Consolidation of a Double-Layered Compressible Foundation Partially Penetrated by Deep Mixed Columns

Linchang Miao¹; Xinhui Wang²; and Edward Kavazanjian Jr.³

Abstract: Deep mixed columns often penetrate partially into the soft soil as floating columns due to the depth of the end-bearing layer. Partially penetrated soft soil by columns and the underlying compressible soft soil create a double-layered compressible foundation. So far, no reasonable solution is available to estimate the consolidation of such a double-layered foundation. This paper proposes an analytical solution for consolidation of a double-layered compressible foundation partially penetrated by deep mixed columns considering one-side or two-side vertical drainage. The Laplace transform method was used to solve the consolidation equation for the double-layered system while Stehfest’s algorithm was used to solve the inverse Laplace transform for time-dependent loading. A consolidation algorithm was used to calculate the time-settlement relationship of an embankment constructed upon the double-layered foundation partially penetrated by deep mixed columns. The calculated settlements were compared well with field measurements.

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Introduction

In the past decades, a number of analytical solutions have been developed for calculating the consolidation of compressible foundations under loading (Barron 1948; Hansbo 1981; Zeng and Xie 1989; Lee and Xie 1996; Tang and Onitsuka 2001; Han and Ye 2001, 2002). These solutions have provided the methods for the design of embankments over soft soils with vertical drains or stone columns. The construction of an embankment and traffic loading are time dependent and would increase total vertical stresses in the soft soils. In practice, however, the rate of consolidation is often calculated based on one-dimensional consolidation theory assuming instantaneous loading. This calculation is then corrected for two- or three-dimensional effects using, for example, the Skempton-Bjerrum method (Skempton and Bjerrum 1957). Terzaghi (1943) proposed a simple method for correcting the time-settlement curve considering the construction period. Olson (1977) obtained a mathematical solution for one-dimensional consolidation of a single compressible layer under simple ramp loading. Furthermore, Schiffman (1958, 2001) developed a consolidation solution of a clay soil layer under time-dependent loading.

However, the soils in the field are often layered and it is unrealistic to simplify the compressible strata into one single soil layer. Schiffman and Stein (1970) developed a solution to handle the layered consolidation problem under depth-independent loading. Xie et al. (1994), Tang and Onitsuka (2001), and Wang and Jiao (2004) presented the consolidation solutions for double-layered soils with vertical drains to accelerate the consolidation. These solutions all assumed that the total load remains constant during the consolidation. Zhu and Yin (1999) presented a solution for consolidation of a double-layered soil system under a ramp load.

Moreover, these solutions are not suitable for calculating the consolidation of a double-layered compressible foundation partially penetrated by deep mixed columns. Detailed discussions on deep mixing technology can be found in many references; for example, Terashi and Tanaka (1983) and Han et al. (2002). The review of this technology is beyond the scope of this study. In this study, the Laplace transform method was used to develop the consolidation equation for a double-layered foundation considering vertical drainage. Stehfest’s algorithm was used to solve the inverse Laplace transform for time-dependent loading. This consolidation algorithm was applied to calculate the settlement of an embankment constructed over a double-layered foundation of Jiangsu marine clay.

Consolidation Theory of Double-Layered Foundations

Basic Assumptions

Consider a double-layered compressible foundation system, in which the upper layer has a thickness \( h_1 \), a constrained modulus \( E_{11} \), and a saturated hydraulic conductivity in the vertical direction \( k_{1v} \); and the lower layer has a thickness \( h_2 \), a constrained modulus \( E_{22} \), and a saturated hydraulic conductivity in the vertical direction \( k_{2v} \). In this study, the upper layer is treated by deep mixed columns with an area replacement ratio \( m \), which is defined as the ratio of the column area to the total treated area.
Analytical Solution

In order to simplify the formulation, the following three dimensionless parameters are defined below:

\[ a = \frac{k_{z2}}{k_{z1}}, \quad b = \frac{m_{z2}}{m_{z1}} = \frac{E_1}{E_2} \text{ (no lateral deformation)} \quad \text{and} \quad c = \frac{h_2}{h_1} \]

where \( m_{z1} \) and \( m_{z2} \) are coefficients of volumetric compressibility of the upper and lower soil layers (equal to the reciprocal of the constrained modulus); \( E_1 = mE_{col} + (1 - m)E_{c1} \), \( E_{col} \) is constrained modulus of deep mixed columns, and \( E_{c1} \) is constrained modulus of the upper soil layer; \( E_2 = E_{c2} \).

