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**Seção 13.C - Signaling**

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**Exercise 1.** Utilize a discussão em small type (pág. 459 e 460) da seção 13.C para ilustrar que Sinalização pode Pareto melhorar uma alocação de Equilíbrio Competitivo.

**Exercise 2 (MWG 13.C.1).** Consider a game in which, first, nature draws a workers type from some continuous distribution on  $[\underline{\theta}, \bar{\theta}]$ . Once the worker observes her type, she can choose whether to submit to a costless test the reveals her ability perfectly. Finally, after observing whether the worker has taken the test and its outcome if she has, two firms bid for the workers services. Prove that in any subgame perfect Nash equilibrium of this model all worker types submit to the test, and firms offer a wage no greater than  $\underline{\theta}$  to any worker not doing so.

**Exercise 3.** Consider the signaling model from section 13.B of MWG (1994). There are two types of workers with productivity  $\theta_L = 1$  and  $\theta_H = 2$ . Suppose the education cost is given by  $c(e, \theta) = \frac{e^2}{\theta}$ . The proportion of type  $\theta_H$  workers is  $\lambda \in (0, 1)$ .

- (a) Define the perfect Bayesian equilibrium of this game.
- (b) Find the set of educational levels  $e$  such that a pooling equilibrium exists. Which pooling equilibrium Pareto dominates the others?
- (c) Find the interval  $[\tilde{e}, e_1]$  such that if  $e \in [\tilde{e}, e_1]$ , a separating equilibrium exists with educational level  $e$  belonging to the high type. Which separating equilibrium Pareto dominates the others?
- (d) Suppose we are in a separating equilibrium that Pareto dominates the others. Which is the lowest  $\lambda = \Pr(\theta = \theta_H)$  such that the most productive workers are worse with signaling than when it is not possible?

**Exercise 4 (MWG 13.C.4).** Reconsider the signaling model discussed in section 13.C, now assuming that worker types are drawn from the interval  $[\underline{\theta}, \bar{\theta}]$  with a density function  $f(\theta)$  that is strictly positive everywhere on this interval. Let the cost function be  $c(e, \theta) = (e^2/\theta)$ . Derive the (unique) separating perfect Bayesian equilibrium.

**Exercise 5.** Consider a model with two periods and without intertemporal discount. Firms can invest in a project that requires an amount of 20 in  $t = 0$  and gives a certain return of 30 in  $t = 1$ . The value of this investment should be obtained in the financial market through the emission of shares. Potential investors are uncertain about the value of the firms assets before the investment:  $A \in \{50, 100\}$  with  $\Pr(A = 100) = 1/10$ .

- (a) Suppose the investors believe that both types of firms invest. Which fraction of the firms should be sold to the new investors? What is the initial shareholders' payoff if they implement the project? Are these beliefs reasonable?
- (b) Suppose the investors believe that only the worst firms issue shares. Answer the same questions in item (a).

Now suppose that the initial shareholders commit in  $t = 0$  to undertake an advertising campaign in  $t = 1$ , after the project is realized, which does not impact on its return. This advertising campaign is an irreversible action. The firm chooses the amount it spends in advertising,  $K$ .

- (c) Define the perfect Bayesian equilibrium of this game.
- (d) Show that a good firm can signal its type through the advertising campaign.