Hyperbolic Method for Prediction of Prefabricated Vertical Drains Performance

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Abstract: For predicting the settlement rate of a ground area that incorporates prefabricated vertical drains, the hyperbolic method is herein developed based on Barron’s solution and validated by using three documented case studies. This method is suitable within the degrees of consolidation of 60–90%. Results indicate that the estimated coefficients of radial consolidation ($C_r$) are slightly larger. In two case studies, the estimated $C_r$ values are closer to the coefficients of vertical consolidation obtained from the standard oedometer tests but differ from those values derived from the cone penetration test (CPT) tests. In another case study, where not laboratory results are available, the estimated $C_r$ values fall within a lower bound determined by CPT results. The reason of these differences is due to various factors such as smear and well resistance, the vertical drain type, and its finite drainage capacity. It also appears that, in contrast to the solution obtained by Barron, the settlement versus time curves from the proposed method concur with the monitored curves. Finally, the applicability of the proposed method is discussed based on the case studies.

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Introduction

In general, even though consolidation parameters are obtained by using sophisticated laboratory and field tests, the settlement and consolidation time of full scale structures are difficult to accurately predict, particularly for thick clay deposits that incorporate vertical drains. For example, at the reclamation works in the Nakdong River estuary in Busan, Korea, the settlement and consolidation time were underestimated by 120–200% and 200–600%, respectively (Chung 1999). Owing to such inaccuracy, it is usual practice for the predictions that are made at the design stage to be corrected based on those obtained from observational procedures in the field.

Various methods that are extensively used in practice are known to provide reasonable estimations of final settlement, such as the hyperbolic method (Tan et al. 1991; Tan 1995), the method proposed by Asaoka 1978, as well as a number of other methods (Chung et al. 1998). However, for estimating the settlement rate of ground that incorporates vertical drains, because of its simplicity, only the theoretical method proposed by Magnan et al. (1983) is extensively used for practical applications. The disadvantage with the method that was developed based on the graphical method of Asaoka (1978) is that it is strongly affected by individual graphic work (Asaoka 1978; Edil et al. 1991). Nevertheless, a method to validate the estimated value by means of the back-analyzed settlement curve has not been proposed. In contrast to the simple Magnan et al. (1983) method, inverse analysis can be applied for estimating the coefficient of radial consolidation ($C_r$) value using simplified theoretical solutions (Lo 1991; Bergado et al. 1992; Stark et al. 1999). Due to many related factors and due to the complexity, the inverse analysis is however difficult to use for practical applications. It is thus necessary to develop an inverse analysis or more simple methods in order to obtain a reliable estimation of the $C_r$ value. It would also be an advantage if the back-analyzed settlement versus consolidation time curve, using the rate of settlement given, could be obtained and then compared with the monitored curve.

The purpose of this study is to develop a theoretical method to better estimate the rate of settlement of ground and to develop a back-analyzed settlement curve, following the selection of the prefabricated vertical drain technology that can be used as a soil improvement technique. In order to validate the proposed method, it has been applied to three case studies (Bo et al. 2003; Choa 1995; Koutsofas et al. 1987; Foott and Handfelt 1987; Lo 1991; and Pusan Newport Co. 2005), where $C_r$ values have been calculated and compared with those obtained from the theoretical method of Magnan et al. (1983) and with values obtained from laboratory and field testing. Settlement curves that are back-analyzed using the proposed method are compared with those obtained from monitoring and those obtained using the Barron solution in order to verify the reliable estimation of $C_r$. Furthermore, overall discussions are presented concerning the applicability of the proposed method.
Existing Methods for Predicting Radial Consolidation Using Field Measured Data

