A

DETERMINAÇÃO DA MATRIZ S DE ELEMENTOS DE CIRCUITOS

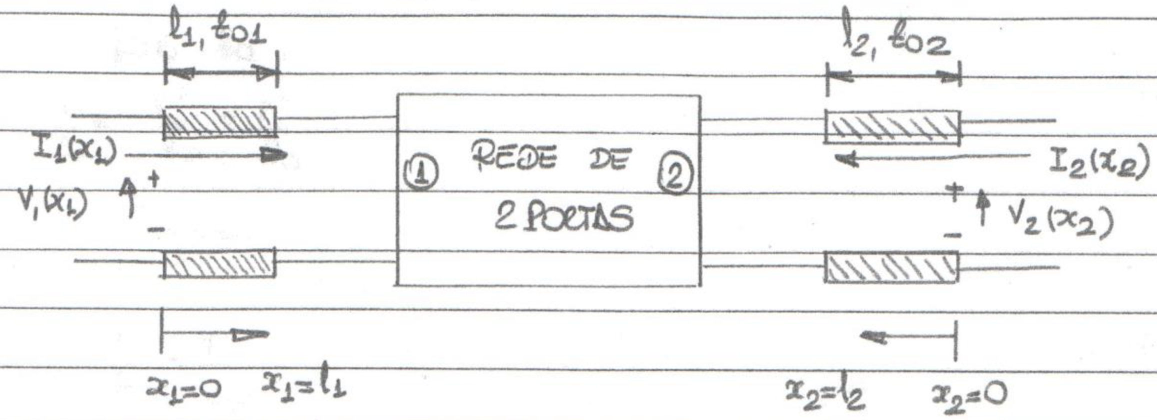

Fig.1. Rede de 2 portas conectada a 2 trechos de linha de transmissão sem perdas.

linha 1: comprimento l_1 e impedância característica Z_{01}, puramente real

linha 2: idem, l_2 e Z_{02}

Tensões e correntes nas linhas l_1 e l_2 : $V_1(x_1)$; $I_1(x_1)$
$V_2(x_2)$; $I_2(x_2)$

reunidas em notação única: $V_i(x_i)$; $I_i(x_i)$

para simplificar a notação ao longo das deduções, vamos utilizar a notação simplificada:

$V_1(x_1), V_2(x_2) \rightarrow V_i(x_i) \rightarrow V(x) \rightarrow V$

$I_1(x_1); I_2(x_2) \rightarrow I_i(x_i) \rightarrow I(x) \rightarrow I$

Para uma linha de transmissão,

$$V = V^+ + V^- \qquad (1)$$

$$I = I^+ - I^- \qquad (2)$$

(+): onda incidente

(−): onda refletida

De (2):

$$I = I^+ - I^- = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \quad (3)$$

÷ (2) por Z_0:

$$\frac{V}{Z_0} = \frac{V^+}{Z_0} + \frac{V^-}{Z_0} \quad (4)$$

Assim, temos:

$$I = \frac{V^+}{Z_0} - \frac{V^-}{Z_0} \quad (5)$$

$$\frac{V}{Z_0} = \frac{V^+}{Z_0} + \frac{V^-}{Z_0} \quad (6)$$

somando (5) e (6):

$$2\frac{V^+}{Z_0} = \frac{V}{Z_0} + I \quad (7)$$

x por Z_0:

$2V^+ = V + Z_0 I$ ou $V^+ = \frac{1}{2}(V + Z_0 I)$

÷ por $\sqrt{Z_0}$

$$\frac{V^+}{\sqrt{Z_0}} = \frac{1}{2\sqrt{Z_0}}(V + Z_0 I) \quad (8)$$

definindo:

$$a = \frac{V^+}{\sqrt{Z_0}} \quad e \quad a = \sqrt{Z_0}\, I^+ \quad (9)$$

(9) em (8):

$$a_i(x_i) = \sqrt{Z_{0i}}\, I_i^+(x_i) = \frac{1}{2\sqrt{Z_{0i}}}\left[V_i(x_i) + Z_{0i}\, I_i(x_i)\right] \quad (10)$$

