326] K. Klöppel, Zur Frage der Vorschriften für geschweißte Fachwerkkrane. *Z. Ver. dtsch. Ing.* **87,** Nr. 31/32, S. 501–503 (1943).

[327] G. Gürtler, Zur Frage der Erhöhung der Dauerfestigkeit von Blechen, Rohren und Profilen aus Elektron. *Zentr. wiss. Ber., Forsch-Ber.* Nr. 590, 24 (1936).

[328] G. Gürtler and W. Schmid, Untersuchungen über die Dauerfestigkeit von Legierungen der Silumingruppe. *Aluminium, Berl.* **20,** Nr. 3, 174–181 (1938).

[329] G. Gürtler, Untersuchungen über die Schadenslinie bei Leichtmetallen. *Z. Metallk.* **32,** Nr. 2, 21–30 (1940).

[330] G. Gürtler, W. Jung-König and E. Schmid, Über die Dauerbewährung der Leichtmetalle bei verschiedenen Temperaturen. *Aluminium, Verl.* **21,** Nr. 3, 202–208 (1939).

[331] E. Lehr, Die Dauerfestigkeit, ihre Bedeutung für die Praxis und ihre kurzfristige Ermittlung mittels neuartiger Prüfmaschinen. *Glasers Ann. Gew.* **99,** Nr. 1184, 1926 S. 109–114, Nr. 1185, 1926, S. 117–122, Nr. 1188, 1926, S. 177–180, und Vol. 100, Nr. 1191, 1927, S. 33–39 (1926).

[332] E. Lehr, Oberflächenempfindlichkeit und innere Arbeitsaufnahme der Werkstoffe bei Schwingungsbeanspruchung. *Z. Metallk.* **20,** Nr. 2, S. 78–90 (1928).

[333] E. Lehr, Minderung der Wechselfestigkeit von Fahrzeugfederblättern durch die Walzhaut. *Forsch.Arb. IngWes.* **3,** S. 54–55 (1932).

[334] E. Lehr and R. Mailänder, Einfluß von Hohlkehlen an abgesetzten Wellen und von Querbohrungen auf die Biegewechselfestigkeit. *Arch. Eisenhüttenw.* **11,** Nr. 11, S. 563–568 (1938).

[335] E. Lehr, Nutzbarmachung der Ergebnisse der neueren Festigkeitsforschung für den Konstrukteur. *Metallwirtschaft* **18,** S. 595–599 and S. 617–623 (1939).

[336] E. Lehr and R. Ruef, Beiträge zur Frage der Dauerhaltbarkeit der Kurbelwellen von Groß-Dieselmotoren. *MTZ* **5,** Nr. 11/12, S. 349–357 (1943).

[337] H. Neuber, Kerbspannungslehre, Grundlage für eine genaue Spannungsrechnung. Springer-Verlag Berlin (1937).

[338] H. Neuber, *Theory of Notch Stresses* (Edited by J.W. Edwards). Ann Arbor, Mich. (1946).

[339] H. Bürnheim, Beitrag zur Frage der Zeit- und Dauerfestigkeit der Leichtmetallnietverbindungen des Flugzeugbaus unter besonderer Berücksichtigung der Werkstoffe, des Nietbildes und einiger sonstiger Besonderheiten. Dissertation TH Darmstadt, 2 Bände (1944).

[340] R. E. Peterson, *Stress Concentration Design Factors*, p. 8. John Wiley, New York, U.S.A. (1953).

[341] R. E. Peterson, Fatigue tests of large specimens. *Proceedings ASTM* **29,** 371–380 (1929).

[342] R. E. Peterson and A. M. Wahl, Fatigue of shafts at fitted members, with a related photoelastic analysis. *Trans. ASME, J. appl. Mech.* **57,** A1–A11, A69–A74 (1935).

[343] R. E. Peterson and A. M. Wahl, Two- and three-dimensional cases of stress concentration and comparison with fatigue tests. *Trans. ASME, J. appl. Mech.* **58,** A15–A22 (1936).

[344] R. E. Peterson, Stress concentration phenomena in fatigue of metals. *Trans. ASME* **55,** 157–71 (1933).

[345] R. Peterson, Stress concentrations and fatigue strength, in *Proc. American Society for Testing and Materials* **30,** Part 1, 298–310 (1930).

[346] R. E. Peterson, Application of stress concentration factors in design. *Proc. Soc. Exp. Stress Anal.* **1,** 118–127 (1943).

[347] R. E. Peterson and J. M. Lessells, Effect of surface strengthening on shafts having a fillet or a transverse hole. *Proc. Soc. Exp. Stress Anal.* **2,** 191–199 (1944).

[348] R. E. Peterson, Fatigue tests of small specimens with particular reference to size effect. *Trans. Amer. Soc. Steel Treat.* **18,** 1041–1056 (1930).

[349] O. J. Horger and H. R. Neifert, Improving fatigue resistance by shot peening. *Proc. Soc. Exp. Stress Anal.* **2,** 178–190 (1944).

[350] O. J. Horger, H. R. Neifert and R. R. Regen, Residual stresses and fatigue studies. *Proc. Soc. Exp. Stress Anal.* **1,** 10–18 (1943).

[351] O. J. Horger and T. V. Buckwalter, Fatigue comparison of 7-in. diameter solid and tubular axles. *Proc. Amer. Soc. Test. Mater.* **41,** 682–695 (1941).

[352] O. J. Horger, Effect of surface rolling on the fatigue strength of steel. *J. appl. Mech.* **2,** A128–A136 (1935).

