9.2-1

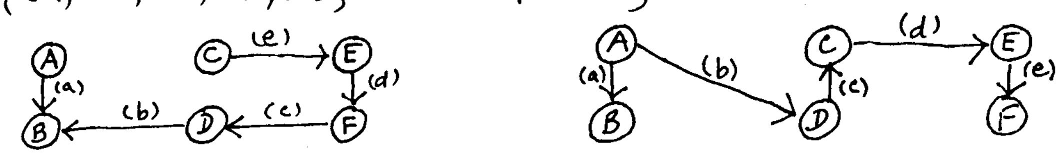
a) AD - DC - CE - EF (A → D → C → E → F) is a directed path from A to F

AD - FD (A → D → F)
CA - CE - EF (A → C → E → F)
AD - ED - EF (A → D – E → F)
} are undirected paths from A to F

b) AD - DC - CA
DC - CE - ED
DC - CE - EF - FD
} are directed cycles

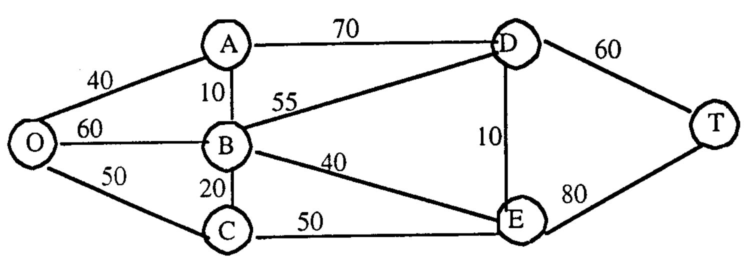
CA - CE - EF - FD - DB - AB is an undirected cycle which includes every node.

c) {CA, CE, DC, FD, DB} is a spanning tree.

d) 

9.3-1

a)



b)

n	Solved nodes connected to unsolved nodes	its closest connected unsolved node	total distance involved	n^{th} nearest node	its minimum distance	its last connection
1	O	A	40	A	40	OA
2,3	O	C	50	C	50	OC
	A	B	40+10=50	B	50	AB
4	A	D	40+70=110	E	90	BE
	B	E	50+40=90			
	C	E	50+50=100			
5	A	D	40+70=110	D	100	ED
	B	D	50+55=105			
	E	D	90+10=100			
6	D	T	100+60=160	T	160	DT
	E	T	90+80=170			

shortest route: O-A-B-E-D-T Total distance = 160

c) Yes d) Yes

e)

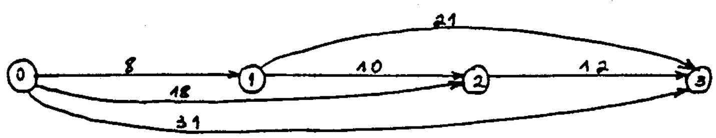
From	To	On Route	Distance
Origin	A	1	40
Origin	B	0	60
Origin	C	0	50
A	B	1	10
A	D	0	70
B	C	0	20
B	D	0	55
B	E	1	40
C	E	0	50
D	E	0	10
D	Destination	1	60
E	Destination	0	80
E	D	1	10
	Total Distance =	160	

Nodes	Net Flow		Supply/Demand
Origin	1	=	1
A	0	=	0
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
Destination	-1	=	-1

9.3-2

a) Nodes are the years

d_{ij} = cost (in \$ thousands) of using same tractor from end of year i to end of year j.



(b)

n	Solved nodes connected to unsolved nodes	its closest connected unsolved node	total distance involved	nth nearest node	its minimum distance	its last connection
1	0	1	8	1	8	01
2	0 1	2 2	18 8+10 = 18	2 2	18	02 12
3	0 1 2	3 3 3	31 8+21 = 29 18+12 = 30	3	29	13

After buying new tractor, replace it at end of year 1 and then keep the new till the end of year 3, for total cost of 29,000

c)