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subtraindo; (6) − (5):

$$\frac{V}{Z_0} - I = 2\frac{V^-}{Z_0} \qquad (11)$$

× por Z_0

$$V^- = \frac{1}{2}\left(V - Z_0 I\right)$$

÷ por $\sqrt{Z_0}$

$$\frac{V^-}{(Z_0)^{1/2}} = \frac{1}{2\sqrt{Z_0}}\left(V - Z_0 I\right)$$

definindo

$$b = \frac{V^-}{\sqrt{Z_0}} \qquad (12)$$

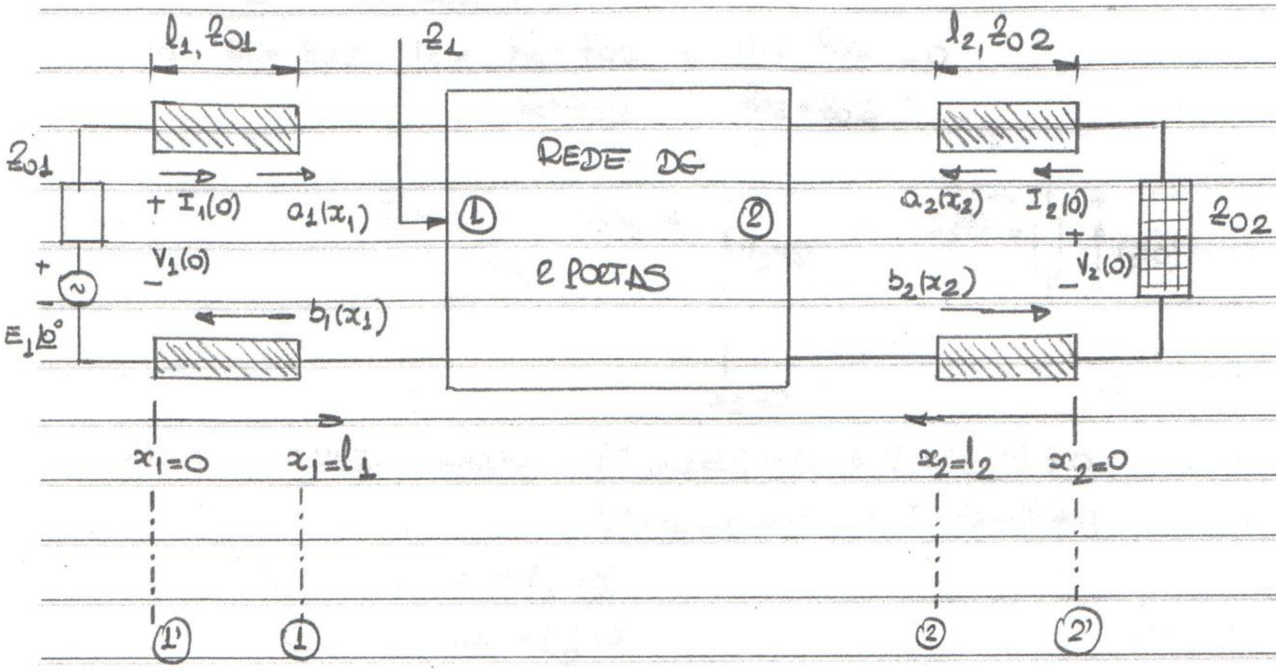
temos;

$$b_i(x_i) = \sqrt{Z_{0i}}\, I_i^-(x_i) = \frac{1}{2\sqrt{Z_{0i}}}\left[V_i(x_i) - Z_{0i}\, I_i(x_i)\right] \qquad (13)$$

RESUMO:

$$a_i(x_i) = \sqrt{Z_{0i}}\, I_i^+(x_i) = \frac{1}{2\sqrt{Z_{0i}}}\left[V_i(x_i) + Z_{0i}\, I_i(x_i)\right] \qquad (14)$$

$$b_i(x_i) = \sqrt{Z_{0i}}\, I_i^-(x_i) = \frac{1}{2\sqrt{Z_{0i}}}\left[V_i(x_i) - Z_{0i}\, I_i(x_i)\right] \qquad (15)$$



