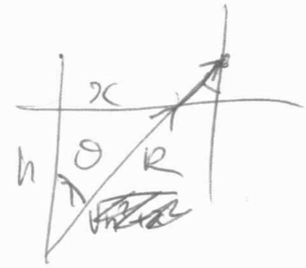
Prova 1 - Física III - IQ - 2014

1) (a) = $\lambda = \frac{q}{L}$ $\quad L = h \, tg\, \theta_0$



$$\lambda = \frac{q}{h \, tg\, \theta_0}$$

(b) $dF_x = \frac{kQ\lambda dx}{R^2} \, sen\, \theta \, d\theta$

$R = \frac{h}{\cos\theta}$

$\frac{x}{h} = tg\, \theta \qquad dx = \frac{h}{\cos^2\theta} d\theta$

$$dF_x = Qk\lambda \cdot \frac{h \, d\theta}{\cos^2\theta} \, \frac{sen\,\theta}{h^2} \cos^2\theta$$

$$dF_x = \frac{Qk\lambda}{h} \, sen\,\theta \, d\theta$$

$$F_x = \frac{Qk\lambda}{h} \int_0^{\theta_0} sen\,\theta \, d\theta = \frac{Qk\lambda}{h}(1 - \cos\theta_0)$$

$dF_y = \frac{Qk\lambda}{h} \cos\theta \, d\theta \qquad F_y = \frac{Qk\lambda}{h} \int_0^{\theta_0} \cos\theta \, d\theta$

$$F_y = \frac{Qk\lambda}{h} \, sen\, \theta_0$$

$$F_y = \frac{Qkq}{h^2 \, tg\,\theta_0} sen\,\theta_0 = \frac{Qkq}{h^2} \cos\theta_0$$

$$F_x = \frac{Qk\lambda q}{h^2 \, tg\,\theta_0} \cos\theta = \frac{Qk\lambda q}{h^2} \frac{\cos^2\theta_0}{sen\,\theta_0}$$

c)
$$\vec{F}_Q = -\vec{F} = \cancel{q}\,\vec{E}_Q$$

$$\vec{E}_Q = -\frac{\vec{F}}{Q} = -\frac{k\lambda}{h}(1-\cos\theta_0)\,\hat{x} - \frac{k\lambda}{h}\sin\theta_0\,\hat{y}$$

d)
$$W_{\infty\to h} = U_{\infty\to h} = Qk\int\frac{dq}{R}$$

$$W = Qk\int_0^{\theta_0}\frac{\lambda h\,d\theta}{\cos^2\theta}\,\frac{\cos\theta}{h}$$

$$W = \frac{kQq}{h\,tg\,\theta_0}\int_0^{\theta_0}\frac{d\theta}{\cos\theta}$$

$$W = \frac{kQq}{h\,tg\,\theta_0}\left[\ln\left(\frac{1}{\cos\theta_0}+tg\,\theta_0\right) - \cancel{\ln 1}\right] \rightarrow 0$$

$$W = \frac{kQq}{h\,tg\,\theta_0}\ln\left(\frac{1}{\cos\theta_0}+tg\,\theta_0\right) \qquad \left(\begin{matrix}0<\theta_0\\ \theta_0<90^\circ\end{matrix}\right)$$

2)(a) $\rho = \rho_0 \left(\frac{R}{R_0}\right)$

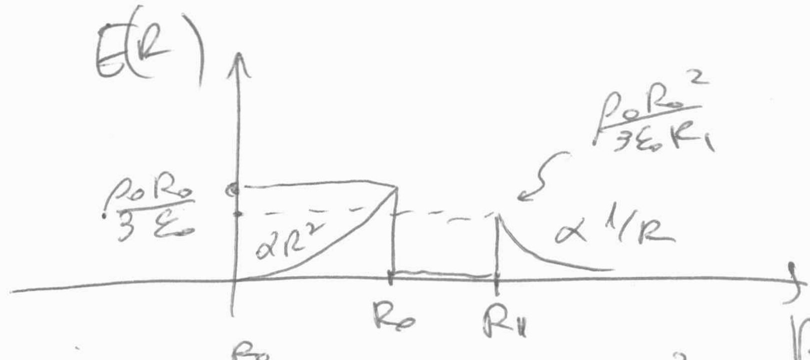
(I) $Q(R) = 2\pi L \frac{\rho_0}{R_0}\int_0^R R^2\, dR = \frac{2\pi L \rho_0}{R_0}\frac{R^3}{3}$

