Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

International Journal of Fatigue 26 (2004) 1189–1196

International Journal of Fatigue

www.elsevier.com/locate/ijfatigue

A study of adhesively bonded joints subjected to constant and variable amplitude fatigue

S. Erpolat [a], I.A. Ashcroft [a,*], A.D. Crocombe [b], M.M. Abdel-Wahab [b]

[a] *Wolfson School of Mechanical and Manufacturing Engineering, Loughborough University, Loughborough, Leicestershire LE11 3TU, UK*
[b] *School of Engineering, University of Surrey, Guildford GU2 7XH, UK*

Received 22 May 2003; received in revised form 19 March 2004; accepted 31 March 2004

Abstract

Double lap joints were tested under constant and variable amplitude loading. Palmgren–Miner's rules were applied to predict the constant amplitude fatigue life using variable amplitude loading. These significantly overestimated the fatigue life, indicating a high crack growth acceleration due to the load transitions in the variable amplitude fatigue spectrum. Crack growth acceleration was mainly attributed to mean stress variations in the spectrum but overloads were shown to be important in crack initiation. A model incorporating a 'cycle mix' factor has been proposed as an improved method of predicting the fatigue life of bonded joints subjected to variable amplitude fatigue and this model has been shown to result in significantly improved fatigue lifetime predictions.

Keywords: Composites; Joints; Fatigue life prediction; Variable amplitude loading

1. Introduction

Adhesive bonding is the most attractive joining technique for many aerospace applications. One of the most important reasons for this is the superior fatigue performance of bonded joints compared to traditional joining techniques and fatigue is considered one of the most important design concerns for aerospace structures. There has been a significant research effort into the analysis of fatigue behaviour in bonded joints in recent years [1–7], predominantly using constant amplitude (CA) sinusoidal waveforms. Although CA loading condition can be assumed for many real structures, such as pressure vessels, a more complex load history is seen in many applications such as aerospace and automotive structures. Many standardised service-simulation load histories, which are characteristic to a specific structure, can be found in the literature, e.g. FALSTAFF for a fighter aircraft wing skin or HELIX/FELIX for helicopter rotor blades. The potential difficulty in analysing structures subjected to such complex loading arises from the fact that materials can behave very differently under variable amplitude (VA) loading than they do under CA conditions, whereas fatigue life data are commonly only available for CA loading. Despite this, there is very little work on the VA fatigue of bonded joints [8–10].

A number of methods of predicting VA fatigue life using the data obtained from CA loading have been proposed, e.g. RMS methods, cumulative damage rules, models based on fracture mechanics, etc. Cumulative damage rules are the most popular methods in current use. The concept of cumulative damage rules is to define damage as a linear or non-linear function of the ratio of the number of stress cycles imposed on a component to the total number of cycles of the same amplitude necessary to cause failure, n/N_f. Both linear and non-linear cumulative damage rules have been proven to be quite satisfactory for many metals and are relatively easy to apply. The most widely used cumulative damage rule is the Palmgren–Miner (P–M) rule [11,12], where damage is considered accumulating in

---

[*] Corresponding author. Tel.: +44-1509-227535; fax: +44-1509-227648.

E-mail address: i.a.ashcroft@lboro.ac.uk (I.A. Ashcroft).

0142-1123/$ - see front matter © 2004 Elsevier Ltd. All rights reserved.
doi:10.1016/j.ijfatigue.2004.03.011

a linear manner without consideration of any load interaction effects. Linear cumulative damage rules become unreliable when there are significant load interaction effects, such as the effect of overloads, load sequencing, etc. If the load interaction effects are significant, the prediction based on such an analysis will be either conservative due to crack growth retardation or unconservative due to acceleration.

In fact, crack growth retardation is the more commonly reported phenomenon. For example, it is well known that overloads generally retard fatigue crack propagation in metals. Some of the proposed mechanisms to account for this phenomenon are the compressive residual stresses in the vicinity of a crack tip as well as crack tip closure and crack tip blunting [13]. Although neglecting such beneficial effects using a non-interactive methodology can deprive us of the opportunity to achieve a lighter structure, at least, the design is on the side of safety. However, if the load interactions cause crack growth acceleration instead, the structure under investigation can fail much earlier than predicted using CA data and a non-interactive prediction methodology. Although most of the studies in this field indicate retardation behaviour, there is also some work in the literature reporting crack growth acceleration for both metals [14–16] and composites [17–19]. These studies report several different mechanisms accounting for the acceleration behaviour.

Chang et al. [14] considered the effect of compressive stresses following an overload in an aluminium alloy. They suggested that compressive loading in the spectrum somehow annihilates the compressive residual stresses built up in the vicinity of the crack tip and thus had an acceleration effect. However, several cases were reported showing the existence of crack growth acceleration even when there is no compressive loading at all.