Linhas de transmissão (referência, $x=0$, sobre a carga)

onda incidente: $V^+ e^{j\beta x}$

onda refletida: $V^- e^{-j\beta x}$

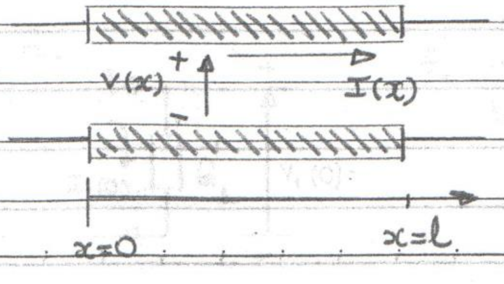
coeficiente de reflexão: $\Gamma(x) = \frac{V^-}{V^+} e^{-j2\beta x} = \Gamma_c \, e^{-j2\beta x}$

tensão em qualquer ponto: $V(x) = V^+ e^{j\beta x} + V^- e^{-j\beta x}$

corrente: $I(x) = \frac{1}{Z_0}\left(V^+ e^{j\beta x} - V^- e^{-j\beta x}\right)$

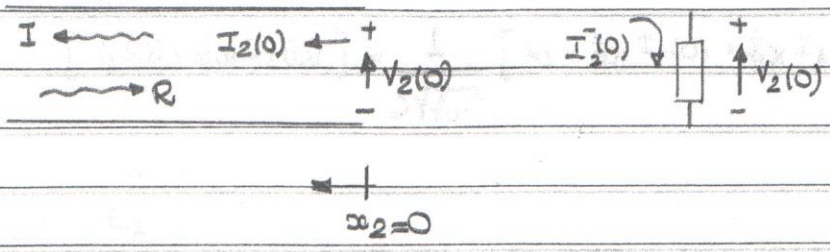
$$Z(x) = \frac{V}{I} = Z_0 \cdot \frac{1+\Gamma(x)}{1-\Gamma(x)} = Z_0 \cdot \frac{1+\Gamma_c e^{-j2\beta x}}{1-\Gamma_c e^{-j2\beta x}}$$

Se $\Gamma_c = 0 \rightarrow Z(x) = Z_0$



1. Linha 2: $Z_L = Z_{02}$ (linha casada)

como $Z_L = Z_{02}$, $\Gamma_c = \dfrac{Z_c - Z_{02}}{Z_c + Z_{02}} = \dfrac{Z_{02} - Z_{02}}{Z_{02} + Z_{02}} = 0$



sobre a carga casada: $V^+(x_2=0) = 0$ e $V^-(x_2=0) \neq 0$

$I^+(x_2=0) = 0$ e $I^-(x_2=0) \neq 0$

como $V_2(0) = V_2^+(0) + V_2^-(0)$

$I_2(0) = I_2^+(0) - I_2^-(0)$

e $V_2(0) = V_2^-(0)$

$I_2(0) = -I_2^-(0)$

mas, $V_2(0) = I_2^-(0) Z_{02}$: condição de contorno em $x_2 = 0$

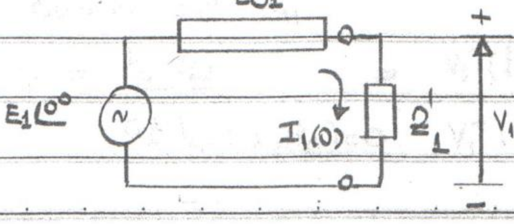
∴ $V_2(x_2=0) = -Z_{02} I_2(x_2=0)$ (16)

De (14):

$$a_2(0) = \frac{1}{2\sqrt{Z_{02}}}\left[V_2(0) + Z_{02} I_2(0)\right] = \frac{1}{2\sqrt{Z_{02}}}\left[-Z_{02} I_2(0) + Z_{02} I_2(0)\right]$$

∴ $a_2(x_2=0) = 0$ (17)

2. Linha 1:



Z_1': impedância na entrada da linha 1 ($x_1 = 0$)

