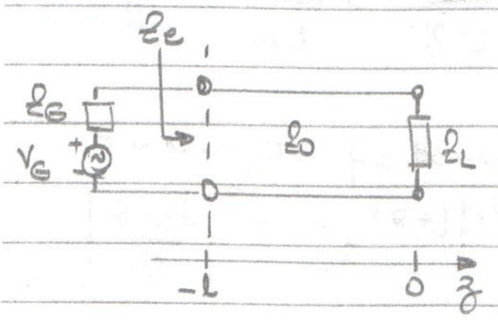
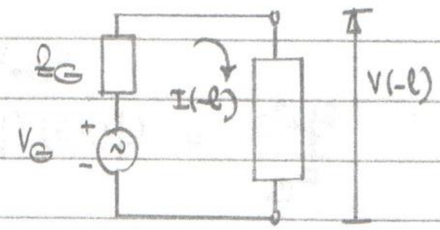
MÁXIMA TRANSFERÊNCIA DE POTÊNCIA

LINHA DE TRANSMISSÃO - FONTE : SISTEMA





COEFICIENTES DE REFLEXÃO

$$\Gamma_G = \frac{Z_G - Z_0}{Z_G + Z_0} \qquad \Gamma_L = \frac{Z_L - Z_0}{Z_L + Z_0}$$

TENSÃO

$$V(-l) = V_G \frac{Z_e}{Z_e + Z_G}$$

CORRENTE

$$I(-l) = \frac{V_G}{Z_G + Z_e}$$

utilizando as relações em linha de transmissão

$$V(-l) = V^+ \left(e^{j\beta l} + \Gamma_L e^{-j\beta l}\right)$$

$$I(-l) = \frac{V^+}{Z_0} \left(e^{j\beta l} - \Gamma_L e^{-j\beta l}\right)$$

POTÊNCIA DISSIPADA em Z_e

$$P_e = \frac{1}{2} Re\{V(-l)\, I^*(-l)\} = \frac{1}{2} Re\left\{V(-l) \frac{V^*(-l)}{Z_e^*}\right\}$$

$$= \frac{|V(-l)|^2}{2} Re\left\{\frac{1}{Z_e^*}\right\} = \frac{|V(-l)|^2}{2} Re\left\{\frac{1}{Z_e}\right\}$$

$$P_e = \frac{1}{2} \left|\frac{V_G Z_e}{Z_e + Z_G}\right|^2 Re\left\{\frac{1}{Z_e}\right\} = \frac{1}{2} |V_G|^2 \left|\frac{Z_e}{Z_e + Z_G}\right|^2 Re\left\{\frac{1}{Z_e}\right\}$$

$$Z_e = R_e + jX_e \quad ; \quad Z_L = R_L + jX_L \quad ; \quad Z_G = R_G + jX_G$$

Sabemos que, se $x = a + jb$ e $y = c + jd$,

$$\frac{x}{y} = \frac{a+jb}{c+jd} = \frac{(a+jb)(c-jd)}{(c+jd)(c-jd)} = \frac{(ac+bd)+j(bc-ad)}{c^2+d^2} = \frac{1}{D}(e+jf)$$

$$e = ac+bd;\ f = bc-ad;\ D = c^2+d^2$$

$$\left|\frac{e+jf}{D}\right|^2 = \frac{1}{D^2}|e+jf|^2 = \frac{1}{D^2}(e+jf)(e-jf) = \frac{e^2+f^2}{D^2}$$

$$= \frac{1}{D^2}\left[(ac+bd)^2+(bc-ad)^2\right] = \frac{1}{D^2}\left[(ac)^2+2acbd+(bd)^2+(bc)^2\right.$$

$$\left.-2bcad+(ad)^2\right] = \frac{1}{D^2}\left[(ac)^2+(ad)^2+(bd)^2+(bc)^2\right]$$

$$= \frac{1}{D^2}\left[a^2(c^2+d^2)+b^2(c^2+d^2)\right] = \frac{1}{D^2}\left[(a^2+b^2)(c^2+d^2)\right]$$

$$= \frac{(a^2+b^2)(c^2+d^2)}{c^2+d^2}$$

$$\left|\frac{e+jf}{D}\right|^2 = \frac{a^2+b^2}{c^2+d^2}$$

Portanto,

$$\frac{Z_e}{Z_e+Z_G} = \frac{R_e+jX_e}{(R_e+R_G)+j(X_e+X_G)} = \frac{x}{y};\ a = R_e;\ b = X_e;\ c = R_e+R_G;\ d = X_e+X_G$$

$$\left|\frac{Z_e}{Z_e+Z_G}\right|^2 = \frac{R_e^2+X_e^2}{(R_e+R_G)^2+(X_e+X_G)^2}$$

$$R_e\left\{\frac{1}{Z_e}\right\} = R_e\left\{\frac{1}{R_e+jX_e}\right\} = R_e\left\{\frac{R_e-jX_e}{R_e^2+X_e^2}\right\} = \frac{R_e}{R_e^2+X_e^2}$$

$$\frac{Z_e}{Z_e+Z_G} \quad Re\left\{\frac{1}{Z_e}\right\} = \frac{R_e^2+x_e^2}{(R_e+R_G)^2+(x_e+x_G)^2} \cdot \frac{R_e}{R_e^2+x_e^2} = \frac{R_e}{(R_e+R_G)^2+(x_e+x_G)^2}$$

$$\tilde{P}_e = \frac{|V_G|^2}{2} \frac{R_e}{(R_e+R_G)^2+(x_e+x_G)^2}$$ potência entregue à linha