Although a number of consolidation theories for vertical drains have been proposed (Indraratna and Bamunawita 2002), there are only a few methods available for determining the \( C_r \) values based on monitored results. Magnan et al. (1983) proposed a method for estimating the \( C_r \) based on the solution proposed by Asaoka (1978), which was developed for the purpose of estimating the consolidation settlement in the absence of drains

\[
C_r = \frac{d_r^2 \cdot F(n)}{8 \Delta t} - \ln \beta_1 = \frac{d_r^2 \cdot F(n)}{8} \tag{1}
\]

where \( \Delta t \) = time interval; \( d_r \) = diameter of the influence zone of each drain; \( n \) = ratio of \( d_r \) to the drain diameter \( d_w \); \( F(n) \) = drain spacing factor as expressed in Eq. (4b); \( \beta_1 \) is the slope of a straight line on the curve that represents the settlements according to time; and \( c \) = coefficient denoted by \( -\ln \beta_1 / \Delta t \). As may be noticed, the \( C_r \) value varies depending on the selected values of \( \Delta t \) and \( \beta_1 \), which should be obtained from individual results. Unfortunately, a method has not been proposed for a time-settlement curve back-analyzed using the estimated \( C_r \) value for comparison with the monitored settlement, for the purpose of validating the value determined from Eq. (1).

Back (or inverse) analysis could be used to reliably estimate the \( C_r \) value. A number of researchers (Lo 1991; Bergado et al. 1992; and Stark et al. 1999) have back-analyzed the \( C_r \) value based on the graphical method of Asaoka (1978) and on simplified theoretical solutions that consider the effects of smear and well resistance (Hansbo 1979) and the vertical and radial consolidation (Carrillo 1942). Since the analysis is complex and is also affected by many factors as described by Eriksson et al. (2000), it has in addition the disadvantages that it is not simple to use and it is unfamiliar to practitioner engineers.

**Hyperbolic Method for Settlement and Consolidation Theories**

**Hyperbolic Method for Predicting Final Settlement**

Tan et al. (1991) proposed a hyperbolic relationship between monitored settlement \( (s) \) and consolidation time \( (t) \), which includes two linear segments

\[
s = \frac{t}{\alpha + \beta t} \quad \text{or} \quad s = \alpha + \beta t \tag{2}
\]

Hence the ultimate (or final) settlement \( s_f \) was defined as

\[
\lim_{t \to \infty} s = \lim_{t \to \infty} \frac{1}{\alpha / t + \beta} = s_f = \frac{1}{\beta} \tag{3}
\]

where \( \alpha \) and \( \beta \) are the intercept and the slope of the initial linear line, respectively, in the \( t/s \) versus \( t \) plot as shown in Figs. 6, 9, and 12.

**Consolidation Theories and Their Limitations**

It is well-known that in time-settlement relationships, the consolidation effect should account for the radial and vertical drainage. Nevertheless, the combined effects of consolidation are strongly dependent on the clay drainage depth and the drain spacing ratio \( (n) \) (Tan 1995). As clay layers are thicker, the drain spacing is less and consequently, the effect of the vertical drain becomes insignificant and it may be negligible for practical purposes.

In order to theoretically verify the vertical drain consolidation, the Barron (1948) solution for an ideal drain (without considering the effects of smear and well resistance) is considered as

\[
U = 1 - \exp \left( -\frac{8 T_r}{F(n)} \right) \tag{4a}
\]

\[
F(n) = \frac{n^2}{n^2 - 1} \ln(n) - \frac{3n^2 - 1}{4n^2} \tag{4b}
\]

\[
n = \frac{d_r}{d_w} \tag{4c}
\]

The Eq. (4a) can be represented by

\[
U = 1 - \exp \left( -\frac{8 T_r}{d_r^2 F(n)} \right) = 1 - \exp \left( -\frac{8 C_r t}{d_r^2 F(n)} \right) = 1 - \exp \left( -\frac{t}{\Lambda} \right) \tag{5a}
\]

\[
t = \Lambda \log \left( \frac{1}{1 - U} \right) \tag{5b}
\]

\[
\Lambda = \frac{d_r^2 F(n)}{8 C_r} \tag{5c}
\]

Fig. 1 shows two examples of monitored settlement versus time curves obtained from observations carried out at Changi Airport, Singapore (Bo et al. 2003). The figure includes a predicted curve using the Hyperbolic method, theoretical curves obtained with Eq. (5b) with varying \( \Lambda \) values from 300–1000, where the initial settlement \( s_0 \) is the settlement that occurred up until the soil placement was complete \( (t_0) \). It appears that while the hyperbolic curve agrees well with the monitored curves, the theoretical curves using the Barron solution are partly consistent with the monitored curve only within a specific time period (first 300 days). It is therefore inferred that the direct application of Eq. (5a) is inappropriate for practical situations.
Extension of Hyperbolic Method for Predicting Prefabricated Vertical Drain Performance

