PTC-2305: Respostas da quarta lista de exercícios

1. (a) $\mathrm{E}[XY] = 2$

   (b) $C_{XY} = \dfrac{2}{49}$

2.
$$p_X(x) = \begin{cases} \dfrac{x+1}{21}, & x = 0, 1, 2, 3, 4, 5 \\ 0, & \text{caso contrário} \end{cases}$$

$$p_Y(y) = \begin{cases} \dfrac{6-y}{21}, & y = 0, 1, 2, 3, 4, 5 \\ 0, & \text{caso contrário} \end{cases}$$

$$C_{XY} = \frac{10}{9}$$

3. (a) $\mathrm{E}[X_1 - X_2] = 0$

   (b) $\mathrm{var}[X_1 - X_2] = 2\sigma_X^2$

4. (a) $\mathrm{E}[X] = 2/3$ e $\mathrm{var}[X] = 1/18$

   (b) $\mathrm{E}[Y] = 2/3$ e $\mathrm{var}[Y] = 1/18$

   (c) $C_{XY} = 0$

5. (a) $\Pr[X > Y] = 3/5$

   (b) $\mathrm{E}[XY] = 6$

   (c) $C_{XY} = 0$

6. (a) $\mathbf{C_X} = \begin{bmatrix} 4 & 3 \\ 3 & 9 \end{bmatrix}$

   (b) $\mathbf{C_Y} = \begin{bmatrix} 28 & -66 \\ -66 & 252 \end{bmatrix}$

7. $\mathrm{E}[\mathbf{Y}] = [\ 1/3 \quad 1/3\ ]^T$

   $\mathbf{R_Y} = \begin{bmatrix} 1/6 & 1/12 \\ 1/12 & 1/6 \end{bmatrix}$

   $\mathbf{C_Y} = \begin{bmatrix} 1/18 & -1/36 \\ -1/36 & 1/18 \end{bmatrix}$

8. (a) $\mathbf{R_X} = \begin{bmatrix} 20 & 30 & 25 \\ 30 & 68 & 46 \\ 25 & 46 & 40 \end{bmatrix}$

   (b) $f_{X_1,X_2}(x_1, x_2) = \frac{1}{2\pi\sqrt{12}} \exp\left(-\frac{(x_1-4)^2 + (x_1-4)(x_2-8) + (x_2-8)^2}{6}\right)$

   (c) $\Pr[X_1 > 8] = 0.0228$.

9. (a) $\boldsymbol{\mu_Y} = [\ 8 \quad 0\ ]^T$

   (b) $\mathbf{C_Y} = \frac{1}{9}\begin{bmatrix} 43 & 55 \\ 55 & 103 \end{bmatrix}$

   (c) $\mathbf{R_Y} = \frac{1}{9}\begin{bmatrix} 619 & 55 \\ 55 & 103 \end{bmatrix}$

   (d) $\Pr[-1 \leq Y_2 \leq 1] = 0.2325$.

10. (a) $\mathrm{E}[X] = 6$ ms

    (b) $\mathrm{var}[X] = 12\ \mu s^2$

    (c) $\mathrm{E}[A] = 72$ ms

    (d) $\mathrm{var}[A] = 144\ \mu s^2$

    (e) $\Pr[A > 75] = 0.4013$

    (f) $\Pr[A < 48] = 0.0227$

11. $\Pr[499000 \leq W \leq 501000] = 0.9544$

12. (a) $\Pr[400 \leq C \leq 600] = \sum_{i=0}^{200} \frac{1000!}{(600-i)!(400+i)!}\left(\frac{1}{2}\right)^{1000} \approx 1$, onde C é a VA que representa o número total de caras.

    (b) $\Pr[400 \leq C \leq 600] = \Phi\left(\frac{600-500}{\sqrt{250}}\right) - \Phi\left(\frac{400-500}{\sqrt{250}}\right) \approx 1$

13. $\Pr[V < 600] = 0.5662$, onde V é a VA que representa a soma das vidas úteis.

14.

$$M_X(j\omega) = \frac{e^{j\omega b} - e^{j\omega a}}{j\omega(b-a)}$$

$$\mathrm{E}[X] = \frac{b+a}{2}$$

$$\mathrm{E}[X^2] = \frac{b^2 + ab + a^2}{3}$$