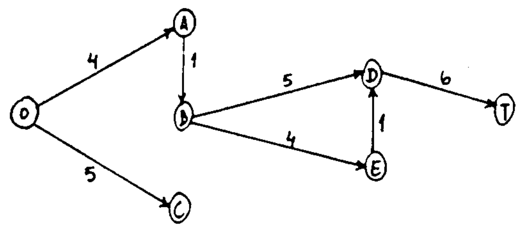
From	To	On Route	Cost
Node 0	Node 1	1	\$8,000
Node 0	Node 2	0	\$18,000
Node 0	Node 3	0	\$31,000
Node 1	Node 2	0	\$10,000
Node 1	Node 3	1	\$21,000
Node 2	Node 3	0	\$12,000
	Total Cost =	\$29,000	

Nodes	Net Flow		Supply/Demand
0	1	=	1
1	0	=	0
2	0	=	0
3	-1	=	-1

9.3-3

(a)

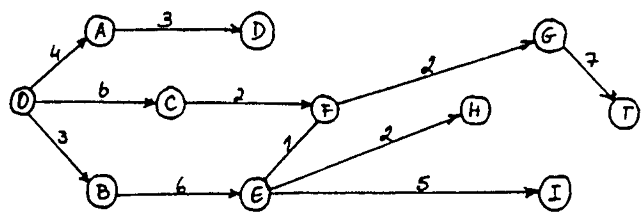
0	4	5	5	10	9	16
(O)	(A)	(B)	(C)	(D)	(E)	(T)
OA-4 OC-5 ~~OD-6~~	AB-1 ~~AD-7~~	~~BA-1~~ ~~BD-2~~ BE-4 BD-5	~~CB-2~~ ~~CE-5~~	~~DE-1~~ ~~DB-5~~ DT-6 DA-7	ED-1 ~~EB-4~~ ~~EC-5~~ ~~ET-8~~	



Lenght of Shortest Path = 16

(b)

0	4	3	6	7	9	8	10	11	14	17
(O)	(A)	(B)	(C)	(D)	(E)	(F)	(G)	(H)	(I)	(T)
OB-3 OA-4 OC-6	AD-3 ~~AC-5~~	~~BC-4~~ BE-6	~~CD-2~~ CF-2 ~~CB-4~~ ~~CE-5~~ ~~CA-5~~	~~DE-2~~ ~~DF-2~~ ~~DA-3~~ ~~DG-4~~	~~EF-1~~ EH-2 EI-5 ~~EC-5~~ ~~EB-6~~	FE-1 ~~FC-2~~ ~~FD-2~~ FG-2 ~~FH-5~~	~~GF-2~~ ~~GH-2~~ ~~GD-4~~ GT-7	~~HE-2~~ ~~HG-2~~ ~~HI-3~~ ~~HF-5~~ ~~HT-8~~	~~IH-3~~ ~~IT-4~~ ~~IE-5~~	



Length of Shortest Path = 17.

9.3-4

This is just the minimum cost flow problem with a unit source at the origin and a unit sink at the destination.

Assume without loss of generality that the origin is node 1 and the destination is node n. The LP formulation is

$$\min. \quad Z = \sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{1j} - \sum_{j=1}^{n} x_{j1} = 1$$

$$\sum_{j=1}^{n} x_{ij} - \sum_{j=1}^{n} x_{ji} = 0 \qquad 2 \le i \le n-1$$

$$\sum_{j=1}^{n} x_{nj} - \sum_{j=1}^{n} x_{jn} = -1$$

$$0 \le x_{ij} \le 1, \quad 1 \le i, j \le n$$

9.3-5

a) Times play the role of distances.

b)

n	solved nodes connected to unsolved nodes	its closest connected unsolved node	total distance involved	nth nearest node	its minimum distance	its last connection
1	SE	C	4.2	C	4.2	SE-C
2	SE C	A F	4.6 4.2+3.4=7.6	A	4.6	SE-A
3	SE C A	B F E	4.7 4.2+3.4=7.6 4.6+3.4=8	B	4.7	SE-B
4	A B C	E E F	4.6+3.4=8 4.7+3.2=7.9 4.2+3.4=7.6	F	7.6	C-F
5	A B C F	E E E LN	4.6+3.4=8 4.7+3.2=7.9 4.2+3.5=7.7 7.6+3.8=11.4	E	7.7	C-E
6	A B F E	D D LN LN	4.6+3.5=8.1 4.7+3.6=8.3 7.6+3.8=11.4 7.7+3.6=11.3	D	8.1	A-D
7	D E F	LN LN LN	8.1+3.4=11.5 7.7+3.6=11.3 7.6+3.8=11.4	LN	11.3	E-LN

optimal path: SE-C-E-LN
minimum distance = 11.3

c)