Nisitani and Nakamura [16] studied the VA fatigue of crack growth behaviour of steel specimens tested under a spectrum composed of a very small number of overloads and a very large number of cycles below or near the fatigue limit. This was a spectrum very similar to the one to be used in the current study. They observed that the application of a linear cumulative damage rule resulted in extremely unconservative results. A possible explanation of this phenomenon is that although understress cycles cannot initiate a crack, they can contribute to existing fatigue damage created by the overloads. They might cause considerable damage just after the initiation of a crack by an overload or when the crack reaches a certain size. However, there may also be other factors at work. For instance, Nisitani and Nakamura [16] observed that overloads enhance the damaging effect of understress cycles for physically short cracks (≤ 1 mm), even when they do not cause a substantial change in the crack size. The crack growth acceleration for such physically short cracks has been observed by several other investigators [20–23].

Farrow [17] and Schaff and Davidson [18,19] studied fatigue crack growth acceleration for composite materials. Farrow [17] found that the fatigue life of composite laminates subjected to small block loading is shorter than that of laminates subjected to large block loadings when the blocks have different mean stress levels. He called this phenomenon the 'cycle mix effect'. Schaff and Davidson [18,19] reviewed the study made by Farrow and tried to develop a strength-based wearout model. They suggested that the 'cycle mix' effect occurred during the transition from a CA stage to another stage having a higher mean stress value, although they did not discuss the reason for the strength degradation during transition. They incorporated a 'cycle mix' factor in their damage accumulation model and proposed that the 'cycle mix' effect should be taken into account when the stages in a spectrum were such that the ratio of cycles in a stage to cyclic life for that stage was less than 0.001, as is the case for the spectrum in this study.

In one of the few works on the VA fatigue of adhesively bonded joints, Jones and Williams [10] studied bonded metal box sections subjected to block-loading spectra with different characteristics including overloads, mean stress jumps, etc. Based on the P–M results, they observed severe fatigue crack growth retardation in all cases. They attributed this to blunting of the crack tip due to yielding. However, it should be noted that the structure they considered was very different from the samples considered in this investigation. In a more relevant work, Yang and Du [9] studied VA fatigue of CFRP-to-metal and metal-to-metal adhesively bonded single lap joints subjected to a random flight-by-flight loading spectrum and observed crack growth acceleration for both combinations. They also suggested a statistical strength-based wearout model, which gave fairly good results.

The main tasks to be undertaken in this work are first to see whether the P–M rule is valid for bonded joints subjected to a representative aerospace fatigue spectrum. If not, the next step is to define the critical load interactions, i.e. the effect of overloads, understress cycles, mean stress variations, etc. and finally to suggest an alternative predictive methodology to account for these load interactions.

2. Theory

For a block-loading spectrum, Palmgren–Miner's (P–M) rule [11,12] can be defined as:

$$N_B \sum_{i=1}^{n_B} \frac{n_i}{N_i} = C \tag{1}$$

where N_B is the number of loading blocks to failure; n_B

is the number of CA stages in a block; n_i is the number of cycles in a stage with a stress level corresponding to a fatigue life of N_i and finally C is called Miner's sum or damage sum. C would be equal to 1.0 for 100% damage if there are no load interactions.

In addition to load interaction effects, another limitation of the P–M rule is that all the cycles applied must lie above the fatigue limit as originally stated by Miner [12]. However, the effect of understress cycles on the fatigue life is not clearly understood, as discussed previously. Schutz and Heuler [24] recommended that S–N curves should be extended below the fatigue limit with a slope 'k' equal to the slope of the high cycle fatigue line (elementary Miner's rule) or with a decreased slope (extended Miner's rule) to take the loading below the fatigue limit into account, as shown in Fig. 1.

As discussed in the previous section, one of the identified load interaction effects is the 'cycle mix' effect. Schaff and Davidson [18,19] proposed a strength-based wearout model to account for the 'cycle mix' effect. Based on their approach, the following model has been developed.

According to this model, the residual strength of a bonded joint is initially equal to the quasi-static failure load, L_u. However, the residual strength continuously decreases during the fatigue cycling. The final failure occurs when the residual strength, R, equals the maximum load of the spectrum, L_{max}, i.e. when $R(N_f) = L_{max}$. Any approach based on this assumption is generally called a 'strength-based wearout model'. Although not physically based, this has been a successful approach for composites [25,26]. Although it was not proven that such a decrease in residual strength really exists in the joints studied here, previous work on bonded composite-to-metal joints [9,27] showed that residual strength does decrease during fatigue cycling.