Derivation of Coefficient of Radial Consolidation

Sridharan et al. (1987) proposed the rectangular hyperbola method, based on the fact that in the equation proposed by Terzaghi the \( U-T_r \) relationship is a rectangular hyperbola over a fairly wide range of \( T_r \). Similarly, the extension of the proposed method is based on the same simple assumptions as those of the rectangular hyperbola method proposed by Sridharan et al. (1987) for prefabricated vertical drains.

Figs. 2(a–c) show plots of \( T_r/U \) versus \( T_r \) for the values of \( n=20, 30, \) and 50, respectively, using Barron’s solution for an ideal condition (Leonards 1962). All the curves also include a straight line for \( 60\% \leq U \leq 90\% \), with a correlation coefficient of 0.998. However, as can be seen in Fig. 2, while the corresponding least-squares lines have equal slopes \( (M=7.495 \times 10^{-3}) \), they also have unequal intercepts for each \( n \) value. Therefore, Barron’s solution can be represented by

\[
T_r/U = D + MT_r = D + (7.495 \times 10^{-3})T_r, \tag{6}
\]

where the intercept \( D \) varies depending on the ratio \( n \).

For the linear part of the curve, as shown in Fig. 2, Barron’s solution can also be represented by

\[
T_r/U = M_1T_r = AMT_r \tag{7}
\]

and hence

\[
A = \frac{M}{M_1} = \frac{1}{MU} \tag{8}
\]

Since \( M_1=1/U \), the ratios \( M_1/M \) and \( M_2/M \) are 222.370 and 148.247 for 60% and 90% of \( U \), respectively, which are independent of the ratio \( n \). In Fig. 2, the \( M_1 \) and \( M_2 \) values represent the slopes of lines that connect the origin to the points of \( U=60\% \) and 90%, respectively, on the curves of \( T_r/U \) versus \( T_r \), and the \( M_1 \) is between \( M_1 \) and \( M_2 \). The coefficient \( A \) can be used to locate any points within the range on the experimental curves.

In the same way, the linear portion of \( t/s \) versus \( t \) plot can be represented by the equation

\[
\frac{t}{s} = A \beta t \tag{9}
\]

where \( A = \text{constant depending on the degree of consolidation, as shown in Eq. (8)}. \) From Eqs. (2) and (9)

\[
t = \frac{\alpha}{\beta(A-1)} \tag{10}
\]

Hence the \( C_r \) value is obtained by

\[
C_r = \frac{T_r d_r^2}{t} = \frac{\beta(A-1)T_r d_r^2}{t} = B \cdot \frac{d_r^2}{\alpha} \tag{11a}
\]

\[
B = (A-1)T_r \tag{11b}
\]

Using Barron’s solution, the coefficient \( B \) can be calculated as shown in Table 1. It can be seen that the coefficient \( B \) varies with the ratio \( n \), irrespective of \( U \). The average values of \( B \) can be adopted for each \( n \) value, since the standard deviations are between 0.002 and 0.011 for the given range. Fig. 3 shows a relationship between the average values of \( B \) and \( n \), with a correlation coefficient of 0.9998, which is given by

\[
B = 0.1333 \log_e(n) - 0.0906 \tag{12}
\]

In Fig. 3, the range of \( n \) that is generally used in practice is marked by the dashed area, i.e., the spacing of 1.0–2.5 m with \( d_u=66.2 \) mm. Therefore, Eq. (11a) becomes

\[
C_r = [0.1333 \log_e(n) - 0.0906] \frac{d_r^2}{\alpha}\tag{13}
\]

It can be seen from Eq. (13) that the \( \alpha \) and \( \beta \) values considerably affect the \( C_r \) value. It is thus recommended that the values of \( \alpha \) and \( \beta \) should be determined to simulate the monitored settlement curve by using a trial-and-error method, rather than directly determining the linear portion in the \( t/s \) versus \( t \) plot (Chung et al. 1999). It also needs to be verified whether the initial linear portion of settlement curve is within the range of 60%–90%, based on the ultimate settlement obtained from Eq. (3) and on Eq. (14a).