[353] O. J. Horger, Fatigue failure of railroad axles and stresses in hollow cylinders. Stephen Timoshenko 60th Anniversary Volume, New York, The Macmillan Co., pp. 73–80 (1938).

[354] O. J. Horger, Photoelastic analysis practically applied to design problems. *J. appl. Phys.* **9,** 457–465 (1938).

[355] O. J. Horger and H. R. Neifert, Fatigue strength of machined forgings 6 to 7 in. diameter. *Proc. Amer. Soc. Test. Mater.* **39,** 723–740 (1939).

[356] O. J. Horger and T. V. Buckwalter, Fatigue strength of axles with surfaces metal coated and flame hardened. *Heat Treat. Forg.* **26,** 321–325, 336 (1940).

[357] M. G. Ros and A. Eichinger, Festigkeitseigenschaften der Stähle bei hohen Temperaturen. *Ber. eidgenöss. MatPrüfAnst.* **87,** 51 (1934).

[358] M. G. Ros und E. Brandenberger, Erfahrungen mit röntgendurchstrahlten, geschweißten Druckleitungen und deren festigkeitstechnische Sicherheit. *Ber. eidgenöss. MatPrüfAnst.* **122,** 26 (1939).

[359] M. G. Ros and A. Eichinger, Festigkeitseigenschaften der Stähle bei hohen Temperaturen. *Ber. eidgenöss. MatPrüfAnst.* **138,** 55 (1941).

[360] M. G. Ros, Festigkeit und Verformung von auf Biegung beanspruchten Eisenbetonbalken, bewehrt mit "TOR-Stahl", hochwertigem Stahl "St52", Normalstahl "St N". *Ber. eidgenöss. MatPrüfAnst.* **141,** 84 (1942).

[361] R. L. Templin, The fatigue properties of light metals and alloys. *Proc. Amer. Soc. Test. Mater.* **33,** 364–386 (1933).

[362] R. L. Templin, Fatigue of light metal alloys. *Metals and Alloys* **10,** 243–245 (1939).

[363] E. C. Hartmann and R. L. Templin, Fatigue data used in design. *Metal. Prog.* **39,** 604, 606, 612 (1941).

[364] F. P. Zimmerli, Shot blasting and its effects on fatigue life surface treatment of metals. 261 Americal Society for Metals (1941), Proc. Symposium Cleveland (1940), American Society for Metals (1941).

[365] Civil aircraft accident — Report of the Court of Enquiry into the Accidents to Comet G-ALYP on 10th January 1954 and Comet G-ALYY on 8th April 1954. HMSO, London (1955).

[366] RAE, Report on Comet Accident Investigation. Part 3, Fatigue Tests on the Pressure Cabin and Wings. RAE Accident Note 263, Royal Aircraft Establishment, Farnborough (1954).

[367] T. Swift, Damage tolerance in pressurized fuselages. 11th Plantema Memorial Lecture, in *New Materials and Fatigue Resistant Aircraft Design*. Proc. of the 14th ICAF Symposium (Edited by D. L. Simpson). Ottawa, Canada, EMAS (8–12 June 1987).

[368] N. I. Bullen, The distribution of gusts in the atmosphere: an integration of U.K. and U.S. data. ARC CP 419 (1959).

[369] R. Hain Taylor, A summary of V-g recorder results for civil aircraft. Royal Aircr. Establ. Tech. Note no. S.M.E. 368 (July 1946).

[370] T. L. Coleman and M. R. Copp, Maneuver accelerations experienced by five types of commercial transport airplanes during routine operations. NACA Tech. Note 3086 (1954).

[371] P. Donely, Summary of information relating to gust loads on airplanes. NACA Rept. 997 (1950).

[372] T. L. Coleman and M. W. Fetner, An analysis of acceleration, airspeed and gust velocity from a few engine transport airplane in operation on an Aaston US route. NACA TN 3483 (1955).

[373] J. K. Oaks and P. H. Townshend, Fluctuating load tests of typhoon tail planes. R.A.E. Report S.M.E. 3354 (1954).

[374] K. D. Raithby and J. Longson, Some fatigue characteristics of a two spar light alloy structure. ARC CP 258 (1956).

[375] M. J. McGuigan, D. F. Bryan and R. E. Whaley, Fatigue investigation of transport-airplane wings. NACA TN 3190 (March 1954). See also NACA TN 2920 (April 1953).

[376] R. E. Whaley and M. J. Guigan, Fatigue crack propagation and residual static strength results on transport aircraft. NACA TN 3847 (December 1956).

[377] A. O. Payne, An investigation into the fatigue characteristics of a typical 24S-T aluminium alloy wing, in *Proceedings of the International Conference on Fatigue of Metals*, pp. 641–649. Institution of Mechanical Engineers, London (1965).

[378] J. L. Kepert, C. A. Patching and M. R. Rice, Fatigue characteristics of a riveted 24S-T aluminium alloy wing. ALR report SM 248 (October 1956).

[379] A. O. Payne, Determination of the fatigue resistance of aircraft wings by full-scale testing, in *Proceedings of the ICAF-Symposium Amsterdam* (Edited by F. J. Plantema and J. Schijve). Pergamon (1961).

[380] W. W. Johnstone, Static and repeated load tests on mosquito wings. CSIR Melbourne, Div. Aeron., Rep. SM 104 (September 1947).

[381] W. W. Johnstone, C. A. Patching and A. O. Payne, Fatigue strength of "Boomerang" wings. ARL rep. SM 160 (September 1950).