From	To	On Route	Time
SE	A	0	4.6
SE	B	0	4.7
SE	C	1	4.2
A	D	0	3.5
A	E	0	3.4
B	D	0	3.6
B	E	0	3.2
B	F	0	3.3
C	E	1	3.5
C	F	0	3.4
D	LN	0	3.4
E	LN	1	3.6
F	LN	0	3.8

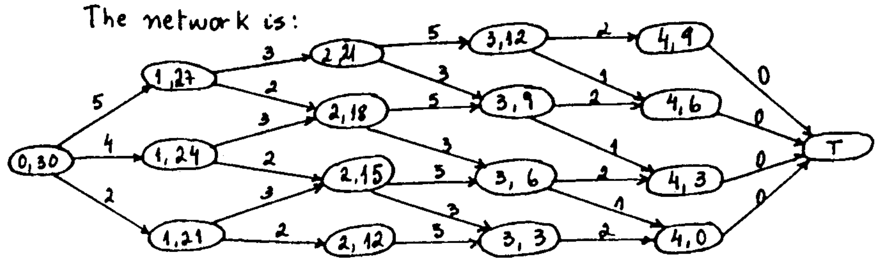
Total Time = 11.3

Nodes	Net Flow		Supply/Demand
SE	1	=	1
A	0	=	0
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
F	0	=	0
LN	-1	=	-1

9.3-6

(a) Let node (i,j) denote phase i being completed with j left to spend. $t_{(i,j),(i+1,k)}$ = the time taken to complete phase $i+1$ if $(j-k)$ million is spent.

The network is:



(b)

n	Solved nodes connected to unsolved nodes	its closest connected unsolved node	total distance involved	nth nearest node	its minimum distance	its last connection
1	(0,30)	(1,21)	2	(1,21)	2	(0,30)-(1,21)
2	(0,30)	(1,24)	4	(1,24)	4	(0,30)-(1,24)
	(1,21)	(2,12)	2+2=4	(2,12)	4	(1,21)-(2,12)
4	(0,30)	(1,27)	5	(1,27)	5	(0,30)-(1,27)
	(1,21)	(2,15)	2+3=5	(2,15)	5	(1,21)-(2,15)
	(1,24)	(2,15)	4+2=6			
	(2,12)	(3,3)	4+5=9			
6	(1,24)	(2,18)	4+3=7	(2,18)	7	(1,24)-(2,18)
	(1,27)	(2,18)	5+2=7	(2,18)	7	(1,27)-(2,18)
	(2,12)	(3,3)	4+5=9			
	(2,15)	(3,3)	5+3=8			
7	(1,27)	(2,21)	5+3=8	(2,21)	8	(1,27)-(2,21)
	(2,12)	(3,3)	4+5=9			
	(2,15)	(3,3)	5+3=8	(3,3)	8	(2,15)-(3,3)
	(2,18)	(3,6)	7+3=10			
9	(2,15)	(3,6)	5+5=10	(3,6)	10	(2,15)-(3,6)
	(2,18)	(3,6)	7+3=10	(3,6)	10	(2,18)-(3,6)
	(2,21)	(3,9)	8+3=11			
	(3,3)	(4,0)	8+2=10	(4,0)	10	(3,3)-(4,0)
11	(2,18)	(3,9)	7+5=12			
	(2,21)	(3,9)	8+3=11			
	(3,6)	(4,3)	10+2=12			
	(4,0)	T	10+0=10	T	10	(4,0)-T

