ADDITIONAL TABLES FOR DESIGN OF OPTIMUM LADDER NETWORKS *

BY

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Part I **

ABSTRACT

In a preceding paper tables were presented for the design of three large classes of ladder networks. These networks had characteristics given by Butterworth, Tschebyscheff, and Bessel polynomials. In the present paper a number of additional tables are presented, the tables now being classified on the basis of the parameter r, which is the input-to-output resistance or conductance ratio. The tables give the element values of normalized low-pass ladders with one of the following characteristics: maximally flat magnitude (Butterworth), equal-ripple magnitude (Tschebyscheff), and maximally flat time delay (Bessel polynomial). By means of frequency transformations the networks given by the tabulated element values for the Butterworth and Tschebyscheff networks may be converted to give high-pass, band-pass, and band-elimination filters. Thus the tables may be used as a handbook for the design of these optimum networks.

INTRODUCTION

Tables for the design of three large classes of networks were presented in a preceding paper (1).[2] It was shown there that by use of these tables the engineer who knows little about the theory of modern synthesis can synthesize useful networks. These networks had characteristics given by Butterworth, Tschebyscheff, and Bessel polynomials.

In this paper the same characteristics are considered. However, many new tables are added. In addition, the basis for classifying the tables has been changed to one that is believed to be more useful for most applications.

The following extensions and changes have been made:

1. The tables are no longer classified in terms of the decrement ratio D, as they were in the preceding paper. The basis for classification in this paper is r, where $r = R_n/R_1$ (or $r = G'_n/G'_1$), the ratio of the input to the output resistance (or input-to-output conductance). These tables give a much larger range of input-to-output terminations than was available in the previously published tables.

* This paper is based on the author's report with the same title, Technical Memorandum No. 434, Hughes Research Laboratories, Culver City, Calif.

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[2] The boldface numbers in parentheses refer to the references appended to Part I of this paper.

2. Tables for the Tschebyscheff characteristic with $\frac{1}{10}$-db and $\frac{1}{4}$-db. ripple have been added.

3. In the preceding paper the Bessel-polynomial networks were given only for a resistance termination at one end. In this paper the networks are given for the same wide range of resistance ratios as are the Butterworth and Tschebyscheff networks.

It is felt that since these tables eliminate tedious computations, they will be of great value to engineers familiar with synthesis theory. But the paper is directed mainly at those practical engineers who know little about synthesis and desire a final working formula and a set of tabulated values. It was therefore decided to make the paper as self-contained as possible, within the space restrictions, by giving an analytical discussion of each type of network. Though the discussion is necessarily brief, it is believed that it is sufficient to acquaint the reader with the characteristics of the networks whose element values are given in the tables.

As mentioned in Sec. V, a number of different networks is possible, each of which realizes the *identical* transfer function (including the constant multiplier). The networks differ because of the different choices for the zeros of the reflection coefficient. The tables in this paper are based on the choice that gives maximum gain-bandwidth product for a specified value of shunt capacitance (2), that is, all the zeros are chosen to lie in one half-plane.

The paper is divided into five main sections. How to use the tables of element values is discussed in the first section. The next two treat Butterworth and Tschebyscheff networks, respectively, while the fourth section treats the maximally flat time-delay networks obtained by the use of Bessel polynomials. In each of these three sections tables of the element values of the normalized low-pass ladder network are given. In the final section it is briefly shown: (*a*) how to transform Butterworth and Tschebyscheff networks to serve high-pass, band-pass, or band-elimination functions; (*b*) how to remove the normalization of the element values—that is, how to change the pass band of the network from $\omega = 1$ to the desired radian frequency, and how to raise the level of the network; (*c*) how to use duality and reciprocity to obtain sets of new networks; and (*d*) how to convert the symmetrical Butterworth and Tschebyscheff networks to unsymmetrical ones with any desired ratio of input-to-output resistance.

I. USE OF THE TABLES OF ELEMENT VALUES

The general form of the low-pass ladder network whose element values are given in the tables is a lossless network terminated in resistance. In all the tables and in the figures the element values are in ohms, henrys, and farads. The tabulated element values are normalized in that the pass band has a cutoff radian frequency ω_c equal to unity and the network has a one-ohm resistance load, that is, in all the tables $R_1 = 1$. As shown in Sec. V and in the illustrative examples the removal of these normalizations requires only simple multiplications.

Six different values of $r = R_n/R_1$ (or $r = G'_n/G'_1$) are included: 0, $\frac{1}{8}$, $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{2}$, and 1. Since $R_1 = 1$ for all the tables, the value of R_n for each table is given by the r of that table and thus need not be tabulated.

The primed and the unprimed values in the tables yield dual networks so that a transfer impedance or a transfer admittance can be realized. The networks for the general ladder, with a resistance termination at both ends and with a current-source input, are shown in Fig. 1 for n odd and in Fig. 2 for n even; n is the degree of the denomina-

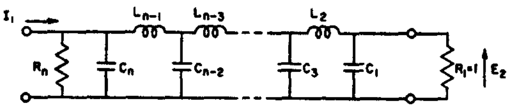

FIG. 1. General form of low-pass ladder network with a current-source input and n odd.

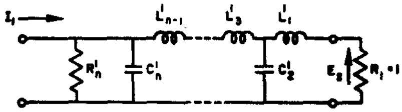

FIG. 2. General form of low-pass ladder network with a current-source input and n even.

tor of the transfer function and is thus also equal to the number of reactances in the ladder. The transfer function realized by these networks is the transfer impedance $Z_{21} = E_2/I_1$.

For a voltage source used as the input, the dual of the above networks can be used. The networks are shown in Figs. 3 and 4 for n odd and

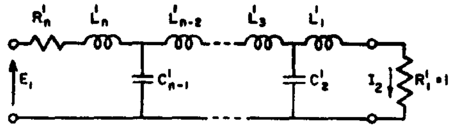

FIG. 3. General form of low-pass ladder network with a voltage-source input and n odd.

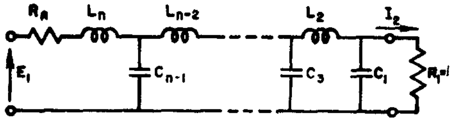

FIG. 4. General form of low-pass ladder network with a voltage-source input and n even.

even, respectively. The transfer function realized by these networks is the transfer admittance $Y_{21} = I_2/E_1$.