Based on Terzaghi's 1D consolidation solution (Terzaghi 1943), the excess pore water pressure for a free draining top surface problem can be obtained as follows:

\[ u_i = \sum_{m=1}^{\infty} \frac{g_m(z)e^{-\theta_m t}}{B_m + C_mT_m(t)} \quad (i = 1, 2) \]  

where \( g_m(z) = \sin(\lambda_m z)/(\lambda_m^2) \), \( g_{m2}(z) = A_m \cos(\mu_1 z)/(\lambda_m^2) \); \( \beta_m = \mu_1 \lambda_m A_m \), \( B_m \), and \( C_m \) are constants; and \( T_m(t) \) = time coefficient due to \( R(t) = [\mu_1 z]/[\lambda_m^2] \). Substituting Eq. (3) into the boundary condition (3), the parameter \( A_m \) can be obtained as follows:

\[ A_m = \sum_{m=1}^{\infty} \frac{\sin(\lambda_m)}{\cos(\mu_1 \lambda_m)} \]

Substituting Eq. (5) into Eq. (1) yields

\[ \beta_m = \frac{c_{zz} \lambda_m^2}{h_1} \]

Using Eq. (5) and the orthogonal relationship formula as follows:

\[ \sum_{m=1}^{\infty} \frac{C_m g_m(z)}{B_m + C_mT_m(t)} = 1 \quad \text{and} \quad \sum_{m=1}^{\infty} \frac{C_m g_{m2}(z)}{B_m + C_mT_m(t)} = 1 \]

According to the initial condition, the following expression can be obtained:

\[ \sum_{m=1}^{\infty} B_m g_m(z) = p(z) \quad (i = 1, 2) \]

Thus, the coefficient \( B_m \) can be derived as follows:

\[ B_m = 2 \left( \int_0^{\lambda_m z} e^{-\theta_m t} \left[ \frac{1}{B_m + C_m \int_0^t R(\tau) e^{\theta_m \tau} d\tau} \right] \right) \]  

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The compression of each soil layer can be calculated as follows:

\[ c_i = a_i c_{vi} \frac{d^2 \sigma_i'(z,t)}{dz^2} \quad (i = 1, 2) \]  

(15)

The initial condition and boundary condition are as follows:

\[ \sigma_i'(0, t) = 0, \quad |z| = H = h_1 + h_2 \]  

(16a)

\[ \sigma_i'(0, t) = q(t), \quad t > 0 \]  

(16b)

where \( \sigma_i'(z, t) \) = effective stress of the soil layer \( i \); and \( c_{vi} \) = coefficient of consolidation of the soil layer \( i \). Laplace’s transform of Eq. (15) yields the following equation:

\[ \tilde{\sigma}_i'(z, s) = A_i e^{r_1 z} + A_2 e^{-r_2 z} \]  

(17)

where \( s \) = parameter of Laplace’s transform, \( r_1 = s/c_{vi} \). The inverse Laplace’s transform of Eq. (17) can be expressed as follows:

\[ \sigma_i'(z, t) = \frac{1}{2\pi i} \int_{-\infty}^{\infty} \tilde{\sigma}_i'(z, s) e^{st} ds \]  

(18)

The compression of each soil layer can be calculated as follows:

### Laplace’s Transform

To obtain a solution for a consolidation problem with a time-dependent load, Laplace’s transform was first used and then Stehfest’s method was used as the inverse transform for the numerical calculation. Eq. (1) can be rewritten in the effective stress form as follows:

\[ \frac{\partial \sigma_i'(z,t)}{\partial t} = c_{vi} \frac{\partial^2 \sigma_i'(z,t)}{\partial z^2} \quad (i = 1, 2) \]  

(15)

The initial condition and boundary condition are as follows:

\[ \sigma_i'(0, t) = 0, \quad |z| = H = h_1 + h_2 \]  

(16a)

\[ \sigma_i'(0, t) = q(t), \quad t > 0 \]  

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### Stehfest’s Inversion Formula

The inverse Laplace transform can be easily used for simple problems. For complicated problems, however, a numerical method must be adopted to solve the inverse Laplace’s transform. The Stehfest algorithm is a numerical solution that has good stability for solving the inverse Laplace transform (Stehfest 1970). Stehfest’s numerical inversion formula of the Laplace transform for a specific time \( t = T \) is as follows:

\[ f(T) = \frac{\ln 2}{T} \sum_{j=1}^{n} V_j \tilde{f}_j \left( \frac{\ln 2}{T} \right) \]  

(21)

where \( \tilde{f}(s) \) = image function of \( f(t) \); \( \tilde{f}(s) = L[f(t)] = \int_0^T f(t) e^{-st}dt \)

\[ V_j = (-1)^{nj + 1} \sum_{k=0}^{n} \frac{k^{(n/2)}(2k)!}{(N/2 - k)! k! (k - 1)! (i - k)! (2k - i)!} \]

and \( N = \) positive even number.