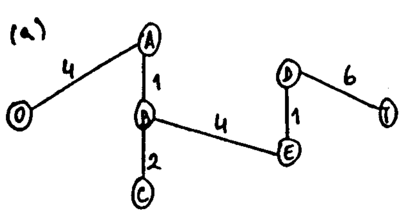
Shortest Route: (0,30) —2→ (1,21) —3→ (2,15) —3→ (3,3) —2→ (4,0) —0→ (T)

Phase	Level	Cost	Time
Research	crash	9	2
Development	priority	6	3
Design	crash	12	3
Production	priority	3	2

Total time = 10

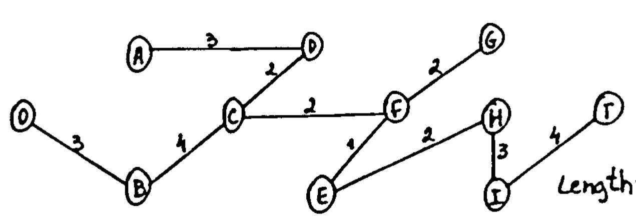
9.4-1

(a)



Length = 18

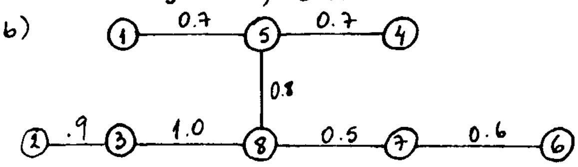
(b)



Length = 26

9.4-2 (a) nodes ~ groves, branches ~ roads

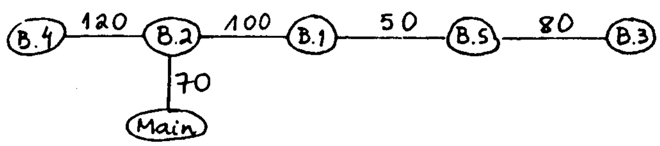
(b)



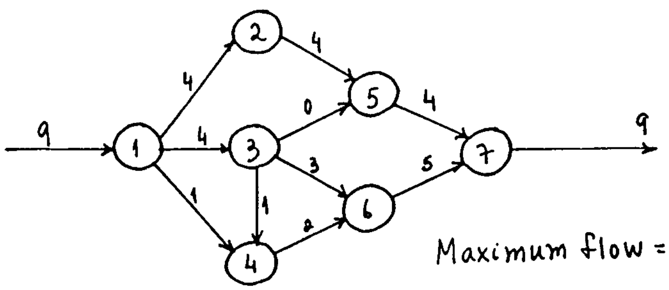
Length = 5.2

9.4-3 (a) nodes ~ {Main office, Branch 1, ..., Branch 5}, branches ~ telephone lines

(b)



9.5-1



Maximum flow = 9

9.5-2 Let node 1 be the source and node N be the sink. Then:

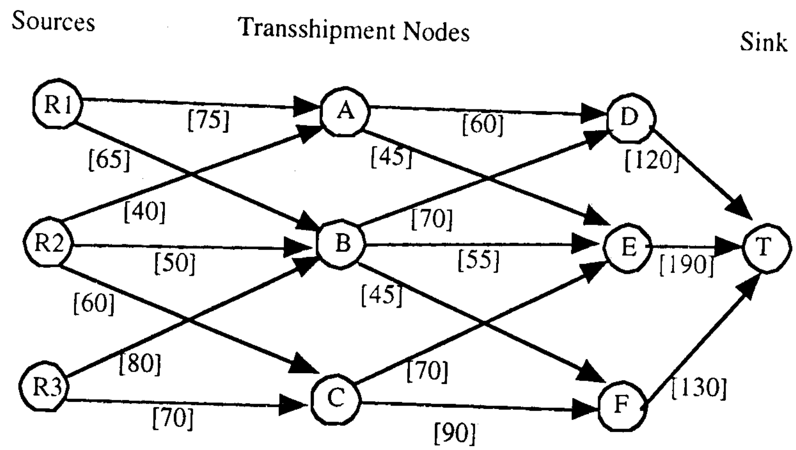
Maximize $\sum_{j=2}^{N} x_{1j}$

subject to $\sum_{\substack{j=1 \\ j\neq i}}^{N} x_{ij} - \sum_{\substack{j=1 \\ j\neq i}}^{N} x_{ji} = 0$ for $i = 2, 3, \ldots, N-1$