$2\pi R L\, E(R) = \frac{Q(R)}{\varepsilon_0}$

$E(R) = \frac{\cancel{2\pi L}\, \rho_0 R^3}{\cancel{2\pi L}\, \varepsilon_0 R_0 3R} = \frac{\rho_0}{3\varepsilon_0 R_0} R^2 \qquad 0<R<R_0$

(II) $E(R) = 0 \qquad R_0 < R < R_1$

(III) $E(R) = \frac{Q(R_0)}{2\pi L R \varepsilon_0} = \frac{\cancel{2\pi L}\, \rho_0 R_0^2}{\cancel{2\pi L}\, \varepsilon_0 3R} = \frac{\rho_0 R_0^2}{3\varepsilon_0}\frac{1}{R}$



$\vec{E}(R) = E(R)\hat{R}$

(b) $V = \varphi(0) - \varphi(R_0) = \int_0^{R_0} E(R)\, dR = \frac{\rho_0}{3\varepsilon_0 R_0}\frac{R_0^3}{3} = \frac{\rho_0 R_0^2}{9\varepsilon_0}$

3) $\cancel{E_x = -\frac{\partial \varphi}{\partial x} =}$ $\qquad \varphi = \cos z\, e^{-xy}$

(a) $E_x = -\frac{\partial \varphi}{\partial x} = +y \cos z\, e^{-xy}$

$E_y = -\frac{\partial \varphi}{\partial y} = +x \cos z\, e^{-xy}$

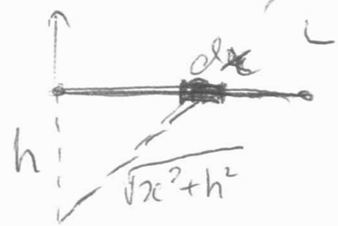
$E_z = -\frac{\partial \varphi}{\partial z} = +\sin z\, e^{-xy}$

$\vec{E} = \cos z\, e^{-xy}\left(y\hat{x} + x\hat{y} + \operatorname{tg} z\,\hat{z}\right)$

(b) $\Phi = e^{-xy}\int_0^1\int_0^1 \sin z\, \hat{z}\cdot\hat{z}\, dx\, dy \Big|_{z=0} = 0$

(2) outra formulação

(b)



$$L = h \, tg\, \theta_0$$

$$dF_x = \frac{k x \lambda dx \, Q}{(x^2+h^2)^{3/2}}$$

$$F_x = k \lambda Q \int_0^L \frac{x\, dx}{(x^2+h^2)^{3/2}} \qquad u = x^2+h^2$$

$$du = 2x\, dx \qquad -\frac{3}{2}+\frac{2}{2} = -\frac{1}{2}$$

$$F_x = k \lambda Q \int_{h^2}^{L^2+h^2} \frac{1}{2} \frac{du}{u^{3/2}} = k \lambda Q \frac{2}{2} \left(\frac{1}{\sqrt{h^2}} - \frac{1}{\sqrt{L^2+h^2}} \right)$$

$$F_x = k \lambda Q \left(\frac{1}{h} - \frac{1}{h\sqrt{1+tg^2\theta_0}} \right) = \frac{k\lambda Q}{h} (1 - \cos\theta_0)$$

$$dF_y = \frac{k Q h \lambda\, dx}{(x^2+h^2)^{3/2}}$$

$$F_y = k h \lambda Q \int_0^L \frac{dx}{(x^2+h^2)^{3/2}} \qquad x = h\, tg\, \theta$$

$$dx = \frac{h}{\cos^2\theta} d\theta$$

$$x^2+h^2 = h^2(1+tg^2\theta) = \frac{h^2}{\cos^2\theta}$$

$$F_y = k h \lambda Q \int_0^{\theta_0} \frac{\cos^3\theta}{h^3} \frac{h}{\cos^2\theta} d\theta = \frac{k \cancel{h}}{h^{\cancel{2}}} \lambda Q \, sen\, \theta_0 = \frac{k\lambda Q \, sen\, \theta_0}{h}$$