[382] A. J. S. Pippard and J. L. Pitchard, *Aeroplane Structures*. Longmans, Green and Co., New York (1919).

[383] E. H. Spaulding, Observations on the design of fatigue resistant and "fail-safe" aircraft structures, in *Int. Conf. Fatigue of Metals*. IMECHE, London (1956).

[384] W. Crichlow, The ultimate strength of damaged structure, in *Full Scale Testing of Aircraft Structures* (Edited by J. Schijve). Plantema, ICAF Symposium, Amsterdam. Pergamon (9–11 June 1959).

[385] M. Habermehl, Fatigue and the day it almost wiped out SAC! Boeing-Journal (1980).

[386] J. W. Lincoln, Damage tolerance — USAF experience, in *Durability and Damage Tolerance in Aircraft Design*. Proc. 13th ICAF Symp. 1985 (Edited by A. Salvetti and G. Cavallino). EMAS, Pisa (1985).

[387] *Proc. of the Symposium on Fatigue of Aircraft Structures*. WADC Tech. Report No 59-507, Dayton, Ohio (1959).

[388] A. W. Dallas, Combating structural fatigue in airline operations. WADC-TR-59-507.

[389] T. P. Gerrity, Maintenance and logistic aspects of structural fatigue problems. WADC-TR-59-507.

[390] W. H. Keen, The aircraft fatigue problem: barely under control. WADC-TR-59-507.

[391] H. B. Lowndes, The WADC fatigue test philosophy. WADC-TR-59-507.

[392] P. A. Parmley, WADC views on establishment of structural life criteria. WADC-TR-59-507.

[393] R. V. Rhode, An approach to improvement of the fatigue situation. WADC-TR-59-507.

[394] H. W. Smith, Aircraft industry philosophy on structural fatigue. WADC-TR-59-507.

[395] J. P. Taylor, Structural integrity program for high performance aircraft. WADC-TR-59-507.

[396] A. A. Vollmecke, FAA philosophy on the problems of fatigue in relation to civil aircraft. WADC-TR-59-507.

[397] H. E. Watkins, Aircraft structural fatigue problems in SAC. WADC-TR-59-507.

[398] R. E. Watson and L. L. Gore, Complete vehicle — full scale — testing techniques. WADC-TR-59-507.

[399] J. D. Caldara, The impact of metal fatigue mishaps in the U.S. Air Force. *Proc. Symposium on Fatigue of Aircraft Structures*, W.A.D.C. Tech. Rep. No. 59-507 (8–30 August 1959).

[400] W. B. Huston and J. F. Ward, Evaluation of fatigue sensitive areas and cumulative damage on full-scale fatigue testing. WADC-TR-59-507.

[401] E. Gaßner, Betriebsfestigkeit Eine Bemessungsgrundlage für Konstruktionsteile mit statistisch wechselnden Betriebsbeanspruchungen. Konstruktion, 6. Jahrgang, Heft 3, S. 97–104 (1954).

[402] E. Gaßner, Effect of variable load and cumulative damage of fatigue in vehicle and airplane structures. *Int. Conf. on Fatigue of Metals*, London and New York 1956, Session 3: Stress Distribution, Institution of Mechanical Engineers, London (1956).

[403] O. Svenson, Unmittelbare Bestimmung der Größe und Häufigkeit von Betriebsbeanspruchungen. Trans. of Instruments and Measurements Conference, Stockholm (1952).

[404] W. Weibull, A statistical distribution function of wide applicability. *Trans. ASME, J. appl. Mech.* **73,** 293–297 (1951).

[405] W. Weibull, A statistical representation of fatigue failures in solids. Trans. of Royal Institute of Technology, Sweden, No. 27 (1947).

[406] W. Weibull, Static strength and fatigue properties of threaded bolts. Report 59, FFA (1955).

[407] W. Weibull, Scatter in fatigue life of 24-St cladded plates. Minutes of the 3rd ICAF-Conference, Cranfield, U.K. (2–5 January 1955).

[408] A. M. Freudenthal and E. J. Gumbel, On the statistical interpretation of fatigue tests. *Proceedings of the Royal Society of London* **216,** 309–332, London (1953).

[409] H. Perret, Erhöhte Sicherheit bei der Berechnung von Wälzlagern. *Konstruktion* **3,** 251 (1951).

[410] A. Erker, Sicherheit und Bruchwahrscheinlichkeit. MAN-Forschungsheft (1958).

[411] A. K. Head and F. H. Hooke, Random noise fatigue testing, in *Proceedings of the International Conference on Fatigue of Metals* (pp. 301–303). Institution of Mechanical Engineers, London (September 1956).

[412] A. M. Freudenthal, A random fatigue testing procedure and machine. *ASTM* **53,** S. 896–910 (1953).

[413] A. M. Freudenthal, Cumulative damage under random loading. *The International Conference on Fatigue of Metals*. London (September 1956).

[414] F. R. Shanley, A theory of fatigue based on unbonding during reverse slip. The Rand Corporation, Santa Monica, 1952, P-350 and D-350 Supplement.

[415] H. T. Corten and T. J. Dolan. Cumulative fatigue damage. *Proceedings of the International Conference on Fatigue of Metals*. Institution of Mechanical Engineers, London (September 1956).

[416] W. Schütz and H. Zenner, Schadensakkumulationshypothesen zur Lebensdauervorhersage bei schwingender

Beanspruchung — ein kritischer Überblick. Zeitschrift für Werkstofftechnik, 4. Jahrgang, 1973 Teil 1 und 2, Heft 1 und 2 (1973).

