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Constant life diagrams — a historical review

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Abstract

A historical review of the early development of constant life diagrams (variously referred to as Goodman, Smith, Haigh, etc. diagrams) is presented. It is shown that there were two distinct approaches to the formulation of constant life diagrams for fatigue design purposes. The first one was based on Wöhler's fatigue experiments and involved engineering curve fits of the fatigue endurance data. The Launhardt–Weyrauch, Gerber and Johnson formulae are the main representatives of this approach. The second approach is based on the dynamic theory used for bridge design. The Fidler–Goodman formula is an example of this approach. The early proponents of the second approach questioned Wöhler's test results and did not believe that they could be used for design purposes. Finally, the first books on fatigue of metals introduced citation inaccuracies, which were propagated by subsequent authors.

Keywords: Constant life diagrams; Fatigue; History

1. Introduction

There is an unfortunate tendency in engineering and the sciences to associate personal names with particular concepts. Even though this is normally done to honor and credit the originators of particular concepts, it often gives credit to the wrong persons through inadequate historical research. Moreover, the original attributions are often improperly interpreted and used by subsequent writers. A case in point are the constant life diagrams, which are now commonly called *Goodman diagrams*. Gough [1] and Moore and Kommers [2] used the phrase *constant life diagram* to refer to any plot that defines a safe operating region in some stress space. When the diagram included a region defined by a formula, they associated the name of the originator of the formula with the diagram. Thus, Gough presented both Launhardt–Weyrauch [3] and Goodman [4] diagrams for the same experimental data.

2. Graphical representations of constant life data

Constant life diagrams are graphical representations of the safe regime of constant amplitude loading for a given specified life, e.g. the endurance limit or infinite life. These diagrams can be drawn in a number of ways, depending on which parameters are selected to describe the constant amplitude cyclic loading. Constant amplitude cyclic loading can be described by specifying any two of the following parameters:

σ_{max} =maximum stress
σ_{min} =minimum stress
σ_m $=1/2(\sigma_{max}+\sigma_{min})$=mean stress
σ_a $=1/2(\sigma_{max}-\sigma_{min})$=alternating stress
R $=\sigma_{min}/\sigma_{max}$=stress ratio.

Apparently the first graphical representation of constant life data was published in 1873 by Müller [5], who plotted σ_{max} vs. σ_{min} to illustrate the similarity in constant life behavior of Phoenix wrought iron and Krupp cast steel. Subsequently, Lippold [6] used this type of plot in comparing Wöhler's test data with the results of various other investigators.

In 1874, Gerber [7] published two graphical representations of Wöhler's fatigue data, without including the

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actual data in the plot. He plotted σ_{max}/σ_u vs. σ_{min}/σ_u, where σ_u is the ultimate tensile strength, as shown by the dashed line in Fig. 1. In this plot, he included the line $\sigma_{max}/\sigma_u=\sigma_{min}/\sigma_u$, defining the lower stress boundary for the admissible stress region. This line is shown as a dotted line in the figure. In 1899, Goodman [4] published a similar graphical representation of available fatigue data. He plotted σ_{max}/σ_u and σ_{min}/σ_u as the ordinates against an arbitrary abscissa (proportional to σ_{min}/σ_u because all the σ_{min}/σ_u points fell on a straight line), which he did not label as shown by the dotted line in Fig. 1. He included a theoretical line (solid lines in Fig. 1) representing the safe operating cyclic stress region according to the *dynamic theory*. Concurrently, Marburg [8] published a similar plot, connecting the end points of the Launhardt [9] and Weyrauch [3] formulae by straight lines to define the safe operating stress regime. Gerber [7] also plotted σ_{range}/σ_u vs. σ_{min}/σ_u, where $\sigma_{range}=2\sigma_a$. Subsequently, Unwin [10] used a similar plot in comparing Wöhler's and Bauschinger's endurance test data.