The first step in the model is to calculate the total strength degradation during a single loading block. The residual strength is assumed to be degraded by two different mechanisms: degradation due to cycles above the fatigue limit and degradation due to the transitions from one CA stage to another having a higher mean load value ($\Delta L_{mn} > 0$), i.e. mean load jumps.

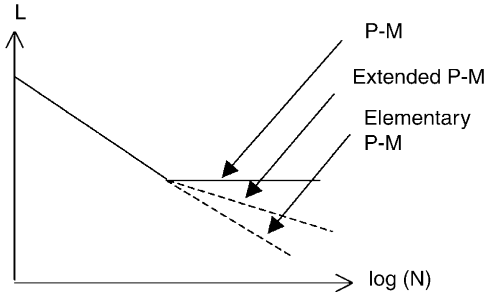

Fig. 1. Modifications to Palmgren–Miner's rules.

Assuming a linear damage accumulation, residual strength degradation by each cycle above the fatigue limit, LD, can be defined as:

$$LD = \frac{L_u - L_{OL}}{N_{f,OL}} \tag{2}$$

where L_{OL} is the maximum load of the cycle and $N_{f,OL}$ is the fatigue life corresponding to this cycle. The residual strength degradation due to mean load jumps, CM, is defined as:

$$CM = \alpha(\Delta L_{mn})^{\beta L_{max}(\Delta L_{max}/\Delta L_{mn})} \tag{3}$$

where ΔL_{mn} and ΔL_{max} are the changes in the mean and maximum load values during the transition; L_{max} is the maximum load in the spectrum; α and β are the 'cycle mix' constants and dependent on material and geometry. They can be determined by comparing VA fatigue lives under different spectra, e.g. two spectra with and without mean load jumps. Assuming that in each loading block, there are a number of cycles above the fatigue limit (OL_1, OL_2,...,) and a number of mean load jumps (CM_1, CM_2,...,), the strength degradation during a single block, ΔR_B, can be defined as:

$$\begin{aligned}\Delta R_B = \alpha\Big[&(\Delta L_{mn,1})^{\beta \cdot L_{max}(\Delta L_{max,1}/\Delta L_{mn,1})}\\ &+ (\Delta L_{mn,2})^{\beta \cdot L_{max}(\Delta L_{max,2}/\Delta L_{mn,2})} + \ldots\Big]\\ &+ \left[\frac{L_u - L_{OL1}}{N_{f,OL1}} + \frac{L_u - L_{OL2}}{N_{f,OL2}} \ldots\right]\end{aligned} \tag{4}$$

and the corresponding number of blocks to failure is:

$$N_B = \frac{L_u - L_{max}}{\Delta R_B} \tag{5}$$

We call the method described above the 'linear cycle mix model' (LCM). The main difference between the model described in this paper and the cycle-mix model proposed by Schaff and Davidson [18,19] is the definition of the 'cycle mix' factor (Eq. (3)). Schaff and Davidson assumed the exponent of ΔL_{mn} to be proportional to the square of $(\Delta L_{max}/\Delta L_{mn})$ and independent of the maximum load of the overall spectrum. In the LCM method, L_{max} was introduced in order to obtain a linear VA L–N curve, whose slope can be adjusted to match the experimental L–N curve. Testing with spectra having different $(\Delta L_{max}/\Delta L_{mn})$ ratios indicated that the exponent for the bonded CFRP joints tested in this programme should be defined as a linear function of $(\Delta L_{max}/\Delta L_{mn})$ rather than a square one.

3. Experimental

3.1. Joint configuration

Double lap joints (DLJ) with a total overlap length of 12.5 mm (Fig. 2.) were used. The adherends were epoxy based carbon fibre reinforced (CFRP) composite, AS4/8552. Laminates were laid up using a quasi-isotropic (0°,+45°,−45°,90°)s stacking sequence. The mechanical properties of the composite are presented in Table 1. The adhesive used, Cytec 4535A, was a single part epoxy paste.

The composite panels were autoclave cured at 180 °C for 130 min and then grit blasted and degreased prior to bonding. The adhesive was cured in a bonding press at 135 °C for 60 min with the panels shimmed to give a 0.5 mm bondline thickness.

3.2. Test conditions

In the CA test programme, the spectra under which the DLJ were tested in load control had sinusoidal waveforms with a frequency of 10 Hz and load ratios of $R = 0.1$ and 0.5. The testing was undertaken in the ambient laboratory environment. A servo-hydraulic, fatigue testing machine with computerised control and data logging was utilised for the testing. The VA tests were also carried out in load control, with the same frequency and ambient laboratory environment. The main spectrum input to the controller was representative of the loading on a joint in a composite wing. In this complex spectrum (Fig. 3), one block is composed of 100 cycles and 17 stages with a variety of $L_{\max}$ and R-values. This spectrum exhibits all the major features responsible for load interaction effects, i.e. overloads, changes in mean load and changes in load amplitude. Several other spectra were designed for the investigation of the ‘cycle mix’ effect. For convenience, load levels in the VA loading spectra are identified in by the highest load value in the spectrum, $L_{\max}$. All other loads are scaled accordingly.

