Leading Indicator Variables, Performance Measurement, and Long-Term Versus Short-Term Contracts

SUNIL DUTTA* AND STEFAN REICHELSTEIN†

Received 24 June 2002; accepted 19 June 2003

ABSTRACT

In this article we develop a multiperiod agency model to study the role of leading indicator variables in managerial performance measures. In addition to the familiar moral hazard problem, the principal faces the task of motivating a manager to undertake “soft” investments. These investments are not directly contractible, but the principal can instead rely on leading indicator variables that provide a noisy forecast of the investment returns to be received in future periods. Our analysis relates the role of leading indicator variables to the duration of the manager’s incentive contract. With short-term contracts, leading indicator variables are essential in mitigating a holdup problem resulting from the fact that investments are sunk at the end of the first period. With long-term contracts, leading indicator variables will be valuable if the manager’s compensation schemes are not stationary over time. The leading indicator variables then become an instrument for matching the future investment return with the current investment expenditure. We identify conditions under which the optimal long-term contract induces larger investments and less reliance on the leading indicator variables as compared with short-term contracts. Under certain conditions, though, the principal does better with a sequence of one-period contracts than with a long-term contract.

*University of California, Berkeley; †Stanford University. We would like to thank seminar participants for their comments at the Universities of British Columbia, California at Los Angeles, Toulouse, Chicago-Minnesota Theory Mini-Conference, and the 2002 Management Accounting Section Conference. We are particularly grateful to Shane Dikolli, Jerry Feltham, Jon Glover, and an anonymous reviewer for useful suggestions.
1. Introduction

In the search for improved managerial performance measures, it appears that an increasing number of firms view accounting data as only one among several sources of information that are useful for aligning the long-run objectives of shareholders and managers.\(^1\) One indication of the trend toward broader performance measures is the increasing reliance on stock grants and stock options in compensation packages for top-level managers.\(^2\) Although stock price is acknowledged to be a noisy and imperfect performance indicator that reflects numerous factors beyond management’s control, the informational appeal of stock prices is that they project future financial outcomes resulting from management’s current and past decisions.\(^3\)

Another illustration of the trend toward broader performance measures is provided by the balanced scorecard concept (see Kaplan and Norton [1996, 2001]). In addition to current accounting data, balanced scorecards report on a range of other variables pertaining to the firm’s customer base, internal process efficiency, and organizational growth. These variables include both financial and nonfinancial indicators, and some are measured subjectively (e.g., customer satisfaction) rather than objectively (e.g., cost reductions, defective output).

Proponents of a broader approach to performance measurement cite several advantages for the inclusion of nonaccounting variables. In particular, they point to the fact that many of the variables typically reported in a balanced scorecard are leading indicators of future profitability.\(^4\) From an optimal contracting perspective, the natural question is whether performance measures become more effective by including imperfect indicators of future profitability.\(^5\)

A standard result in the agency literature is that any informative signal is valuable for contracting purposes (Holmstrom [1979]). This result suggests that irrespective of their noisiness, leading indicator variables, such as stock or nonfinancial variables, are useful for providing managerial incentives.\(^6\) In a multiperiod agency setting, however, this argument requires further

---

\(^1\) For a survey on trends in performance measurement, see Ittner and Larcker [1998a].


\(^3\) The agency models of Paul [1992], Bushman and Indjejikian [1993], Kim and Suh [1993], and Dutta and Reichelstein [2002a] analyze the relative weights to be placed on accounting income and stock price in a manager’s performance measure.

\(^4\) Kaplan and Norton [1996] also argue that balanced scorecards are a means of quantifying particular goals at different levels of the organization and that such quantification facilitates the implementation of corporate strategy.

\(^5\) Hauser, Simester, and Wernerfelt [1994], Ittner, Larcker and Rajan [1997], and Banker, Potter, and Srinivasan [2000] provide evidence on the use of various nonfinancial variables in managerial incentive schemes.

\(^6\) When different signals are aggregated linearly, it is generally optimal to choose the relative weights on the signals according to their signal-to-noise ratio (see Banker and Datar [1989], Lambert [2001], Datar, Kulp, and Lambert [2001]).
examination: leading indicator variables may provide additional information at an early stage but this information is supplanted by the actual results at a later stage. Thus, we ask whether an optimal performance measure must include a noisy forecast of future cash flows if receipt of the actual cash flows in the future can be adequately rewarded at that time.

One would expect the need for leading indicator variables to depend on the parties’ planning horizons and on their abilities to make long-term contractual commitments. To address this question formally, we develop a two-period model in which both investments and managerial effort are hidden information variables. Although investments do not impose a personal cost on the manager, the principal nonetheless faces an induced incentive problem because the periodic cash flows reflect both the investment decision and the agent’s managerial effort.

We find that leading indicators can be valuable even under ideal contracting circumstances: the principal and the agent are equally patient and they can commit to long-term contracts. The role of the leading indicator variable then is to provide matching between the first-period investment and the future cash returns of that investment. Such intertemporal matching is useful for the principal to separate the investment problem from the periodic moral hazard problems. In the extreme case in which the leading indicator variable provides a perfect forecast of the second-period cash return, the principal would obtain perfect matching by recognizing the net present value of the investment entirely in the first-period performance measure. As a consequence, the agent would have the desired investment incentives irrespective of the compensation schemes used in the two periods. Clearly, such incentives could not be attained by contracts based only on cash flows.

Our analysis predicts that leading indicator variables are not needed for performance-evaluation purposes when the parties enter into long-term contracts and the underlying moral hazard problems are stationary over time. The principal can then choose identical compensation schemes to address optimally the moral hazard problem in each period. As a consequence, the manager will correctly internalize the future cash returns of current investment expenditures without reliance on the noisy leading indicator variable.

---

7 Sliwka [2002] concludes that nonfinancial performance indicators are not needed with full commitment to a long-term contract. Dikolli [2001] demonstrates a need for nonfinancial variables in a setting where the manager is more impatient than the principal. We discuss these studies in more detail later.

8 This matching function of leading indicator variables is related to recent work on the role of accruals in performance measures. For instance, Reichelstein [2000] and Dutta and Reichelstein [2002b] show that it is preferable to generate investment incentives by using performance measures based on depreciation charges versus performance measures based only on cash flows. Although these studies consider long-term contracts, Wagenhofer [2003] obtains a similar conclusion in a setting in which the manager quits his or her job before all investment returns are received.
When the principal does not commit to a long-term contract but instead offers a sequence of one-period contracts, leading indicator variables become crucial for generating any investment incentives. Short-term contracting leads the principal to act in a sequentially rational fashion. In particular, the second-period incentive contract will be chosen opportunistically to appropriate all returns stemming from investments made in the first period. Anticipating this, the agent would not want to make any investments if the first-period incentive scheme were based on cash flow only. Inclusion of the leading indicator variable allows the principal to mitigate the holdup problem. Because of the noisiness of this variable, however, the optimal short-term contract does not completely alleviate the underinvestment problem.

Fudenberg, Holmstrom, and Milgrom [1990] identify sufficient conditions for short-term contracts to achieve the same performance as long-term contracts. The nature of our investment variable violates their sufficient conditions because the impending investment return is not public information at the beginning of the second period. We find that a potential advantage of long-term commitments is that investment incentives can be generated more cheaply. With a sequence of one-period contracts, any investment incentives derive from the weight placed on the leading indicator variable. In contrast, with a long-term contract managers are motivated to invest partly because their bonus in the second period is responsive to the investment returns received in that period.

It is somewhat surprising that under certain conditions the principal is better off not entering into a long-term contract with the manager. When long-term contracts entail overinvestment due to less severe moral hazard problems in future periods, the principal does better with a sequence of one-period contracts that entail managerial rotation; that is, a new manager is hired in the second period. As a consequence, it becomes possible to contain the agent's tendency to overinvest without having to compromise the periodic incentive provisions. We note that the usual replication argument, according to which the principal makes a long-term commitment to the same actions and contracts he or she would have chosen in equilibrium under short-term contracting, cannot be made in our setting. Short-term contracting requires managerial rotation, yet, by definition, this strategy is

---

9 In all other respects our setting conforms to that of Fudenberg, Holmstrom, and Milgrom [1990], though our analysis is more restrictive because we confine attention to a multiperiod LEN model (i.e., linear contracts, exponential utility, and normally distributed noise terms).

10 One would also expect that, as compared with short-term contracting, long-term contracting results in larger investments and less reliance on the leading indicator variable. We confirm this intuition for settings in which the underlying moral hazard problems become more severe over time. For the opposite setting, we obtain the counterintuitive finding that short-term contracting may lead to larger investments. This possibility again reflects the interaction between the investment and the moral hazard problems. Under long-term contracting the principal may find it preferable to focus on the periodic moral hazard and, in the process, to neglect the investment incentives.
not feasible if the parties commit to a long-term contract at the outset.

Among earlier agency models addressing the need for nonfinancial performance indicators, Dikolli [2001] examines a one-period setting in which some component of the agent’s action results in delayed outcomes. If the agent discounts later payments more heavily than the principal, it becomes advantageous to reward the agent based on a current but noisy indicator of the future outcome. In the context of a two-period model, Śliwka [2002] argues that optimal contracts cannot be based on noisy leading indicator variables provided the principal can make long-term commitments. However, leading indicator variables may become useful if the principal is confined to renegotiation-proof contracts.11

The remainder of the paper is organized as follows. Following the description of the model in the next section, we examine full-commitment contracts in section 3. Our results show that optimal contracts must include the leading indicator variable and that the agent’s ability to commit to long-term contracts is of no consequence to the principal. We analyze short-term contracts in section 4, and we compare the performance attainable under the two contracting scenarios in section 5. We examine the effects of renegotiation of long-term contracts in section 6 and conclude in section 7.