In 1880, R.H. Smith [11] published a graphical interpretation of the Launhardt [9] and Weyrauch [3] formulae, in which he plotted σ_{max} and σ_{min} vs. R, as shown in Fig. 2. In 1884, Kennedy [12] published some of Wöhler's data on a plot of σ_{max} and σ_{min}, as the ordinates against an arbitrary abscissa, which he did not label as illustrated in Fig. 3. As can be seen from the figure, the abscissa used by Kennedy was proportional to σ_{max}. In 1910, Smith [13] plotted σ_{max} and σ_{min} vs. σ_m in discussing the fatigue behavior of various steels as shown in Fig. 4. In 1917, Haigh [14,46] plotted σ_{max} and σ_{min} vs. σ_m, and σ_{range} vs. σ_m to illustrate a point about the fatigue behavior of brasses. This seems to be the earliest example of a σ_a vs. σ_m plot. In 1923, Wilson and Haigh [15] presented the advantages of using σ_a vs. σ_m in plotting constant life data. Fig. 5 illustrates a Haigh plot of the Gerber and Goodman diagrams.

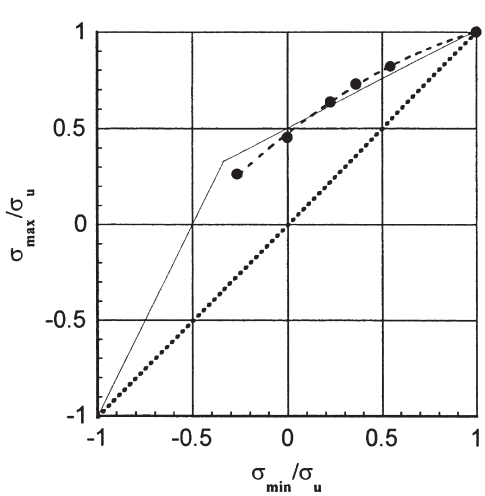

Fig. 1. Gerber (dashed line) and Goodman (solid line) diagrams for Wöhler's data.

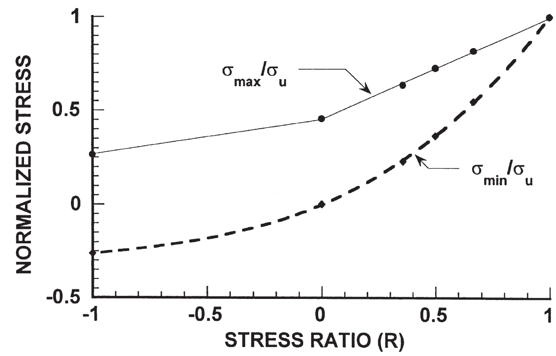

Fig. 2. R.H. Smith form [11] of Launhardt–Weyrauch diagram for Krupp steel.

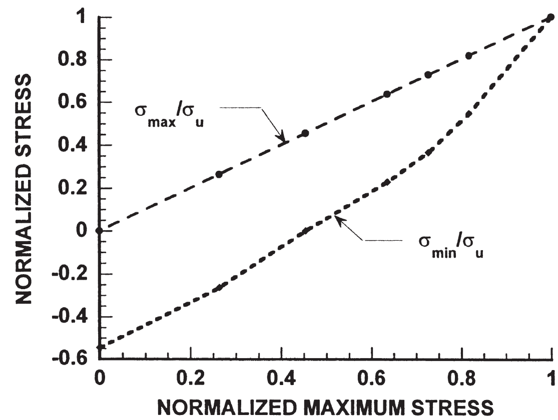

Fig. 3. Normalized Kennedy plot [12] of Wöhler's data for Krupp steel.

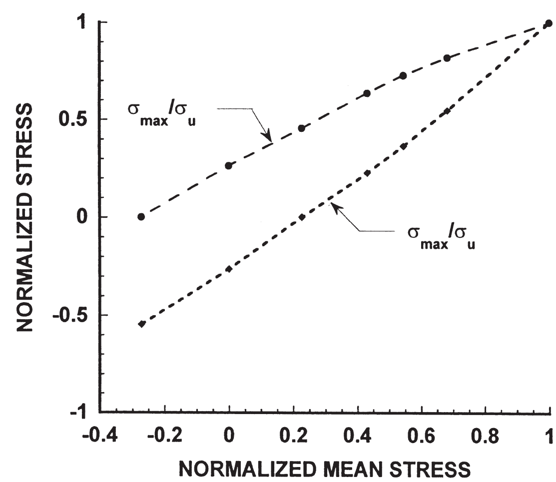

Fig. 4. Normalized J.H. Smith plot [13] of Wöhler's data for Krupp steel.