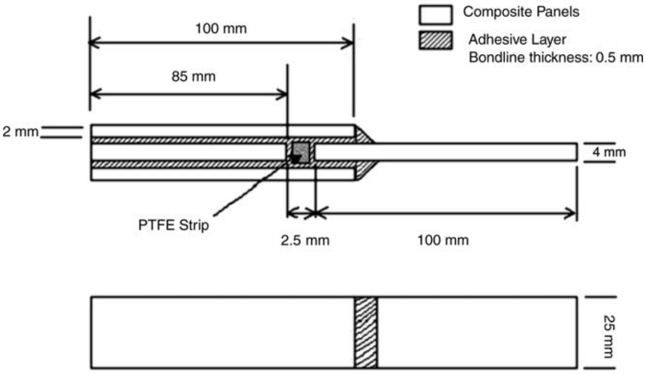

Fig. 2. Specimen geometry.

Table 1
Mechanical properties of multi-directional composite

E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}	v_{32}
57.8	57.8	19.9	0.3	0.3

The test spectra were programmed in the test machine software using a complex function generator prior to the testing. The multiple input minimal integral control synthesis (MIMICS) control system [28] was used to improve control in the spectrum testing.

In some polymers, cyclic loading can result in significant hysteretic heating. In order to check if the adhesive used in this testing programme was susceptible to this effect, the temperature of one of the joints was monitored using thermocouples attached to the specimen during fatigue testing. One of the thermocouples was attached to the adhesive near the fillet area and the other was attached to the adhesive near the PTFE block. No increase in temperature was observed with either thermocouple during fatigue cycling, even when a peak load of 9 kN and a frequency of 30 Hz was applied. This is probably because heat generation is localised at points of high stress in the adhesive and any heat generated is quickly dissipated in the surrounding material.

3.3. P–M rules results

All joints failed predominantly in the composite adherends, in the 0°-ply adjacent to the bondline. On one side of the joint, failure was in the outer adherend, and on the other side, failure was in the middle adherend as shown in Fig. 4. Loci of failure were the same for the specimens tested under both CA and VA loading, an important condition that should be satisfied in order to predict the VA fatigue life from CA data.

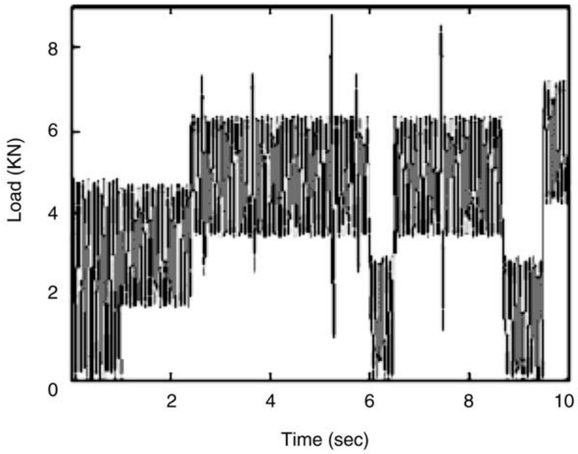

Fig. 3. VA spectrum for a maximum load of 9 kN.

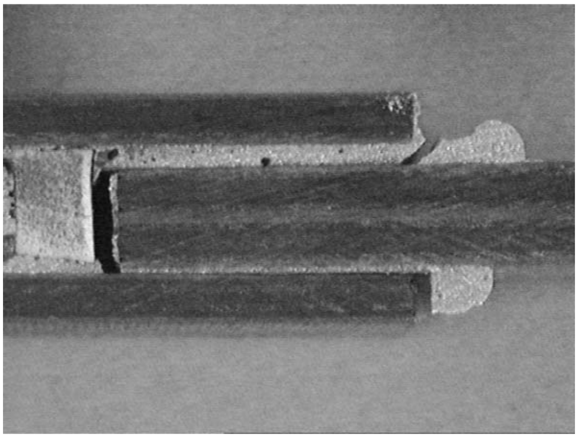

Fig. 4. Locus of failure for the joints tested under both CA and VA loading.