2. The Model

We consider a two-period contracting problem between a risk-neutral principal and a risk-averse agent (manager). In each period $t \in \{1, 2\}$, the manager contributes to the firm’s current operating cash flow through personally costly effort $a_t$. In addition, the manager makes an investment decision in the first period. This investment requires a cash outflow of $b$ in the first period and results in a cash inflow of $m(b)$ in the second period. The investment payoff function $m(\cdot)$ is assumed to be concave such that $m'(b) \to \infty$ as $b \to 0$, and $m'(b) \to 0$ as $b \to \infty$. The observed cash flows in the two periods are:

$$
\tilde{c}_1 = a_1 - b + \tilde{\epsilon}_1,
$$

and

$$
\tilde{c}_2 = a_2 + m(b) + \tilde{\epsilon}_2,
$$

where the random variables $\tilde{\epsilon}_1$ and $\tilde{\epsilon}_2$ represent uncertain events beyond the manager’s control.

For contracting purposes, the principal can rely on the realized cash flow in each period. We assume that the principal does not observe the agent’s choice of investment expenditures $b$. This assumption reflects the notion

11 Because Śliwka’s [2002] findings are seemingly at odds with our results, we discuss the differences between the models and the analysis in section 6.
that for certain soft investments, such as product and process improvements or personnel training, it is frequently difficult for the accounting system to separate investment expenditures from ordinary operating expenses.\textsuperscript{12} At the end of the first period, the parties can rely on an unbiased forecast of the investment payoff $m(b)$ to be realized in the second period. This leading indicator, $f$, which is assumed to be verifiable and contractible, can be thought of as a nonfinancial performance variable. Applicable examples include measures of customer satisfaction, product quality, product awareness, on-time delivery, and so on.\textsuperscript{13} For simplicity, we assume that $f$ takes the form:

$$\bar{f} = m(b) + \bar{\delta}. \tag{3}$$

The unbiased noise term $\bar{\delta}$ reflects residual uncertainty about the future investment returns.

For reasons of tractability, we adopt a multiperiod LEN framework; that is, we confine attention to linear contracts, exponential utility, and normally distributed noise terms. In particular, the random variables $\bar{\delta}$ and $\tilde{\varepsilon}_t$ are assumed to be independent and normally distributed such that $\tilde{\varepsilon}_t \sim N(0, \sigma^2_t)$ and $\bar{\delta} \sim N(0, \mu^2)$.

The risk-neutral principal seeks to maximize the present value of future expected cash flows net of compensation payments. The manager is risk averse and his or her preferences can be described by an additively separable exponential utility function of the form:

$$U_0 = -\sum_{i=1}^{\infty} \gamma^i \cdot \exp\{-\hat{\rho} \cdot (\phi_t - e(a_t))\}, \tag{4}$$

where $\gamma \equiv \frac{1}{1+r}$ denotes the discount factor. In each period, the manager’s current utility depends on his or her current consumption of money $\phi_t$ and the cost of current effort $e(a_t)$. The function $e(\cdot)$ is increasing and convex with $e'(0) = 0$ and $e'(a) \to \infty$ as $a \to \infty$. Finally, the coefficient $\hat{\rho}$ represents the agent’s degree of absolute risk aversion.

It should be noted that in our model formulation the moral hazard problems are stationary except for possible differences in the variances of $\tilde{\varepsilon}_1$ and $\tilde{\varepsilon}_2$. The difference in these variances is, in effect, our proxy variable for intertemporal changes in the moral hazard problem.\textsuperscript{14}

Consistent with the earlier literature on repeated agency models, the agent is assumed to have access to third-party banking. Specifically, the agent

\textsuperscript{12} Kaplan and Norton [1996] cite the increasing need for soft investments (as opposed to hard investments in plant, property, and equipment) as one reason current accounting data are insufficient for performance measurement.

\textsuperscript{13} Recent empirical work by Ittner and Larcker [1998] and Nagar and Rajan [2001] documents that improvements in customer satisfaction or product quality do indeed translate into higher future revenues.

\textsuperscript{14} Alternatively, we could have allowed for intertemporal variations in the agent’s productivity of effort or for intertemporal variations in the cost of effort.
can borrow and lend in each period at the principal’s interest rate $r$. Denoting the compensation payment in period $t$ by $s_t$ and the agent’s savings at date $t$ by $W_t$, consumption is given by

$$\phi_t = s_t + (1 + r) \cdot W_{t-1} - W_t.\quad (5)$$

Access to third-party banking for the agent ensures that the choice of incentive scheme does not need to be concerned with smoothing the agent’s consumption over time. In combination with additively separable exponential utility, these two assumptions imply that in any period $t$ the agent’s preferences over alternative incentive schemes are independent of his or her current wealth $W_{t-1}$. We therefore set $W_0 = 0$, without loss of generality. On leaving the firm, the agent is assumed to be able to earn a net wage of zero; that is, he or she can earn a fixed wage $\hat{s}_t$ by exerting effort $\hat{a}_t$, such that $\hat{s}_t - \epsilon_t(\hat{a}_t) = 0$.

As part of the LEN framework, we restrict our attention to linear compensation schemes of the form:

$$s_t = \alpha_t + \beta_t \cdot \pi_t,\quad (6)$$

where $\pi_t$ is the manager’s performance measure in period $t$. The performance measure can be based on any linear combination of the available information variables at date $t$. In particular, $\pi_1$ can depend on $c_1$ and $f$, and $\pi_2$ can depend on $c_1$, $c_2$, and $f$. The parameters $\alpha_2$ and $\beta_2$ of the second-period compensation scheme can then without loss of generality be chosen independently of the history.

The weight on the nonfinancial indicator $f$ in an optimal performance measure is likely to depend on the abilities of the principal and the manager to make long-term contractual commitments. Our analysis considers three commitment scenarios: (1) both the agent and the principal commit to long-term contracts, (2) the principal commits to long-term contracts but the agent does not, and (3) both the principal and the agent commit only to short-term (i.e., one-period) contracts. The time line in figure 1 illustrates the sequence of events when the two parties sign short-term contracts. Under long-term contracting, the sequence of events is also described by figure 1 except that no new contract would be offered at date 1.

3. Long-Term Contracts

We suppose initially that the parties sign a two-period contract at the outset without the possibility of renegotiation. Given such lock-in contracts
and our LEN framework, there is no loss of generality in restricting attention to performance measures of the form:

$$\pi_t = c_t + u_t \cdot f.$$  \hfill (7)

Here, $u_t$ denotes the coefficient on the leading indicator $f$, whereas the coefficients for the cash flows $c_t$ are normalized to one. We note that the performance measures in (7) do not allow for $c_1$ to be included in $\pi_2$. This specification is without loss of generality because the agent has equal access to credit, and with full commitment both parties are indifferent as to whether the agent’s reward for $c_1$ is partly delayed to the second period.\(^{15}\)

**Lemma 1.** For any long-term contract of the form in equations (6) and (7), the certainty equivalent of the agent’s expected utility at date $0$ is given by:

$$CE_0 = \gamma \cdot \left[ \alpha_1 + \beta_1 \cdot (a_1 - b + u_1 \cdot m(b)) - e(a_1) - \rho \cdot \beta_1^2 \cdot \sigma_1^2 \right. \\
\left. + \gamma^2 \cdot \left[ \alpha_2 + \beta_2 \cdot (a_2 + (1 + u_2) \cdot m(b)) - e(a_2) - \rho \cdot \beta_2^2 \cdot \sigma_2^2 \right], \right.$$

where $\rho \equiv \frac{1}{2} \cdot \hat{\rho} \cdot (1 - \gamma).$\(^{16}\)

Lemma 1 shows that the certainty equivalent of the agent’s expected utility is given by the present value of mean-variance expressions generated by the incentive schemes for each period. In particular, we note that the manager has the same time horizon as the principal and the parties are equally patient. These model specifications make the strongest possible case in favor of waiting for the actual investment returns in the second period rather than including the noisy leading indicator variable $f$ in the performance measure.\(^{17}\)

The risk-neutral principal seeks to maximize the present value of future cash flows net of compensation expenses, and therefore the principal’s optimization problem is:

$$\max_{[\alpha_t, \beta_t, u_t]} \sum_{t=1}^{2} E[\tilde{c}_t - \tilde{s}_t] \cdot \gamma^t$$

subject to:

\(^{15}\) The same argument can be made regarding the use of $f$ in $\pi_2$. However, our results in Proposition 2 not only show that the coefficient $u_2$ could be set equal to zero but that in fact it must be zero if the agent’s participation constraint is to hold at the beginning of the second period.

\(^{16}\) All proofs are provided in the Appendix.

\(^{17}\) In contrast, Dikolli [2001] examines a setting in which the agent is intrinsically more impatient than the principal. Similarly, in Wagenhofer [2003], the agent has a relatively short horizon because he or she leaves the firm for exogenous reasons before the completion of the project.
LEADING INDICATOR VARIABLES

(i) \( CE_0 \geq 0 \),

(ii) \( a_t \in \text{argmax}_\beta \{ \beta_t \cdot \tilde{a}_t - e(\tilde{a}_t) \} \) for each \( t \),

(iii) \( b \in \text{argmax}_\beta \{ \beta_1 \cdot (u_1 \cdot m(b) - \tilde{b}) + \gamma \cdot \beta_2 \cdot (1 + u_2) \cdot m(b) \} \).

The constraints in this optimization program reflect the agent’s initial participation constraint and the incentive compatibility conditions with regard to the effort and investment choices, respectively. In the current LEN framework, these incentive compatibility constraints can be replaced with the corresponding first-order conditions:

\[
\beta_t = e'(a_t) \tag{9}
\]

for each \( t \), and

\[
\begin{bmatrix}
  u_1 + \gamma \cdot \frac{\beta_2}{\beta_1} \cdot u_2
\end{bmatrix} = \frac{1}{m'(b)} - \gamma \cdot \frac{\beta_2}{\beta_1}. \tag{10}
\]

The left-hand side in (10) is the effective coefficient on the leading indicator variable. Because the parties are assumed to commit to a long-term contract, only the effective coefficient matters while the individual \( u_1 \) and \( u_2 \) remain indeterminate.