[417] M. A. Melcon and W. Chrichlow, An engineering evaluation of methods for the prediction of fatigue life in airframe structures. Technical Report No. ASD-TR-61-434.

[418] M. A. Melcon and W. Chrichlow, Investigation of the representation of aircraft service loadings in fatigue tests. Technical Report No. ASD-TR-61-435.

[419] J. Kowalewski, On the relation between fatigue lifes under random loading and under corresponding programme loading. *Proceedings of the ICAF-Symposium Amsterdam* (Edited by F. J. Plantema and J. Schijve). Pergamon (1961).

[420] W. Weibull, The propagation of fatigue cracks in light alloy plates. Minutes of the Third ICAF Conference, Cranfield, U.K. (2–5 January 1955).

[421] N. E. Frost, Crack formation and stress-concentration effects in direct stress fatigue. Parts I and II. *Engineer* **200** (1955).

[422] N. E. Frost, A relation between the critical alternating propagation stress and crack length for mild steel. The Institution of Mechanical Engineers, 7/59, London.

[423] N. E. Frost and D. S. Dugdale, Propagation of fatigue cracks in sheet specimens. *J. Mech. Phys. Solids* **6**, No. 2 (1958).

[424] McEvily and W. Illg, The rate of fatigue-crack propagation in two aluminium alloys. NACA TN 4394, Nat. Advisory Committee for Aeronautics (September 1958).

[425] C. M. Hudson and H. F. Hardrath, Effect of changing stress amplitude on the rate of fatigue-crack propagation in two aluminium alloys. NASA TN D-960 (September 1961).

[426] P. Kuhn and H. F. Hardrath, An engineering method for estimating notch size effect in fatigue tests of steel. NACA TN 2805 (1952).

[427] P. Kuhn, Effect of geometric size on notch fatigue, in *Fatigue in Aircraft Structures* (Edited by H.M. Freudenthal). Academic Press, New York (1956).

[428] H. J. Grover, W. S. Hyler, P. Kuhn, B. Landers and F. M. Howell, Axial-load fatigue properties of 24S-T and 75S-T aluminium alloys as determined in several laboratories. NACA TN 2928 (May 1953).

[429] S. S. Manson, Behaviour of materials under conditions of thermal stress. *Heat Transfer Symposium*, University of Michigan Engineering Research Institute, pp. 9–75 (1953).

[430] L. F. Coffin, The problem of thermal stress fatigue in austenitic steels at elevated temperatures. *ASTM STP* No. 165, Am. Soc. Testing Mat., Symposium on Effect of Cyclic Heating and Stessing on Metals at Elevated Temperatures, p. 31 (1954).

[431] L. F. Coffin, A study of the effects of cyclic thermal stresses on a ductile metal. *Trans. American Society for Testing and Materials* **76**, 931–950 (1954).

[432] G. R. Irwin, Fracture. *Handbuch der Physik*, Vol. IV, pp. 551–590. Springer (1958).

[433] K. D. Raithby, Minutes of the third conference of the ICAF College of Aeronautics, Cranfield (2–5 January 1955).

[434] N. N., Minutes of the sixth conference of the ICAF (Edited by F. J. Plantema and J. Schijve), Amsterdam, NLL (June 1959).

[435] Full scale fatigue testing of aircraft structures. *First ICAF Symposium* (Edited by F. J. Plantema and J. Schijve), Amsterdam. Pergamon (1961).

[436] J. Schijve, The fatigue strength of riveted joints and lugs. NLL rep. 1952 (May 1954), NACA TM 1395 (August 1956).

[437] J. Schijve and F. A. Jacobs, The fatigue strength of aluminium alloy lugs. NLL TN M 2024 (January 1957).

[438] S. Luthander and G. Wallgren, Determination of fatigue life with stress cycles of varying amplitude. FAA Rep. 18 (1946), Trans. Soy. Swedish Air Board, No. 10 (1949) and also RAE Lib. Trans. 421 (1953).

[439] G. Wallgren, Fatigue tests with stress cycles of varying amplitude. FFA rep. 28 (1949).

[440] B. Lundberg and S. Eggwertz, The relationship between load spectra and fatigue life. FFA rep. 67 (March 1956).

[441] Manual on fatigue testing. American Society for Testing Materials, STP No. 91 (1949).

[442] Symposium on Statistical Aspects of Fatigue. ASTM STP No. 121 (1952).

[443] Large fatigue testing machines and their results. ASTM STP 216 (1957).

[444] Fatigue of aircraft structures. American Society for Testing Materials, ASTM-STP No. 203 (1957).

[445] *Symposium on Fretting Corrosion*. ASTM STP 144 (1952).

[446] Effect of cyclic heating and stressing on metals at elevated temperatures. ASTM STP 165 (1954).

[447] The failure of metals by fatigue. Proceedings of a Symposium, University of Melbourne (2–6 December 1946).

[448] Fatigue and fracture of metals. Symposium at Massachusetts Institute of Technology (Edited by W. M. Murray). Wiley, New York (1952).

[449] Colloquium on Fatigue, Stockholm 1956 (Edited by W. Weibull and F. K. G. Odquist). Springer Verlag, Berlin (1956).

[450] Fatigue in aircraft structures. Proceedings of the International Conference, Columbia University (Edited by A. M. Freudenthal). Academic Press, New York (1956).

[451] *Proceedings of the International Conference on Fatigue of Metals*. The Institution of Mechanical Engineers, London (1956).

