
Seção 13.B - Adverse Selection

Exercise 1 (MWG 13.B.2). Suppose that $r(\cdot)$ is a continuous and strictly increasing function and that there exists $\hat{\theta} \in (\underline{\theta}, \bar{\theta})$ such that $r(\theta) > \theta$ for $\theta > \hat{\theta}$ and $r(\theta) < \theta$ for $\theta < \hat{\theta}$. Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$. Show that a competitive equilibrium with unobservable worker types necessarily involves a Pareto inefficient outcome.

Exercise 2 (MWG 13.B.3). Consider a *positive selection* version of the model discussed in Section 13.B in which $r(\cdot)$ is a continuous, strictly *decreasing* function of θ . Let the density of workers of type θ be $f(\theta)$, with $f(\theta) > 0$ for all $\theta \in [\underline{\theta}, \bar{\theta}]$.

- (a) Show that the *more capable* workers are the ones choosing to work at any given wage.
- (b) Show that if $r(\theta) > \theta$ for all θ , then the resulting competitive equilibrium is Pareto efficient.
- (c) Suppose that there exists a $\hat{\theta}$ such that $r(\theta) < \theta$ for $\theta > \hat{\theta}$ and $r(\theta) > \theta$ for $\theta < \hat{\theta}$. Show that any competitive equilibrium with strictly positive employment necessarily involves *too much* employment relative to the Pareto Optimal allocation of workers.

Exercise 3. Let $\Theta = [a, b]$ and assume that the productivity θ is distributed according to a density f with $f(\theta) > 0, \forall \theta \in \Theta$. Let $r : [a, b] \mapsto [0, \infty)$ be a strictly increasing and continuous function, with $r(a) < a$. In this case, the function $w \mapsto E[\theta|r(\theta)w]$ is continuous in $[r(a), \infty)$. Use the Intermediate Value Theorem to demonstrate that a competitive equilibrium exists.

Exercise 4. Consider $\Theta = [0, 1]$. Suppose that the population of workers is uniformly distributed in this interval. Let $r(\theta) = \alpha\theta$ with $0 < \alpha < 1$ be the opportunity cost of a worker with productivity θ . Show that:

- (a) If $\alpha > 1/2$, there is no competitive equilibrium.
- (b) If $\alpha = 1/2$, there are an infinite number of competitive equilibriums. Which one employs a bigger set of workers?
- (c) If $\alpha < 1/2$, there is only one competitive equilibrium.

Exercise 5 (MWG 13.B.7). Suppose that it is impossible to observe worker types and consider a competitive equilibrium with wage rate w^* . Show that there is a Pareto-improving market intervention $(\tilde{w}_e, \tilde{w}_u)$ that reduces employment if and only if there is one of the form $(w_e, w_u) = (w^*, \hat{w}_u)$ with $\hat{w}_u > 0$. Similarly, argue that there is a Pareto-improving market intervention $(\tilde{w}_e, \tilde{w}_u)$ that increases employment if and only if there is one of the form $(w_e, w_u) = (\hat{w}_e, 0)$ with $\hat{w}_e > w^*$. Can you use these facts to give a simple proof of Proposition 13.B.2?

Exercise 6 (MGW 13.B.8). Consider the following alteration to the adverse selection model in Section 13.B. Imagine that when workers engage in home production, they use product x . Suppose that the amount consumed is related to a worker's type, with the relation given by the increasing function $x(\theta)$. Show that if a central authority can observe purchases of good x but not worker types, then there is a market intervention that results in a Pareto improvement even if the market is at the highest-wage competitive equilibrium.