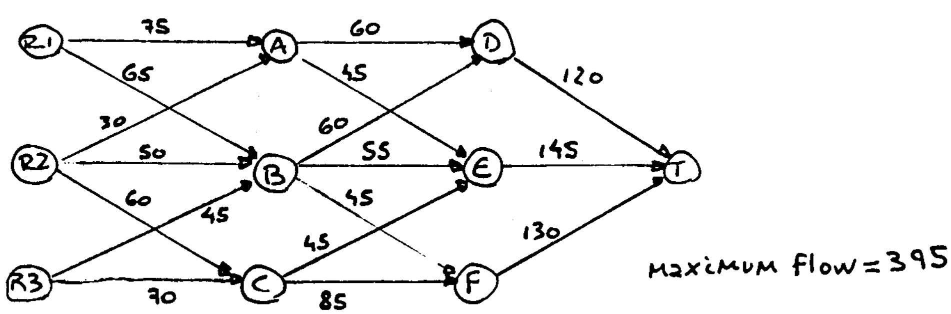
$0 \le x_{ij} \le c_{ij}$ where $c_{ij} = 0$ if (i,j) is not a branch

9.5-3

a)



b)



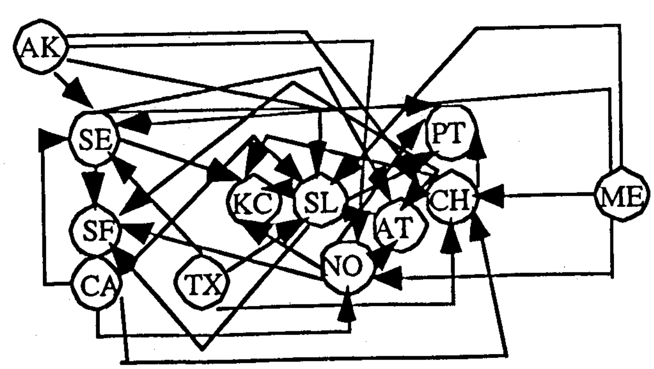
c)

From	To	Ship	Capacity
R1	A	75	75
R1	B	65	65
R2	A	30	40
R3	B	45	80
R3	C	70	70
A	D	60	60
A	E	45	45
B	D	60	70
C	E	45	70
C	F	85	90
D	T	120	120
E	T	145	190
F	T	130	130

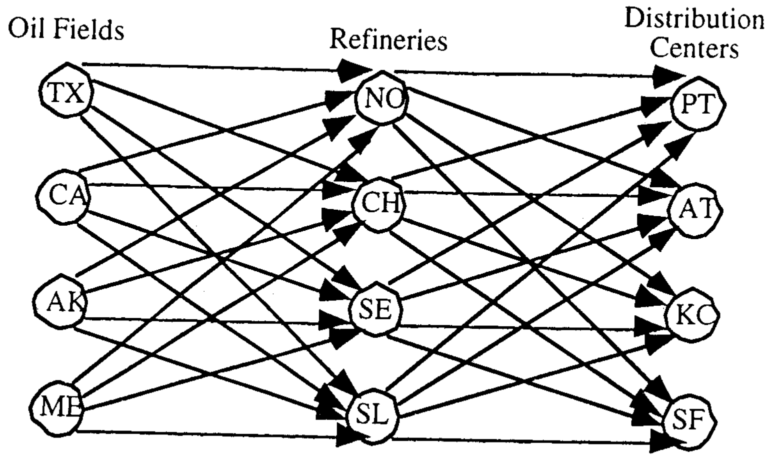
Maximum Flow = 395

Nodes	Net Flow		Supply/Demand
R1	140		
R2	140		
R3	115		
C	0	=	0
D	0	=	0
E	0	=	0
F	0	=	0
T	-395		