[452] Höhere Gestaltfestigkeit durch Oberflächenverfestigung. Vorträge auf der Tagung der Wnitomasch vom 21. bis 24. April 1954 in Moskau. Gesamtredaktion von S.W. Serensen, VEB Verlag Technik Berlin (1954).

[453] Surface stressing of metals. Five Educational Lectures. Cleveland, American Society of Metals (1946).

[454] Interpretation of tests and correlation with service. Four Education Lectures. American Society for Metals, Chicago (October 1952).

[455] R. Cazaud, *La Fatigue des Métaux*. 4. Auflage Dunod, Paris (1959).

[456] R. Cazaud, *La Fatigue des Métaux*. 5. Auflage Dunod, Paris (1969).

[457] R. Cazaud, *Fatigue of Metals*. Chapman and Hall, London (1953).

[458] *Battelle Memorial Institute*, *Prevention of the Fatigue of Metals under Repeated Stress*. John Wiley, New York (1949).

[459] D. N. Kimmelmann, Berechnung von Maschinenteilen auf Dauer- und Zeitschwingfestigkeit. Übersetzung aus dem Russischen. VEB Verlag Technik Berlin (1953).

[460] G. N. Savin, Spannungserhöhung am Rande von Löchern. VEB Verlag Technik Berlin (1956).

[461] G. N. Savin, Stress concentration around holes. Trans. by E. Gros, Pergamon (1961).

[462] P. Ye Kravchenko, *Fatigue Resistance*. Pergamon (1964).

[463] R. Hänchen, Berechnung und Gestaltung der Maschinenteile auf Dauerhaltbarkeit. Pädagogischer Verlag Bernhard Schulz (1950).

[464] H. Tauscher, Berechnung der Dauerfestigkeit. Einfluß von Werkstoff und Gestalt. Fachbuchverlag Leipzig, Sechste Auflage (1960).

[465] C. Lipson, G. C. Noll and L. S. Clock, *Stress and Strength of Manufactured Parts*. McGraw Hill, New York (1950).

[466] L. Locati, La fatica dei materiali metallici. Hoepli, Mailand (1950).

[467] S. Berg, Gestaltfestigkeit Versuche mit Schwingern. Deutscher Ingenieur-Verlag GmbH, Düsseldorf (1952).

[468] W. Kloth, Atlas der Spannungsfelder in technischen Bauteilen. Verlag Stahl und Eisen, Düsseldorf (1961).

[469] H. Wiegand and B. Haas, Berechnung und Gestaltung von Schraubverbindungen. 2. Auflage, Springer-Verlag Berlin (1958).

[470] *Metal Fatigue* (Edited by J. A. Pope). Chapman and Hall (1959).

[471] H. J. Grover, S. A. Gordon and L. R. Jackson, *Fatigue of Metals and Structures*. Thames and Hudson, London (1956).

[472] *Metal Fatigue* (Edited by G. Sines and J L. Waisman). McGraw-Hill Book Co. (1959).

[473] J. Schijve and D. Broek, Crack propagation tests based on a gust spectrum with variable amplitude loading. *Aircraft Engineering* **34**, 314–316 (1962).

[474] J. Schijve and A. H. W. Hoeymakers, Fatigue crack growth in lugs and the stress intensity factor. *Fatigue of Engng Materials and Structures* **1**, 185–201 (1971).

[475] J. Schijve, F. A. Jacobs and P. J. Tromp, Crack propagation in 2024-T3 Alclad under flight-simulation loading. Effect of truncating high gust loads. Nat. Aerospace Lab. NLR, Amsterdam, Report TR 69050 (1969).

[476] J. Schijve, F. A. Jacobs and P. J. Tromp, Flight simulation tests on notched elements. NLR-Report TR 74033 (1974).

[476a] J. Schijve, The analysis of random load–time histories with relation to fatigue tests and calculations, in *Fatigue of Aircraft Structures*. Proceedings of the Second ICAF-Symposium 1961 (Edited by W. Barrois and E. L. Ripley). Pergamon (1963).

[477] J. Schijve and H. T. M. van Lipzig, Investigation on the fail safe properties of laminated sheet material (in Dutch). Delft University of Technology, Report VTH-188 (1974).

[478] L. B. Vogelesang, R. Marissen and J. Schijve, A new fatigue resistant material: aramid aluminium laminate. *Proc. 11th ICAF Symposium*, Nordwijkerhout, paper 3.4 (1981).

[479] J. Schijve, Fatigue of aircraft materials and structures, in *Fatigue of Aircraft Materials*, Delft Structures and Materials Laboratory, Delft University Press (1992).

[480] L. Schwarmann, N. Ohrloff and T. Beumler, On the application of GLARE for airbus fuselage structures, in *Fatigue of Aircraft Materials*, Delft Structures and Materials Laboratory, Delft University Press (1992).

[481] J. Branger, Full-scale test and service experience in the Swiss Venom case. *Proceedings of the ICAF Technical Session* (Stockholm, May 1969), Aeronautical Research Institute of Sweden (FFA), (1969).

[482] P. C. Paris, The growth of cracks due to variations in load. Dissertation, Lehigh University (1962).

[483] P. C. Paris, M. P. Gomez and W. E. Anderson, A rational analytic theory of fatigue. *The Trend in Engineering* **13**, 9 (1961).

[484] P. C. Paris, The fracture mechanics approach to fatigue, in *Fatigue — An Interdisciplinary Approach* (Edited by J. H. Burke, N. L. Reed and V. Weiss). Syracuse University Press (1964).