It is also pointed out that Thévenin's or Norton's theorem can be used to effect a source conversion and thus yield new network configurations. For example, Thévenin's theorem applied to R_n and the current source in Fig. 1 yields a voltage source and a series resistance. In this way a transfer admittance or transfer voltage ratio may be realized with a shunt capacitance branch at both ends of the coupling network.

The primed and unprimed elements shown in Figs. 1–4 are intended to correspond to the primed and unprimed values given in the tables.

Thus for a specified Z_{21} and *n odd* the *unprimed* tabulated values yield the network of Fig. 1, whereas for *n even* the *primed* tabulated values yield the network of Fig. 2. When a transfer admittance Y_{21} is required, the *primed* tabulated values yield the network shown in Fig. 3 for *n odd*, and for *n even* the *unprimed* values yield the network of Fig. 4.

The tables are divided as follows:

(*a*) Table I gives the element values for the Butterworth filter, where (*a*) applies for $r = 0$, (*b*) for $r = \frac{1}{8}$, (*c*) for $r = \frac{1}{4}$, (*d*) for $r = \frac{1}{3}$, (*e*) for $r = \frac{1}{2}$, and (*f*) for $r = 1$.

(*b*) Tables II–VII apply to the Tschebyscheff filter, with the subdivisions being necessary to provide for the different ripple factors. For example, Tables II and III give the element values for a $\frac{1}{10}$-db ripple and a $\frac{1}{4}$-db ripple, respectively. The alphabetical subdivisions are the same as for the Butterworth case.

(*c*) Tables VIII and IX apply to the Bessel-polynomial networks. Table VIII gives the frequencies at which significant values of time delay and loss occur. The variable u is the normalized frequency ω/ω_0, where $\omega_0 = 1/t_0$ and t_0 is the desired time delay, that is, the time delay occurring at zero frequency. Table IX gives the element values; the alphabetical subdivisions are the same as for the Butterworth and Tschebyscheff cases.

As has been mentioned, the parameter r is equal either to the ratio of the input to the output *resistance* or to the input-to-output *conductance* ratio. A little reasoning always suffices to determine which applies, but for convenience the practical rules are expressed explicitly below.

(*a*) Except in the case of $r = 0$, for networks formed from the *unprimed* values in the tables, r is equal to the *resistance ratio* R_n/R_1. For networks formed from the *primed* values r is equal to the *conductance ratio* G'_n/G'_1 (which is of course equal to R'_1/R'_n).

(*b*) For $r = 0$ the parameter r is equal to the *resistance ratio* for the combinations: (1) *unprimed* values and *n even*, and (2) *primed* values and *n odd*. It is equal to the *conductance ratio* for (1) *unprimed* values and *n odd*, and (2) *primed* values and *n even*. This is merely a detailed way of stating that for $r = 0$, *R_n becomes a short* for networks with a *series input*, that is, those in Figs. 3 and 4, and *R'_n becomes an open circuit* for networks with a *shunt input*, that is, those in Figs. 1 and 2.

Inspection of the tables for the Tschebyscheff networks shows that for a number of tables element values are not given for n even. For these cases the specified resistance (or conductance) ratio is too large to be physically realizable. This occurs for:

(*a*) $r = 1$ for all ripples.
(*b*) $r = \frac{1}{2}$ for 1-db, 2-db, and 3-db ripples.
(*c*) $r = \frac{1}{3}$ for 2-db and 3-db ripples.
(*d*) $r = \frac{1}{4}$ for 2-db and 3-db ripples.

With the above preliminary remarks the steps in the procedure for using the tables follow:

1. Determine from the specifications of the problem whether a Butterworth, Tschebyscheff, or Bessel-polynomial network is to be used.
2. Calculate the value of n that gives the required degree of the denominator polynomial of the transfer function and consequently the required complexity of the network. For the Tschebyscheff characteristic it is first necessary to calculate the ripple factor ϵ.
3. Using this value of n look up the element values in the appropriate table.
4. Remove the normalizations as shown in Sec. IV. The bandwidth is thus changed from $\omega_c = 1$ to the desired cutoff value, and the load resistance and the network level are changed to the required values.

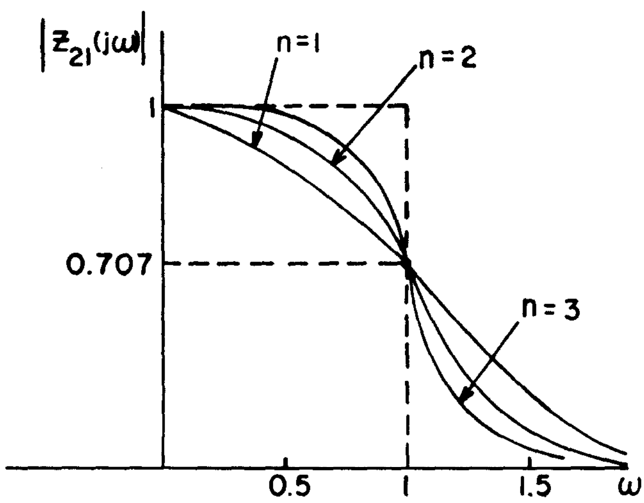

FIG. 5. Sketches of the first three orders of the Butterworth approximation to the low-pass filter.

5. If a high-pass, band-pass, or band-elimination network is desired, convert the element values by means of the frequency transformations of Sec. V.

In carrying out the first step, if we require a filter whose magnitude characteristic is specified, then the Butterworth or Tschebyscheff characteristic may be used. For the same value of n, the Tschebyscheff filter gives a better coverage of the pass band and a faster drop-off outside the band than any other possible transfer function that is also a constant divided by a polynomial; its phase characteristic, however, is more nonlinear than that of the Butterworth filter. Thus the choice between the two will depend on the importance of the phase characteristic. Neither of these filters gives a linear phase characteristic, that is, a pure time delay, over a specified frequency range. For this purpose the Bessel polynomials are used. The filters obtained by use of the Bessel polynomials also have a low-pass magnitude characteristic so that they may also be used in those problems where the magnitude characteristic is specified.

The method for calculating the value of n and, in the case of the Tschebyscheff filter, the value of ϵ, is shown in the respective sections.