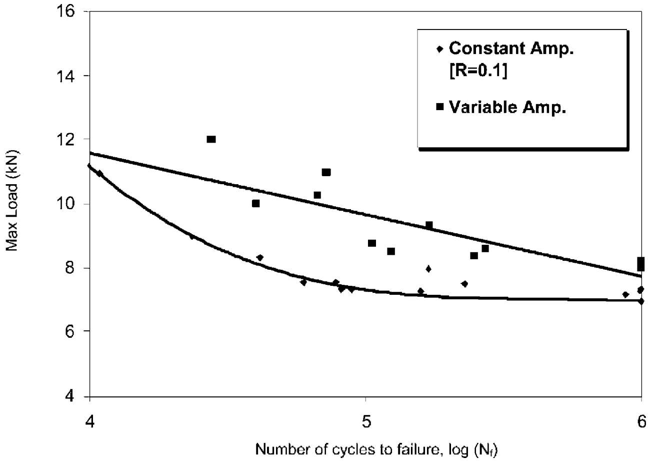

Fig. 5. L–N plots for CA and VA fatigue of DLJs.

L–N curves (maximum load, $L_{\max}$, against number of cycles to failure, N) for variable and constant amplitude spectra were constructed. The fatigue threshold for VA loading was defined as the highest maximum load in a loading block at which a sample could survive 10^6 cycles with no observable damage using an optical microscope. The results are presented in Table 2. It can be seen that the CA fatigue threshold increases as R increases. However, this depends on how the loading spectrum is characterised. If the threshold value is defined in terms of ΔL instead of $L_{\max}$, it can be said that the threshold value decreases as R increases.

Another observation is that the threshold value for the VA spectrum is almost the same as that for $R = 0.5$ CA loading. This could not be anticipated, even though most of the cycles in the VA spectra had a load ratio, $R = 0.5$, as the threshold value for VA loading was defined in terms of the maximum load in the spectrum. The cycles of $R = 0.5$ had a much smaller maximum load value than this peak load, by at least 20%, therefore, it can be said that the fatigue threshold value for VA, in fact, turned out to be much smaller than expected. The L–N curves for the VA spectrum and the CA spectrum corresponding to the overloads, i.e. for $R = 0.1$, are presented together in Fig. 5.

The P–M rule was used to predict the VA fatigue life from the CA results. Most of the cycles in the spectrum had a load ratio close to either 0.1 or 0.5. Therefore, CA data were used directly without any modification for R-ratio effect. The cycles below the fatigue limit, which are assumed not to contribute to the fatigue crack growth, were not taken into consideration. At the end of the analysis, the total damage sum, C, which ideally should have been equal to 1.0, proved to be at most 0.25. This means that the P–M rule overestimates the fatigue life, leading to unconservative life prediction. The extended Miner's rule was, therefore, applied in order to take the cycles below the fatigue limit into account. This had only a modest effect, C increasing to at most 0.3. The Miner sums obtained by both methods are presented in Fig. 6.

Table 2
Summary of results for DLJs

R	Static failure load (kN)	Fatigue threshold $L_{\max}$ (kN)	Fatigue threshold ΔL (kN)
0.1		6.8	6.1
0.5	15.8 (±1.4)	7.8	3.9
VA		8	7.2

4. The 'cycle mix' effect

An experimental testing programme was undertaken to investigate the existence of the 'cycle mix' effect. In

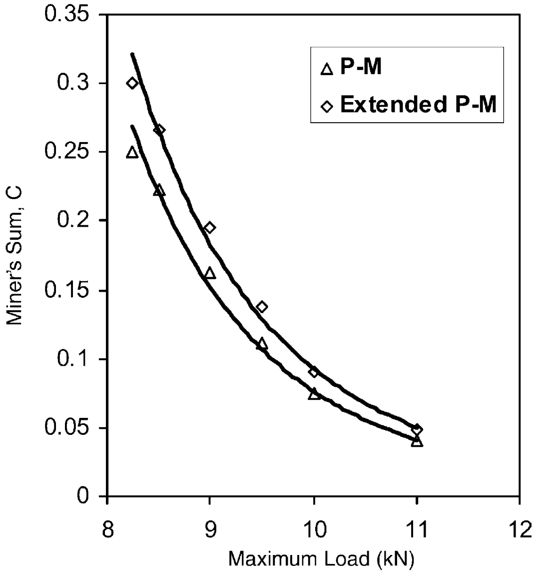

Fig. 6. Plot of C vs. $L_{\max}$ for VA loaded specimens.

this programme, double lap joints were tested using a number of modified fatigue spectra. Four to five specimens were tested for each load spectra. Specimens tested under constant and variable amplitude spectra for an $L_{\max}$ of 9 kN resulted in fatigue lives of 22.5 and 200 K, respectively. Given that only two out of 100 cycles were distinctly above the threshold value in the VA spectrum (Fig. 7(a)), it can be said that the fatigue life was significantly shorter than expected and thus the acceleration effect due to load interactions was high. The first modification to this spectrum is shown in Fig. 7(b), where the ratio of number of understress cycles to overstress cycles is 10 and the mean load is kept constant ($\Delta L_{\text{mean}} = 0$). The Miner sum for this spectrum was approximately 0.9, which suggested that the acceleration effect had practically disappeared.