Do circuito temos que:

$$V_1(0) = E_1 - Z_{01}\, I_1(0) \qquad (18)$$

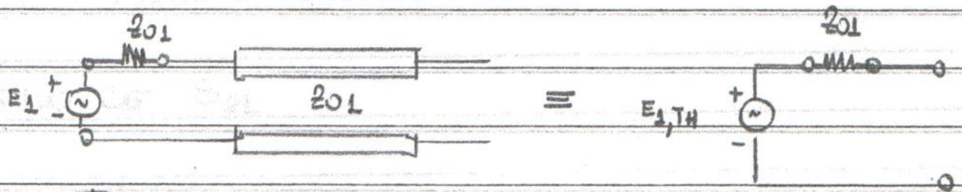
De (14):

$$a_1(x_1=0) = \frac{1}{2\sqrt{Z_{01}}}\left[V_1(0) + Z_{01} I_1(0)\right] = \frac{1}{2\sqrt{Z_{01}}}\left[E_1 - Z_{01} I_1(0) + Z_{01} I_1(0)\right]$$

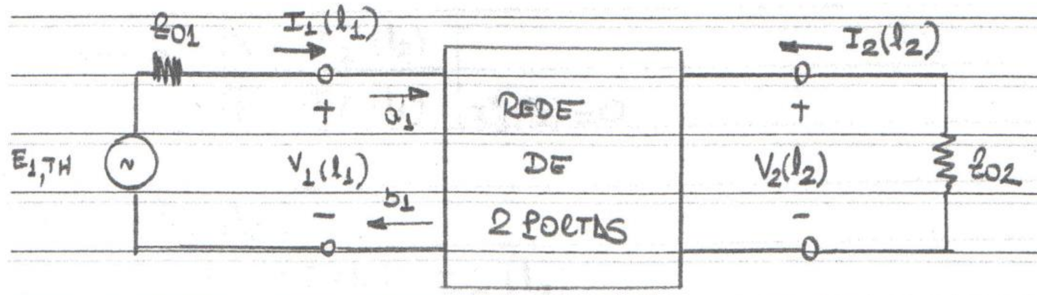
e $$a_1(x_1=0) = \frac{E_1}{2\sqrt{Z_{01}}} \qquad (19)$$

CÁLCULO DOS PARÂMETROS S NAS PORTAS 1 e 2

Equivalente de Thevenin referente à fonte e linha de transmissão 1



fonte de Tensão: curto-circuito
fonte de corrente: circuito aberto

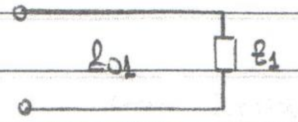


1. CÁLCULO DE S_{11}

$$S_{11} = \left.\frac{b_1(l_1)}{a_1(l_1)}\right|_{a_2(l_2)=0} = \left.\frac{V_1^-(l_1)/\sqrt{Z_{01}}}{V_1^+(l_1)/\sqrt{Z_{02}}}\right|_{a_2(l_2)=0} = \left.\frac{V_1^-(l_1)}{V_1^+(l_1)}\right|_{a_2(l_2)=0}$$

mas, $V_1^-(l_1) / V_1^+(l_1) = \Gamma(x = l_1)$

Situação:



$x_1 = l_1$

Z_1: impedância vista na entrada 1 da rede de 2 portas

$$\therefore \quad V_1^-(l_1) / V_1^+(l_1) = \Gamma(x = l_1) = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}}$$

Assim,

$$\boxed{S_{11} = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}}} \qquad (20)$$

S_{11}: coeficiente de reflexão na porta 1 com a linha 2 casada ($a_2(l_2) = 0$)

2. CÁLCULO DE S_{21}

$$S_{21} = \left. \frac{b_2(l_2)}{a_1(l_1)} \right|_{a_2(l_2) = 0}$$

$$S_{21} = \left. \frac{\sqrt{Z_{02}}\, I_2^-(l_2)}{\sqrt{Z_{01}}\, I_1^+(l_1)} \right|_{I_2^+(l_2) = 0}$$

em $x_2 = l_2$:

$I_2(l_2) = I_2^+(l_2) - I_2^-(l_2)$

como $a_2(l_2) = 0 \rightarrow I_2^+(l_2) = 0$ e $I_2^-(l_2) = -I_2(l_2)$

$$S_{21} = -\left. \frac{\sqrt{Z_{02}}\, I_2(l_2)}{\sqrt{Z_{01}}\, I_1^+(l_1)} \right|_{I_2^+(l_2) = 0} \qquad (21)$$