TABLE I.—*Element Values (in ohms, henrys, farads) for a Normalized Butterworth Filter.*

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	1.0000									
2	0.7071	1.4142								
3	0.5000	1.3333	1.5000							
4	0.3827	1.0824	1.5772	1.5307						
5	0.3090	0.8944	1.3820	1.6944	1.5451					
6	0.2588	0.7579	1.2016	1.5529	1.7593	1.5529				
7	0.2225	0.6560	1.0550	1.3972	1.6588	1.7988	1.5576			
8	0.1951	0.5776	0.9370	1.2588	1.5283	1.7287	1.8246	1.5607		
9	0.1736	0.5155	0.8414	1.1408	1.4037	1.6202	1.7772	1.8424	1.5628	
10	0.1564	0.4654	0.7626	1.0406	1.2921	1.5100	1.6869	1.8121	1.8552	1.5643
b) r = 1/8										
1	9.0000									
2	11.9764	0.0939								
3	12.4442	0.1735	4.1674							
4	12.5685	0.2032	8.9296	0.0493						
5	12.6076	0.2169	11.3305	0.1146	2.5343					
6	12.6190	0.2243	12.6794	0.1533	6.1898	0.0330				
7	12.6199	0.2287	13.5040	0.1778	8.5907	0.0835	1.8121			
8	12.6166	0.2314	14.0417	0.1940	10.2279	0.1190	4.6929	0.0248		
9	12.6117	0.2333	14.4102	0.2053	11.3856	0.1446	6.8248	0.0653	1.4086	
10	12.6064	0.2346	14.6730	0.2135	12.2305	0.1635	8.4293	0.0965	3.7699	0.0198
c) r = 1/4										
1	5.0000									
2	6.2741	0.1992								
3	6.3870	0.3608	2.1699							
4	6.3840	0.4180	4.6024	0.1018						
5	6.3636	0.4435	5.8036	0.2350	1.2992					
6	6.3425	0.4567	6.4673	0.3130	3.1601	0.0675				
7	6.3238	0.4641	6.8671	0.3618	4.3727	0.1700	0.9225			
8	6.3078	0.4687	7.1244	0.3940	5.1943	0.2417	2.3838	0.0503		
9	6.2941	0.4716	7.2984	0.4162	5.7720	0.2932	3.4607	0.1325	0.7143	
10	6.2825	0.4735	7.4209	0.4321	6.1916	0.3312	4.2683	0.1955	1.9091	0.0401
d) r = 1/3										
1	4.0000									
2	4.8284	0.2761								
3	4.8473	0.4934	1.6725							
4	4.8105	0.5676	3.5233	0.1386						
5	4.7743	0.5997	4.4239	0.3186	0.9912					
6	4.7446	0.6156	4.9155	0.4233	2.4042	0.0913				
7	4.7206	0.6244	5.2085	0.4882	3.3200	0.2294	0.7006			
8	4.7012	0.6295	5.3950	0.5308	3.9376	0.3258	1.8075	0.0678		
9	4.6853	0.6326	5.5200	0.5601	4.3702	0.3948	2.6209	0.1785	0.5410	
10	4.6720	0.6346	5.6071	0.5809	4.6833	0.4454	3.2293	0.2630	1.4445	0.0540
e) r = 1/2										
1	3.0000									
2	3.3461	0.4483								
3	3.2612	0.7789	1.1811							
4	3.1868	0.8826	2.4524	0.2175						
5	3.1331	0.9237	3.0510	0.4955	0.6857					
6	3.0938	0.9423	3.3687	0.6542	1.6531	0.1412				
7	3.0640	0.9513	3.5532	0.7512	2.2726	0.3536	0.4799			
8	3.0408	0.9558	3.6678	0.8139	2.6863	0.5003	1.2341	0.1042		
9	3.0223	0.9579	3.7426	0.8565	2.9734	0.6046	1.7846	0.2735	0.3685	
10	3.0072	0.9588	3.7934	0.8864	3.1795	0.6808	2.1943	0.4021	0.9818	0.0825
f) r = 1										
1	2.0000									
2	1.4142	1.4142								
3	1.0000	2.0000	1.0000							
4	0.7654	1.8478	1.8478	0.7654						
5	0.6180	1.6180	2.0000	1.6180	0.6180					
6	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176				
7	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450			
8	0.3902	1.1111	1.6629	1.9616	1.9616	1.6629	1.1111	0.3902		
9	0.3473	1.0000	1.5321	1.8794	2.0000	1.8794	1.5321	1.0000	0.3473	
10	0.3129	0.9080	1.4142	1.7820	1.9754	1.9754	1.7820	1.4142	0.9080	0.3129

II. BUTTERWORTH CHARACTERISTIC (3,4,5)

The Butterworth function is used to approximate the squared magnitude of a transfer function. For the transfer impedance it is given by

$$|Z_{21}(j\omega)|^2 = \frac{1}{1 + \omega^{2n}}. \tag{1}$$

This function gives an approximation to a low-pass filter characteristic; sketches of the Butterworth approximation for the first three values of n are shown in Fig. 5. The Butterworth function is said to have a *maximally flat* magnitude characteristic.

By use of Eq. 1 the complete transfer function is given as

$$Z_{21}(s) = \frac{H}{B_n(s)}, \tag{2}$$

where H is a constant multiplier. The polynomials B_n are called the Butterworth polynomials; these polynomials have a unity coefficient for s^n and their zeros are the nth roots of unity that lie in the left half-plane.

The element values are given in Table I; the resulting networks realize the transfer function within a constant multiplier. To obtain the constant multiplier we let $s = 0$ in the network and in the transfer function.

An example of the use of the tables to design a Butterworth filter is presented below.

Example 2.1. We wish to design a low-pass filter that has a resistance termination at the output only. The cutoff frequency is $\omega_c = 10{,}000$ radians/sec and the output resistance is to be 750 ohms. At a frequency $\omega = 3\omega_c$ the magnitude response is to be down at least 50 db. The input source is a cathode follower which approximates a true voltage source.

First we determine the value of n.

$$\left.\frac{1}{1+\omega^{2n}}\right|_{\omega=3} = 10^{-5}$$

$$(1+\omega^{2n})\,|_{\omega=3} = 10^{5}$$

$$3^{2n} \cong 10^{5}$$

$$n = \frac{1}{2}\,\frac{5}{\log 3}$$

$$= 5.23.$$

The next larger integer $n = 6$ must be used.

Since no input resistance is required, the table for $r = 0$ is used, namely, Table I(a). Since the input is a voltage source and n is even, the network form of Fig. 4 (with R_n omitted) is applicable; that is, the unprimed element parameters are used.

Consulting the tables yields the element values

$$R_1 = 1 \qquad L_4 = 1.553$$
$$C_1 = 0.2588 \qquad C_5 = 1.759$$
$$L_2 = 0.7579 \qquad L_6 = 1.553$$
$$C_3 = 1.202$$

To obtain a load resistance of 750 ohms, we multiply R_1 and all L's and divide all C's by 750. To change the cutoff frequency to 10,000 rad/sec every L and C must be divided by this value.