Hyperbolic Settlement versus Time Curve

As shown in Fig. 1, the curves for monitored settlement (or \( U \)) versus time do not concur with Barron’s solution. If the \( C_r \) value and the condition of drains are provided, it is likely that the actual settlement curve can be produced using the above derivation. For this, Eqs. (2) can be represented by

\[
U = \frac{s}{s_f} = \frac{t}{\alpha s_f + \beta t} = \frac{t}{\kappa + t} \tag{14a}
\]
The appropriateness of the estimated curve with the monitored curves. It is therefore possible to validate the method of Asaoka (1991) as shown in Fig. 6. In the estimation, the initial time, the estimation indicates that the settlements predicted by the hyperbolic method are 1.10–1.12 times larger than those obtained by the Asaoka method. Fig. 5 shows that, in general, the \( C_r \) values obtained from the laboratory and dissipation tests (CPTu) are 4–6 times larger than the \( C_r \) values from the laboratory.

Two sets of monitored data obtained from trial embankments were chosen, for which the Colbond drains (100 × 6 mm) and the Mebra drains (93 × 4 mm) were installed by using a square drain spacing of 1.5 m. The average installed depth was approximately 43 m. The monitoring was performed using surface settlement monitoring that was carried out at the east reclamation project of Changi Airport, Singapore. For this study, Phase 1C construction of the embankment was completed. In the Asaoka method, the time interval that strongly affects the prediction of final settlement was chosen within the range recommended by Edil et al. (1991). As a general trend, the estimation indicates that the settlements predicted by the hyperbolic method are 1.10–1.12 times larger than those obtained by the Asaoka method. Fig. 5 also shows that the predicted settlement curves obtained from the hyperbolic method (or Eq. (16)) concur with the measured curves. The predicted settlement curves from Barron’s solution were obtained using the \( C_r \) values which were acquired from the method proposed by Magnan et al. (1983) [Eq. (11)] and from another value chosen to be close to the measured curves. Nevertheless,

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**Table 1. Coefficients \( A \) and \( B \) Using Barron’s Solution (Data from Leonards 1962)**

<table>
<thead>
<tr>
<th>( U ) (%)</th>
<th>( A )</th>
<th>( n=10 )</th>
<th>( n=20 )</th>
<th>( n=30 )</th>
<th>( n=40 )</th>
<th>( n=50 )</th>
<th>( n=80 )</th>
<th>( n=100 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>60</td>
<td>2.223</td>
<td>0.220</td>
<td>0.316</td>
<td>0.372</td>
<td>0.412</td>
<td>0.443</td>
<td>0.509</td>
<td>0.540</td>
</tr>
<tr>
<td>65</td>
<td>2.053</td>
<td>0.218</td>
<td>0.312</td>
<td>0.366</td>
<td>0.406</td>
<td>0.437</td>
<td>0.502</td>
<td>0.533</td>
</tr>
<tr>
<td>70</td>
<td>1.906</td>
<td>0.209</td>
<td>0.299</td>
<td>0.352</td>
<td>0.391</td>
<td>0.419</td>
<td>0.482</td>
<td>0.511</td>
</tr>
<tr>
<td>75</td>
<td>1.779</td>
<td>0.213</td>
<td>0.305</td>
<td>0.358</td>
<td>0.397</td>
<td>0.427</td>
<td>0.490</td>
<td>0.520</td>
</tr>
<tr>
<td>80</td>
<td>1.668</td>
<td>0.212</td>
<td>0.302</td>
<td>0.357</td>
<td>0.395</td>
<td>0.425</td>
<td>0.487</td>
<td>0.518</td>
</tr>
<tr>
<td>85</td>
<td>1.570</td>
<td>0.212</td>
<td>0.304</td>
<td>0.358</td>
<td>0.397</td>
<td>0.427</td>
<td>0.490</td>
<td>0.521</td>
</tr>
<tr>
<td>90</td>
<td>1.485</td>
<td>0.219</td>
<td>0.313</td>
<td>0.369</td>
<td>0.409</td>
<td>0.440</td>
<td>0.504</td>
<td>0.536</td>
</tr>
</tbody>
</table>

Note: AVE. = average value; STDEV = standard deviation.