Optimality requires that the agent’s participation constraint hold as an equality; therefore, the present value of the fixed payments, \( \gamma \cdot \alpha_1 + \gamma^2 \cdot \alpha_2 \), is chosen so that \( CE_0 = 0 \). By \( a_t(\beta_t) \) we denote the effort choice induced by \( \beta_t \), that is, \( \beta_t = e'(a_t(\beta_t)) \). The principal’s problem in (8) can therefore be restated as an unconstrained optimization problem in which \( \beta_1, \beta_2 \), and \( b \) are chosen to maximize:

\[
V^*(\beta_1, \beta_2, b) \equiv Y_1(\beta_1) + Z^*(\beta_1, \beta_2, b) + \gamma \cdot Y_2(\beta_2), \tag{11}
\]

where

\[
Y_t(\beta_t) \equiv a_t(\beta_t) - e(a_t(\beta_t)) - \rho \cdot (e'(a_t(\beta_t)))^2 \cdot \sigma_i^2
\]

and

\[
Z^*(\beta_1, \beta_2, b) \equiv \gamma \cdot m(b) - b - \rho \cdot \beta_2^2 \cdot \left( \frac{1}{m'(b)} - \gamma \cdot \frac{\beta_2}{\beta_1} \right)^2 \cdot \mu^2.
\]

The function \( Y_t(\cdot) \) represents the principal’s expected net return from the agent’s productive effort in period \( t \), and \( Z^*(\cdot) \) represents the net present value of investments including the risk premium required to induce those investments.\(^\text{18}\) The first-best investment level will be denoted by \( b^0 \); that is, \( b^0 \) maximizes \( \{ \gamma \cdot m(b) - b \} \). Clearly, \( Z^*(\cdot) \) is maximized at \( b^0 \) if \( \beta_1 = \beta_2 \). Conversely, if \( \beta_1 \neq \beta_2 \) the optimal investment does not coincide with \( b^0 \) because \( \gamma \cdot m'(b^0) = 1 \). Thus, investments do entail an indirect agency cost except for the special case in which the bonus coefficients are constant over time.

\(^{18}\)We assume throughout that \( Y_t(\cdot) \) is a single peaked function of \( \beta_t \). This condition will be satisfied provided the effort function \( e(\cdot) \) has a non-negative third derivative.
With long-term contracts, one may suspect that there is no need to include the leading indicator variable \( \tilde{f} = m(b) + \tilde{\delta} \) in the agent’s performance measure. Because this signal provides a noisy forecast of the future cash flow, it may be preferable for the principal to wait for the actual, and noiseless, cash return \( m(b) \) in the second period. The following result shows that this intuition holds only when the underlying moral hazard problems do not change over time. Otherwise, the noisy leading indicator variable is indeed valuable because it would be more costly for the principal to disentangle the investment problem from the moral hazard problems using a performance measure based only on cash flows.

**Proposition 1.** An optimal long-term contract sets \((u_1^*, u_2^*) = (0, 0)\) if and only if the noise terms \( \tilde{\epsilon}_1 \) and \( \tilde{\epsilon}_2 \) have the same variance.

Because investments are not personally costly to the manager, the investment decision is subject to an induced rather than an intrinsic incentive problem. When the moral hazard problems in the two periods are identical, in the sense that \( \tilde{\epsilon}_1 \) and \( \tilde{\epsilon}_2 \) have the same variance, the principal can choose identical bonus coefficients for the two periods with the consequence that the manager will make first-best investments. For such stationary problems, optimal performance measures can thus be based on the realized cash flows only.

In contrast, if the moral hazard problems differ across the two periods (because of different variances for \( \tilde{\epsilon}_1 \) and \( \tilde{\epsilon}_2 \)), incentive schemes based only on cash flows will become overloaded. Irrespective of its noisiness, the leading indicator variable now becomes valuable because it provides an additional instrument for separating the investment problem from the moral hazard problems.

The leading indicator variable may be viewed as an accrual that matches in the first period the investment expenditure with the second-period cash return. This matching function becomes perfect in the extreme case where \( f \) is noiseless, that is, \( \text{Var}(\tilde{\delta}) = 0 \). The optimal weights on \( f \) then are \((u_1, u_2) = (\gamma, -1)\). Irrespective of the choice of bonus coefficients, the agent will now invest the optimal amount: the performance measure in the first period, \( \pi_1 = a_1 - b + \epsilon_1 + \gamma \cdot m(b) \), is congruent with the principal’s objective, whereas the second-period performance measure is independent of \( b \).

To explore the importance of bilateral commitments to long-term contracts, we first consider a scenario in which the principal can credibly commit to a two-period contract, but the agent is unable to do so. Instead, the agent can seek alternative employment at the end of the first period. A contract that induces the agent to stay must then also satisfy the date 1 interim participation constraint:

\[
CE_1(b, f) = \gamma \cdot \left[ \alpha_2 + \beta_2 \cdot (a_2 + m(b) + u_2 \cdot f) - e(a_2) - \rho \cdot (\beta_2 \cdot \sigma_2)^2 \right] \geq 0.
\]

Here, \( CE_1(b, f) \) denotes the certainty equivalent of the manager’s date 1 expected utility as a function of his or her investment choice \( b \) and the
realized value of the leading indicator \( f \). Because \( \tilde{f} = m(b) + \tilde{\delta} \) and \( \tilde{\delta} \) is normally distributed, the interim participation constraint requires that \( CE_1(b, f) \geq 0 \) for any realization of the signal \( f \). That implies \( u_2 = 0 \), yet this restriction is of no consequence to the principal. It is immediately seen from the incentive compatibility condition in (10) and the agent’s mean-variance preferences in Lemma 1 that only the aggregate coefficient \( u_1 + \gamma \cdot \beta_{1}^{*} \cdot u_2 \) matters, whereas the individual values of \( u_1 \) and \( u_2 \) remain indeterminate when both parties commit to a two-period contract.

**COROLLARY TO PROPOSITION 1.** Imposing an interim participation constraint changes neither the principal’s expected payoff nor the induced level of investment \( b^* \) under long-term contracting. Given the interim participation constraint, the coefficient on the leading indicator variable \( u_2 \) must be zero.

In many situations it will be difficult for managers to commit at the outset to staying in jobs they would rather quit after the realization of unfavorable events. Because the corresponding interim participation constraint is costless for the principal, we focus on long-term contracts that satisfy this constraint.

A standard finding in hidden action models is that optimal incentive provisions induce action choices that are less than first best. Similarly, the primary issue in incomplete contracting problems with noncontractible investments is that agents have a tendency to underinvest.\(^{19}\) In our setting, distortions in the second-best investment can go either way because investments are not personally costly to the manager. The following result shows that the direction of any distortions depends on intertemporal changes in the moral hazard problem.

**PROPOSITION 2.** The optimal coefficient on the leading indicator variable in the first period satisfies:

\[
    u_1^* = \frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_{2}^*}{\beta_{1}^*}.
\]

If \( \text{Var}(\tilde{\varepsilon}_1) < \text{Var}(\tilde{\varepsilon}_2) \), long-term contracting results in underinvestment and the optimal coefficient on the leading indicator \( u_1^* \) is positive. If \( \text{Var}(\tilde{\varepsilon}_1) > \text{Var}(\tilde{\varepsilon}_2) \), long-term contracting results in overinvestment and the optimal \( u_1^* \) is negative.

When the hidden action problem is relatively more severe in the second period (\( \tilde{\varepsilon}_2 \) has higher variance than \( \tilde{\varepsilon}_1 \)), an optimal contract will set \( \beta_{1}^* > \beta_{2}^* \). This difference in bonus coefficients leaves the agent with a tendency to underinvest. The principal can mitigate this bias by attaching a positive weight to the leading indicator variable. Because of the noisiness of \( \tilde{f} \), it would be too costly to alleviate the underinvestment problem entirely.

The interaction between the investment and moral hazard problems makes it desirable for the principal to compromise on both problems. To

\(^{19}\) See Tirole (1999) for a survey of recent work on incomplete contracts.
illustrate, let $\beta^0_t$ denote the maximizer of the function $Y_t(\beta_t)$. Absent the investment problem, $\beta^0_t$ would therefore be the optimal bonus coefficient in period $t$. If the principal were to set $\beta_1 = \beta^0_1 > \beta_2 = \beta^0_2$, a small decrease in $\beta_1$ would result in a second-order loss for $Y_t(\cdot)$ but in a first-order gain for $Z^*(\beta_1, \beta_2, b)$. Thus, it would also be too costly to implement second-best solutions to the moral hazard problems. The case where $\text{Var}(\tilde{\varepsilon}_1) > \text{Var}(\tilde{\varepsilon}_2)$ can be argued symmetrically.

When the hidden action problems are stationary over time—that is, when $\text{Var}(\tilde{\varepsilon}_1) = \text{Var}(\tilde{\varepsilon}_2)$—the optimal contract induces the first-best investment without reliance on the leading indicator. It is instructive to interpret this result in relation to the multitask agency literature that has examined the congruency and precision of performance measures (see Datar, Kulp, and Lambert [2001], Feltham and Xie [1994]). We note that in our long-term contracting framework, a perfectly congruent performance measure, namely the compounded value of the cash flows (i.e., $(1 + r) \cdot c_1 + c_2$) is always available for incentive contracting purposes. Furthermore, because the investment incentive problem is an induced one, the manager will completely internalize the principal’s investment objectives if he or she is compensated solely on the basis of the compounded value of future cash flows. This implies that whenever $(1 + r) \cdot c_1 + c_2$ is a sufficient statistic for $(c_1, c_2)$ with respect to the manager’s action choices $(a_1, a_2)$, it is also a sufficient statistic for $(c_1, c_2, f)$ with respect to $(a_1, a_2, b)$.