The final values are therefore

$$R = 750\,R_1 = 750 \qquad L_d = \frac{RL_4}{\omega_c} = 1.16 \times 10^{-1}$$
$$C_a = \frac{C_1}{\omega_c R} = 3.45 \times 10^{-8}$$
$$L_b = \frac{RL_2}{\omega_c} = 5.68 \times 10^{-2} \qquad C_e = \frac{C_5}{R\omega_c} = 2.34 \times 10^{-7}$$
$$C_c = \frac{C_3}{\omega_c R} = 1.60 \times 10^{-7} \qquad L_f = \frac{RL_6}{\omega_c} = 1.16 \times 10^{-1}$$

and the network is shown in Fig. 6.

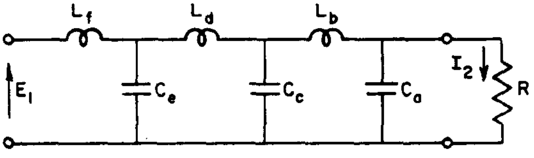

FIG. 6. Final network achieved for Example 2.1.

At $s = 0$ the network becomes a pure resistance and therefore

$$Z_{21}\big|_{s=0} = \frac{H}{B_6(0)} = R = 750.$$

Since the constant term of every B_n is unity, the constant multiplier H is 750. The transfer voltage ratio E_2/E_1, since $E_2 = 750\,I_2$, is given by

$$\frac{E_2}{E_1} = \frac{1}{B_6(s)}.$$

III. TSCHEBYSCHEFF CHARACTERISTIC (6)

The Tschebyscheff approximation to the magnitude characteristic of a low-pass filter is given by

$$|Z_{21}(j\omega)|^2 = \frac{1}{1 + [\epsilon T_n(\omega)]^2}. \tag{3}$$

TABLE II.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with $\frac{1}{10}$-db Ripple (ϵ = 0.1526, ϵ^2 = 0.0233).*

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.1526									
2	0.4215	0.7159								
3	0.5158	1.0864	1.0895							
4	0.5544	1.1994	1.4576	1.2453						
5	0.5734	1.2490	1.5562	1.5924	1.3759					
6	0.5841	1.2752	1.5999	1.6749	1.7236	1.4035				
7	0.5906	1.2908	1.6236	1.7107	1.7987	1.7395	1.4745			
8	0.5949	1.3008	1.6380	1.7302	1.8302	1.8070	1.8163	1.4660		
9	0.5978	1.3076	1.6476	1.7423	1.8473	1.8343	1.8814	1.7991	1.5182	
10	0.6000	1.3124	1.6542	1.7503	1.8579	1.8489	1.9068	1.8600	1.8585	1.4964
b) r = 1/8										
1	1.3736									
2	5.9892	0.0567								
3	8.9466	0.1403	4.3787							
4	10.0512	0.1866	9.8722	0.0733						
5	11.1128	0.2008	12.7123	0.1602	4.8368					
6	11.2235	0.2179	13.5071	0.2041	10.4600	0.0770				
7	11.8455	0.2179	14.5445	0.2153	13.2359	0.1652	4.9726			
8	11.6822	0.2287	14.4866	0.2312	13.9349	0.2087	10.6567	0.0783		
9	12.1681	0.2248	15.1596	0.2297	14.9251	0.2191	13.4207	0.1672	5.0298	
10	11.9040	0.2336	14.8765	0.2401	14.8149	0.2346	14.0887	0.2106	10.7462	0.0790
c) r = 1/4										
1	0.7631									
2	3.0912	0.1220								
3	4.5446	0.2886	2.3272							
4	5.0046	0.3815	5.0696	0.1559						
5	5.5547	0.4037	6.4880	0.3281	2.5577					
6	5.5377	0.4403	6.7916	0.4167	5.3580	0.1632				
7	5.8904	0.4353	7.3489	0.4330	6.7472	0.3379	2.6256			
8	5.7441	0.4601	7.2417	0.4675	7.0032	0.4258	5.4537	0.1659		
9	6.0374	0.4478	7.6338	0.4594	7.5404	0.4404	6.8374	0.3419	2.6542	
10	5.8435	0.4690	7.4193	0.4839	7.4072	0.4742	7.0778	0.4295	5.4970	0.1672
d) r = 1/3										
1	0.6105									
2	2.3497	0.1712								
3	3.4253	0.3914	1.8216							
4	3.7120	0.5161	3.8671	0.2172						
5	4.1422	0.5389	4.9341	0.4442	1.9966					
6	4.0823	0.5906	5.1046	0.5639	4.0826	0.2272				
7	4.3777	0.5785	5.5512	0.5786	5.1294	0.4572	2.0481			
8	4.2247	0.6155	5.4212	0.6284	5.2644	0.5760	4.1536	0.2308		
9	4.4804	0.5942	5.7535	0.6119	5.6978	0.5885	5.1967	0.4623	2.0697	
10	4.2930	0.6266	5.5455	0.6490	5.5485	0.6374	5.3199	0.5810	4.1857	0.2325
e) r = 1/2										
1	0.4579									
2	1.5715	0.2880								
3	2.2746	0.6035	1.3341							
4	2.3545	0.7973	2.6600	0.3626						
5	2.6921	0.8042	3.3882	0.6853	1.4572					
6	2.5561	0.8962	3.3962	0.8761	2.8071	0.3785				
7	2.8260	0.8560	3.7594	0.8685	3.5246	0.7050	1.4932			
8	2.6324	0.9285	3.5762	0.9619	3.5095	0.8950	2.8547	0.3843		
9	2.8839	0.8762	3.8788	0.9121	3.8660	0.8836	3.5703	0.7127	1.5084	
10	2.6688	0.9429	3.6461	0.9887	3.6707	0.9765	3.5472	0.9027	2.8761	0.3870
f) r = 1										
1	0.3052									
2	- - -	- - -								
3	1.0316	1.1474	1.0316							
4	- - -	- - -	- - -	- - -						
5	1.1468	1.3712	1.9750	1.3712	1.1468					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	1.1812	1.4228	2.0967	1.5734	2.0967	1.4228	1.1812			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	1.1957	1.4426	2.1346	1.6167	2.2054	1.6167	2.1346	1.4426	1.1957	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

Here the parameter ϵ is the ripple factor and $T_n(\omega)$ is the Tschebyscheff polynomial of order (and degree) n; $T_n(\omega)$ is defined by $\cos(n \cos^{-1} \omega)$. The role played by ϵ and the equal-ripple quality of the Tschebyscheff approximation are illustrated in Fig. 7, where $n = 3$ and a 1-db ripple are used.