P2 - Física III p/ IQ - 2014

Gabarito

① (a) $B_v = 2\times10^{-3}\,T$ $\quad L = 4\times10^{-3}\,m$

$$d\vec{B}_v = \frac{\mu_0 I}{4\pi}\frac{d\vec{l}\times\vec{r}}{r^3} \qquad |d\vec{l}\times\vec{r}| = dl\,r\cos\theta$$
$$= dl\,R_0$$

$$R_0 = L\cos 60^\circ = \frac{L\sqrt{3}}{2} = 2\sqrt{3}\times10^{-3}\,m$$

$$B_v = \frac{\mu_0 I}{4\pi}R_0\int_{-L/2}^{L/2}\frac{dl}{r^3} \qquad r = \frac{R_0}{\cos\theta}$$

$$l = R_0 \cancel{tg}\,\theta \qquad dl = \frac{R_0}{\cos^2\theta}d\theta$$

$$B_v = \frac{\mu_0 I}{4\pi}R_0\int_{-\theta_0}^{\theta_0}\frac{R_0}{\cos^2\theta}\frac{\cos^3\theta}{R_0^3}d\theta = \frac{\mu_0 I}{4\pi R_0}\int_{-\theta_0}^{\theta_0}\cos\theta\, d\theta$$

$$B_v = \frac{\mu_0 I}{4\pi R_0}2\,\text{sen}\,\theta_0 \qquad \text{sen}\,\theta_0 = \text{sen}\,30^\circ = \frac{1}{2}$$

$$B_v = \frac{\mu_0 I}{4\pi R_0}$$

$$I = \frac{4\pi R_0 B_v}{\mu_0 \cancel{3}} = \frac{2\sqrt{3}\times10^{-3} \times 2\times10^{-3}}{10^{-7}}$$

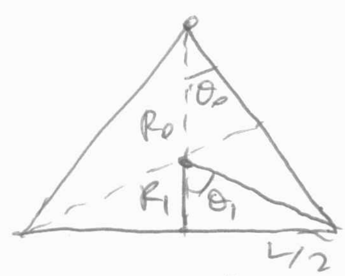
$$I = 40\sqrt{3}\ A = 69{,}3\ A$$

$$\vec{\mu} = I a \hat{n}$$

$$a = \frac{L R_0}{2} \qquad \mu = Ia = \frac{40\sqrt{3} \times 4 \times \cancel{2}\sqrt{3}\times10^{-6}}{\cancel{2}}$$

$$\mu = 160 \times 3 \times 10^{-6} = 480\times10^{-6} = 4{,}8\times10^{-4}\ Am^2$$

(b) $$\frac{B_o}{B_v} = 3\,\frac{\sin\theta_1}{\sin\theta_0}\,\frac{R_0}{R_1}$$



$\theta_1 = 60^\circ \quad tg60^\circ = \sqrt{3}$

$\theta_0 = 30^\circ$

$$R_1 = \frac{L}{2\,tg60^\circ} \cancel{= \frac{4\times10^{-3}}{6}}$$

$$\frac{R_0}{R_1} = \frac{L\sqrt{3}}{2} \Big/ \frac{L}{2\sqrt{3}} = 3$$

$$\frac{B_o}{B_v} = 3\left(\frac{\sqrt{3}}{\cancel{2}}\times\cancel{2}\right)(3) = 9\sqrt{3} = 15.6$$