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$\to$ (16): $V_2(l_2) = -Z_{02}\, I_2(l_2) \;\to\; I_2(l_2) = -\dfrac{V_2(l_2)}{Z_{02}}$ (22)

(22) em (21):

$$S_{21} = -\frac{\sqrt{Z_{02}}\; I_2(l_2)}{\sqrt{Z_{01}}\; I_1^+(l_1)} = -\frac{\sqrt{Z_{02}}}{\sqrt{Z_{01}}} \cdot \left[-\frac{V_2(l_2)}{Z_{02}}\right] \frac{1}{I_1^+(l_1)}$$

$$S_{21} = \left.\frac{\sqrt{Z_{02}}\; V_2(l_2)}{\sqrt{Z_{01}}\; Z_{02}\; I_1^+(l_1)}\right|_{I_2^+(l_2)=0} \qquad (23)$$

Determinação de $I_1^+(l_1)$:

De (10) temos que

$$a_i(x_i) = \sqrt{Z_{0i}}\, I_i^+(x_i) = \frac{1}{2\sqrt{Z_{0i}}}\left[V_i(x_i) + Z_{0i}\, I_i(x_i)\right]$$

e

$$\sqrt{Z_{01}}\, I_1^+(l_1) = \frac{1}{2\sqrt{Z_{01}}}\left[V_1(l_1) + Z_{01}\, I_1(l_1)\right]$$

ou

$$I_1^+(l_1) = \frac{1}{2\sqrt{Z_{01}}}\left[\sqrt{Z_{01}}\, I_1(l_1) + \frac{V_1(l_1)}{\sqrt{Z_{01}}}\right] \qquad (24)$$

Do circuito equivalente,

$$E_{1,TH} = V_1(l_1) + Z_{01}\, I_1(l_1)$$

e

$$I_1(l_1) = \frac{E_{1,TH}}{Z_{01}} - \frac{V_1(l_1)}{Z_{01}} \qquad (25)$$

(25) em (24),

$$I_1^+(l_1) = \frac{1}{2\sqrt{Z_{01}}}\left[\sqrt{Z_{01}}\,\frac{E_{1,TH}}{Z_{01}} - \sqrt{Z_{01}}\,\frac{V_1(l_1)}{Z_{01}} + \frac{V_1(l_1)}{\sqrt{Z_{01}}}\right]$$

$$I_1^+(l_1) = \frac{1}{2\sqrt{z_{01}}}\left[\sqrt{z_{01}}\,\frac{E_{1,TH}}{z_{01}} - \frac{V_1(l_1)}{\sqrt{z_{01}}} + \frac{V_1(l_1)}{\sqrt{z_{01}}}\right] = \frac{E_{1,TH}}{2\,z_{01}} \qquad (26)$$

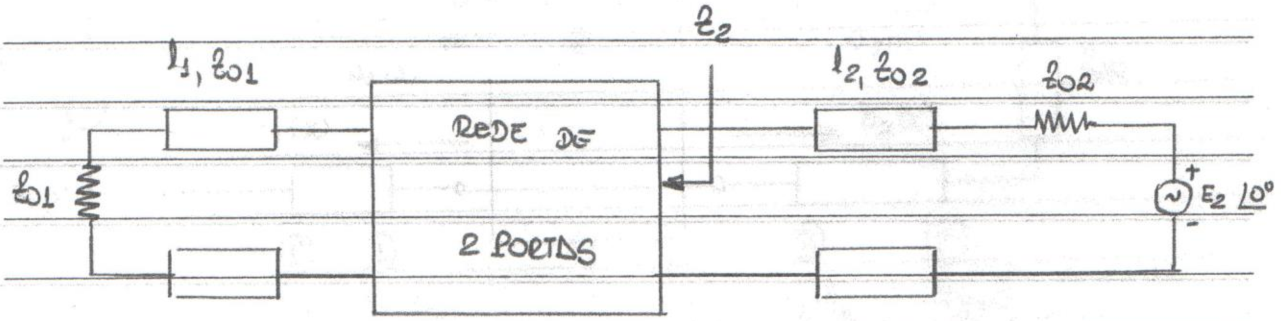
(26) em (23):

$$S_{21} = \frac{\sqrt{z_{02}}}{\sqrt{z_{01}}} \cdot \frac{1}{z_{02}} \, \frac{V_2(l_2)}{E_{1,TH}} \cdot 2\,z_{01} = \frac{2\sqrt{z_{02}}\,\sqrt{z_{01}}\,\sqrt{z_{01}}}{\sqrt{z_{01}}\,\sqrt{z_{02}}\,\sqrt{z_{02}}} \cdot \frac{V_2(l_2)}{E_{1,TH}}$$

e

$$S_{21} = \frac{2\sqrt{z_{01}}}{\sqrt{z_{02}}}\,\frac{V_2(l_2)}{E_{1,TH}} \qquad (27)$$