Because the performance measures $(c_1, c_2)$ are informationally equivalent to $((1 + r) \cdot c_1 + c_2, c_2)$, the question becomes when is the second-period cash flow incrementally useful given the measure $(1 + r) \cdot c_1 + c_2$? In a multitask setting, an additional signal can be useful for two reasons: (1) it makes the performance measure more congruent with the principal’s objective, and (2) it reduces risk in the manager’s compensation. Given that the compounded value of cash flows $(1 + r) \cdot c_1 + c_2$ is perfectly congruent with the principal’s objective, the additional signal $c_2$ will be useful if and only if the principal seeks to induce different amount of efforts in the two periods, which will be the case if $c_1$ and $c_2$ have different variances. In contrast, when $\text{Var}(\tilde{\varepsilon}_1) = \text{Var}(\tilde{\varepsilon}_2)$, the compounded value of cash flows $(1 + r) \cdot c_1 + c_2$ constitutes a sufficient statistic for the pair $(c_1, c_2)$ because the principal optimally seeks to induce identical efforts across two periods.20

Proposition 2 shows that the optimal weight on the leading indicator can be negative. This result hinges on our implicit assumption that the realized value of the leading indicator depends only on the chosen level of investment $b$ but is not subject to other manipulation by the manager. To conclude this section, we consider a scenario in which the manager can costlessly manipulate the leading indicator downward; that is, the manager can always

---

20 Datar, Kulp, and Lambert [2001] examine the optimal weight on an incongruent signal in the presence of a perfectly congruent signal. Our findings are consistent with their characterization of the optimal weights on the two signals.
take measures that lead to underreporting of the leading indicator variable. This alternative scenario appears to be descriptive in many instances. For example, suppose the leading indicator variable represents customer satisfaction as measured through post-sales surveys. It is then conceivable that the manager can easily bias the survey results downward.

**Corollary to Proposition 2.** Suppose the manager can costlessly underreport the leading indicator variable. If \( \text{Var}(\tilde{\varepsilon}_1) < \text{Var}(\tilde{\varepsilon}_2) \), long-term contracting results in underinvestment and the optimal coefficient on the leading indicator \( u^*_1 \) is positive. If \( \text{Var}(\tilde{\varepsilon}_1) > \text{Var}(\tilde{\varepsilon}_2) \), long-term contracting results in overinvestment and the optimal \( u^*_1 \) is zero.

When \( \text{Var}(\tilde{\varepsilon}_1) > \text{Var}(\tilde{\varepsilon}_2) \), the principal will set \( \beta^*_2 > \beta^*_1 \), which induces overinvestment. Given the manager’s ability to bias the outcome of the leading indicator variable, however, the principal can no longer counteract the manager’s overinvestment incentives by putting a negative weight on \( f \). Instead, the principal must now rely exclusively on the choice of bonus parameters \( \beta_1 \) and \( \beta_2 \). As a consequence, the optimal effort incentives across the two periods will be more symmetric (and hence more distorted) than they were in a setting where the manager could not manipulate the realized value of the leading indicator variable.

### 4. Short-Term Contracts

We now turn to a setting in which the principal cannot commit to a two-period contract. In the first period, the principal hires an agent and offers a contract of the form:

\[
s_1 = \alpha_1 + \beta_1 \cdot [c_1 + u_1 \cdot f].
\]

At the end of the first period, the principal is free to contract either with the same or a different agent.\(^{21}\) The second-period contract is chosen optimally given the principal’s information at that stage and his or her conjectures about the first-period decisions. Furthermore, the first-period contract is chosen in anticipation that the second-period contract will be sequentially optimal.

To characterize optimal short-term contracts, it is useful to first consider a managerial rotation setting in which a different agent is hired in the second period. The second agent is offered a contract of the form:

\[
s_2 = \alpha_2 + \beta_2 \cdot c_2.
\]

The second-period contracting problem is a standard one-shot moral hazard problem in the LEN framework with one exception: the performance

\(^{21}\) We assume implicitly that the principal can choose from a competitive fringe of identical agents whose preferences are again described by negative exponential utility.
measure \( c_2 \) depends on the investment decision \( b \) undertaken by the previous agent. The principal and the new agent share identical conjectures about the investment decision made in the first period. Denoting the conjectured investment by \( b^c \), the second agent’s preferences over alternative linear incentive contracts can be represented by the following certainty equivalent expression:

\[
CE(a_2) = \alpha_2 (b^c) + \beta_2 \cdot [a_2 + m(b)] - e(a_2) - \rho \cdot \beta_2^2 \cdot \sigma_2^2,
\]

(12)

The principal adjusts the fixed payment \( \alpha_2 (b^c) \) to account for the fact that the second-period cash flow will be shifted by \( m(b^c) \). Provided that the conjecture is rational (i.e., \( b^c = b \)), both the principal and the first agent are indifferent as to whether the second-period contract is offered to the incumbent agent or to a different agent.

Because managerial rotation is rationally anticipated by both parties, any investment incentives have to be generated in the first period by means of the leading indicator variable. In our LEN framework the first-period agent will again have mean-variance preferences over alternative linear incentive schemes. In particular, the first agent will choose \( a_1 \) and \( b \) to maximize:

\[
\alpha_1 + \beta_1 \cdot [a_1 - b + u_1 \cdot m(b)] - e(a_1) - \rho \cdot \beta_1^2 \cdot [\sigma_1^2 + u_1^2 \cdot \mu^2].
\]

(13)

In contrast to the long-term contracting scenario analyzed in the previous section, the bonus coefficient \( \beta_1 \) now has no bearing on the agent’s investment decision. Obviously, this would not be true if the principal were to retain the same agent for both periods. The following result, however, shows that a managerial rotation policy is essential under short-term contracts.

**Lemma 2.** Under short-term contracting, the principal hires a different agent in the second period.

To see why managerial rotation is essential, consider a case when the same agent is retained over two periods. A solution to short-term contracting must induce agent responses \((a_1, a_2, b)\) satisfying the following conditions:

(i) Given a conjecture \( b^c \) regarding the agent’s actual investment choice, the contract \( s_2(\cdot) \) maximizes the principal’s expected payoff subject to the second-period incentive and participation constraints.

(ii) The principal’s conjecture is rational in the sense that \( b^c = b \). The fixed payment \( \alpha_2 \) is chosen so that the agent breaks even provided the actual level of investment \( b \) is equal to the conjectured level \( b^c \).

(iii) The response \((a_1, b)\) maximizes the agent’s expected utility given the current contract \( s_1(\cdot) \) and the anticipated contract \( s_2(\cdot) \) in the second period.

Because the second-period contract parameters \( \alpha_2 \) and \( \beta_2 \) must be based on the conjectured investment level \( b^c \) rather than the actual \( b \), the incentive
compatibility condition for $b$ becomes:

$$
\beta_1 \cdot [-1 + u_1 \cdot m'(b)] + \gamma \cdot \beta_2 \cdot m'(b) = 0.
$$

As in the long-term contracting problem, the agent’s investment return will again be partly provided through the second-period cash flow. If the first-period contract $s_1(\cdot)$ maximizes the principal’s expected payoff (among all first-period contracts that induce agent responses and second-period contracts that satisfy the preceding sequential rationality requirements), the agent must break even in the first period provided he or she chooses $(a_1, b)$.

For any such arrangement, however, the agent could do better by the following deviation: choose a level of investment $\tilde{b}$ that is myopically optimal for the current period, that is,

$$
-1 + m'(\tilde{b}) \cdot u_1 = 0,
$$

and thereafter reject the second-period contract. Because $\alpha_1$ is chosen to compensate agents for the higher investment level $b$, they would achieve more than their reservation utility in the first period.\(^{22}\) We conclude that there is no solution to the principal’s problem in which the same agent is retained for both periods when $b$ is unobservable.\(^{23}\)

To characterize further the solution to short-term contracting, we note that the second-period compensation parameters $(\alpha_2, \beta_2)$ can be chosen to maximize:

$$
a_2 - E[\tilde{s}_2]
$$

subject to the second-period agent’s incentive and participation constraints. Therefore, the fixed payment $\alpha_2$ will be chosen so that the agent’s certainty equivalent in (12) is zero at the conjectured level of investment $b^c$. Furthermore, the second-period bonus parameter $\beta_2$ will be chosen to maximize $Y_2(\beta_2)$, as defined in connection with the objective function in (11).

For the first-period contract, the certainty equivalent expression in (12) shows that the desired investment expenditure must meet the incentive compatibility condition:

$$
u_1 \cdot m'(b) = 1,
$$

---

\(^{22}\) We note that if it were costly to find a replacement for the incumbent manager, a pure strategy equilibrium would fail to exist. (We thank Jon Glover for bringing this to our attention.) The unique equilibrium then would entail randomization, i.e., the incumbent manager would be replaced with probability less than one. However, it can be shown that as the replacement cost approaches zero, the mixed strategy equilibrium converges to the pure strategy equilibrium in Lemma 2.