[485] W. Elber, Fatigue crack propagation: some effects of crack closure on the mechanisms of fatigue crack propagation and cyclic tensile loading. Ph. D. Thesis, University of New South Wales (1968).

[486] USAF, Military specification airplane damage tolerance requirements. Mil-A-83444 (July 1974).

[487] N. N., *Proceedings of the Air Force Conference on Fatigue and Fracture of Aircraft Structures and Materials*. Tech. Editors: H. A. Wood, R. M. Bader, W. J. Trapp, R. F. Hoener, R. C. Donat. Miami Beach, Florida, AFFDL TR 70-144 (15–18 December 1969).

[488] USAF, military standard; engine structural integrity program (ENSIP). MIL-STD-1783, USAF (1984).

[489] J. W. Lincoln, Damage tolerance for helicopters, in *Aeronautical Fatigue in the Electronic Era*. Proceedings of the 15th ICAF Symposium (Edited by A. Berkovits). Jerusalem, EMAS (13–21 June 1989).

[490] *Damage Tolerance Design Handbook*, Vol. 1–4 (Edited by J. Gallagher). Metals and Ceramics Information Center (December 1983).

[491] R. M. Engle, J. P. Gallagher, F. J. Giessler and A. P. Berens, USAF DT Design Handbook. Guidelines for the Analysis and Design of Damage Tolerant Aircraft Structures. AFWAL-TR-82-3073 (May 1984).

[492] *Proceedings of the* 1991 *USAF Structural Integrity Program Conference* (Edited by T. Cooper, J. Lincoln and R. Bader). WL-TR-92-4045 (1992).

[492a] *Proceedings of the* 1992 *USAF Structural Integrity Program Conference* (Edited by T. Cooper, J. Lincoln and R. Bader). WL-TR-93-480 (September 1993).

[492b] *Proceedings of the* 1990 *USAF Structural Integrity Program Conference* (Edited by T. Cooper and J. Lincoln). AD-A 242943 (August 1991).

[492c] J. W. Lincoln, Overview of the structural integrity process, in *An Assessment of Fatigue Damage and Crack Growth Proediction Techniques*. AGARD Report 797 (March 1994).

[493] JAR 25, Chapter 7, Amend. 7, Nov. 1980, ACF 25.571 (a) Damage tolerance and fatigue evaluation of structure (acceptable means of compliance).

[494] Damage-tolerance and fatigue evaluation of structure. Draft Advisory Circular, AC No. 25.571-1A, US Department of Transportation, Federal Aviation Administration (March 1985).

[494a] Aircraft accident report, Aloha Airlines Flight 243, Boeing 737–200, April 28, 1988. NTSB/AAR-89/03 (June 1989).

[495] A. J. Emmerson, New lamps for old-safety regulation through structural airworthiness standards, in *Aeronautical Fatigue: Key to Safety and Structural Integrity*. Proceedings of the 16th ICAF Symposium (Edited by A. Kobayashi), EMAS (1991).

[495a] *Structural Integrity of Aging Airplanes* (Edited by S. N. Atluri, S. N. Sampath and R. Tong). International Symposium, Atlanta, GA, U.S.A. (20–2 March 1990).

[495b] C. E. Harris, 1991 *International Conference on Aging Aircraft and Structural Airworthiness*. International Conference, Washington DC, U.S.A. (19–21 November 1991).

[495c] *International Workshop on Structural Integrity of Aging Airplanes*, Atlanta, Georgia (31 March–2 April 1992).

[495d] *5th International Conference on Structural Airworthiness of New and Aging Aircraft*. Proceedings. DGLR, Hamburg. DGLR-Bericht 93-02 (16–18 June 1993).

[496] W. Schütz and R. Weber, Steuerung von schwingprüfmaschinen durch prozessrechner. *Materialprüfung* **12**, S. 369–377 (1970).

[497] W. Schütz, Qualitätssicherung im modernen Schwingfestigkeitslabor – Anforderungen an Personal und Ausrüstung, in *Werkstoffprüfung*. Bad Nauheim, DVM (1991).

[498] W. Schütz, Standardized stress time histories, an overview, in *Development of Fatigue Loading Spectra*, ASTM STP 1006 (1989).

[499] Short crack behaviour of various aircraft materials (Edited by P.R. Edwards and J.C. Newman). AGARD Report No. 767 (1990).

[500] N. N., Gemeinschaftsarbeit BMFT-LBF-Industrie. Stand (1978).

[501] M. Hück, P. Heuler and J. Bergmann, Gemeinschaftsarbeit Pkw-Industrie-IABG. Relative Miner-Regel. Teil II, Ergebnisse der Bauteilversuche, IABG-Bericht TF-2904 (1991).

[501a] W. Schütz and P. Heuler, Miner's rule revisited, in: *An Assessment of Fatigue Damage and Crack Growth Prediction Techniques*. AGARD Report 797 (March 1994).

[501b] K. G. Eulitz, K. L. Kotte and A. Esderts, Datensammlung für Betriebsfestigkeitsversuche zur Bewertung von Auswerte- und Rechenverfahren, in *Bauteillebensdauer Rechnung und Versuch*. DVM (October 1993).

[502] W. Schütz, The fatigue life under three different load spectra tests and calculations. AGARD CP 118, Symposium on Random Load Fatigue. Lyngby, Dänemark (1972).