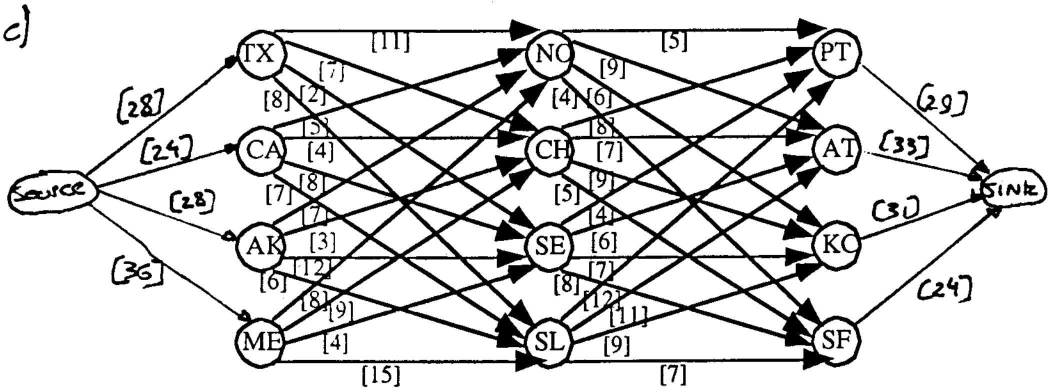
9.5-4 a)



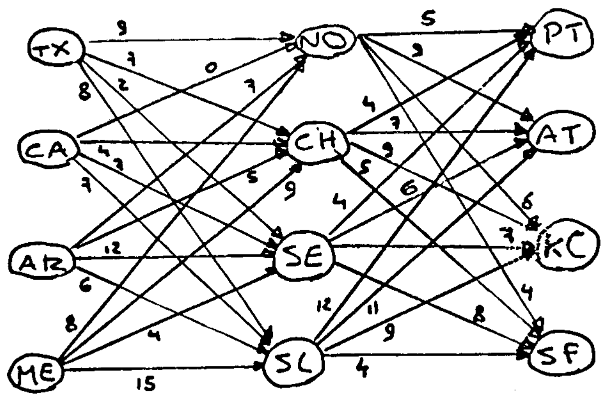
9.5-4 b)


Oil Fields
Refineries
Distribution Centers
TX
CA
AK
ME
NO
CH
SE
SL
PT
AT
KC
SF

c)


Source
[28]
[24]
[28]
[36]
TX
CA
AK
ME
NO
CH
SE
SL
PT
AT
KC
SF
[11]
[5]
[15]
[7]
[29]
[33]
[30]
[24]
Sink

d)


TX
CA
AK
ME
NO
CH
SE
SL
PT
AT
KC
SF
9
5
15
4

e)

From	To	Ship	Capacity
TX	NO	9	11
TX	CH	7	7
TX	SE	2	2
TX	SL	8	8
CA	NO	0	5
CA	CH	4	4
CA	SE	7	8
CA	SL	7	7
AK	NO	7	7
AK	CH	5	5
AK	SE	12	12
AK	SL	6	6
ME	NO	8	8
ME	CH	9	9
ME	SE	4	4
ME	SL	15	15
NO	PT	5	5
NO	AT	9	9
NO	KC	6	6
NO	SF	4	4
CH	PT	4	8
CH	AT	7	7
CH	KC	9	9
CH	SF	5	5
SE	PT	4	4
SE	AT	6	6
SE	KC	7	7
SE	SF	8	8
SL	PT	12	12
SL	AT	11	11
SL	KC	9	9
SL	SF	4	7

Maximum Flow = 110

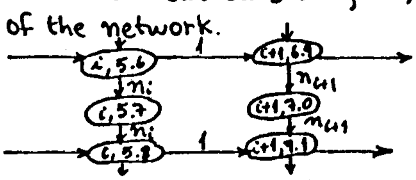
Nodes	Net Flow		Supply/Demand
TX	26		
CA	18		
AK	30		
ME	36		
NO	0	=	0
CH	0	=	0
SE	0	=	0
SL	0	=	0
PT	-25		
AT	-33		
KC	-31		
SF	-21		

9.5-5 For convenience, call Faireparc station siding 0 and the Portstown station siding $s+1$. Let node (i,j) represent siding i at time j for $i=0,1,\dots,s,s+1$ and $j=0,.1,.2,\dots,23.9$. Node $(0,0)$ is the source and node $(s+1,23.9)$ the sink.