In the second modified spectrum, Fig. 7(c), the understress cycles were at a lower maximum load level than those in the first modified spectrum and mean load jumps, ΔL_{mean}, were introduced. For this case, the Miner sum turned out to be as low as 0.2. This gives a strong indication that the changes in mean load are more damaging than the overloads for fatigue crack growth.

The third spectrum used is shown in Fig. 7(d). This spectrum looks the same as our initial VA spectrum, except for one very important difference in terms of the ‘cycle mix’ effect. Some of the understress cycles were raised to a higher load level and hence all ΔL_{mean} values were reduced to half of their original values. The change in fatigue life was quite radical. The joint did not fail even after 10^6 cycles. This confirms that ΔL_{mean} is one of the main factors leading to crack growth acceleration.

The last spectrum applied is shown in Fig. 7(e). In this spectrum, overload cycles were trimmed leaving only understress cycles with mean load jumps. Loading

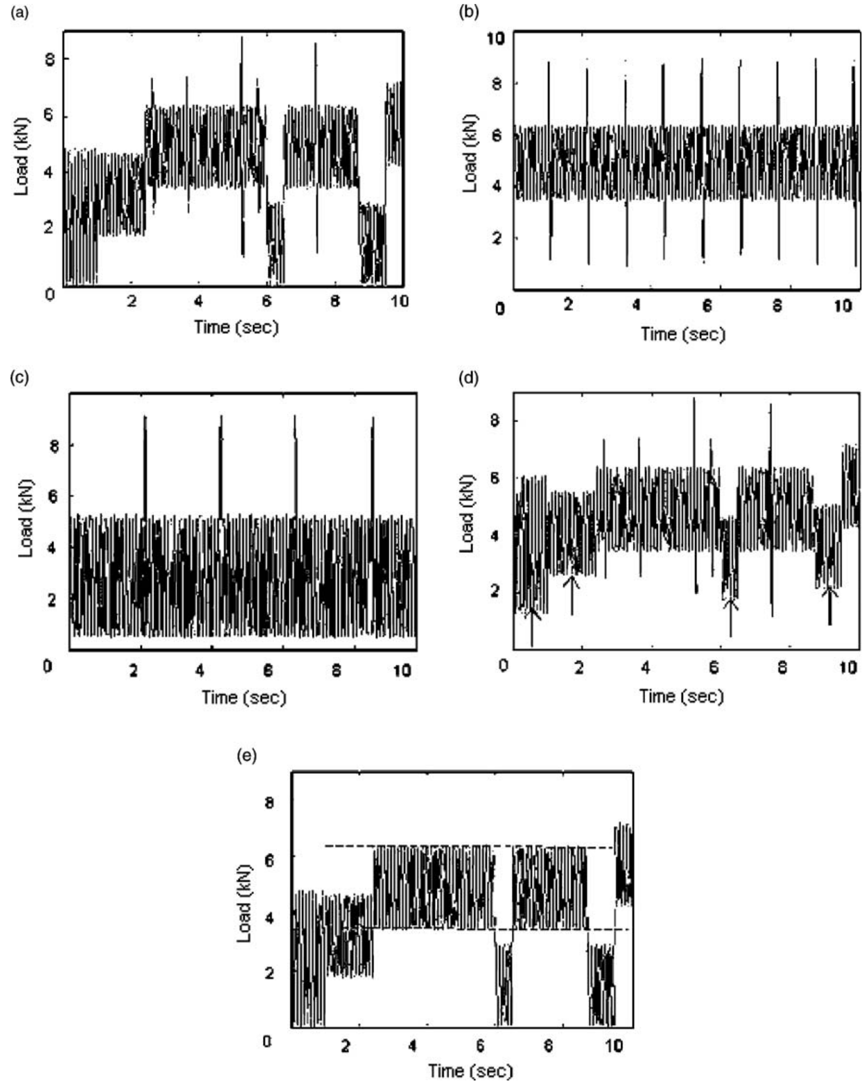

Fig. 7. ‘Cycle mix effect’ testing programme.

was such that the understress cycles coincided with our original spectrum with an L_{max} of 9 kN. The elimination of the overloads reduces L_{max} and hence after 10^6 cycles occasional overloads were applied to check if the residual strength had decreased to L_{max} associated with the overloads. Testing continued to 1.5×10^6 cycles with no failure. The sample was then examined using optical microscopy and there was no evidence of any sort of damage. This suggests that changes in ΔL_{mean} activate accelerated crack growth during the cycles slightly below the threshold or increase the damage of the overload cycles. However, $L_{max,}$ and hence overloads may be more important in initiating damage in an uncracked sample.