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Application to Case Records

In order to validate the proposed method, three well-documented case studies were used for which the soil type and prefabricated vertical drains adopted for each case are summarized in Table 2.

**Changi Airport Site, Singapore**

Choa (1995) and Bo et al. (2003) reported a well-documented field monitoring that was carried out at the east reclamation project of Changi Airport, Singapore. For this study, Phase 1C was considered at the site and the soil profile and \( C_r \) values are shown in Fig. 4. It appears that, in general, the \( C_r \) values obtained from the laboratory and dissipation tests (CPTu) are 4–6 times larger than the \( C_r \) values from the laboratory.

Two sets of monitored data obtained from trial embankments were chosen, for which the Colbond drains (100 × 6 mm) and the Mebra drains (93 × 4 mm) were installed by using a square drain spacing of 1.5 m. The average installed depth was approximately 43 m. The monitoring was performed using surface settlement gauges. Fig. 5 shows the time line of construction and measured settlement for both cases.

The final consolidation settlements for the first stage of loading were estimated by using the hyperbolic method [Eq. (3)] and the method of Asaoka (1978), as shown in Fig. 6. In the estimation, the initial time, \( t_0, \) and settlement, \( s_0, \) were chosen once the construction of the embankment was completed. In the Asaoka method, the time interval that strongly affects the prediction of final settlement was chosen within the range recommended by Edil et al. (1991). As a general trend, the estimation indicates that the settlements predicted by the hyperbolic method are 1.10–1.12 times larger than those obtained by the Asaoka method. Fig. 5 also shows that the predicted settlement curves obtained from the hyperbolic method [or Eq. (16)] concur with the measured curves. The predicted settlement curves from Barron’s solution were obtained using the \( C_r \) values which were acquired from the method proposed by Magnan et al. (1983) [Eq. (11)] and from another value chosen to be close to the measured curves. Nevertheless,
the curves obtained from the Barron solution also tend to differ from the measured curves. It can therefore be inferred that the Barron solution with only an ideal drain is insufficient to represent the actual settlement behavior.

Table 3 shows \( C_r \) values estimated from both the proposed method [Eq. (13)] and the method proposed by Magnan et al. (1983) [Eq. (1)]. It appears that the estimated values obtained from the proposed method are 1.87–2.08 times larger than those obtained from the Magnan et al. (1983) method. The estimated values obtained from the proposed method are significantly close to the coefficients of vertical consolidation obtained from the laboratory test (Fig. 4).

**Chek Lap Kok Airport, Hong Kong**

Fig. 7 shows a typical soil profile and the \( C_r \) values obtained for the construction site at Chek Lap Kok Airport, Hong Kong. These were estimated from the CPT dissipation tests and using the method proposed by Baligh and Levadoux (1980) (Koutsoftas et
al. 1987). It appears that the uppermost marine clay is softer and has lower \( C_v \) values compared to those of the lower layers.

A trial embankment of 100 m × 100 m in plane was established in order to provide various types of drains and their spacings in four different zones. Herein, the settlements that were monitored at the Haul Road site for the stabilization of embankment are chosen (Lo 1991). The 100 mm × 7 mm prefabricated vertical drains, named Alidrain, were installed up to a depth of approximately 7.0 m with the triangular drain spacings of 1.0 m (SP-17) and 2.0 m (SP-27). Fig. 8 shows the settlement monitored using the surface settlement gauges at the points.