Arcs with capacity 1 will exist between nodes (i,j) and $(i+1, j+t_i)$ if and only if a freight train leaving siding i at time j could not be overtaken by a scheduled passenger train before it reached siding $i+1$. Arcs with capacity n_i will exist between nodes (i,j) and $(i,j+1)$ for $j=0,.1,.2,\dots,23.8$

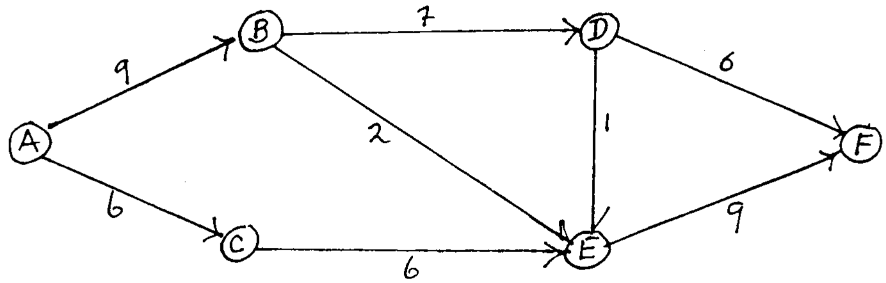
No other arcs exist.

For example, if $t_i=1.3$ and a scheduled passenger train could overtake a freight train leaving siding i at time 5.7 before it reached siding $i+1$, the following would be a portion of the network.



Solving the maximal flow problem will maximize the number of freight trains that are sent.

9.5-6

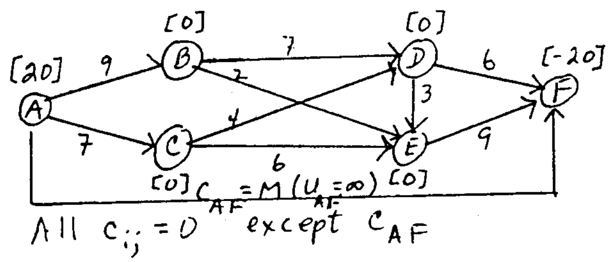


From	To	Ship	Capacity
A	B	8	9
A	C	7	7
B	D	7	7
B	E	1	2
C	D	2	4
C	E	5	6
D	E	3	3
D	F	6	6
E	F	9	9

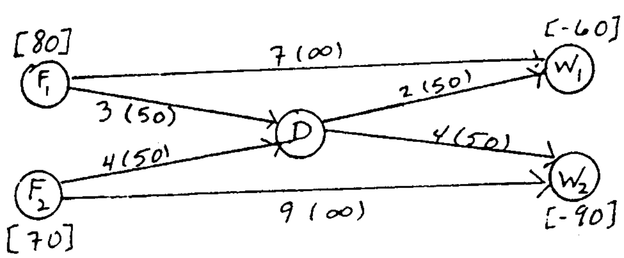
Maximum Flow = 15

Nodes	Net Flow		Supply/Demand
A	15		
B	0	=	0
C	0	=	0
D	0	=	0
E	0	=	0
F	-15		

9.6-1



9.6-2 a)

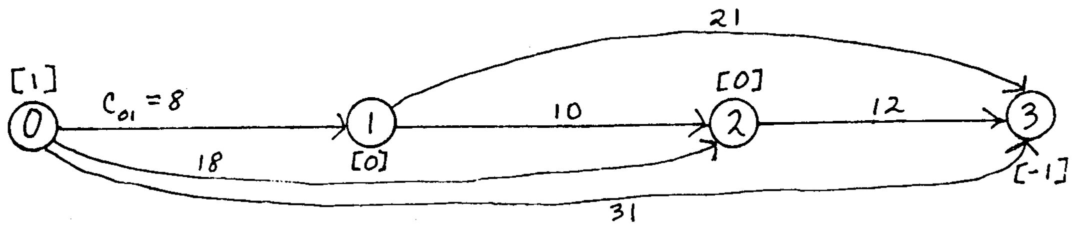


9.6-2 b) Min $7x_{F_1W_1} + 3x_{F_1D} + 2x_{DW_1} + 4x_{F_2D} + 4x_{DW_2} + 9x_{F_2W_2}$