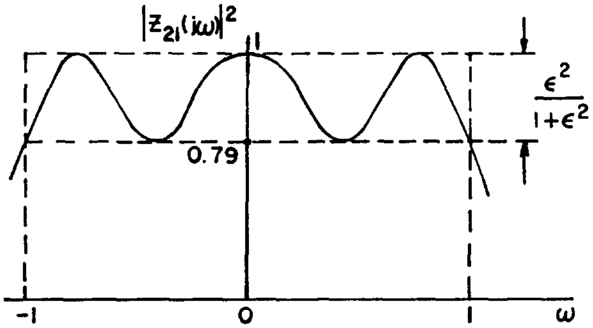

FIG. 7. Low-pass filter obtained by using the Tschebyscheff approximation with $n = 3$ and a 1-db ripple.

From the magnitude given by Eq. 3 the complete transfer function can be found. It is given by

$$Z_{21}(s) = \frac{H}{V_n(s)}, \tag{4}$$

where H again is a constant and V_n is formed from the left half-plane zeros of the denominator of Eq. 3; V_n is a polynomial of degree n with the coefficient of s^n equal to unity and with its zeros lying on an ellipse.

The element values of the ladder networks for values of ϵ corresponding to $\frac{1}{10}$-, $\frac{1}{4}$-, $\frac{1}{2}$-, 1-, 2-, and 3-db ripple are presented in Tables II through VII.

After the value of ϵ has been calculated from a specified ripple factor, it is necessary to determine the required value of n. Formulas useful for this purpose are

$$T_n(\omega) = \frac{(\omega + \sqrt{\omega^2 - 1})^n + (\omega + \sqrt{\omega^2 - 1})^{-n}}{2} \tag{5}$$

or

$$T_n(\omega) = \frac{(\omega - \sqrt{\omega^2 - 1})^n + (\omega - \sqrt{\omega^2 - 1})^{-n}}{2}. \tag{6}$$

TABLE III.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with $\frac{1}{4}$-db Ripple ($\epsilon = 0.2434$, $\epsilon^2 = 0.0593$).*

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.2434									
2	0.5566	0.8499								
3	0.6517	1.2198	1.2248							
4	0.6891	1.3215	1.5979	1.3003						
5	0.6912	1.3538	1.6741	1.6371	1.4480					
6	0.7173	1.3868	1.7271	1.7144	1.8105	1.4193				
7	0.7234	1.3999	1.7475	1.7450	1.8816	1.7497	1.5323			
8	0.7274	1.4083	1.7598	1.7612	1.9099	1.8124	1.8806	1.4647		
9	0.7302	1.4140	1.7678	1.7711	1.9248	1.8365	1.9439	1.7927	1.5648	
10	0.7322	1.4180	1.7733	1.7776	1.9338	1.8490	1.9676	1.8505	1.9119	1.4864
b) r = 1/8										
1	2.1908									
2	7.0446	0.0755								
3	10.0648	0.1562	5.5746							
4	10.4126	0.2051	10.8046	0.0922						
5	11.7227	0.2052	13.6987	0.1726	5.8758					
6	11.2734	0.2298	13.7315	0.2211	11.3063	0.0958				
7	12.3557	0.2179	15.2643	0.2183	14.2825	0.1780	6.1472			
8	11.5987	0.2378	14.4405	0.2423	14.0925	0.2250	11.4700	0.0970		
9	12.5942	0.2228	15.7237	0.2287	15.6054	0.2214	14.4391	0.1797	6.2011	
10	11.7539	0.2414	14.7148	0.2489	14.7268	0.2452	14.2166	0.2267	11.5438	0.0976
c) r = 1/4										
1	1.2171									
2	3.5907	0.1647								
3	5.1234	0.3182	2.9867							
4	5.1282	0.4214	5.4989	0.1990						
5	5.8822	0.4097	7.0092	0.3509	3.1328					
6	5.5080	0.4669	6.8423	0.4540	5.7437	0.2062				
7	6.1786	0.4322	7.7470	0.4354	7.3060	0.3611	3.2774			
8	5.6503	0.4815	7.1583	0.4930	7.0209	0.4618	5.8228	0.2087		
9	6.2867	0.4410	7.9582	0.4543	7.9206	0.4415	7.3830	0.3643	3.3046	
10	5.7179	0.4880	7.2798	0.5050	7.3032	0.4989	7.0810	0.4650	5.8583	0.2099
d) r = 1/3										
1	0.9737									
2	2.6983	0.2337								
3	3.8716	0.4279	2.3508							
4	3.7640	0.5729	4.1610	0.2812						
5	4.4034	0.5439	5.3427	0.4720	2.4596					
6	4.0210	0.6299	5.1006	0.6179	4.3438	0.2910				
7	4.6162	0.5713	5.8745	0.5783	5.5708	0.4851	2.5735			
8	4.1167	0.6480	5.3173	0.6668	5.2361	0.6285	4.4023	0.2945		
9	4.6917	0.5822	6.0240	0.6017	6.0086	0.5864	5.6288	0.4893	2.5944	
10	4.1620	0.6561	5.4002	0.6817	5.4294	0.6749	5.2808	0.6328	4.4285	0.2962
e) r = 1/2										
1	0.7303									
2	1.7288	0.4104								
3	2.5965	0.6465	1.7402							
4	2.2884	0.9039	2.7832	0.4930						
5	2.8983	0.8007	3.6926	0.7162	1.8152					
6	2.4162	0.9771	3.2941	0.9837	2.9094	0.5100				
7	3.0294	0.8341	4.0204	0.8546	3.8585	0.7340	1.9007			
8	2.4631	1.0000	3.4072	1.0463	3.3925	1.0015	2.9490	0.5161		
9	3.0724	0.8478	4.1088	0.8843	4.1199	0.8670	3.8989	0.7404	1.9157	
10	2.4852	1.0100	3.4501	1.0651	3.4927	1.0606	3.4235	1.0085	2.9666	0.5190
f) r = 1										
1	0.4868									
2	- - -	- - -								
3	1.3034	1.1463	1.3034							
4	- - -	- - -	- - -	- - -						
5	1.3824	1.3264	2.2091	1.3264	1.3824					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	1.4468	1.3560	2.3476	1.4689	2.3476	1.3560	1.4468			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	1.4604	1.3704	2.3800	1.5000	2.4414	1.5000	2.3800	1.3704	1.4604	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

TABLE IV.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter* ($\frac{1}{2}$*-db Ripple* ($\epsilon = 0.3493$, $\epsilon^2 = 0.1220$).