② $J_a = 3{,}5 \times 10^6\ A/m^2$

$R_1 = \frac{2}{\sqrt{\pi}} \times 10^{-3} m$

$R_2 = \frac{3}{\sqrt{\pi}} \times 10^{-3} m$

$R_3 = \frac{4}{\sqrt{\pi}} \times 10^{-3} m$

(a) $I = J_a \pi R_1^2$

$I = 3{,}5 \times 10^{+6} \times \cancel{\pi} \times \frac{4}{\cancel{\pi}} \times 10^{-6} = 14\ A$

$J_c = \frac{I}{A_c} = \frac{I}{\pi(R_3^2 - R_2^2)} = \frac{14}{\cancel{\pi}\left(\frac{16}{\cancel{\pi}} - \frac{9}{\cancel{\pi}}\right) \times 10^{-6}}$

$J_c = \frac{14}{7} \times 10^6 = 2 \times 10^6\ A/m^2$

(b) $V_a = V_c = R_{TOT} I \Rightarrow E_a = E_c = E$

$J_a = \frac{1}{\rho_a} E \quad J_c = \frac{1}{\rho_c} E$

$\rho_c = \rho_a \frac{J_a}{J_c} = \frac{2 \times 10^{-7} \times 3{,}5}{2} = 3{,}5 \times 10^{-7}\ \Omega m$

(c) $\oint_c \vec{B} \cdot d\vec{l} = 2\pi R\, B(R) = \mu_0 I_{S(c)}$

a: $B(R) = \frac{\mu_0}{2\pi R} J_a \pi R^2 = \frac{\mu_0 J_a}{2} R$

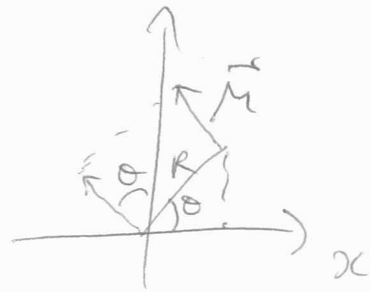
b: $B(R) = \frac{\mu_0 I}{2\pi R}$

c: $B(R) = \frac{\mu_0}{2\pi R}\left(I - J_c \pi (R^2 - R_2^2)\right)$

d: $B(R) = 0$

$M = \chi B/\mu_0 \quad a:$

(d) $M = \dfrac{\chi \mu_0 J_a R}{2\mu_0} = \dfrac{\chi J_a}{2} R$



$\vec{M} = -M \operatorname{sen}\theta\, \hat{x} + M \cos\theta\, \hat{y}$

$\operatorname{sen}\theta = \dfrac{y}{R}$

$\cos\theta = \dfrac{x}{R}$

$\vec{M} = -\dfrac{\chi J_a}{2} y\, \hat{x} + \dfrac{\chi J_a}{2} x\, \hat{y}$

$$\vec{\nabla}\times\vec{M} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & 0 \\ -\frac{\chi J_a y}{2} & \frac{\chi J_a x}{2} & 0 \end{vmatrix} = \hat{z}\left(\frac{\chi J_a}{2} + \frac{\chi J_a}{2}\right)$$

indep. z.

$\vec{J}_m = \vec{\nabla}\times\vec{M} = \chi J_a \hat{z} = cte$

$\dfrac{I_m}{I} = \dfrac{\chi J_a}{J_a} = \chi = 1{,}7\times 10^{-5}$

(área igual)

P3 - I Q 2014 - Física III

(a) $\mathcal{E} = -\dfrac{d\Phi}{dt} = -\pi R_0^2 \dfrac{\partial \vec{B}}{\partial t} = +\pi R_0^2 B_0 \omega \operatorname{sen}(\omega t)$

(b) $i = \dfrac{\mathcal{E}}{r_e} = \dfrac{\pi R_0^2 B_0 \omega \operatorname{sen}(\omega t)}{r_e}$

$P = \mathcal{E} i = \pi^2 \dfrac{R_0^4 B_0^2 \omega^2}{r_e} \operatorname{sen}^2(\omega t)$