FONTE NO LADO 2



3. CÁLCULO DE S_{22}

$$S_{22} = \left.\frac{b_2(l_2)}{a_2(l_2)}\right|_{a_1(l_1)=0} = \frac{z_2 - z_{02}}{z_2 + z_{02}} \qquad (28)$$

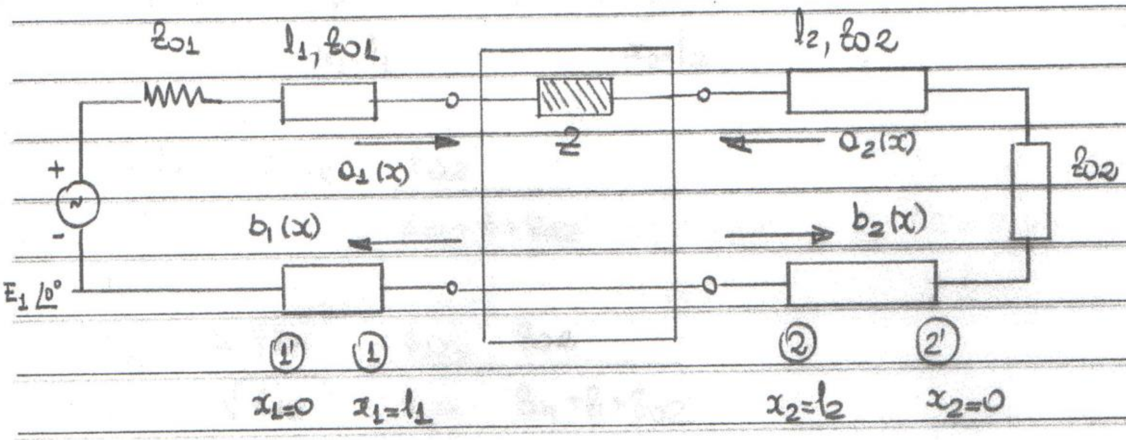
$$S_{12} = \left.\frac{b_1(l_1)}{a_2(l_2)}\right|_{a_1(l_1)=0} = \frac{2\sqrt{z_{02}}}{\sqrt{z_{01}}} \cdot \frac{V_1(l_1)}{E_{2,TH}} \qquad (29)$$

RESUMO

$$S_{11} = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}} \qquad S_{12} = \frac{2\sqrt{Z_{02}}}{\sqrt{Z_{01}}} \frac{V_1(l_1)}{E_{2,TH}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}}}{\sqrt{Z_{02}}} \frac{V_2(l_2)}{E_{1,TH}} \qquad S_{22} = \frac{Z_2 - Z_{02}}{Z_2 + Z_{02}}$$

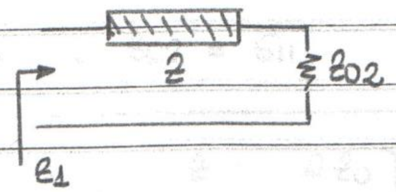
EXEMPLO 1: Determinar os parâmetros S de uma impedância em série.



1. cálculo de S_{11}

$$S_{11} = \left.\frac{b_1}{a_1}\right|_{a_2=0} \quad ; \quad a_2=0 \rightarrow \text{l.t. 2 terminada por } Z_{02} \text{ e } Z(x_2=l_2)=Z_{02}$$

$$S_{11} = \frac{Z_1 - Z_{01}}{Z_1 + Z_{01}}$$



$$Z_1 = Z + Z_{02}$$

$$S_{11} = \frac{Z + Z_{02} - Z_{01}}{Z + Z_{02} + Z_{01}}$$