\(^{23}\) We note that our short-term contracting framework reflects a complete lack of commitment from either party. In contrast, Feltham, Indjejikian, and Nanda [2003] consider an alternative commitment scenario. In their setting, the principal cannot make long-term commitments but the agent can commit not to leave provided that he or she receives a fair contract in the second period.
and the participation constraint must hold with equality for an optimal contract:

\[ \alpha_1 + \beta_1 [a_1 - b + u_1 \cdot m(b)] - e(a_1) - \rho \cdot \beta_2^2 \left[ \sigma_1^2 + u_1^2 \cdot \mu_1^2 \right] = 0. \]  

(15)

Substituting (14) and (15) into the principal’s objective function yields an unconstrained optimization problem in which \((\beta_1, \beta_2, b)\) are chosen to maximize:

\[ \hat{V}(\beta_1, \beta_2, b) \equiv Y_1(\beta_1) + \hat{Z}(\beta_1, b) + \gamma \cdot Y_2(\beta_2), \]  

(16)

where

\[ \hat{Z}(\beta_1, b) \equiv \gamma \cdot m(b) - b - \rho \cdot \beta_1^2 \cdot \left( \frac{1}{m'(b)} \right)^2 \cdot \mu_1^2. \]

By \((\hat{\beta}_1, \hat{\beta}_2, \hat{b})\) we denote the optimal choice variables for the objective function in (16). Clearly, \(\hat{b} < b^0\) because \(1/m'(b)\) is increasing in \(b\) because of the concavity of the function \(m(\cdot)\). We obtain the following result:\(^{24}\)

**Proposition 3.** Short-term contracting results in underinvestment, that is, \(\hat{b} < b^0\). The optimal coefficient on the leading indicator variable, \(f\), is given by:

\[ \hat{u}_1 = \frac{1}{m'(b)}. \]  

(17)

In sum, we find that short-term contracts differ in two major ways from the long-term incentive provisions characterized in the previous section. First, the bonus parameter of the second-period contract is chosen opportunistically to maximize the expected payoff attainable in that period. Second, the investment incentives have to be provided exclusively through the leading indicator variable (rather than through a combination of the leading indicator and the second-period cash return). In particular, the bonus coefficients now have no bearing on the investment decision.\(^{25}\)

5. *Performance Comparison of Long-Term and Short-Term Contracts*

This section compares long-term and short-term contracts along the following dimensions: (1) the principal’s expected payoff, (2) the

\(^{24}\) The proof of this result is omitted.

\(^{25}\) We note that if the investment decision were observable to the principal but remained unverifiable for contracting purposes, the principal’s expected payoff would remain unchanged. Sequential rationality again compels the principal to select a second-period contract that appropriates the entire investment return to be realized in that period. However, unlike the preceding scenario in which the payments were based on the conjectured investment level, the principal can now rely on his or her observation of the actual investment choice. As a consequence, the same agent can be retained for both periods. In either observability scenario, the investment incentives must be generated exclusively through the leading indicator variable. Thus, the observability of \(b\) does not change the principal’s expected payoff under short-term contracting. However, it is an open question whether the principal can improve his or her expected payoff attainable with long-term commitments when investments are observable but remain unverifiable.
induced level of investment, and (3) the weight on the leading indicator variable in the agent’s performance measure. With regard to the principal’s expected payoff, the comparison between the two contracting scenarios might appear obvious. After all, with long-term contracts the principal always has the option of committing to the same course of action he or she would have taken in equilibrium under short-term contracting. This argument, however, does not apply in our model because the optimal short-term contract requires hiring a different agent in the second period. By definition, this scenario is ruled out under long-term contracting.

Fudenberg, Holmstrom, and Milgrom [1990] identify two sufficient conditions under which long-term and short-term contracts are performance equivalent (see also Salanie and Rey [1990], Chiapori et al. [1994]). First, all public information is contractible, and second, there is no information asymmetry at the time contracts are renegotiated. Because the principal does not observe the agent’s investment choice \( b \), the agent does acquire private information in our model.

Consider first the case where the moral hazard problem is relatively more severe in the second period, that is, \( \text{Var}(\tilde{\epsilon}_1) < \text{Var}(\tilde{\epsilon}_2) \). Propositions 2 and 3 show that in this case both long-term and short-term contracting result in underinvestment. Because long-term contracts can rely on both the leading indicator and the second-period cash returns in generating investment incentives, one might expect long-term contracts to perform better. The following result confirms this intuition.

**Proposition 4.** If \( \text{Var}(\tilde{\epsilon}_1) < \text{Var}(\tilde{\epsilon}_2) \), long-term contracting strictly dominates short-term contracting.

The proof of Proposition 4 is constructive in the sense that any short-term contract can be improved on with a suitable two-period commitment. Given any \( (\hat{\beta}_1, \hat{\beta}_2, \hat{u}) \), suppose first that \( \hat{\beta}_1 > \hat{\beta}_2 \). Under long-term contracting, the principal can always induce the same effort choices and a more efficient investment decision by setting \( \beta^*_1 = \hat{\beta}_1 \) and reducing the weight on the noisy leading indicator to \( \max\{0, \hat{u} - \gamma \cdot \hat{\beta}_2 \} \). Such a compensation scheme strictly dominates the original incentive scheme because, in addition to the more efficient investment choice, the new contract reduces the amount of risk imposed on the agent. Similarly, when \( \hat{\beta}_1 < \hat{\beta}_2 \) the principal can achieve a higher expected payoff by setting \( \beta_1 = \beta_2 = \hat{\beta}_2 \), and \( u = 0 \). This new contract is superior in two respects: (1) it induces first-best investments without imposing risk associated with the noisy signal \( f \), and (2) it improves the agent’s first-period effort choice because \( \hat{\beta}_1 < \beta_1 = \hat{\beta}_2 < \hat{\beta}_1 \).

Because long-term contracts generate investment incentives via the leading indicator as well as the second-period cash returns, one might expect that long-term contracting induces more investment (i.e., less underinvestment). This intuition, however, turns out not to be correct. One can find parameterizations for which the optimal investment under
long-term contracting is less than that under short-term contracting, and vice versa.

To illustrate, we consider the following numerical example. Suppose that $e(a_t) = \frac{1}{2} \cdot a_t^2$, $m(b) = 2 \cdot \gamma^{-1} \cdot \ln(1 + b)$, $\gamma = 0.9$, $\rho = 0.5$, $\sigma_1^2 = 0.3$, and $\sigma_2^2 = 1$. The first-best investment level then is $b^0 = 1$ and the optimal bonus parameters in the absence of the investment incentive problem are $\beta_1^0 = 0.77$ and $\beta_2^0 = 0.5$. When $\mu^2 = 1$, the numerical calculations show that the optimal compensation parameters are $\beta_1^* = 0.68$, $\beta_2^* = 0.49$, and $b^* = 0.92$ under long-term contracting, and $\hat{\beta}_1 = 0.54$, $\hat{\beta}_2 = 0.5$, and $\hat{b} = 0.64$ under short-term contracting. In contrast, when $\mu^2 = 20$, we find that $(\beta_1^*, \beta_2^*, b^*) = (0.34, 0.27, 0.81)$ and $(\hat{\beta}_1, \hat{\beta}_2, \hat{b}) = (0.06, 0.5, 0.87)$. Thus, $b^* > \hat{b}$ when $\mu^2 = 1$, but $b^* < \hat{b}$ when $\mu^2 = 20$.

The surprising finding that short-term contracting can result in larger investments reflects the interaction between the investment and the moral hazard problems in our model. The principal’s marginal cost of inducing investment is lower with long-term contracts for given bonus parameters $\beta_1$ and $\beta_2$. For fixed bonus coefficients, it would therefore indeed be optimal to induce larger investment under long-term contracting. The countervailing effect, however, is that the marginal cost of providing effort incentives is decreasing in $b$. Therefore, it may become preferable to overcompensate along one dimension, for example, set $\beta_1^* > \hat{\beta}_1$ but induce $b^* < \hat{b}$.

To summarize, the advantage of long-term contracts is that the principal can commit to ex post inefficient contracts for the second period to maximize his or her ex ante expected payoff. Specifically, the second-period bonus parameter $\beta_2^*$ under long-term contracting is generally different from its myopically optimal value of $\beta_2^0$. In addition, long-term contracts allow the principal to commit not to appropriate the impending investment returns in the second period. Taken together, these effects explain the dominance result in Proposition 4. With regard to the induced investment level and the weight on the leading indicator variable, however, the comparison is ambiguous across the two contracting scenarios.

Consider now the opposite case where the first-period moral hazard problem is more severe, that is, $\text{Var}(\tilde{\varepsilon}_1) > \text{Var}(\tilde{\varepsilon}_2)$. Propositions 3 and 4 show that long-term contracting unambiguously induces more investment because $b^* > b^0 > \hat{b}$. Furthermore, the optimal weight on the leading indicator is always less under long-term contracting than under short-term contracting because $u^* < 0 < \hat{u}$. Our next result, though, shows that for this setting the principal may be worse off with long-term commitments.26

---

26 Similar effects arise in Feltham, Indjejikian, and Nanda [2003] and Christensen, Feltham, and Sabac [2002]. In the contexts of their models, though, the advantage of short-term contracting is that the first-period outcome is informative about the desired bonus coefficient for the second period.
Proposition 5. For any given $\text{Var}(\tilde{\epsilon}_2)$, short-term contracting strictly dominates long-term contracting for values of $\text{Var}(\tilde{\epsilon}_1)$ sufficiently large.

The marginal cost of providing effort incentives is increasing in $\text{Var}(\tilde{\epsilon}_1)$. Holding all else constant, the optimal bonus parameter $\beta_1$ will therefore be decreasing in $\text{Var}(\tilde{\epsilon}_1)$. As $\beta_1$ is lowered, however, the manager views investments as less expensive. To control the resulting overinvestment problem, the principal may distort the choice of bonus parameters and put a negative weight on the leading indicator variable. Under short-term contracting, in contrast, the agent’s investment incentives are independent of the choice of the bonus parameters $\beta_1$ and $\beta_2$. The principal therefore can contain the tendency to overinvest without having to compromise on the choice of the bonus coefficients in the two periods. Therefore, short-term contracts dominate when the overinvestment problem under long-term contracting becomes sufficiently severe, that is, when $\text{Var}(\tilde{\epsilon}_1)$ becomes sufficiently large.

It is instructive to ask whether the insights of Propositions 4 and 5 are specific to a multiperiod setting or whether our findings are also applicable to coordination issues across multiple organizational subunits within a firm. Specifically, suppose that $c_1$ and $c_2$ are the cash flows obtained by two separate operations. Investment can then be interpreted as the type of cooperative investments analyzed in the incomplete contracting literature (e.g., see Che and Hausch [1999]). The investment creates an externality between the two operations, and the principal could seek to internalize this externality by putting one manager in charge of both operations. The alternative job design structure would put separate managers in charge of each operation (for simplicity, we suppose that the effort costs are additively separable across the two operations).

