
Seção 23.B - Mechanism Design

Exercise 1. Considere a Função de Escolha Social (FES) $f : \Theta \mapsto \mathbb{X}$ para o exemplo de Projeto Público (exemplo 23.B.3), ou seja, para cada $\theta \in \Theta$

$$f(\theta) = [k(\theta), t_1(\theta), \dots, t_I(\theta)],$$

$k(\theta) \in \{0, 1\}$ e $\sum_{i=1}^I t_i(\theta) \leq -ck(\theta)$. Mostre que $f(\cdot)$ é *ex post* eficiente se $\forall \theta \in \Theta$,

(i) $k(\theta)$ satisfaz

$$k(\theta) = \begin{cases} 1 & \text{se } \sum_i \theta_i \geq c \\ 0 & \text{caso contrário} \end{cases} \quad (23.B.1)$$

(ii) $t_i(\theta)$ satisfaz

$$\sum_i t_i(\theta) = -ck(\theta) \quad (23.B.2)$$

Exercise 2. Suppose that two agents collectively choose from $X = \{x, y, z\}$. Each agent can be of two types, so $\Theta_1 = \{\theta'_1, \theta''_1\}$ and $\Theta_2 = \{\theta'_2, \theta''_2\}$. Preferences are given by:

$$\begin{array}{ll} x \succ_{\theta'_1} y \succ_{\theta'_1} z & y \succ_{\theta'_1} z \succ_{\theta'_1} x \\ z \succ_{\theta'_2} x \succ_{\theta'_2} y & y \succ_{\theta'_2} x \succ_{\theta'_2} z \end{array}$$

Find all ex-post efficient social choice function. Which of those are truthfully implementable?

Exercise 3 (MWG 23.B.2).

Exercise 4 (MWG 23.B.3).

Exercise 5 (MWG 23.B.4).

Exercise 6. Consider the allocation model of a single unit of an indivisible private good among two agents. One of them, the seller, possesses the good. The other agent is the buyer. The seller can have two valuations for the good, c_0 and c_1 , with equal probabilities ($c_1 > c_0 > 0$). When the seller has valuation c_i , the buyer has valuation v_i , with $v_i > c_i$ for $i \in \{0, 1\}$. The seller knows his and the buyer's type as well. The buyer does not know his type (or only knows it after the mechanism is defined). Both are risk neutral with relation to monetary values and the good. So, given a probability x of the buyer getting the good, and a transfer t made by him, the utilities of a buyer and a seller of a specific type are, respectively:

$$v_i - t$$

$$t - c_i$$

Consider that $\frac{v_1 + v_0}{2} < c_1$. Show that incentive compatibility and individual rationality are not consistent with the ex-post efficiency of the mechanism.