[502a] W. Ludwig, K. F. Sahm and E. Steinheil, Fatigue measuring gauges FMG, a new system measuring fatigue loads. *Structural Fatigue as a Design Factor*. Proceedings of the 10th ICAF-Symposium (Edited by A. Maenhard), Brussells (1979).

[502b] C. M. Sonsino, Limitations in the use of RMS values and equivalent stresses in variable amplitude loading. *Int. J. Fatigue* **11**, 142–152 (1989).

[502c] W. Schütz, Über eine Beziehung zwischen der Lebensdauer bei konstanten und bei veränderlichen Beanspruchungsamplituden und ihre Anwendbarkeit auf die Bemessung von Flugzeugbauteilen. Ph.D. Thesis, Technical University of Munich (1966).

[503] Fatigue under complex loading, analyses and experiments (Edited by R.M. Wetzel). Society of Automotive Engineers (1977).

[504] V. Grubisic and A. Rupp, Gegenüberstellung der experimentell und rechnerich bestimmten Lebensdauer von Bauteilen und Probestäben, in *Bauteillebensdauer Rechnung und Versuch*. DVM (October 1993).

[504a] T. Seeger and P. Heuler, Generalized application of Neuber's rule. *J. Test. Eval.* **8**, 199–204 (1980).

[504b] M. Vormwald, P. Heuler and T. Seeger, A fracture mechanics based model for cumulative damage assessment as part of fatigue life prediction, in *Advances in Fatigue Life Prediction Techniques* (Edited by M.R. Mitchell and R.W. Landgraf). San Francisco American Society for Testing and Materials, Philadelphia, STP 1122 (1992).

[504c] M. Hoffmann and T. Seeger, Local strain aproach in multiaxial fatigue: feasibilities and current limitations, in *Low Cycle Fatigue and Elasto–Plastic Behaviour of Materials* (Edited by K.-T. Rie). Elsevier Applied Science, London (1987).

[504d] M. Hoffmann and T. Seeger, Estimating multiaxial elastic–plastic notch stresses and strains in combined loading, in *Biaxial and Multiaxial Fatigue* (Edited by M. W. Brown and K. J. Miller). Proceedings Sec. Int. Conf., Sheffield, U.K., 1985, MEP, London, U.K. EGF 3 (1989).

[505] E. C. Naumann, Fatigue under random and programmed loads. NASA TN D-2629 (1964).

[506] J. Schijve and F. A. Jacobs, Program fatigue tests on notched light alloy specimens of 2024 and 7045 material. NLR-Report M 2070 (1960).

[507] J. Schijve, Effect of load sequences on crack propagation under random and program loading. *Engng Fracture Mech.* **5**, 245–252 (1974).

[508] E. Gaßner, W. Lipp and V. Dietz, Schwingfestigkeitsverhalten von Bauteilen im Betrieb und im Betriebslastennachfahrversuch. LBF-TM Nr. 76/76 (1976).

[509] E. Gaßner and W. Lipp, Long life random fatigue behaviour of notched specimens in service, in service duplication tests and in program tests. ASTM **671**, 222–239 (1979).

[510] E. Gaßner and G. Jacoby, Betriebsfestigkeit zur Ermittlung zulässiger Entwurfsspannungen für die Flügelunterseite eines Transportflugzeuges. *Luftfahrttechnik-Raumfahrttechnik* 9 Nr. 1, S. 6–19 (1964).

[511] E. Gaßner and G. Jacoby, Experimentelle und rechnerische Lebensdauerbeurteilung von Bauteilen mit Start-Lande-Lastwechsel. *Luftfahrttechnik* 11 Nr. 6, S. 138–148 (1965).

[512] L. Krüger, K. Boß and S. Angerer, Bremsensprüfstand für Betriebslastensimulation. *ATZ* 94, Nr. 1, S. 222–230 (1992).

[513] S. Angerer and H. Naundorf. Fahrdynamischer Räderprüfstand. *ATZ* 85, Nr. 2 (1983).

[514] S. Angerer, P. Hutmann and H. Naundorf, Zweiaxialer Räderprüfstand. *ATZ* 88, Heft 10 (1986).

[515] K.-P. Weibel, Mehr als 10 Jahre Betriebsfestigkeitsprüfung mit den Mehrkomponenten-Prüfständen der BMW AG. *ATZ* 89, Heft 4 (1987).

[516] S. Angerer, S. Gruber and H. Naundorf, Fahrdynamischer Räderprüfstand Messungen, Ergebnisse und neue Prüfmethoden. *ATZ* 87, Heft 11, S. 571–578 (1985).

[517] Acoustical fatigue in aerospace structures. *Proceedings of the 2nd Int. Conf.* (Edited by W. J. Trapp and D. M. Forney). Dayton, Syracuse University Press, New York (1965).

[518] *International Conference on Structural Safety and Reliability* (Edited by A. M. Freudenthal). Washington, DC (1969). Pergamon Press, New York (1972).

[519] W. J. Harris, *Metallic Fatigue*. Pergamon Press (1961).

[520] W. Weibull, *Fatigue Testing and Analysis of Results*. Pergamon Press (1961).

[521] J. Taylor, Manual on aircraft loads. AGARDograph 83, Pergamon Press (1965).

[522] N. E. Frost, K. J. March and L. P. Pook, *Metal Fatigue*. Clarendon Press, Oxford, pp. 370–397 (1974).

[523] H. O. Fuchs and R. I. Stephens, *Metal Fatigue in Engineering*. John Wiley, U.S.A. (1980).

[524] P. G. Forrest, *Fatigue of Metals*. Pergamon Press (1962).