$U_T = \int_0^T P(t)\,dt = \pi^2 \dfrac{R_0^4 B_0^2 \omega^2}{r_e} \int_0^T \operatorname{sen}^2(\omega t)\,dt$

$\theta = \omega t \qquad d\theta = \omega dt$

$\int_0^T \operatorname{sen}^2(\omega t)\,dt = \dfrac{1}{\omega}\int_0^{2\pi} \operatorname{sen}^2(\theta)\,d\theta = \dfrac{\pi}{\omega}$

$U_T = \pi^3 R_0^4 B_0^2 \dfrac{\omega}{r_e}$

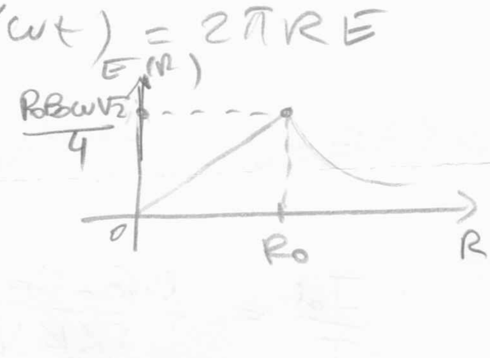
(c) $\mathcal{E} = \oint \vec{E} \cdot d\vec{l} = 2\pi R E = -\dfrac{d\Phi}{dt}$

(I) $R > R_0 \qquad \mathcal{E} = \pi R_0^2 B_0 \omega \operatorname{sen}(\omega t) = 2\pi R E$

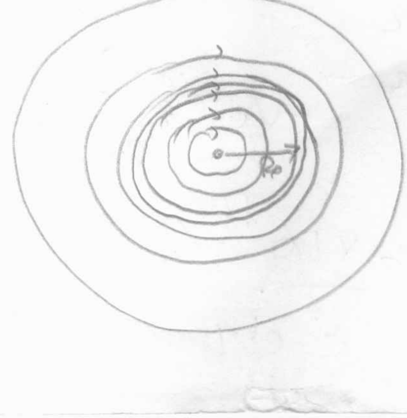
$E = \dfrac{\mathcal{E}}{2\pi R} = \dfrac{R_0^2}{2R} B_0 \omega \operatorname{sen}(\omega t)$

(II) $R < R_0 \qquad \mathcal{E} = \pi R^2 B_0 \omega \operatorname{sen}(\omega t) = 2\pi R E$

$E = \dfrac{R}{2} B_0 \omega \operatorname{sen}(\omega t)$



$t = \dfrac{\pi}{4\omega} \quad \operatorname{sen}(\omega t) = \operatorname{sen}\left(\dfrac{\pi}{4}\right) = \dfrac{\sqrt{2}}{2}$



(d) $u = \frac{1}{2}\varepsilon_0 E^2 + \frac{1}{2\mu_0} B^2$

$E = \frac{R B_0 \omega \sqrt{2}}{4}$ $\quad B = B_0 \frac{\sqrt{2}}{2}$ $\quad B^2 = \frac{B_0^2}{2}$

$$U = \int_0^{R_0} \frac{\cancel{2}\pi R L \varepsilon_0 E^2}{\cancel{2}} dR + \frac{\pi R_0^2 L}{2\mu_0}\frac{B_0^2}{2}$$

$$U_e = \frac{\pi L \varepsilon_0 B_0^2 \omega^2}{8}\int_0^{R_0} R^3 dR = \frac{\pi L \varepsilon_0 B_0^2 \omega^2 R_0^4}{32}$$

$$U = \frac{\varepsilon_0 \omega^2}{32}\pi L B_0^2 R_0^4 + \frac{1}{4\mu_0}\pi L B_0^2 R_0^2$$

$$U = \left(\frac{\varepsilon_0 \omega^2}{32} R_0^2 + \frac{1}{4\mu_0}\right)\pi L B_0^2 R_0^2$$