3. Formulae for endurance limit

3.1. Launhardt, Weyrauch and similar formulae

In 1870, Wöhler [3,16] gave a general law which may be stated as: "Rupture may be caused, not only by a

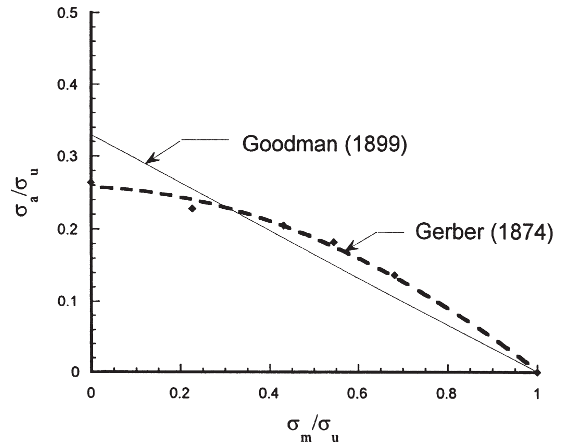

Fig. 5. Haigh [14] form of normalized Gerber and Goodman diagrams for Wöhler's data for Krupp steel.

steady load which exceeds the carrying strength, but also by repeated application of stresses, none of which are equal to this carrying strength. The differences of these stresses are measures of the disturbance of the continuity, in so far as by their increase the minimum stress which is still necessary for rupture diminishes." This may be written as

$$\sigma_{max}=f(\sigma_{range}) \quad (1)$$

In 1873, Launhardt [9] assumed the simplest case of Wöhler's law and took

$$\sigma_{max}=C\sigma_{range} \quad (2)$$

where C is a constant. Launhardt expressed C as

$$C=(\sigma_u-\sigma_o)/(\sigma_u-\sigma_{max})$$

where σ_o is the value of σ_{max} for R=0. Hence, Eq. (2) can be rewritten as

$$\sigma_{max}=\sigma_o+(\sigma_u-\sigma_o)R \quad (3)$$

This equation is known as the *Launhardt formula*. As derived by Launhardt, Eq. (3) only holds for $0\leq R\leq 1$. In 1877, Weyrauch [3] extended the Launhardt formula to the case $-1\leq R\leq 0$. The *Weyrauch formula* is

$$\sigma_{max}=\sigma_o+(\sigma_o-\sigma_{-1})R \quad (4)$$

where σ_{-1} is the value of σ_{max} for $R=-1$. These two formulae, Eqs. (3) and (4), are always used together and are commonly referred to as the *Launhardt–Weyrauch Formula*. For wrought iron, Weyrauch [3,17] built in additional conservatism into the formulae and arrived at

$$\sigma_{max}=\sigma_o(1+R/2) \quad (5)$$

This version of the Launhardt–Weyrauch formula was given by Wilson [18] in 1885, who pointed out that the formula does not provide for impact loading and adopted

$$\sigma_{max}=\sigma_o(1+R) \text{ for } 0\leq R\leq 1 \quad (6a)$$

$$\sigma_{max}=\sigma_o(1+R/2) \text{ for } -1\leq R\leq 0 \quad (6b)$$

In 1878, Cain [19] proposed Eq. (6a) as a modification of Launhardt's formula to account for impact loading. In 1888, Johnson [20] used an expended work argument to justify the Launhardt and Weyrauch formulae.

In 1885, Merriman [21] argued that the end points of the Launhardt and Weyrauch formulae should be connected by a smooth curve. Based on this argument, he proposed

$$\sigma_{max}=\sigma_o+\frac{\sigma_u-\sigma_{-1}}{2}R+\frac{\sigma_u+\sigma_{-1}-2\sigma_o}{2}R^2 \quad (7)$$

for use for design purposes. Subsequently, Bach [22] proposed the same equation, but with arbitrary coefficients.

In 1889, Fowler [23] derived the following formula for dimensioning bridge members:

$$\sigma_{max}=0.5\sigma_y(1+R) \quad (8)$$

where σ_y is the yield stress.