[525] R. B. Heywood, *Designing against Fatigue*. Chapman and Hall, London, pp. 168–247 (1962).
[526] H. Hertel, Ermüdungsfestigkeit der Konstruktionen. Springer-Verlag Berlin (1969).
[526a] Leitfaden für eine Betriebsfestigkeitsrechnung. VDEH. A.G. Betriebsfestigkeit, Bericht ABF 01, Korrigierter Neudruck der ersten Auflage, Düsseldorf (April 1979).
[527] L. Sors, Berechnung der Dauerfestigkeit von Maschinenteilen. Verlag der ungarischen Akademie der Wissenschaften Budapest (1963).
[528] K. J. Marsh, *Full-Scale Fatigue Testing of Components and Structures*. Butterworth (1988).
[529] T. R. Gurney, *Fatigue of Welded Structures*. Cambridge University Press, Cambridge (1979).
[530] Schwingfestigkeit. VEB Deutscher Verlag für Grundstoffindustrie (1973).
[531] D. Cottin and E. Puls, Angewandte Betriebsfestigkeit. VEB Deutscher Verlag für Grundstoffindustrie, Leipzig (1985).
[532] *New Materials and Fatigue Resistant Aircraft Design*. Proceedings of the 14th ICAF-Symposium (Edited by D. L. Simpson). Ottawa, Canada, EMAS (8–12 June 1987).
[533] *Aeronautical Fatigue in the Electronic Era*. Proceedings of the 15th ICAF-Symposium (Edited by A. Berkovits). Jerusalem, Israel, EMAS (21–23 June 1989).
[534] *Aeronautical Fatigue, Key to Safety and Structural Integrity*. Proceedings of the 16th ICAF-Symposium (Edited by A. Kobayashi). Tokyo, Japan, EMAS (22–24 May 1991).
[535] W. Hanel, Beitrag zur Lebensdauervorhersage regellos beanspruchter Bauteile auf der Grundlage einer zweiparametrigen Klassierung. Dissertation TU Dresden (1978).
[536] L. Kotte, Lebensdauerberechnung stochastisch belasteter Bauteile unter besonderer Berücksichtigung zweiparametrig ausgewerteter Beanspruchungen. Dissertation TU Dresden (1987).
[537] British Standards Institution: Steel, Concrete and Composite Bridges. BS 5400, Part 10, Code of Practice Fatigue. British Standard Institution, London (1980).
[538] Eurocode 3, Design of Steel Structures. Part 1 General Rules and Rules for Buildings. Provisional, Commission of the European Communities. EC3-88-C11-D4.
[539] O. Buxbaum, Betriebsfestigkeit. Sichere und wirtschaftliche Bemessung schwingbruchgefährdeter Bauteile. Verlag Stahl und Eisen, Düsseldorf, 1986, 2. Auflage (1992).
[540] E. Haibach, Betriebsfestigkeit, Verfahren und Daten zur Bauteilberechnung. VDI-Verlag Düsseldorf (1989).
[541] E. Haibach, Betriebsfeste Bauteile. Ermittlung und Nachweis der Betriebsfestigkeit, konstruktive und unternehmerische Gesichtspunkte. Springer Verlag, Berlin (1992).
[542] *Data Book on Fatigue Strength of Metallic Materials*. Vols 1–3, The Society of Material Science, Japan (1982).
[543] H. E. Boyer, *Atlas of Fatigue Curves*. American Society for Metals (1986).
[544] D. Taylor, *A Compendium of Fatigue Thresholds and Growth Rates*. EMAS (1985).
[545] R. Olivier and W. Ritter, Wöhlerlinienkatalog für Schweißverbindungen aus Baustählen. DVS-Berichte 56/I –56/V, Deutscher Verlag für Schweißtechnik GmbH, Düsseldorf (1979).
[546] *Wöhlerlinienkatalog*. Institut für Leichtbau, Dresden (1964).
[547] Dauerfestigkeitskatalog. Technisch-wissenschaftliche Abhandlungen des Zentralinstituts für Schweißtechnik der DDR, Halle (Saale).
[548] Engineering Sciences Data Sheets, Fatigue Endurance Data. 1–8, ESDU International London (1992).
[549] Engineering Sciences Data Sheets, Fatigue Fracture Mechanics Data. 1–4, ESDU International London (1992).
[550] M. Hück, L. Thrainer and W. Schütz, Berechnung von Wöhlerlinien für Bauteile aus Stahl, Stahlguß und Grauguß Synthetische Wöhlerlinien. Bericht Nr. ABF 11, Verein Deutscher Eisenhüttenleuten, Düsseldorf (1983).
[551] W. Schütz, Über die Übertragbarkeit von Probestabdaten auf das Bauteil, in *Kerben und Betriebsfestigkeit*. DVM, Ingolstadt (October 1989).
[552] W. Schütz, *Corrosion Fatigue*. Proceedings of the 2nd Int. Conf. on Structural Failure, Product Liability and Technical Insurance (Edited by H. P. Rossmanith), Vienna. Inderscience Publishers, London (1987).
[553] H. Zenner, R. Heidenreich and I. Richter, Dauerschwingfestigkeit bei nichtsynchroner mehrachsiger Beanspruchung. *Z. Werkstofftech.* **16**, S. 101–112 (1985).
[554] R. Heidenreich, I. Richter and H. Zenner, Schubspannungsintensitätshypothese weitere experimentelle und theoretische Untersuchungen. *Konstruktion* **36**, Heft 3, S. 99–104 (1984).

(*Received* 31 *July* 1994)