Determinação da máxima potência entregue à linha

Devemos calcular $\frac{\partial P_e}{\partial R_e} = 0$ e $\frac{\partial P_e}{x_e} = 0 \rightarrow$ sistema de 2 equações

$$\frac{\partial P_e}{\partial R_e} = \frac{1}{(R_e+R_G)^2+(x_e+x_G)^2} - \frac{2R_e(R_e+R_G)}{[(R_e+R_G)^2+(x_e+x_G)^2]^2}$$

$$= \frac{(R_e+R_G)^2+(x_e+x_G)^2 - 2R_e(R_e+R_G)}{D_1^2}$$

$$= \frac{R_e^2+2R_eR_G+R_G^2+x_e^2+2x_ex_G+x_G^2-2R_e^2-2R_eR_G}{D_1^2}$$

$$\frac{\partial P_e}{\partial R_e} = \frac{-R_e^2+R_G^2+(x_e+x_G)^2}{(R_e+R_G)^2+(x_e+x_G)^2} = 0 \rightarrow R_G^2-R_e^2+(x_e+x_G) = 0 \quad (1)$$

$$\frac{\partial P_e}{\partial x_e} = -\frac{2x_e(x_e+x_G)}{D_1^2} = 0 \rightarrow x_e(x_e+x_G) = 0 \quad (2)$$

$$\therefore \begin{cases} R_G^2-R_e^2+(x_e+x_G) = 0 & (S1) \\ x_e(x_e+x_G) = 0 & (S2) \end{cases}$$

Supondo $x_e \neq 0$, de S2, $x_e = -x_G$

Substituindo em S1, $R_G^2-R_e^2 = 0$ e $R_e = R_G$

$\therefore$ $R_e = R_G$ e $x_e = -x_G$ ou $Z_e = Z_G^*$: condição de casamento conjugado

$$P_e = \frac{1}{2}|V_G|^2 \frac{R_G}{(2R_G)^2+(0)^2} \quad \text{ou} \quad \boxed{P_e = \frac{|V_G|^2}{8R_G}}$$ potência máxima entregue à linha

VALOR DA TENSÃO

$$V(-l) = V_G \frac{Z_e}{Z_e+Z_G} = V^+ (e^{j\beta l} + \Gamma_L e^{-j\beta l})$$

$$V^+ = V_G \frac{Z_e}{Z_e+Z_G} \cdot \frac{1}{e^{j\beta l}+\Gamma_L e^{-j\beta l}} = V_G \frac{Z_e}{Z_e+Z_G} \frac{1}{e^{j\beta l}} \frac{1}{1+\Gamma_L e^{-j2\beta l}}$$

mas,

$$Z_e = Z_0 \frac{1+\Gamma_L e^{-j2\beta l}}{1-\Gamma_L e^{-j2\beta l}} \longrightarrow 1+\Gamma_L e^{-j2\beta l} = \frac{Z_e}{Z_0}(1-\Gamma_L e^{-j2\beta l})$$

$$V^+ = V_G \frac{Z_e}{Z_e+Z_G} \cdot \frac{e^{-j\beta l}}{\frac{Z_e}{Z_0}(1-\Gamma_L e^{-j2\beta l})} = V_G \cdot \frac{Z_0}{Z_e+Z_G} \cdot \frac{e^{-j\beta l}}{1-\Gamma_L e^{-j2\beta l}}$$

$$\frac{Z_0}{Z_e+Z_G} = \frac{Z_0}{Z_G + Z_0 \cdot \frac{1+\Gamma_L e^{-j2\beta l}}{1-\Gamma_L e^{-j2\beta l}}} = \frac{Z_0}{Z_G(1-\Gamma_L e^{-j2\beta l}) + Z_0(1+\Gamma_L e^{-j2\beta l})}$$

e $$\frac{Z_0}{Z_e+Z_G} = \frac{Z_0(1-\Gamma_L e^{-j2\beta l})}{Z_0(1+\Gamma_L e^{-j2\beta l}) + Z_G(1-\Gamma_L e^{-j2\beta l})}$$

Portanto,

$$V^+ = V_G \cdot \frac{Z_0(1-\Gamma_L e^{-j2\beta l})}{Z_0(1+\Gamma_L e^{-j2\beta l}) + Z_G(1-\Gamma_L e^{-j2\beta l})} \cdot \frac{e^{-j\beta l}}{1-\Gamma_L e^{-j2\beta l}}$$

e $$V^+ = V_G \frac{e^{-j\beta l}}{Z_0(1+\Gamma_L e^{-j2\beta l}) + Z_G(1-\Gamma_L e^{-j2\beta l})}$$

Desenvolvendo o denominador:

$$Z_0 + Z_0 \Gamma_L e^{-j2\beta l} + Z_G - Z_G \Gamma_L e^{-j2\beta l} = (Z_0+Z_G)(Z_0-Z_G)\Gamma_L e^{-j2\beta l}$$

ou $$(Z_G+Z_0) - (Z_G-Z_0)\Gamma_L e^{-j2\beta l}$$