In 1897, Johnson [24,25] criticized the Launhardt–Weyrauch formula and proposed using

$$\sigma_{max}=\sigma_u/(2-R) \text{ for } -1\leq R\leq 1 \quad (9a)$$

in terms of σ_{max} and R as its replacement. He justified using Eq. (9a) by showing that it can be derived from the *old design formula*

$$\sigma_{min}+2\sigma_{range}-\sigma_u \quad (9b)$$

which can be rewritten as

$$\sigma_a=(\sigma_u/3)[1-\sigma_m/\sigma_u] \quad (9c)$$

in terms of σ_a and σ_m, and as

$$\sigma_{max}=(\sigma_u/2)[1+\sigma_{min}/\sigma_u] \quad (9d)$$

in terms of σ_{max} and σ_{min}.

Johnson recommended using Eq. (9a) because it was a single equation replacing the Launhardt and Weyrauch formulae and could be easily used with a slide rule for dimensioning structural members. In 1901, Barr [26] published a nomograph based on Johnson's formula to simplify its use in design. It should be noted that Barr did not simplify Johnson's formula as claimed by Moore and Kommers [2].

3.2. Gerber formula

According to Weyrauch [3], Gerber was the first to use Wöhler's experimental results to prepare specifications for allowable stresses for iron and steel railroad bridge construction, which were adopted by the Bavarian Government in 1872 and published in 1874 [7]. Letting

$$\alpha=\sigma_{min}/\sigma_u,\ \eta=\sigma_{range}/\sigma_u,\ \beta=\sigma_{max}/\sigma_u$$

Gerber assumed that Wöhler's experimental data can be represented by the parabola

$$\alpha^2+\eta^2/4+\alpha\eta+\eta k=\eta_{-1}k \quad (10a)$$

where k is a constant and η_{-1} is the endurance limit stress range for $\alpha=-\beta$. He assumed that $k=-1/\eta_{-1}$, and found that k is 1.5 and 1.8 for wrought iron and steel, respectively. It should be noted that $\sigma_{\min}$ and σ_{range} were the natural variables, since they correspond to the dead and live loads on bridge structures. Eq. (10a) can be rewritten in terms of σ_a and σ_m

$$\sigma_a=\sigma_{-1}[1-(\sigma_m/\sigma_u)^2] \quad (10b)$$

where σ_{-1}, is the endurance limit for $\sigma_m=0$, and as

$$\beta=2\sqrt{1+2\alpha/\eta_{-1}+1/\eta_{-1}^2}-\alpha-2/\eta_{-1} \quad (10c)$$

in terms of normalized $\sigma_{\max}$ and $\sigma_{\min}$. After deriving the parabolic representation of Wöhler's data, Gerber presented procedures for dimensioning bridge members. Eqs. (10a)–(10c) are called the *Gerber formula*. Using somewhat different notation, Schäffer [27] also obtained Eq. (10a) and presented different procedures for dimensioning bridge members.

3.3. Dynamic theory based formula

In 1899, Goodman [4] proposed that the maximum safe operating loads on structures can be determined using the *dynamic theory*. The dynamic theory assumes "that the varying loads … were equivalent to suddenly applied loads, and consequently a piece of material will not break under repeated loadings unless the 'momentary' stress, due to sudden applications, does not exceed the statical breaking strength of the material. … If the dynamic theory were perfectly true, … then the minimum stress (taken as being due to a dead load) *plus* twice the range of stress (i.e. maximum stress−minimum stress) taken as being due to a live load should together be equal to the statical breaking strength of the material" [4]. This is a statement in words of Eq. (9b).

Goodman justified the use of the dynamic theory on the basis that it was easy to remember, simple to use, and gave results as good or better than the other available design formulae. Moreover, Goodman was familiar with Gerber's paper [7] and Johnson's book [24]. Reference to Gerber's paper and Johnson's book were dropped by the time of the 9th edition of the book [28].

The graphical representation of the safe operating cyclic stress region according to the dynamic theory, defined by Eq. (9b), was called the Goodman diagram by Gough [1] and Moore and Kommers [2], who criticized the applicability of Eq. (9b). In response to the criticism by Gough [1], Goodman [29] stated that the dynamic theory was not his innovation, but had been available for some time. In fact, Thurston [30] recommended using Eq. (9b) in 1883.

It turns out that Fidler [31] published in 1877 a derivation of the dynamic theory and proposed its use for the design of bridge structures. He compared the results based on the dynamic theory with Wöhler's experimental data, showing that it gave a good fit to the data. Moreover, he pointed out that the use of the Launhardt–Weyrauch formula required imposing an additional factor of safety to take into account the dynamic nature of the impact loading on bridge structures, while the dynamic theory did not require such a factor. Goodman [4] was familiar with Fidler's book. It should be noted that Fidler was not the originator of the dynamic theory, but seems to be the first one to clearly propose its use for dimensioning bridge members.

