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Microeconomia II
Programa de Pós-Graduação em Economia
FEARP - USP

Seção 23.D e 23.E - Bayesian Implementation and Participation Constraints

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Exercise 1. Considere a Função de Escolha Social (FES) $f : \Theta \mapsto \mathbb{X}$, ou seja, para cada $\theta \in \Theta$

$$f(\theta) = [k(\theta), t_1(\theta), \cdots, t_I(\theta)],$$

$k(\theta) \in \mathbb{K}$ e $\sum_{i=1}^{I} t_i(\theta) \leqslant 0$. Mostre que

(a) Se $f(\cdot)$ é *ex post* eficiente, então $\forall \theta \in \Theta$,

(i) $k(\theta)$ satisfaz

$$\sum_{i=1}^{I} v_i(k(\theta), \theta_i) \geqslant \sum_{i=1}^{I} v_i(k, \theta_i), \quad \forall k \in \mathbb{K} \tag{23.C.7}$$

(ii) $t_i(\theta)$ satisfaz

$$\sum_{i=1}^{I} t_i(\theta) = 0. \tag{23.C.12}$$

(b) Se $f(\cdot)$ satisfaz (23.C.7) e (23.C.12), então $f(\cdot)$ é *ex post* eficiente.

Exercise 2. Consider a sealed bid first price auction with *iid* valuations uniformily distributed on $[0, 1]$. Compute the optimal reservation price and show that it is independent of the number of bidders. What is the optimal reservation price for a second-price auction in the same environment?

Exercise 3. Assuming *iid* uniformily distributed valuations, consider an auction in which the highest bidder pays 50% of his bid plus 50% of the bid of the second highest bidder. Solve for a symmetric BNE.

Exercise 4 (**MWG 23.D.2**).

Exercise 5 (**MWG 23.D.5**).

Exercise 6 (**MWG 23.D.6**).

Exercise 7. The federal government wants to build a new hospital and sets up an auction. The specific format chosen is a first-price closed-bid procurement auction, meaning that each bidder submits a price proposal for committing to construct the hospital and the contract is awarded to any of the bidders submitting a lowest price.
To be more concrete, suppose there are $N \geqslant 2$ companies bidding for the contract. Each bidder is characterized by a number θ which measures the quality of the hospital she can construct and is private information at the time of the auction. The types are independently and uniformily distributed over $[0, 1]$. There is a common cost function $c(\theta) = \theta^2$ for constructing a hospital of quality θ.

(a) Derive a symmetric Bayesian Nash equilibrium (be careful while writing the equilibrium probability of winning for a given type).

(b) Determine the equilibrium expected cost for the government and the equilibrium expected quality of the hospital.

Exercise 8 (**MWG 23.E.2**).

Exercise 9 (**MWG 23.E.6**).

Exercise 10 (**MWG 23.E.7**).