These results were broadly in agreement with the observations of Farrow [17] and Schaff and Davidson [18,19] for polymer composite samples. They also demonstrated the existence of the 'cycle mix' effect but did not suggest any physical mechanisms associated with the phenomenon. However, it is clear that the 'cycle mix' effect is crucial for the VA analysis of bonded joints and that a general predictive method should include ΔL_{mean} as well as L_{max} effects.

The next step in our work was a quantitative analysis of the 'cycle mix' effect. The LCM model developed earlier in this paper was implemented. The 'cycle mix' constants, α and β, were determined by fitting a straight line between two points on the VA L–N curve (Fig. 8). They were calculated as 2.348×10^{-6} and 0.514, respectively. For these values of α and β, there was a very good agreement with the rest of the experimental points on the L–N curve, as shown in Fig. 8.

The next step was to apply the method to different spectra to see if the experimentally determined values of α and β were valid for any kind of block-loading spectra. For example, for joints tested under the spectrum shown in Fig. 7(c), the experimental fatigue life was 80 K cycles. The fatigue life predicted by the P–M rule was 390 K cycles, i.e. almost five times overestimated. When the cycle mix model was applied, the predicted fatigue life proved to be 62 K, i.e. the error was only about 20%. The model also accounted for the drastic change in the experimental fatigue life when the stress cycles were raised to a higher load level to reduce the mean load jumps as shown in Fig. 7(d) as such a change increased the fatigue life to over 1 M cycles, as predicted. For a spectrum without any mean load variations, as shown in Fig. 7(b), the LCM model predicts a fatigue life almost the same as the P–M rule. These results are summarised in Table 3.

Table 3
Comparison of the P–M rule and the LCM model

Spectra	Fatigue life (cycles at $L_{max} = 9$ kN)			Damage sum, C	
	Experimental	P–M	LCM	P–M	LCM
7 (a)	200 K	$>10^6$	200 K	0.18	1
7 (b)	180 K	200 K	200 K	0.9	0.9
7 (c)	80 K	390 K	62 K	0.21	1.28
7 (d)	$>10^6$	$>10^6$	$>10^6$	–	–
7 (e)	$>10^6$	$>10^6$	$>10^6$	–	–

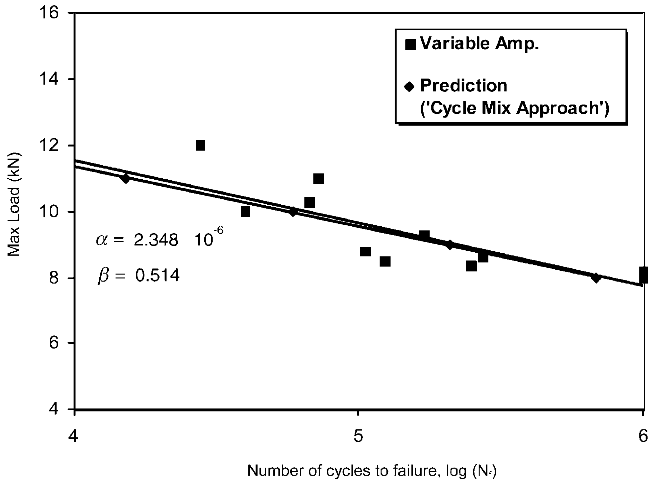

Fig. 8. Experimental and predicted VA L–N curves.

5. Conclusions

(i) The Palmgren–Miner rule, which gives quite satisfactory results in many cases, failed to predict the fatigue life of bonded composite joints subject to a block-loading VA spectrum. The results indicated a severe fatigue growth acceleration due to load interactions. This is of concern to analysts using CA fatigue data to predict variable amplitude fatigue life.

(ii) It was shown that mean stress variations can be more damaging than the overloads with respect to fatigue crack growth accelerations but that L_{max}, and hence overloads, may be important in fatigue crack initiation. It was thus considered that any general model for fatigue life prediction in bonded joints should incorporate both ΔL_{mean} and L_{max} effects.

(iii) A simple predictive model incorporating a 'cycle mix' factor has been proposed. This method proved to be a considerable improvement on traditional cumulative damage laws for predicting the fatigue life of bonded lap joints subjected to variable amplitude fatigue.

References

[1] Xu XX, Crocombe AD, Smith PA. Fatigue crack growth rates in adhesive joints at different frequencies. J Adhes 1996;58: 191–204.

[2] Dessureault M, Spelt JK. Observations of fatigue crack initiation and propagation in an epoxy adhesive. Int J Adhes Adhes 1997;17:183–95.

[3] Curley AJ, Jethwa JK, Kinloch AJ, Taylor AC. The fatigue and durability behaviour of automotive adhesives. Part III: predicting the service life. J Adhes 1998;66:39–59.

