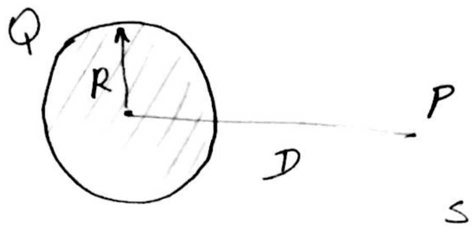
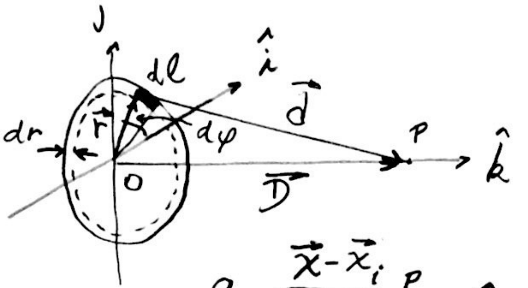
Disco de raio R com carga Q:



calcule $\vec{E}(P)$.

solução:

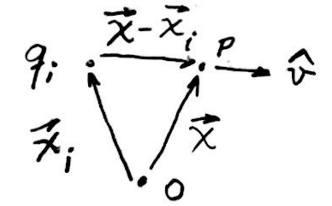


(I) consideramos o anel de espessura dr:

$$\begin{cases} \lambda = \dfrac{Q(dr)}{l} \; ; \; dQ(dr) = \lambda \, dl \\ \text{com:} \\ l = 2\pi r \quad \text{e} \quad dl = r\,d\varphi \end{cases}$$

$$dQ(dr) = \left(\frac{Q(dr)}{2\pi r}\right) \cdot r\,d\varphi$$

Seja:



$$\vec{E}(P) = \frac{q_i}{4\pi\epsilon_0} \cdot \frac{1}{(|\vec{x}-\vec{x}_i|)^2} \cdot \frac{\vec{x}-\vec{x}_i}{|\vec{x}-\vec{x}_i|}$$

no anel de espessura dr do disco:

$$\left.\begin{aligned} \vec{x} &= \vec{D} = D\hat{k} \\ \vec{x}_i &= \vec{r} \end{aligned}\right\} \begin{aligned} &\vec{x}-\vec{x}_i = D\hat{k} - \vec{r} = \vec{d} \\ &|\vec{x}-\vec{x}_i| = \sqrt{D^2+r^2} = |\vec{d}| = d \end{aligned}$$

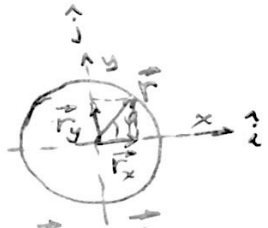
na equação $d\vec{E}(dr)$:

$$d\vec{E}(dr) = \frac{dQ(dr)}{4\pi\epsilon_0} \cdot \frac{1}{(D^2+r^2)} \cdot \frac{D\hat{k}-\vec{r}}{\sqrt{D^2+r^2}}$$

$$d\vec{E}(dr) = \frac{1}{4\pi\epsilon_0} \cdot \frac{Q(dr)}{2\pi} d\varphi \cdot \frac{(D\hat{k}-\vec{r})}{(D^2+r^2)^{3/2}}$$

$$d\vec{E}(dr) = \frac{Q(dr)}{8\pi^2\epsilon_0} \frac{(D\hat{k}-\vec{r})}{(D^2+r^2)^{3/2}} d\varphi$$

e



$$\vec{r} = \vec{r}_x + \vec{r}_y$$

$$\vec{r} = r\cos(\varphi)\hat{i} + r\,\mathrm{sen}(\varphi$$

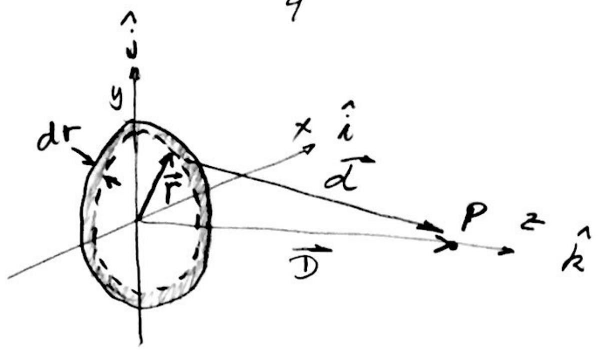
$$\vec{E}(dr) = \frac{Q(dr)}{8\pi^2\epsilon_0} \cdot \frac{1}{(D^2+r^2)^{3/2}} \int_0^{2\pi} (D\hat{k} - \vec{r})\, d\varphi$$

$$\vec{E}(dr) = \frac{Q(dr)}{8\pi^2\epsilon_0} \frac{1}{(D^2+r^2)^{3/2}} \left[D\hat{k} \int_0^{2\pi} d\varphi - r \int_0^{2\pi} (\cos\varphi\, \hat{i} + \mathrm{sen}\varphi\, j) \right.$$