In 1858 and possibly earlier, Rankine [32] stated that "a bar, to resist with safety the sudden application of a given pull, requires to have twice the strength that is necessary to resist the gradual application and steady action of the same pull." This reduces to Eq. (9b). In 1865 and possibly earlier, Rankine [33] stated that "The additional strain arising, whether from the sudden application or swift motion of the load is sufficiently provided for in practice by the method already so frequently referred to, of making the factor of safety for the travelling part of the load about double the factor of safety for the fixed part." Again, this reduces to Eq. (9b). In 1899, Seaman [34] compared Wöhler's data with predictions using the Launhardt [9] formula and Eq. (9b), which he indicated as being based on the theory of work. He showed that Eq. (9b) gives a better representation of the experimental data than the Launhardt formula.

Finally, according to Unwin [10], the Royal Commission, appointed in 1847 to inquire into the conditions which should be observed in the application of iron to railway structures, recommended that "the breaking weight of a cast-iron bridge was to be six times the live load added to three times the dead load." Interpreting the live and dead loads as σ_{range} and $\sigma_{\min}$, this recommendation can be expressed by Eq. (9b) with a factor of safety of 3. It should be noted that the commission report [35] did not make this recommendation as attributed to it by Unwin, but stated that "It may, on the whole, therefore be said, that as far as the effects of reiterated flexure are concerned, cast-iron beams should be so proportioned as scarcely to suffer a deflection of one-third of their ultimate deflection. And as it will presently appear, that the deflection produced by a given load, if laid on the beam at rest, is liable to be considerably increased by the effect of percussion, as well as by motion imparted to the load, it follows, that to allow the greatest load to be one-sixth of the breaking weight is hardly a sufficient limit for safety even upon the supposition that the beam is perfectly sound."

3.4. Haigh formula and its modifications

In 1917, Haigh [14,46] showed that the constant life data can be represented by

$$\sigma_a = \sigma_{-1}[1 - (\sigma_m/\sigma_u)] \quad (11)$$

where σ_{-1} is the endurance limit for fully reversed cyclic loading. This equation has become erroneously known as the *generalized Goodman equation* and a diagram containing it as the *generalized Goodman diagram*. This distinction has been dropped and Eq. (11) is now referred to as the *Goodman equation* and the associated diagram as the *Goodman Diagram*.

In 1922, Moore [36] modified the Johnson formula, retaining the requirement that the endurance limit at R=0 is 1.5σ_{-1}, and dropping the requirement that $\sigma_{-1}=\sigma_u/3$, that is,

$$\sigma_{max} = 3\sigma_{-1}/(2 - R) \text{ for } -1 \leq R \leq 1 \quad (12)$$

In 1923, Wilson and Haigh [15] extended the σ_a vs. σ_m diagram by including the line of constant yield stress

$$\sigma_a + \sigma_m = \sigma_y \quad (13)$$

as an additional limit on the safe design stress region. They also discussed the consequences of the ratio of the yield to ultimate strength of the material. Haugen and Hritz [37] called Eq. (13) the Langer modification of the modified Goodman line.

In 1930, Soderberg [38] suggested modifying the generalized Goodman formula by connecting the endurance limit, σ_{-1}, with the yield point by a straight line, that is,

$$\sigma_a = \sigma_{-1}[1 - (\sigma_m/\sigma_y)] \quad (14a)$$

He also indicated how to use this equation for multiaxial fatigue. In 1938, Kommers [39] proposed using

$$\sigma_{max} = 2\sigma_{-1}/[(1-R)+(1+R)\sigma_{-1}/\sigma_y] \quad (14b)$$

which is Eq. (14a) rewritten in terms of σ_{max} and R. Kommers also stated that “It should be understood clearly that the use of this … formula is advocated only when the designer is working with materials for which complete information regarding the various fatigue limits for different ratios of R is not available.”

According to Kravchenko [40], Kinasoshivili in 1943 advocated the use of two fatigue characteristics for constant life diagrams, namely, σ_{-1}, and σ_0 (the endurance limits for R=−1 and R=0). A straight line is drawn through these two points. The safe region is bounded by this straight line and the yield line, Eq. (13).