Fig. 9 and Table 4 show the final consolidation settlements estimated by using the hyperbolic method and the Asaoka method. It can be seen here that, as expected, the values obtained from the hyperbolic method are 1.09–1.13 times those obtained from the Asaoka method. In Fig. 8, it can again be seen that the predicted settlement curves obtained from the hyperbolic method concur well with the measured curves, and relatively well with those obtained from Barron’s solution. Based on the assumption that the consolidation occurs at the upper marine clay, the \( C_v \) values estimated from the proposed method are more than 1.18–1.64 times those obtained from the method proposed by Magnan et al. (1983) as shown in Table 4. It can also be seen that the values from both methods are approximately identified with the lower bound of the CPT test results.

### Busan New Port, Busan Korea

During late 1990’s, the development of a new port was initiated at the west part of the Nakdong River mouth, in Busan Korea, in order to cope with the growth of the number of cargo containers. For the north container terminal, sand fill of about 8 m high was initially placed (Chung et al. 2007). The typical soil profile and geotechnical properties for this site are shown in Fig. 10. Generally, the deposit consists of an upper layer of soft-to-firm silty clay of approximately 30–35 m and a lower layer of stiff-to-hard silty clay, which at times reaches a depth of up to 70 m. The \( C_v \) values were determined from the standard oedometer test and the \( C_v \) values were determined from the CPT dissipation tests, which were obtained with the aid of the method proposed by Randolph and Wroth (1979). The 100 mm × 4 mm prefabricated vertical drains, known as the kolon drain board PVD (KDBP) drains, with various rectangular drain spacings, were installed up to a depth at which the SPT \( N \) value is larger than 8, which was approximately a maximum of 50 m deep. For this installation, a 150 mm wide diamond-type mandrel, and a 100 mm × 170 mm steel anchor plate were used. Fig. 11 shows the time line of construction and the settlement monitored using the screw-type surface settlement gauges and extensometers at SP-21 (\( s=1.0 \) m) and II-3 (\( s=1.5 \) m).

Comparisons were made between each of the larger final consolidation settlements that were estimated by the observational methods, as shown in Fig. 12 and Table 5. It can be seen that the values from the hyperbolic method are 1.04–1.08 times those obtained by Asaoka’s method at the points. Due to the thick clay deposit, it is likely that the estimated values from this site are larger than those obtained from the other sites. It can also be seen that the predicted settlement curves obtained from the hyperbolic method concur well with the measured curves, and relatively well with those obtained from Barron’s solution.
method concur with the measured curves, as shown in Fig. 11. However, while the curves obtained from Barron’s solution roughly match the monitored curve at II-3, they do not match that obtained at SP-21. Such trend shown at II-3 appears as the spacing of PVD is wider and the duration of fill placement is longer.

The $C_v$ values for the Busan New Port case study are summarized in Table 5. The values obtained from the hyperbolic method are 1.48–2.88 times larger than those obtained from the method of Asaoka et al. (1983), in which the former values are within the range of the $C_v$ values obtained from the laboratory. As seen from the CPT-based $C_v$ values, the estimated $C_v$ values at this site also tend to be slightly larger than those obtained from the above sites. The reason that the estimated $C_v$ value at SP-21 ($s=1.0$ m) is larger than that at II-3 ($s=1.5$ m) can be explained by the difference in soil properties, rather than the effects of drain spacing.

**Discussion**

The case studies indicated that, in general, the predicted ultimate (or final) settlements obtained from the hyperbolic method are slightly larger than those obtained from the method of Asaoka (1978), irrespective of the time interval adjustment made in the latter method. Based on the final settlement, while the curves for settlement versus time predicted from the hyperbolic method (or the proposed method) concur well with the measured curves, the curves obtained from Barron’s solution, using the $C_v$ values obtained from the method of Magnan et al. (1983), differ from those obtained from the hyperbolic method. Specifically, the curves obtained from Barron’s solution are partly consistent with the measured curves. Unfortunately, the back-analyzed settlement versus time curve is unable to be directly obtained using the results obtained from the Asaoka method. It can therefore be assumed that the hyperbolic method (or the proposed method) is more proficient for simulating actual settlement curves than the Barron solution.