The preceding analysis suggests that appointing two separate managers might be preferable if the second operation involves higher powered incentives and therefore a single manager might be prone to overinvestment. Yet, the results are not directly applicable if the investment and effort decisions are made simultaneously (if they are made sequentially, the problem again assumes a multiperiod character). In contrast to our analysis, the principal now has the option of using the cash flow from the second operation, $c_2$, as an instrument for providing investment incentives for the manager of the first operation. Like the leading indicator variable $f$, this cash flow provides a noisy measure of the investment return and the findings of the multitask agency literature suggest that both signals are valuable for the principal. It remains to be seen in future research which of these organizational structures emerges as superior depending on the underlying parameters of the problem.

27 We thank an anonymous reviewer for posing this question.

6. Renegotiation of Long-Term Contracts

Our analysis thus far has contrasted long-term with short-term contracts and the role played by leading indicators in the two contracting scenarios. An intermediate case that has received considerable attention in the contracting literature is one in which the principal offers long-term contracts, but he or she cannot commit not to renegotiate such a contract at the beginning of the second period. Whenever the initial contract specifies a bonus coefficient different from $\beta_2^0$ for the second period, the parties could sign a Pareto-improving contract at date 1, because at this stage the investment choice has been made and an optimal contract for the second period requires that $\beta_2 = \beta_2^0$. Following the approach of Fudenberg and Tirole [1990], we assume that the principal has the entire bargaining power and therefore makes a take-it-or-leave-it offer at date 1. Because this renegotiation is anticipated by both parties at the initial date, the principal cannot do better than to offer a renegotiation-proof contract; that is, one that sets $\beta_2 = \beta_2^0$.

**Proposition 6.** Proposition 2 remains valid if the principal is constrained to offer renegotiation-proof long-term contracts.

The renegotiation proofness constraint is costly for the principal because $\beta_2$ is no longer chosen long-term optimal. Yet, our findings in Proposition 2 are unaffected because, despite the renegotiation proofness constraint, it is still true that $\beta_1^* > \beta_2^0$ whenever $\text{Var}(\tilde{\epsilon}_1) < \text{Var}(\tilde{\epsilon}_2)$.

It is instructive to compare our findings with those of Sliwka [2002], who argues that leading indicator variables are not valuable in a full-commitment setting but that they may be useful if contracts are required to be renegotiation proof. In Sliwka’s model the agent exerts operational effort, which affects only the current-period cash flow, and strategic effort, which contributes to the firm’s cash flows in each of the two periods. In addition, Sliwka’s model is structured so that the first-period cash flow is a sufficient statistic for the leading indicator variable with respect to the agent’s strategic effort.

Because of the interactions between the operational and the strategic incentive problems, the optimal performance measure in the full-commitment scenario puts a negative weight on the leading indicator to filter out unwarranted noise from the cash flows. Sliwka [2002], however, confines attention to performance measures that assign a non-negative weight to the leading indicator variable. Relative to the optimal effort choices in the full commitment setting, the optimal renegotiation-proof contract induces a higher level of operational effort but a smaller level of strategic effort.

---

30 As a consequence, the leading indicator would be entirely useless from an incentive contracting perspective if there were no incentive problem with regard to the agent’s operational action choice.
Because the leading indicator variable reflects only the strategic effort, this variable becomes valuable for realigning the allocation of efforts. The latter result is consistent with our finding in Proposition 6.

Finally, we note that taken together Propositions 4 through 6 show that the performance comparison between renegotiation-proof long-term contracts and short-term contracts can go either way. Recent work by Christensen, Feltham, and Sabac [2002] focuses on a class of multiperiod contracting problems with serially correlated performance measures (but without investment decisions) in which renegotiation-proof long-term contracts are in fact equivalent to short-term contracts provided such contracts are constrained to satisfy a certain fairness condition.

7. Conclusion

In this article we examine an agency model in which a manager makes an investment in the first period and contributes personally costly effort in both the first and second period. The investment decision is not directly contractible, but the principal can rely on a leading indicator variable that provides a noisy estimate of future investment-related cash flows. Our analysis shows that the duration of the manager’s incentive contract determines the need for including the leading indicator variables in the managerial performance measure. The optimal weight on the leading indicator variable, relative to the weight on cash flow, depends crucially on whether the principal offers a long-term contract or a sequence of short-term contracts.

With a sequence of one-period contracts, the leading indicator variable becomes the only instrument for motivating the manager to make any investments. With long-term commitments, in contrast, the leading indicator variable is valuable if and only if the underlying moral hazard problems differ across the two periods. For a stationary agency problem, the principal can choose incentive schemes that are identical over time and are based only on cash flows. As a consequence, the manager fully internalizes the net return of investments. However, we also find that whenever the optimal long-term incentive schemes vary over time, the leading indicator variable becomes valuable for matching the future investment return with the first-period expenditure.

Although short-term contracting always results in underinvestment, the optimal long-term contract may lead to under- or overinvestment depending on the relative severity of the periodic agency problems. When the induced overinvestment problem is sufficiently severe, we find that the principal is better off not making long-term commitments but instead offering a sequence of one-period contracts.

The main part of our analysis assumes that the investment decision is not observable to the principal. As noted in section 4, the performance of short-term contracting is unchanged if the investment choice is in fact observed by the principal, yet it remains unverifiable and therefore noncontractible. The natural question is whether for this alternative observability scenario
the efficiency of long-term contracting can be improved by considering a larger class of contracts that allow the principal some discretion in selecting future incentive schemes depending on his or her observation of the initial investment decision.

The model developed in this article studies the use of a single leading indicator variable in a multiperiod context. Earlier studies by Feltham and Xie [1994] and Datar, Kulp, and Lambert [2001] examine the aggregation of multiple signals in managerial performance measures, albeit only in a one-period context. To gain a better understanding of the use of nonfinancial performance indicators and the construction of balanced scorecards, it is desirable to combine the existing approaches to include multiple, and possibly correlated, leading indicator variables and to characterize the relative weights that should be placed on these variables both cross-sectionally and over time.

APPENDIX

Proof of Lemma 1. We first solve for the agent’s optimal consumption decisions using backward induction. After the second period, the agent will optimally consume the interest on his or her date 2 wealth (i.e., $\phi_t = r \cdot W_2$ for all $t > 2$) to smooth consumption over time. The manager’s utility from future consumption thus becomes $\frac{1}{r \cdot \gamma} \cdot U(r \cdot W_2)$, where for brevity we define $U(\phi) \equiv -\exp(-\hat{\rho} \cdot \phi)$.

At the end of the second period, the agent chooses his or her savings $W_2$ to maximize:

$$U_2(W_2, s_2) = U(s_2 - e(a_2) + (1 + r) \cdot W_1 - W_2) + \frac{1}{r} \cdot U(r \cdot W_2).$$

The first-order condition for the optimal $W_2$ is:

$$W_2^* = \gamma \cdot (s_2 - e(a_2) + (1 + r) \cdot W_1).$$

Let $U_2^*(s_2)$ denote the maximized value of $U_2(W_2, s_2)$. Substituting for $W_2^*$ and simplifying yields:

$$U_2^*(s_2) = \frac{1}{r \cdot \gamma} \cdot U[r \cdot \gamma \cdot (s_2 - e(a_2) + (1 + r) \cdot W_1)].$$

The agent’s date 1 expected utility from future consumption is therefore given by

$$EU_1 = \gamma \cdot E[U_2^*(s_2) | \tilde{f} = f],$$

because at date 1 the second-period compensation $s_2 = a_2 + \beta_2 \cdot (e_2 + w_2 \cdot f)$ is random.
By the mean-variance lemma:\footnote{We are using the fact that for a normally distributed random variable,  
\[ EU[(\tilde{s})] = U(E[\tilde{s}] - \hat{\rho} \cdot \frac{1}{2} \cdot \text{Var}[\tilde{s}]). \]}
\[
EU_1 = \frac{1}{r} \cdot U[r \cdot CE_1(f) + W_1] \]
where
\[
CE_1(f) \equiv \gamma \cdot [\alpha_2 + \beta_2 \cdot (a_2 + m(b) + u_2 \cdot f) - e(a_2) - \rho \cdot \beta_2^2 \cdot \sigma_2^2],
\]
and \( \rho \equiv \frac{1}{2} \cdot \hat{\rho} \cdot (1 - \gamma). \)

At the end of the first period, the agent will make his or her optimal savings decision to maximize:
\[
U_1(W_1 | s_1, f) = U(s_1 - W_1 - e(a_1)) + EU_1(W_1 | f).
\]

Let \( U_1^*(s_1, f) \) denote the maximized value of \( U_1(W_1 | s_1, f) \). Solving for the optimal savings \( W_1^* \) and simplifying yields:
\[
U_1^*(s_1, f) = \frac{1}{r} \cdot U[r \cdot CE_0]
\]
where \( CE_0 \) is as given in the statement of Lemma 1.