3.5. Jasper formula

In 1923, Jasper [41] adopted Haigh’s suggestion that the strain energy absorbed within the elastic limit may be used as a failure criterion and suggested that the change in strain energy density per cycle is a constant at the endurance limit. This gives

$$\sigma_{max} = \sigma_{-1}\sqrt{\frac{2}{1-|R|R}} \quad (15)$$

as the equation for the endurance limit.

3.6. Other formulae and generalizations

In 1930, Haigh [42] pointed out that experimental data indicate that the constant life diagram is not symmetric with respect to σ_m=0 as required by the Gerber and generalized Goodman formulae. He suggested that the data can be represented by the generalized parabolic relation

$$\sigma_a = k_0[1 - k_1(\sigma_m/\sigma_u) - k_2(\sigma_m/\sigma_u)^2] \quad (16)$$

where the constants k_0, k_1, and k_2 are selected to give the best fit of the data. He also discussed how to use the constant life diagram for smooth specimens to predict the life of notched specimens.

In 1962, Heywood [43] proposed using an empirical cubic equation for representing constant life data. His equation can be written as

$$\sigma_a = [1 - (\sigma_m/\sigma_u)][(\sigma_{-1} + \gamma(\sigma_u - \sigma_{-1})] \quad (17)$$

where

$$\gamma = (\sigma_m/\sigma_u)[e + g(\sigma_m/\sigma_u)], \quad (18)$$

where e and g are either positive or negative constants. Most experimentally determined constant life data may be represented by proper selection of the constants.

4. Discussion and conclusions

The first graphical representations of constant life data were introduced by railroad and bridge engineers, who were interested in safely accounting for the dead (or static) and live (or moving) loads in bridge design. To these engineers, the most useful graphical representations of experimental data consisted of plots of σ_{max} vs. σ_{min} or σ_{range} vs. σ_{min}, which were used until after 1900. As the nature of engineering changed and new design needs arose in the early 1900s, other graphical representations were introduced to represent constant life data.

The methods for designing safe bridge structures, developed in the second half of the nineteenth century, were based either on empirical representations of Wöhler’s fatigue data or on theoretical attempts to incorporate the effect of impact and other dynamic loads into the design process. Gerber, Launhardt, Schäffer, Weyrauch and Merriman used the first approach and developed empirical formulae based on experimentally determined endurance data. As can be seen from Figs.

1–5, the then available experimental data justified using a parabolic equation to represent the constant life data. As more experimental data were developed, it became obvious that the parabolic representations of endurance data were neither correct nor conservative. Moreover, they were relatively hard to use. Johnson [24,25] linearized the Launhardt and Weyrauch formulae and derived a straight line representation of the constant life data. He showed that the straight line equation gave a good representation of the then available constant life data and was consistent with the design formula used to incorporate impact loads in design.

The second approach used the dynamic theory as a basis of a formula for dimensioning bridge structures. This approach has apparently been around since before the 1850s. Fidler [31] was apparently the first to publish a good derivation and discussion of the dynamic theory, and a bridge design formula based on it. Goodman wrote a very popular engineering book, which was familiar to Gough [1] and Kommers and Moore [2]. As a result, he was credited with developing the dynamic theory. This formula, stated in words, was used by Fairbairn [44,45] and others in the 1850s to design bridges. Moreover, it should be noted that the proponents of the use of the dynamic theory for bridge design did not believe in fatigue. This can be inferred from the discussion of the Launhardt formula and bridge specifications [34]. Finally, it is ironic that the currently most referenced constant life diagram, the so-called Goodman diagram, is based on an impact criterion for the design of bridge structures.

Citation inaccuracies were introduced in the first books on fatigue of metals [1,2]. These resulted in Goodman receiving credit for being the first to propose a straight line representation of constant life data and the so-called Goodman formula. It is actually due to Fidler, who apparently published it first. Johnson was given credit for generalizing the Goodman equation, which he did not do in the first edition of his book. Haigh was apparently the first to use the straight line representation of constant life data, that has been called the generalized Goodman equation and eventually the Goodman equation. Barr was erroneously given credit for simplifying Johnson's formula, which required no simplification.

Many modifications and generalizations of the first representations of constant life data have been proposed. Some of them have been mentioned herein, while some more recent ones have been omitted.

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