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.3493									
2	0.7014	0.9403								
3	0.7981	1.3001	1.3465							
4	0.8352	1.3916	1.7279	1.3138						
5	0.8529	1.4291	1.8142	1.6426	1.5388					
6	0.8627	1.4483	1.8494	1.7101	1.9018	1.4042				
7	0.8686	1.4596	1.8675	1.7371	1.9712	1.7254	1.5982			
8	0.8725	1.4666	1.8750	1.7508	1.9980	1.7838	1.9571	1.4379		
9	0.8752	1.4714	1.8856	1.7591	2.0116	1.8055	2.0203	1.7571	1.6238	
10	0.8771	1.4748	1.8905	1.7645	2.0197	1.8165	2.0432	1.8119	1.9816	1.4539
b) r = 1/8										
1	3.1438									
2	7.6905	0.0965								
3	11.1053	0.1646	6.8796							
4	10.3991	0.2234	11.2532	0.1135						
5	12.5367	0.2039	14.9223	0.1801	7.3211					
6	11.0346	0.2434	13.5532	0.2386	11.6839	0.1170				
7	12.9745	0.2131	16.0900	0.2154	15.3437	0.1837	7.4478			
8	11.2694	0.2497	14.0704	0.2556	13.8641	0.2422	11.8216	0.1182		
9	13.1608	0.2167	16.4502	0.2230	16.4112	0.2180	15.4833	0.1852	7.5006	
10	11.3804	0.2524	14.2670	0.2607	14.3247	0.2583	13.9670	0.2436	11.8832	0.1188
c) r = 1/4										
1	1.7466									
2	3.8432	0.2145								
3	5.6859	0.3308	3.7139							
4	5.0293	0.4646	5.6377	0.2504						
5	6.3476	0.4023	7.6862	0.3613	3.9411					
6	5.2985	0.5009	6.6492	0.4963	5.8461	0.2576				
7	6.5476	0.4187	8.2283	0.4252	7.8993	0.3683	4.0061			
8	5.3972	0.5122	6.8706	0.5272	6.8027	0.5035	5.9120	0.2602		
9	6.6323	0.4250	8.3941	0.4389	8.3940	0.4303	7.9689	0.3710	4.0331	
10	5.4437	0.5172	6.9544	0.5364	6.9993	0.5328	6.8522	0.5064	5.9413	0.2614
d) r = 1/3										
1	1.3972									
2	2.8282	0.3109								
3	4.3200	0.4405	2.9371							
4	3.6172	0.6399	4.1985	0.3620						
5	4.7896	0.5293	5.8898	0.4809	3.1130					
6	3.7922	0.6851	4.8770	0.6852	4.3536	0.3722				
7	4.9305	0.5495	6.2770	0.5603	6.0535	0.4901	3.1632			
8	3.8560	0.6990	5.0230	0.7235	4.9937	0.6953	4.4022	0.3759		
9	4.9901	0.5572	6.3947	0.5770	6.4061	0.5671	6.1064	0.4936	3.1841	
10	3.8860	0.7051	5.0780	0.7348	5.1229	0.7314	5.0307	0.6993	4.4237	0.3776
e) r = 1/2										
1	1.0479									
2	1.5132	0.6538								
3	2.9431	0.6503	2.1903							
4	1.8158	1.1328	2.4881	0.7732						
5	3.2228	0.7645	4.1228	0.7116	2.3197					
6	1.8786	1.1884	2.7589	1.2403	2.5976	0.7976				
7	3.3055	0.7899	4.3575	0.8132	4.2419	0.7252	2.3566			
8	1.9012	1.2053	2.8152	1.2864	2.8479	1.2628	2.6310	0.8063		
9	3.3403	0.7995	4.4283	0.8341	4.4546	0.8235	4.2795	0.7304	2.3719	
10	1.9117	1.2127	2.8366	1.2999	2.8964	1.3054	2.8744	1.2714	2.6456	0.8104
f) r = 1										
1	0.6986									
2	- - -	- - -								
3	1.5963	1.0967	1.5963							
4	- - -	- - -	- - -	- - -						
5	1.7058	1.2296	2.5408	1.2296	1.7058					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	1.7373	1.2582	2.6383	1.3443	2.6383	1.2582	1.7373			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	1.7504	1.2690	2.6678	1.3673	2.7239	1.3673	2.6678	1.2690	1.7504	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

TABLE V.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 1-db Ripple* ($\epsilon = 0.5088$, $\epsilon^2 = 0.2589$).