$D\hat{k}$ não é função de φ!

$$\int_0^{2\pi} (\cos\varphi\,\hat{i} + \operatorname{sen}\varphi\,\hat{j})\,d\varphi = \cancel{\operatorname{sen}\varphi\,\hat{i}\Big]_0^{2\pi}}^{\,0} - \cancel{\cos\varphi\,\hat{j}\Big]_0^{2\pi}}^{\,0}$$

$$\vec{E}(dr) = \frac{Q(dr)}{\cancel{8}_4\pi^{\cancel{2}}\epsilon_0}\cdot\frac{\cancel{2}\cancel{\pi}D}{(D^2+r^2)^{3/2}}\cdot\hat{k} = \boxed{\frac{Q(dr)}{4\pi\epsilon_0}\,\frac{D}{(D^2+r^2)^{3/2}}\cdot\hat{k}}$$



$$d\vec{E}(p) = \vec{E}(dr)$$

$$d\vec{E}(p) = \frac{dQ}{4\pi\epsilon_0}\cdot\frac{D}{(D^2+r^2)^{3/2}}\,\hat{k}$$

no disco:

$$\sigma = \frac{Q}{S} = \frac{Q}{\pi r^2}$$

$S = \pi r^2$ c/ $dr \to 0$, $r \to R$

no anel:

$$dS = 2\pi r\,dr$$

$$Q(dr) = dQ = \sigma\,dS$$

$$dQ = \sigma\cdot 2\pi r\,dr$$

$$d\vec{E}(p) = \frac{\sigma\,\cancel{2\pi}\,r\,dr}{\cancel{4\pi}_2\,\epsilon_0}\,\frac{D}{(D^2+r^2)^{3/2}}\,\hat{k} = \frac{D\hat{k}}{2\epsilon_0}\,\sigma\,\frac{r}{(D^2+r^2)^{3/2}}\cdot dr$$

$$\vec{E}(p) = \frac{D\hat{k}}{2\epsilon_0}\,\sigma\left[\int_0^R \frac{r\,dr}{(D^2+r^2)^{3/2}}\right]$$

$$\boxed{\begin{aligned} Z &= D^2 + r^2 \\ dZ &= 2r\,dr \\ r\,dr &= \frac{dZ}{2} \end{aligned}}$$

$$\frac{1}{2}\int Z^{-3/2}\,dZ = \frac{1}{2}\,\frac{Z^{(-3/2+1)}}{(-3/2+1)}$$

$$= -\frac{1}{\sqrt{Z}}$$

$$\vec{E}(p) = \frac{\sigma}{2\epsilon_0}D\left[-\frac{1}{\sqrt{D^2+r^2}}\right]_{r=0}^{r=R}\cdot\hat{k}$$

$$\vec{E}(p) = \frac{\sigma}{2\epsilon_0} D \left[\frac{-1}{\sqrt{D^2+R^2}} - \left(\frac{-1}{\sqrt{D^2}} \right) \right]$$

$$\vec{E}(p) = \frac{\sigma}{2\epsilon_0} \left[\frac{\cancel{D}}{\sqrt{\cancel{D^2}}} - \frac{\cancel{D}}{\sqrt{\cancel{D^2}\left(1+\frac{R^2}{D^2}\right)}} \right]$$

$$\vec{E}(p) = \frac{\sigma}{2\epsilon_0} \left[1 - \frac{1}{\sqrt{1+\frac{R^2}{D^2}}} \right] \hat{k}$$

(1º caso) : se $D >> R$:

$$\left[1 - \frac{1}{\sqrt{1+\frac{R^2}{D^2}}} \right] \cong \frac{R^2}{2D^2}$$

$$\vec{E}(p) = \frac{1}{2\epsilon_0} \left(\frac{Q}{\pi \cancel{R^2}} \right) \cdot \frac{\cancel{R^2}}{2D^2} = \boxed{\frac{Q}{4\pi\epsilon_0} \frac{1}{D^2} \cdot \hat{k}}$$

lei de Coulomb.

(2º caso) : se $D \to 0$

$$\left(\frac{1}{\sqrt{1+\frac{R^2}{D^2}} \to \infty} \right) \to 0$$

$$\boxed{\vec{E}(p) = \frac{\sigma}{2\epsilon_0}}$$