\textbf{Proof of Proposition 1.} For any given \( \beta_1 \) and \( \beta_2 \), the principal’s optimal investment choice \( b^* \) satisfies the first-order condition:
\[
\frac{\partial}{\partial b} Z^*(\cdot) = \gamma \cdot m'(b^*) - 1 - \rho \cdot \mu^2 \cdot H(b^*) \cdot \beta_1^2 \left[ \frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_2}{\beta_1} \right] = 0,
\]
where \( H(b) \equiv -2 \cdot \frac{m^2(b)}{(m'(b))^2} \). We note that if \( \beta_1 = \beta_2 \), the optimal \( b^* \) is equal to the first-best level \( b^0 \) (which maximizes \( \gamma \cdot m(b) - b \)). Conversely, it follows that
\[
\frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_2}{\beta_1} \neq 0
\]
whenever \( \beta_1 \neq \beta_2 \). From the agent’s incentive compatibility condition in (10), we thus obtain that \( \beta_1 \neq \beta_2 \) is equivalent to \( (u_1, u_2) \neq 0 \).
To complete the proof, it remains to show that the optimal bonus coefficients \( \beta_1^* \) and \( \beta_2^* \) are equal if and only if \( \sigma_1^2 = \sigma_2^2 \). Let \( \beta^0_i \) denote the maximizer of \( Y_i(\beta_i) \). Clearly, if \( \sigma_1^2 = \sigma_2^2 \), then \( \beta^0_1 = \beta^0_2 \) and furthermore \( Z^*(\beta^0_1, \beta^0_2, b^0) \geq Z^*(\beta_1, \beta_2, b) \) for all \( \beta_1, \beta_2, \) and \( b \).

Therefore \( V^*(\cdot) \) is maximized at \((\beta^0_1, \beta^0_2, b^0)\) if \( \sigma_1^2 = \sigma_2^2 \). Conversely, suppose that \( \sigma_1^2 > \sigma_2^2 \) (without loss of generality), yet \( \beta^* = \beta^*_1 = \beta^*_2 \). The principal therefore still chooses \( b^* = b^0 \). It follows that:

\[
\frac{\partial}{\partial \beta_i} Z^*(\beta^*, \beta^*, b^*) = 0
\]

for both \( i = 1 \) and \( i = 2 \). Because \( \beta^0_1 < \beta^0_2 \), and each function \( Y_i(\cdot) \) is single peaked with its maximum at \( \beta^0_i \), it follows that either \( Y'_1(\beta^*) \neq 0 \) or \( Y'_2(\beta^*) \neq 0 \); therefore, the principal’s objective function \( V^*(\cdot) \) cannot be maximized at \((\beta^*, \beta^*, b^0)\).

**Proof of Corollary to Proposition 1.** At date 1, the certainty equivalent of the agent’s future expected utility is given by:

\[
CE_1(b, f) = \gamma \cdot \left[ a_2 + \beta_2 \cdot [a_2 + m(b) + w_2 \cdot f] - e(a_2) - \rho \cdot (\beta_2 \cdot \sigma_2)^2 \right].
\]

The interim participation constraint requires that \( CE_1 \geq 0 \) for all realizations of \( \tilde{f} \). Therefore, \( w_2 = 0 \) and, by the incentive constraint in (10), the agent’s investment choice satisfies:

\[
u_1 = \frac{1}{m'(b)} - \gamma \cdot \frac{\beta_2}{\beta_1}.\]

Given these choices for \( u_1 \) and \( u_2 \), the certainty equivalent of the agent’s expected utility at date 0, as identified in Lemma 1, reduces to:

\[
CE_0 = \gamma \left\{ a_1 + \beta_1 \cdot a_1 - e(a_1) + \gamma [a_2 + \beta_2 \cdot a_2 - e(a_2)]
\right.
\]

\[
- \rho \cdot \beta^2_1 \cdot \left[ \left( \frac{1}{m'(b)} - \gamma \cdot \frac{\beta_2}{\beta_1} \right)^2 \cdot \mu^2 + \sigma_1^2 \right] - \gamma \cdot \rho \cdot \beta^2_2 \cdot \sigma_2^2 \}.
\]

(A2)

For an optimal long-term contract the constants \( a_1 \) and \( a_2 \) must be chosen so that \( CE_0 = 0 \); therefore, the principal’s expected payoff is again given by the function \( V^*(\beta_1, \beta_2, b) \) in (11) corresponding to long-term contracts that satisfy only the initial participation constraint.

**Proof of Proposition 2.** Step 1: \( \beta^*_1 \geq \beta^*_2 \) if and only if \( \text{Var}(\tilde{e}_1) < \text{Var}(\tilde{e}_2) \).

Following the notation in the proof of Proposition 1, the principal’s objective is:

\[
V^*(\beta_1, \beta_2, b) \equiv Y_1(\beta_1) + Z^*(\beta_1, \beta_2, b) + \gamma \cdot Y_2(\beta_2).
\]
Suppose that contrary to the claim, \( \text{Var}(\hat{e}_1) < \text{Var}(\hat{e}_2) \), yet \( \beta_1^* \leq \beta_2^* \). We know that the values of \( \beta_1^0 \), which are the unconstrained maximizers of \( Y_i(\cdot) \), satisfy \( \beta_1^0 > \beta_2^0 \) when \( \text{Var}(\hat{e}_1) < \text{Var}(\hat{e}_2) \).

**Case I:** \( \beta_1^* \geq \beta_2^* \).

The principal could then decrease \( \beta_2^* \) to the value of \( \beta_1^* \) with the consequence that:

\[
V^*(\beta_1^*, \beta_2^*, b^*) > Y_1(\beta_1^*) + Z^*(\beta_1^*, \beta_1^*, b^0) + \gamma \cdot Y_2(\beta_1^*).
\]

This inequality is based on the fact that \( Y_2(\cdot) \) is a single-peaked function with its maximum to the left of \( \beta_1^*(\cdot) \) and \( \beta_2^*(\cdot) \). Furthermore,

\[
Z^*(\beta_1^*, \beta_2^*, b^0) \geq Z^*(\beta_1^*, \beta_2^*, b^*)
\]

for all values of \( \beta_1^*, \beta_2^*, \) and \( b^* \). This contradicts the hypothesis \( \beta_2^* > \beta_1^* \geq \beta_2^0 \).

**Case II:** \( \beta_1^* < \beta_2^0 < \beta_2^* \).

For bonus coefficients satisfying these inequalities, the principal could increase \( \beta_1^* \) to \( \beta_2^0 \) and decrease \( \beta_2^* \) to \( \beta_2^0 \). The values \( Y_2(\cdot) \) and \( Z^*(\cdot) \) would be increased and so would \( Y_1(\cdot) \) because \( \beta_1^0 > \beta_2^0 \).

**Case III:** \( \beta_1^* < \beta_2^* < \beta_2^0 \).

The principal could then do better by increasing \( \beta_1^* \) and \( \beta_2^* \) to \( \beta_2^0 \), improving the value of both \( Y_i(\cdot) \) and \( Z^*(\cdot) \) functions.

This completes the argument that \( \text{Var}(\hat{e}_1) < \text{Var}(\hat{e}_2) \) implies \( \beta_1^* > \beta_2^* \). In the reverse case of \( \text{Var}(\hat{e}_1) > \text{Var}(\hat{e}_2) \), the same line of arguments establishes that \( \beta_1^* < \beta_2^* \).

**Step 2:** \( u_1^* > 0 \) and \( b^* < b^0 \) whenever \( \beta_1^* > \beta_2^* \).

Suppose \( b^* \geq b^0 \), which implies \( \gamma \cdot m'(b^*) - 1 \leq 0 \). The optimal choice of \( b^* \) satisfies

\[
\frac{\partial}{\partial b} Z^*(\beta_1^*, \beta_2^*, b^*) = \gamma \cdot m'(b^*) - 1 - \rho \cdot (\beta_1^*)^2 \cdot m^2 \cdot H(b^*) \left[ \frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_2^*}{\beta_1^*} \right] = 0,
\]

where \( H(b) \equiv \frac{-m'(b)}{(m(b))^2} > 0 \). Thus,

\[
\frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_2^*}{\beta_1^*} < \frac{1}{m'(b^*)} - \gamma < 0,
\]

which contradicts the first-order condition for the optimal \( b^* \) in (A3). We conclude that \( b^* > b^0 \) when \( \beta_1^* > \beta_2^* \). Because

\[
u_1^* \equiv \frac{1}{m'(b^*)} - \gamma \cdot \frac{\beta_2^*}{\beta_1^*},
\]

it also follows that \( u_1^* > 0 \) when \( \beta_1^* > \beta_2^* \).
Step 3: $u_1^* < 0$ and $b^* > b^0$ when $\beta_1^* \beta_2^*$. The arguments here mirror those in step 2, thus completing the proof of the claim.

Proof of Lemma 2. We show that contracts $\{s_i\}_{i=1}^2$ that retain the same agent in both periods cannot be part of a sequentially optimal solution. Suppose to the contrary that the contracts $s_1 = \alpha_1 + \beta_1 [c_1 + u_1 \cdot f]$ and $s_2 = \alpha_2 + \beta_2 \cdot c_2$ satisfy sequential optimality and elicit the responses $(\hat{a}_1, \hat{b})$ and $\hat{a}_2$ from the agent.

At the beginning of the second period, the principal chooses $(\alpha_2, \beta_2)$ to maximize:

$$a_2 - E[\hat{s}_2],$$

subject to the incentive compatibility and the participation constraints. At date 1, the agent’s preferences over alternative contracts are given by

$$CE_1(a_2, b) = \gamma \{\alpha_2 + \beta_2 \cdot [a_2 + m(b)] - e(a_2) - \rho \cdot \beta_2 \cdot \sigma_2^2\},$$

where $b$ is the actual level of investment undertaken by the agent in the previous period. The incentive compatibility condition in the second period is $\beta_2 = e'(a_2)$. We denote the optimal action choice in response to $\beta_2$ by $a_2(\beta_2)$. The fixed payment $\alpha_2$ is chosen so that the agent’s certainty equivalent is zero at the conjectured level of investment $\hat{b}$. Thus, $a_2(\hat{b})$ satisfies:

$$a_2(\hat{b}) + \beta_2 \cdot [a_2(\beta_2) + m(\hat{b})] - e(a_2(\beta_2)) - \rho \cdot \beta_2^2 \cdot \sigma_2^2 = 0.$$

To check incentive compatibility in the first period, we define the function:

$$CE_1(a_2, b, \hat{b}) = a_2(\hat{b}) + \beta_2 \cdot [a_2 + m(b)] - e(a_2) - \rho \cdot \beta_2^2 \cdot \mu^2.$$

Thus, $CE_1(a_2(\beta_2), \hat{b}, \hat{b}) = 0$. We note that the sequentially optimal contact for the second period is independent of the outcomes $c_1$ and $f$ in the first period. Instead, the principal relies on his or her conjecture $\hat{b}$ to set the fixed payment $a_2(\hat{b})$. Furthermore, the payoffs resulting from the second-period contract are independent of the amount of investment induced in the first period.