In all the above cases, the $C_v$ values calculated from the proposed method were 1.18–2.88 times those obtained from the Magnan et al. (1983) method (based on Asaoka’s method), while the settlements predicted from the hyperbolic method were 1.04–1.13 times those obtained from the Asaoka method. It is interesting to note that the trend of $C_v$ is not consistent with the trend observed in the comparative results of final settlement. Indeed, in the Asaoka method, it is difficult to adequately determine a time interval and a straight line in the $s_t$ versus $s_{t-1}$ plot, which should depend on individual results. If this were the case, then the $C_v$ value from the Magnan et al. (1983) method would also be affected by the results from the Asaoka graphic method, because the former was developed based on the latter. Moreover, in common practice, it is preferable to make a comparison between the back-analyzed values that are obtained from various methods, and for one of these methods to then be appropriately determined for a site. For this situation, the advent of a new method is quite useful.

**Table 4. Coefficients of Radial Consolidation Estimated for Chek Lap Kok Airport, Hong Kong**

<table>
<thead>
<tr>
<th>Point number</th>
<th>$c$ ($10^{-3}$) (day$^{-1}$)</th>
<th>$s_f$ (m)</th>
<th>$C_v$ ($10^{-3}$) (m$^2$/day)</th>
</tr>
</thead>
<tbody>
<tr>
<td>SP-17</td>
<td>6.99</td>
<td>1.916</td>
<td>1.90</td>
</tr>
<tr>
<td>SP-27</td>
<td>3.41</td>
<td>1.853</td>
<td>5.05</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>$\alpha$</td>
<td>$\beta$</td>
<td>$s_f$ (m)</td>
</tr>
<tr>
<td></td>
<td>83.30</td>
<td>0.869</td>
<td>2.089</td>
</tr>
<tr>
<td></td>
<td>246.46</td>
<td>0.908</td>
<td>2.103</td>
</tr>
</tbody>
</table>
because the Magnan et al. (1983) method has been exclusively used in practice thus far.

Generally, it is known that the rates of settlement of full scale structures are several times greater than those predicted using laboratory testing results due to fabric effects such as laminations, layers of silt and fine sand, silt-filled fissures, organic inclusions and root-holes in the clay deposits (e.g., Craig 1997). However, this is not the case for all of the results obtained from the case studies. Specifically, it appears that, while most of the back-analyzed $C_r$ values are slightly less than the in situ testing values, they are closer to the coefficients of vertical consolidation obtained from the standard oedometer tests. The underestimation is probably due to the fact that, as described previously, the analytical solutions proposed herein were developed under an ideal condition, i.e., without taking the smear effect into consideration and well resistance as well as the vertical drain.

Meanwhile, the grounds are partially disturbed during the installation of prefabricated vertical drains and the fill placement. Therefore, the occurrence of consolidation always includes the effects of smear and well resistance (Olson 1998). For example, according to the monitoring of the site at Salt Lake City, Utah (Saye 2001), the $r_{50}$ values are approximately identical within the range of triangular drain spacings of less than 1.5 m. It was reported that this was due to the disturbance effects. It is likely that the smear zone is affected by the types and sizes of mandrel and anchor plate or rod that is used, the soil type and stiffness, etc. Therefore, it is not intended for the back-analyzed coefficients of consolidation to represent the original ground, but to represent the approximate values including the construction-induced effects and other factors such as the vertical drain type and its finite drainage capacity. That is, the values reflect the actual ground condition, i.e., the partially disturbed ground, rather than the conditions from laboratory and field soil testing. Nevertheless, it is likely that the estimated $C_r$ value is used for better predicting of the actual time-settlement curve by using the proposed method [Eq. (16)]. It is therefore expected that this method will be useful in applications in other fields under different site conditions (i.e., soil properties, different length, and/or spacing etc.).