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.5088									
2	0.9110	0.9957								
3	1.0118	1.3332	1.5088							
4	1.0495	1.4126	1.9093	1.2817						
5	1.0674	1.4441	1.9938	1.5908	1.6652					
6	1.0773	1.4601	2.0270	1.6507	2.0491	1.3457				
7	1.0832	1.4694	2.0437	1.6736	2.1192	1.6489	1.7118			
8	1.0872	1.4751	2.0537	1.6850	2.1453	1.7021	2.0922	1.3691		
9	1.0899	1.4790	2.0601	1.6918	2.1583	1.7213	2.1574	1.6707	1.7317	
10	1.0918	1.4817	2.0645	1.6961	2.1658	1.7306	2.1803	1.7215	2.1111	1.3801
b) r = 1/8										
1	4.5796									
2	7.9318	0.1286								
3	12.5563	0.1657	8.8038							
4	9.9024	0.2517	11.1584	0.1467						
5	13.7259	0.1945	16.5650	0.1789	9.2596					
6	10.3304	0.2677	12.7878	0.2668	11.5115	0.1503				
7	14.0719	0.2009	17.5013	0.2045	16.9660	0.1819	9.3890			
8	10.4856	0.2725	13.1313	0.2802	13.0465	0.2701	11.6220	0.1516		
9	14.2174	0.2033	17.7827	0.2097	17.8168	0.2066	17.0949	0.1830	9.4427	
10	10.5585	0.2746	13.2602	0.2842	13.3503	0.2828	13.1287	0.2715	11.6709	0.1522
c) r = 1/4										
1	2.5442									
2	3.7779	0.3001								
3	6.5048	0.3264	4.7927							
4	4.5699	0.5428	5.3680	0.3406						
5	7.0522	0.3776	8.6301	0.3520	5.0313					
6	4.7366	0.5716	6.0240	0.5764	5.5353	0.3486				
7	7.2126	0.3888	9.0689	0.3973	8.8368	0.3577	5.0989			
8	4.7966	0.5803	6.1592	0.6005	6.1501	0.5836	5.5869	0.3515		
9	7.2800	0.3930	9.2001	0.4064	9.2344	0.4015	8.9024	0.3598	5.1270	
10	4.8247	0.5841	6.2098	0.6076	6.2689	0.6063	6.1890	0.5864	5.6096	0.3528
d) r = 1/3										
1	2.0354									
2	2.5721	0.4702								
3	4.9893	0.4286	3.8075							
4	3.0355	0.7929	3.7589	0.5347						
5	5.3830	0.4915	6.6673	0.4622	3.9944					
6	3.1307	0.8287	4.1451	0.8467	3.8812	0.5475				
7	5.4978	0.5050	6.9839	0.5177	6.8280	0.4696	4.0473			
8	3.1647	0.8395	4.2237	0.8764	4.2404	0.8580	3.9186	0.5520		
9	5.5459	0.5101	7.0783	0.5288	7.1141	0.5232	6.8785	0.4724	4.0693	
10	3.1806	0.8442	4.2532	0.8851	4.3088	0.8857	4.2691	0.8623	3.9349	0.5541
e) r = 1/2										
1	1.5265									
2	- - -	- - -								
3	3.4774	0.6153	2.8540							
4	- - -	- - -	- - -	- - -						
5	3.7211	0.6949	4.7448	0.6650	2.9936					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	3.7916	0.7118	4.9425	0.7348	4.8636	0.6757	3.0331			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	3.8210	0.7182	5.0013	0.7485	5.0412	0.7429	4.9004	0.6797	3.0495	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
f) r = 1										
1	1.0177									
2	- - -	- - -								
3	2.0236	0.9941	2.0236							
4	- - -	- - -	- - -	- - -						
5	2.1349	1.0911	3.0009	1.0911	2.1349					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	2.1666	1.1115	3.0936	1.1735	3.0936	1.1115	2.1666			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	2.1797	1.1192	3.1214	1.1897	3.1746	1.1897	3.1214	1.1192	2.1797	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

TABLE VI.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 2-db Ripple ($\epsilon = 0.7648$, $\epsilon^2 = 0.5849$).*

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.7648									
2	1.2441	0.9766								
3	1.3553	1.2740	1.7717							
4	1.3962	1.3389	2.2169	1.1727						
5	1.4155	1.3640	2.3049	1.4468	1.9004					
6	1.4261	1.3765	2.3383	1.4974	2.3304	1.2137				
7	1.4328	1.3836	2.3551	1.5159	2.4063	1.4836	1.9379			
8	1.4366	1.3881	2.3645	1.5251	2.4332	1.5298	2.3646	1.2284		
9	1.4395	1.3911	2.3707	1.5304	2.4463	1.5495	2.4386	1.4959	1.9553	
10	1.4416	1.3932	2.3748	1.5337	2.4538	1.5536	2.4607	1.5419	2.3794	1.2353
b) r = 1/8										
1	6.8830									
2	7.2895	0.1875								
3	14.9900	0.1541	11.9205							
4	8.5051	0.3093	9.9546	0.2088						
5	15.9745	0.1729	19.4874	0.1646	12.4250					
6	8.7527	0.3224	10.9256	0.3260	10.2196	0.2130				
7	16.2581	0.1769	20.2574	0.1811	19.8989	0.1669	12.5671			
8	8.8412	0.3263	11.1205	0.3368	11.1251	0.3295	10.3007	0.2144		
9	16.3765	0.1784	20.4848	0.1844	20.5920	0.1828	20.0271	0.1678	12.6259	
10	8.8824	0.3280	11.1932	0.3399	11.2954	0.3396	11.1858	0.3308	10.3363	0.2151
c) r = 1/4										
1	3.8239									
2	- - -	- - -								
3	7.9106	0.2955	6.5423							
4	- - -	- - -	- - -	- - -						
5	8.3859	0.3285	10.3300	0.3156	6.8118					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	8.5220	0.3354	10.7009	0.3443	10.5472	0.3199	6.8877			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	8.5787	0.3380	10.8103	0.3499	10.8798	0.3475	10.6142	0.3215	6.9191	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
d) r = 1/3										
1	3.0591									
2	- - -	- - -								
3	6.1471	0.3816	5.2161							
4	- - -	- - -	- - -	- - -						
5	6.4974	0.4219	8.0681	0.4076	5.4294					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	6.5974	0.4302	8.3415	0.4425	8.2389	0.4131	5.4893			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	6.6391	0.4334	8.4220	0.4493	8.4834	0.4467	8.2913	0.4152	5.5141	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
e) r = 1/2										
1	2.2943									
2	- - -	- - -								
3	4.3975	0.5326	3.9184							
4	- - -	- - -	- - -	- - -						
5	4.6265	0.5835	5.8503	0.5698	4.0790					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	4.6917	0.5941	6.0293	0.6136	5.9780	0.5776	4.1242			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	4.7187	0.5980	6.0821	0.6220	6.1370	0.6195	6.0168	0.5805	4.1429	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
f) r = 1										
1	1.5296									
2	- - -	- - -								
3	2.7107	0.8327	2.7107							
4	- - -	- - -	- - -	- - -						
5	2.8310	0.8985	3.7827	0.8985	2.8310					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	2.8650	0.9120	3.8774	0.9537	3.8774	0.9120	2.8650			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	2.8790	0.9171	3.9056	0.9643	3.9597	0.9643	3.9056	0.9171	2.8790	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

TABLE VII.—*Element Values (in ohms, henrys, farads) for a Normalized Tschebyscheff Filter with 3-db Ripple* ($\epsilon = 0.9976$, $\epsilon^2 = 0.9953$).