Contingent on the contract $s_1 = \alpha_1 + \beta_1 [c_1 + u_1 \cdot f]$, the agent expects to be offered the contract $s_2 = a_2(\hat{b}) + \beta_2 \cdot c_2$ in the second period. By Lemma 1, the agent’s preferences for such a contract are given by:

$$CE_0 = U_1(a_1, b) + CE_1(a_2, b, \hat{b}),$$

where

$$U_1(a_1, b) = \alpha_1 + \beta_1 [a_1 - b + u_1 \cdot m(b)] - e(a_1) - \rho \cdot \beta_1^2 \cdot [\sigma_1^2 + u_1^2 \cdot \mu^2].$$
The incentive compatibility condition for an interior $\hat{b}$ is:

$$\beta_1 \cdot [-1 + u_1 \cdot m'(\hat{b})] + \gamma \beta_2 \cdot m'(\hat{b}) = 0,$$

and the participation constraint in the first period requires that:

$$U_1(a_1(\beta_1), \hat{b}) \geq 0,$$

because $CE_1(a_2(\beta_2), \hat{b}, \hat{b}) = 0$ implies that the agent would not accept the first-period contract if $U_1(a_1(\beta_1), \hat{b}) < 0$. The preceding contracts and actions would be optimal if the agent were forced to stay in the second period. However, the agent could do better by choosing $b'$, where $b'$ maximizes:

$$-b + u_1 \cdot m(b),$$

and thereafter rejecting the second period contract. Note that:

$$U_1(a_1(\beta_1), b') > U_1(a_1(\beta_1), \hat{b}) = 0,$$

and the agent would do better by the preceding deviation.

**Proof of Proposition 4.** Let $(\hat{u}_1, \hat{\beta}_1, \hat{\beta}_2)$ denote the optimal parameters under short-term contracting. As argued in connection with Proposition 3, $\hat{\beta}_2 = \beta_2^0$ and $\hat{\beta}_1 > \hat{\beta}_1$, where $\beta_i^0$ is the maximizer of the single-peaked function:

$$Y_i(\beta_i) = a_i(\beta_i) - e(a_i(\beta_i)) - \rho \cdot \beta_i^2 \cdot \sigma_i^2.$$

As argued in Proposition 2, the principal’s expected payoff under long-term contracting is given by:

$$V^*(b, \beta_1, \beta_2) = Y_1(\beta_1) + Z^*(b, \beta_1, \beta_2) + \gamma \cdot Y_2(\beta_2),$$

where

$$Z^*(b, \beta_1, \beta_2) = \gamma \cdot m(b) - b - \rho \cdot \left(\frac{\beta_1}{m'(b)} - \gamma \cdot \beta_2^2\right) \cdot \mu^2.$$

Under short-term contracting the principal’s expected payoff is:

$$\hat{V}(b, \beta_1, \beta_2) = Y_1(\beta_1) + \hat{Z}(b, \beta_1) + \gamma \cdot Y_2(\beta_2),$$

with

$$\hat{Z}(b, \beta_1) = \gamma \cdot m(b) - b - \rho \cdot \left(\frac{\beta_1}{m'(b)}\right)^2 \cdot \mu^2.$$

Because $\text{Var}(\varepsilon_1) < \text{Var}(\varepsilon_2)$, we know that $\beta_1^0 \geq \beta_2^0$. Suppose first that $\hat{\beta}_1 < \hat{\beta}_2 = \beta_2^0$. For this level of $\hat{\beta}_1$, let $\hat{b}$ be the corresponding investment that maximizes $\hat{Z}(b, \hat{\beta}_1)$. Under long-term contracting, the principal could then choose $\beta_1 = \beta_2^0$ and $b = b^0$ (i.e., the first-best level of investment). This
would increase both \( Y_1(\cdot) \) and \( Z^*(\cdot) \) relative to the short-term contracting solution. Formally:

\[
\hat{V}(\hat{b}, \hat{\beta}_1, \hat{\beta}_2) = \hat{V}(\hat{b}, \hat{\beta}_1, \beta_2^0) \leq Y_1(\beta_2^0) + \hat{Z}(\hat{b}, \hat{\beta}_1) + \gamma \cdot Y_2(\beta_2^0) \\
< Y_1(\beta_2^0) + Z^*(b^0, \beta_2^0) + \gamma \cdot Y_2(\beta_2^0) \\
\leq V^*(b^*, \beta_1^*, \beta_2^*). \tag{A5}
\]

The first inequality reflects that \( Y_1(\cdot) \) is single peaked and, by hypothesis, \( \hat{\beta}_1 < \beta_2^0 \). The second inequality follows directly from the definitions of \( \hat{Z}(\cdot) \) and \( Z^*(\cdot) \).

Suppose now that \( \beta_1^0 > \hat{\beta}_1 > \hat{\beta}_2 = \beta_2^0 \). Under long-term contracting, the principal has the option of leaving \( \hat{\beta}_1 \) and \( \hat{\beta}_2 \) unchanged but choosing \( u_1^* \) as:

\[
u_1^* = \max\{0, \hat{u}_1 - \gamma \cdot \hat{\beta}\},
\]

where \( \hat{\beta} = \hat{\beta}_2 \). Clearly, \( |u_1^*| < |\hat{u}_1| \). Therefore, \( u_1^* \) entails a smaller risk premium, that is, \( \rho \cdot [\hat{\beta}_1 \cdot u_1^* \cdot \mu]^2 < \rho \cdot [\hat{\beta}_1 \cdot \hat{u}_1 \cdot \mu]^2 \), and the principal will be better off under long-term contracting provided the induced level of investment \( b^* \) exceeds \( \hat{b} \) without resulting in overinvestment, that is, \( \hat{b} < b^* < b_0 \).

The agent’s investment choice \( b^* \) then satisfies:

\[
\hat{\beta}_1 \cdot u_1^* \cdot m'(b^*) + \gamma \cdot \hat{\beta}_2 \cdot m'(b^*) = \hat{\beta}_1.
\]

Because \( \gamma \cdot m'(b^0) = 1 \), by definition, it suffices to show that:

\[
\gamma \geq u_1^* + \gamma \cdot \hat{\beta} \geq \hat{u}_1.
\]

By construction:

\[
u_1^* + \gamma \hat{\beta} = \max\{\gamma \cdot \hat{\beta}, \hat{u}_1\},
\]

and therefore it remains to show that

\[
\max\{\gamma \cdot \hat{\beta}, \hat{u}_1\} \leq \gamma.
\]

This inequality in turn follows from \( \hat{\beta} \leq 1 \) and the observation that \( \gamma \cdot m'(\hat{b}) > 1 \) and \( \hat{u}_1 \cdot m'(\hat{b}) = 1 \).

**Proof of Proposition 5.** For a given \( \sigma_2^2 \), we show that:

\[
\hat{V}(\hat{\beta}_1, \hat{\beta}_2, \hat{b}) > V^*(\beta_1^*, \beta_2^*, \hat{b})
\]

for values of \( \sigma_2^2 \) sufficiently large. Here, the functions \( \hat{V}(\cdot) \) and \( V^*(\cdot) \) are as defined in Propositions 3 and 1, respectively.

Proposition 3 shows that \( \hat{\beta}_2 = \beta_2^0 \). Furthermore, \( \beta_1^0 \to 0 \) as \( \sigma_1^2 \) becomes large. Therefore,
\[ \hat{V}(\hat{\beta}_1, \hat{\beta}_2, b^*) \rightarrow \gamma \cdot m(b^0) - b^0 + \gamma Y_2(b^0_2). \] (A6)

The limit in (A6) reflects that the principal moves \( \beta_1 \) to zero as \( \sigma_1^2 \) becomes large and \( \hat{Z}(\beta_1, b) \) is maximized at a value close to \( b^0 \) as \( \beta_1 \) becomes small.

In the limit, \( \hat{Z}(0, b^0) = \gamma \cdot m(b^0) - b^0 \).

It remains to show that for values of \( \sigma_1^2 \), sufficiently large,

\[ V^*(\beta_1^*, \beta_2^*, b^*) \leq K < \gamma \cdot m(b^0) - b^0 + \gamma \cdot Y_2(b^0_2). \] (A7)

Consider an increasing sequence of \( \sigma_1^2 \) such that the corresponding \( (\beta_1^*, \beta_2^*) \) converge to \( (\beta_1^{**}, \beta_2^{**}) \). Suppose first that \( \frac{\beta_2^{**}}{\beta_1^{**}} \neq 1 \). For \( \sigma_1^2 \) sufficiently large we have:

\[ Y_1(\beta_1^{**}) < Y_1(0), \]
\[ Y_2(\beta_2^{**}) \leq Y_2(b^0_2), \]

and

\[ Z^*(\beta_1^{**}, \beta_2^{**}, b) < \gamma \cdot m(b^0) - b^0 \]

for all \( b \). Therefore, the inequality in (A7) holds if \( \frac{\beta_2^{**}}{\beta_1^{**}} \neq 1 \). Alternatively, if \( \beta_1^{**} = \beta_2^{**} \), we find:

\[ Z^*(\beta_1^{**}, \beta_2^{**}, b^*) = \gamma \cdot m(b^0) - b^0, \]

but \( Y_1(\beta_1^{**}) < Y_1(0) \) and \( Y_2(\beta_2^{**}) \leq Y_2(b^0_2) \). Therefore, the inequality in (A7) again holds, completing the proof.

REFERENCES