(e) $\vec{J}_d = \varepsilon_0 \frac{\partial \vec{E}}{\partial t}$ $\quad \frac{\partial E}{\partial t} = \frac{R}{2} B_0 \omega^2 \cancel{\cos}(\omega t)$

$$I_d = \int \vec{J}_d \cdot \hat{n}\, da = L\varepsilon_0 \int_0^{R_0} \frac{\partial E}{\partial t} dR = \frac{L\varepsilon_0 B_0 \omega^2}{2} \cancel{\cos}(\omega t) \int_0^{R_0} R\, dR$$

$$I_d = \frac{L\varepsilon_0 B_0 \omega^2}{\cancel{4}} \cancel{\cos}(\omega t) R_0^2 = \frac{L\varepsilon_0 B_0 \omega^2 \sqrt{2}}{8} R_0^2 \quad \left(t = \frac{\pi}{4\omega}\right)$$

$B = \mu_0 n I_e$ $\quad I = n I_e L = \frac{B_0 L}{\mu_0}\frac{\sqrt{2}}{2}$

$$\frac{I_d}{I} = \frac{\cancel{\mu_0}\varepsilon_0 \cancel{B_0} \omega^2 R_0^2}{\cancel{B_0} \cancel{L} \frac{\sqrt{2}}{2}}\frac{\sqrt{2}}{8} = \frac{\mu_0 \varepsilon_0 \omega^2 R_0^2}{4} = \frac{(2\pi)^2 R_0^2}{4 c^2 T^2}$$

$$\boxed{\text{\& } R > R_0 \quad E^2 = \frac{R_0^4}{8}\frac{B_0^2 \omega^2}{R^2} \quad U_e = \frac{\varepsilon_0}{2}\int_{R_0}^{\infty} \frac{L 2\pi R\, dR}{R^2}\frac{R_0^4 B_0^2 \omega^2}{8} \to \infty}$$

$$\omega^2 \ll \frac{4}{\mu_0 \varepsilon_0 R_0^2} = 4c^2 = 36 \times 10^{16} /s \qquad \omega \ll 36 \times 10^8 \text{ Hz}$$

$R_0 \ll cT$ $\qquad f \ll 6 \times 10^8$ Hz

P. Sub. Física III – IQ 2d4

1) $\rho(r) = \rho_0 \left(\frac{R_0}{r}\right)^2$

est. $\oint \vec{E}\cdot\hat{n}\,da = 4\pi r^2 E$

(a) $\oint_r \vec{E}\cdot\hat{n}\,da = \frac{q(r)}{\varepsilon_0}$ $q(r) = 4\pi \int \rho(r)\, r^2 dr$

I – $R < R_0$ $q(r) = 0$ $E(r) = 0$

II – $R_0 < R < R_1$ $q(r) = 4\pi \int_{R_0}^{R} \rho_0 \frac{R_0^2}{\cancel{r^2}} \cancel{r^2}\, dr = 4\pi \rho_0 R_0^2 (R - R_0)$

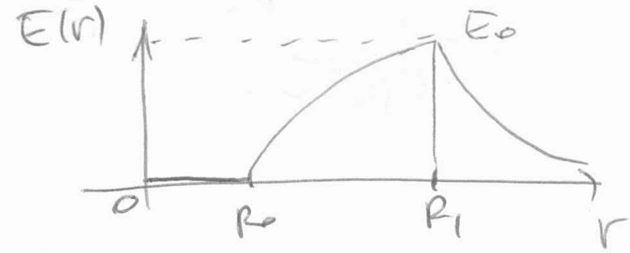
$E(R) = \frac{\rho_0 R_0^2 (R - R_0)}{\varepsilon_0 R^2}$

III – $R > R_0$ $q(r) = Q = 4\pi \rho_0 R_0^2 (R_1 - R_0)$

$E(R) = \rho_0 \frac{R_0^2 (R_1 - R_0)}{\varepsilon_0\, R^2}$

OBS.:

$\left(\text{II}\ \ \frac{dE}{dR} = \frac{\rho_0 R_0^2}{\varepsilon_0}\left(\frac{1}{R^2} - \frac{2R_0}{R^3}\right) \right)$



$E_0 = \frac{\rho_0 R_0^2}{\varepsilon_0}\left(\frac{R_1 - R_0}{R_1^2}\right)$

(b) $Q = 4\pi \rho_0 R_0^2 (R_1 - R_0)$

(c) $\varphi = \frac{Q}{4\pi\varepsilon_0 R_1} = \frac{\cancel{4\pi}\rho_0}{\cancel{4\pi}\varepsilon_0} R^2 \left(1 - \frac{R_0}{R_1}\right) = \frac{\rho_0}{\varepsilon_0} R_0^2 \left(1 - \frac{R_0}{R_1}\right)$

(d) $\varphi(R_0) - \varphi(R_1) = -\int_{R_1}^{R_0} \vec{E}\cdot d\vec{\ell} = \int_{R_0}^{R_1} E(R)\, dR$

$\Delta\varphi = \int_{R_0}^{R_1} \frac{\rho_0 R_0^2 (R - R_0)}{\varepsilon_0 R^2}\, dR = \frac{\rho_0 R_0^2}{\varepsilon_0}\left(\ln\frac{R_1}{R_0} + \frac{R_0}{R_1} - 1\right)$

2) (a) $C_p = \varepsilon_0 \frac{\pi R_0^2}{2R_0} = \frac{\varepsilon_0 \pi}{2} R_0$

$$r_L = \rho_{res} \frac{2R_0}{\pi\left(R_0^2 - (R_0-\delta)^2\right)} = \frac{2R_0\rho_{res}}{\pi\left(2R_0\delta - \delta^2\right)}$$

$$r_L \simeq \frac{2R_0\rho_{res}}{2\pi R_0 \delta} \approx \frac{\rho_{res}}{\pi \delta}$$

(b) $Q = C_p V \qquad V = r_L i \qquad i = -\frac{dQ}{dt}$

$$Q = -C_p r_L \frac{dQ}{dt} \qquad \frac{dQ}{dt} = -\frac{1}{C_p r_L} Q(t)$$

$$Q(t) = Q_0 e^{-\frac{t}{C_p r_L}}$$

(c) $E = \frac{\sigma}{\varepsilon_0} = \frac{Q(t)}{\pi R_0^2 \varepsilon_0} = \frac{Q_0 e^{-\frac{t}{C_p r_L}}}{\pi R_0^2 \varepsilon_0} \qquad \vec{E} = E\hat{z}$

(d) $\oint_C \vec{B}\cdot d\vec{\ell} = \mu_0 \int_{S(C)} (i + i_d)\, da \qquad da = 2\pi r\, dr$

$$j_d = \varepsilon_0 \frac{\partial E}{\partial t} = -\frac{Q_0}{C_p r_L} \frac{e^{-\frac{t}{C_p r_L}}}{\pi R_0^2 \varepsilon_0}$$

$$i_d = \int_0^r j_d\, da = j_d \pi r^2 = -\frac{Q_0}{C_p r_L} \frac{e^{-\frac{t}{C_p r_L}}}{\cancel{\pi} R^2 \cancel{\varepsilon_0}} \cancel{\pi} r^2$$

$$2\pi r B_1 = -\frac{\mu_0 Q_0}{C_p r_L} \frac{e^{-\frac{t}{C_p r_L}}}{R_0^2} r^2$$

$r < R_0 - \delta \qquad B = B_1 = -\frac{\mu_0 Q_0}{C_p r_L} \frac{e^{-\frac{t}{C_p r_L}}}{R_0^2} \frac{r}{2\pi}$

$r > R_0 \quad i = -\frac{dQ}{dt} = +\frac{Q_0}{C_p r_L} e^{-\frac{t}{C_p r_L}} \qquad i_d = -i \qquad i_{Tot} = 0$

$\Rightarrow B = 0$