Value of n	C_1 or L_1'	L_2 or C_2'	C_3 or L_3'	L_4 or C_4'	C_5 or L_5'	L_6 or C_6'	C_7 or L_7'	L_8 or C_8'	C_9 or L_9'	L_{10} or C_{10}'
a) r = 0										
1	0.9976									
2	1.5506	0.9109								
3	1.6744	1.1739	2.0302							
4	1.7195	1.2292	2.5272	1.0578						
5	1.7409	1.2501	2.6227	1.3015	2.4491					
6	1.7522	1.2606	2.6578	1.3455	2.6309	1.0876				
7	1.7591	1.2666	2.6750	1.3614	2.7141	1.3282	2.1827			
8	1.7638	1.2701	2.6852	1.3690	2.7436	1.3687	2.6618	1.0982		
9	1.7670	1.2726	2.6916	1.3733	2.7577	1.3827	2.7414	1.3380	2.1970	
10	1.7692	1.2744	2.6958	1.3761	2.7655	1.3893	2.7683	1.3774	2.6753	1.1032
b) r = 1/8										
1	8.9787									
2	6.1219	0.2596								
3	17.4070	0.1392	14.8205							
4	6.9104	0.3884	8.2760	0.2861						
5	18.3377	0.1530	22.4760	0.1481	15.3856					
6	7.0661	0.4011	8.8887	0.4087	8.4796	0.2913				
7	18.6027	0.1559	23.1920	0.1600	22.9195	0.1499	15.5441			
8	7.1213	0.4048	9.0096	0.4190	9.0452	0.4128	8.5411	0.2931		
9	18.7129	0.1569	23.4023	0.1623	23.5592	0.1614	23.0554	0.1506	15.6097	
10	7.1470	0.4064	9.0546	0.4219	9.1496	0.4223	9.0917	0.4144	8.5679	0.2939
c) r = 1/4										
1	4.9881									
2	- - -	- - -								
3	9.3059	0.2625	8.1669							
4	- - -	- - -	- - -	- - -						
5	9.7676	0.2866	12.0571	0.2791	8.4724					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	9.8986	0.2915	12.4111	0.2998	12.2946	0.2826	8.5581			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	9.9530	0.2934	12.5151	0.3037	12.6097	0.3024	12.3669	0.2839	8.5935	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
d) r = 1/3										
1	3.9905									
2	- - -	- - -								
3	7.2903	0.3358	6.5207							
4	- - -	- - -	- - -	- - -						
5	7.6371	0.3652	9.4808	0.3571	6.7635					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	7.7352	0.3712	9.7463	0.3822	9.6687	0.3615	6.8315			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	7.7760	0.3734	9.8243	0.3870	9.9043	0.3856	9.7256	0.3632	6.8597	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
e) r = 1/2										
1	2.9929									
2	- - -	- - -								
3	5.2910	0.4618	4.8991							
4	- - -	- - -	- - -	- - -						
5	5.5259	0.4993	6.9460	0.4917	5.0821					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	5.5922	0.5069	7.1256	0.5235	7.0869	0.4979	5.1335			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	5.6197	0.5098	7.1785	0.5296	7.2454	0.5282	7.1292	0.5002	5.1547	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -
f) r = 1										
1	1.9953									
2	- - -	- - -								
3	3.3487	0.7117	3.3487							
4	- - -	- - -	- - -	- - -						
5	3.4813	0.7619	4.5375	0.7619	3.4813					
6	- - -	- - -	- - -	- - -	- - -	- - -				
7	3.5185	0.7722	4.6390	0.8038	4.6390	0.7722	3.5185			
8	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -		
9	3.5339	0.7760	4.6691	0.8118	4.7270	0.8118	4.6691	0.7760	3.5339	
10	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -	- - -

The use of the tables is illustrated in the example below.

Example 3.1. Determine a ladder network that has the following characteristics:

1. Low-pass filter with a peak-to-peak ripple in the squared magnitude characteristic not exceeding 15 per cent of the maximum value.
2. A cutoff radian frequency $\omega_c = 5000$ (the bandwidth being measured at the minimum value of the ripple).
3. Resistance terminations at both ends with the load and input resistances equal to 1000 ohms and 500 ohms, respectively.
4. The response is to be down at least 50 db at $\omega = 4\omega_c$.
5. The network is to be driven by a current source.

We first calculate the required value of ϵ^2. At a trough of the ripple we have

$$\frac{1}{1 + \epsilon^2 T_n^2(1)} = 1 - 0.15 = 0.85$$

$$1 + \epsilon^2 = \frac{20}{17}$$

$$\epsilon^2 = 0.176.$$

Since this value lies between $\frac{1}{2}$-db and 1-db ripple we must use Table IV.

Now we calculate n. At $\omega = 4$

$$\frac{1}{1 + \epsilon^2 T^2_n(4)} = 10^{-5}$$

$$1 + \epsilon^2 T^2_n(4) = 10^5$$

$$\epsilon^2 T^2_n(4) \cong 10^5$$

$$T_n(4) = 753.$$

Now using Eq. 5, we have

$$\left.\frac{(\omega + \sqrt{\omega^2 - 1})^n + (\omega + \sqrt{\omega^2 - 1})^{-n}}{2}\right|_{\omega=4} = 753$$

$$(\omega + \sqrt{\omega^2 - 1})^n|_{\omega=4} \cong 1506$$

$$(7.88)^n = 1506$$

$$n = 3.58.$$

Therefore $n = 4$ will be more than satisfactory.

Since the specification calls for $r = \frac{1}{2}$, we use Table IV(e). Since the input is a current source the unprimed values are used. Removing the normalization by multiplying all C's by $\frac{1}{R\omega_c} = \frac{1}{5 \times 10^6}$, all L's by

$\frac{R}{\omega_c} = \frac{1}{5}$, and the resistances by $R = 1000$, we obtain the final element values

$$\frac{1}{5 \times 10^6} C_1 = 0.363 \times 10^{-6}$$

$$\frac{1}{5} L_2 = 0.227$$

$$\frac{1}{5 \times 10^6} C_3 = 0.498 \times 10^{-6}$$

$$\frac{1}{5} L_4 = 0.155$$

$$1000 \, R_n = 500.$$

The network is shown in Fig. 8.

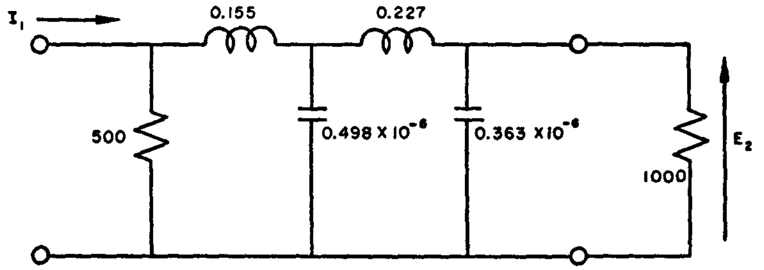

FIG. 8. Network obtained in Example 3.1 (values in ohms, henrys, and farads).

(To be continued)

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