**Conclusions**

A method was proposed in this study for estimating the coefficient of radial consolidation, as well as the consolidation times that are only suitable to a 60–90% degree of consolidation, based

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Table 5. Coefficients of Radial Consolidation Estimated for Busan New Port Site

<table>
<thead>
<tr>
<th>Point number</th>
<th>Magnan et al. (1983)</th>
<th>Hyperbolic method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$c$ ($10^{-3}$)</td>
<td>$s_f$</td>
</tr>
<tr>
<td>SP-21 (s=1.0 m)</td>
<td>13.71</td>
<td>2.204</td>
</tr>
<tr>
<td>II-3 (s=1.5 m)</td>
<td>4.78</td>
<td>2.146</td>
</tr>
</tbody>
</table>

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Fig. 11. Time line of construction and monitored settlement at SP-21 and II-3 (data obtained from Pusan Newport Co. 2005)

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Fig. 12. $t/s$ versus $t$ and $s_f$ versus $s_{r-1}$ plots for Busan New Port site
on the hyperbolic relationship of settlement versus time and on the Barron’s solution. The method was also applied to well-documented case studies and the results were then compared with those obtained from other methods. The conclusions drawn from this study are as follows.

In all cases, the estimated coefficients of radial consolidation, \( C_r \), obtained from the proposed method, are in the range of 1.18–2.88 times those obtained from the Magnan et al. (1983) method, which is based on the Asaoka graphical method. The former results for two of the cases are approximately closer to the \( C_r \) values obtained from the standard oedometer tests than they are to the \( C_r \) values obtained from in-situ tests. However, in another case where only the field test results are available, the estimated \( C_r \) values fall within the lower bound of the tested values. Such results are possibly due to various factors such as smear and well resistance, the vertical drain type and its finite drainage capacity. The estimated \( C_r \) value is therefore considered to be an approximate value that reflects the actual ground condition followed by the PVD installation. The final settlements estimated from the hyperbolic method are 1.04–1.13 times those obtained from the graphical method of Asaoka (1978), which is not consistent with the trend observed in the comparative results of \( C_r \) values.

The results also indicate that the monitored settlement versus time plots for the various cases are governed by the hyperbolic relationship, as well as by the curves obtained from the proposed method. However, most of the curves obtained from the Barron solution show a different trend from the measured curves.

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**Notation**

The following symbols are used in this paper:

\[
\begin{align*}
A &= \text{coefficient depending on degree of consolidation}; \\
B &= \text{coefficient} = (A-1)T_r; \\
c &= \text{coefficient used for Magnan et al. (1983) method} = -\ln \beta_1/\Delta t; \\
C_r &= \text{coefficient of radial consolidation}; \\
C_v &= \text{coefficient of vertical consolidation}; \\
D &= \text{intercept of a straight line in } T_r/U \text{ versus } T_r \text{ plot}; \\
d_e &= \text{diameter of influence zone of each drain}; \\
d_w &= \text{equivalent diameter of drain}; \\
F(n) &= \text{factor to account for drain spacing}; \\
M &= \text{slope of a straight line in } T_r/U \text{ versus } T_r \text{ plot}; \\
M_1 &= \text{slope of a straight line passing the origin for } 60\% \leq U \leq 90\%; \\
M_2 &= \text{slope of a straight line connecting the origin to the points at } U=60\% \text{ and } 90\%; \\
M_s &= \text{drain spacing ratio} = d_e/d_w; \\
s &= \text{drain spacing ratio}; \\
s &= \text{settlement}; \\
s_i, s_{i-1} &= \text{settlements at time } t_i \text{ and } t_{i-1}; \\
s_0, s_f &= \text{initial settlement at } t_0 \text{ and final settlement}; \\
t &= \text{time}; \\
t_0 &= \text{initial time used for analysis};
\end{align*}
\]

\( t_{05} \) = time at \( U=95\% \); \\
\( T_r \) = time factor for radial consolidation; \\
\( T_v \) = time factor for vertical consolidation; \\
\( U \) = average degree of consolidation; \\
\( \alpha \) = intercept of a straight line in \( t/s \) versus \( t \) plot; \\
\( \beta \) = slope of a straight line in \( t/s \) versus \( r \) plot; \\
\( \beta_1 \) = slope of a straight line in \( s_i \) versus \( s_{i-1} \) plot; \\
\( \Delta t \) = time interval; \\
\( \kappa \) = coefficient=\( \alpha \cdot \beta \); and \\
\( \Lambda \) = coefficient=\( d_e^2 F(n) \)/\( 8C_r \).

**References**


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