[4] Ashcroft IA, Abdel Wahab MM, Crocombe AD, Hughes DJ, Shaw SJ. The effect of environment on the fatigue of bonded composite joints. Part 1: testing and fractography. Composites Part A 2001;32:45–58.

[5] Ashcroft IA, Abdel Wahab MM, Crocombe AD, Hughes DJ, Shaw SJ. Effect of temperature on the quasi-static strength and fatigue resistance of bonded composite double lap joints. J Adhes 2001;75:61–88.

[6] Ashcroft IA, Shaw SJ. Mode I fracture of epoxy bonded composite joints. Part 2: fatigue loading. Int J Adhes Adhes 2002;21:151–67.

[7] Crocombe AD, Ong AD, Chan CY, Abdel-Wahab MM, Ashcroft IA. Investigating fatigue damage evolution in adhesively bonded structures using backface strain measurement. J Adhes 2002;78:745–78.

[8] Jeans LL, Grimes GC, Kan HP. Fatigue sensitivity of composite structure for fighter aircraft. 22nd Structures, Structural Dynamics and Materials Conference. Technical Papers, Part 1. 1981, p. 50–60.

[9] Yang JN, Du S. An exploratory study into the fatigue of composites under spectrum loading. J Compos Mater 1983;17: 511–526.

[10] Jones TB, Williams NT. Influence of section design and loading regime on the fatigue properties of spot welded/adhesive bonded box sections. Fifth IAVD Congress: Vehicle Design and Components, Materials Innovation and Automotive Technology. 1989, p. 263–88.

[11] Palmgren A. Durability of ball bearings. Zeitschrift des Vereins Deutscher Ingenieure 1924;68:339–41.

[12] Miner MA. Cumulative damage in fatigue. J Appl Mech 1945;12:159–64.

[13] Chen D, Nisitani H. Analysis of the delaying effects of overloads on fatigue crack propagation. Eng Fract Mech 1991;39(2): 287–98.

[14] Chang JB, Szamossi M, Liu KW. Random spectrum fatigue crack life predictions with or without considering load interactions. In: Chung JB, Hudson CM, editors. Methods and models for predicting fatigue crack growth under random loading. ASTM STP 748. USA: ASTM; 1979, p. 115–32.

[15] Agerskov H. Fatigue in steel structures under random loading. J Constr Steel Res 2000;53:283–305.

[16] Nisitani H, Nakamura K. Trans Japan Soc Mech Eng 1982;48:990 [in Japanese].

[17] Farrow IR. Damage accumulation and degradation of composite laminates under aircraft service loading: assessment and prediction. PhD thesis. vols. 1 and 2, Cranfield Institute of Technology, 1989.

[18] Schaff JR, Davidson BD. Life prediction methodology for composite structures (Part I—constant amplitude and two-stress level fatigue). J Compos Mater 1997;31(2):128–57.

[19] Schaff JR, Davidson BD. Life prediction methodology for composite structures (Part II—spectrum fatigue). J Compos Mater 1997;31(2):158–81.

[20] McClung RC, Sehitoglu H. Closure behaviour of small cracks under high strain fatigue histories. Mechanics of fatigue crack closure. ASTM STP 982. USA: ASTM; 1988, p. 279–99.

[21] Vormwald M, Heuler P. Examination of short crack behaviour and modelling under cyclic inelastic conditions. Fatigue Fract Eng Mater Struct 1993;16:693–706.

[22] Frost NE, Marsh KJ, Pook LP. Metal fatigue. Oxford: Clerendon Press; 1974, p. 307–30.

[23] Gurney TR. Fatigue of welded structures. 2nd ed. Cambridge: Cambridge University Press; 1979, p. 384–94.

[24] Schutz W, Heuler P. A review of fatigue life prediction models for the crack initiation and propagation phases. In: Branco CM, Rosa LG, editors. Advances in Fatigue Science and Technology. 1989, p. 177–219.

[25] Yao WX, Himmel N. A new cumulative damage model for fibre-reinforced plastics. Compos Sci Technol 2000;60:59–64.

[26] Lee CH, Jen HR. Fatigue response and modelling of variable stress amplitude and frequency in AS-4/PEEK composite laminates (Part 2—analysis and formulation). J Compos Mater 2000;34:930–53.

[27] Wolff RV, Lemon GH. Lifetime variability characterization of adhesively bonded composite-to-metal joints. Materials Review '75: Proceedings of the Seventh National Technical Conference. Society for the Advancement of Material and Process Engineering; 1975, p. 230–43.

[28] Stoten DP, Benchoubane H. Robustness of a minimal controller synthesis algorithm. Int J Control 1990;51(4):851–61.