### Parametric Study

The solution to the consolidation equation for a double-layered foundation (shown in Fig. 1) subjected to a time-dependent load was implemented in a computer program. To demonstrate the influence of various parameters on the consolidation behavior of...
the double-layered foundation, this program was used to analyze a double-layered foundation subjected to two-stage loading. In this case, the upper layer is treated by deep mixed columns with an area replacement ratio of 0.15. The two-stage loading assumed in the analysis is shown in Fig. 2. The two ramp loads are assumed to have the same magnitude (i.e., $q_1 = q_2$). The parameters of the soil layers are as follows: $c_{v1} = 0.76 \times 10^{-3}$ cm$^2$/s, $k_{v1} = 3.65 \times 10^{-7}$ cm/s, $k_{v2} = 4.3 \times 10^{-7}$ cm/s, $E_{s1} = 1.9$ MPa, $E_{col} = 120$ MPa, $m = 0.15$, $E_{s2} = 2.0$ MPa, $h_1 = 12$ m, $h_2 = 28$ m, $q_1 = 50$ kPa, and $q_2 = 30$ kPa. Fig. 3 shows the effect of the coefficient of consolidation of the second soil layer on the degree of consolidation of the double-layered foundation for the case of $t_1 = t_3 - t_2 = 90$ days, which shows that $T_v$ increases as $c_{v2}$ increases. Fig. 4 presents the effect of $t_1$, the duration over which the first load is applied, on the degree of consolidation for the double-layered foundation with $c_{v2} = 0.076 \times 10^{-3}$ cm$^2$/s. The results show that $T_v$ increases as $t_1$ increases. Fig. 5 shows how the constrained modulus of the double-layered foundation influences the consolidation. The harder the soil layers, the faster the rate of consolidation for the double-layered foundation.

**Case Study**

**Site Conditions**

The highway from Lianyungang to Xuzhou (named Lian-Xu Expressway) is one of the main trunk expressways in China. The Lianyungang section of the highway is located at a short distance away from the coast of the Yellow Sea of China. Along this section, the crown width of the expressway is 28 m and contains an embankment varying in height from 4 to 7 m. The thickness of the Jiangsu soft marine clay beneath the highway embankment is approximately 12 to 16 m. The clay has high compressibility, low shear strength, and low permeability. The representative engineering parameters of the Jiangsu soft marine clay are summarized in Table 1. To simplify the soil profile of the Jiangsu soft marine clay, its properties are averaged.

**Table 1. Physical and Mechanical Parameters of Jiangsu Soft Marine Clay**

<table>
<thead>
<tr>
<th>Specific gravity ($G_s$)</th>
<th>Saturated unit weight (kN/m$^3$)</th>
<th>Natural water content (%)</th>
<th>Initial void ratio (%)</th>
<th>Liquid limit (%)</th>
<th>Plastic limit (%)</th>
<th>Plastic index (%)</th>
<th>Coefficient of consolidation ($c_{v}$) $\times 10^{-3}$ cm$^2$/s</th>
<th>Sensitivity ($S_t$)</th>
<th>$c_s$ (kPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.75</td>
<td>15.2–16.9</td>
<td>52.1–92.0</td>
<td>1.33–2.56</td>
<td>53.0–85.0</td>
<td>18.0–37.2</td>
<td>17.0–38.0</td>
<td>0.17–2.41</td>
<td>2.0–12.0</td>
<td>5.0–20.0</td>
</tr>
</tbody>
</table>
Settlement Calculation

The settlement calculation was conducted for an embankment constructed over the double-layer system as discussed above using the solution proposed in this paper. Fig. 6 presents the calculated settlement results as compared with the measured ones. However, Xie’s solution (Xie et al. 1994) cannot predict well the consolidation performance of layered soils because his method assumed instantaneous loading. Zhu’s solution (Zhu and Yin 1999) cannot predict well the consolidation performance of layered soils because his method assumed ramp loading.

Conclusions

This paper presents the development of an analytical solution for consolidation of a double-layered compressible foundation formed by partial penetration of deep mixed columns with one- or two-way drainage boundaries. The Laplace transform method was used to solve the consolidation equations while the Stehfest algorithm was used to solve the inverse Laplace transform for time-dependent loading. Parametric study based on a two-stage ramp loading demonstrates an obvious effect of the first loading time on the degree of consolidation of the double-layered soil system. The constrained moduli of two layers within the foundation also significantly influence the consolidation of the double-layered foundation. The stiffness of the double-layered soil profile plays an important role in the rate of consolidation. The harder the soil layers, the faster the rate of consolidation for the double-layered foundation. A case study was selected to demonstrate that the calculated settlements using the proposed solution are in good agreement with the measured ones.

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