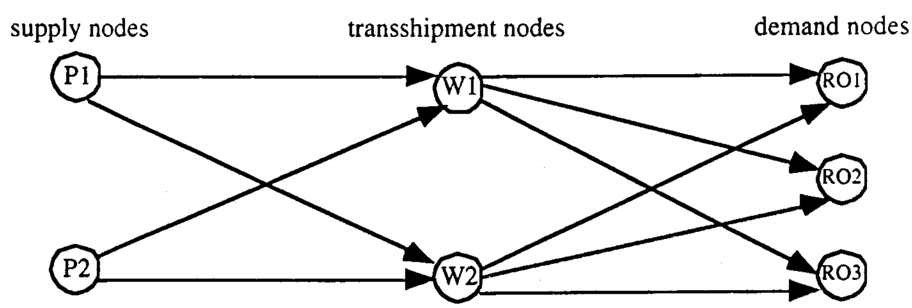
subject to

$$x_{F_1W_1} + x_{F_1D} = 80$$
$$x_{F_2D} + x_{F_2W_2} = 70$$
$$x_{F_1W_1} + x_{DW_1} = 60$$
$$x_{DW_2} + x_{F_2W_2} = 90$$
$$x_{F_1D} - x_{DW_1} + x_{F_2D} - x_{DW_2} = 0$$
$$0 \le x_{F_1D},\ x_{F_2D},\ x_{DW_1},\ x_{DW_2} \le 50$$

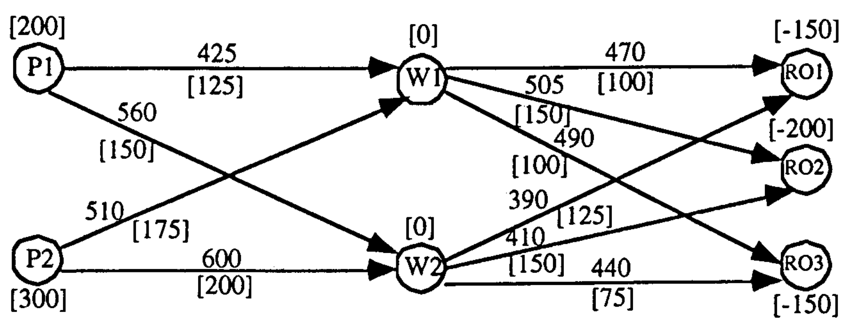
9.6-3



9.6-4 a)



b)



9.6-4

c)

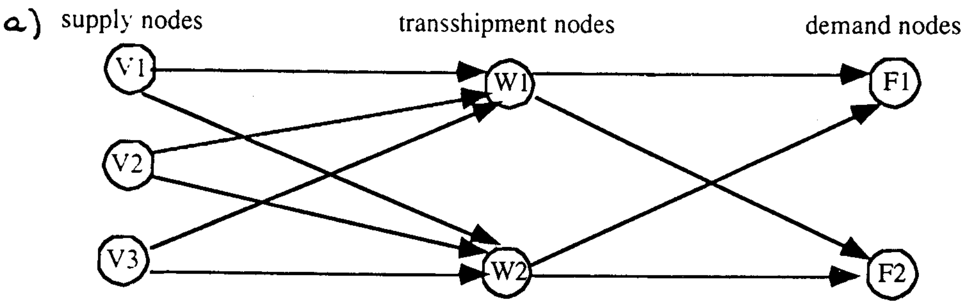
From	To	Ship		Capacity	Unit Cost
P1	W1	125	≤	125	$425
P1	W2	75	≤	150	$560
P2	W1	125	≤	175	$510
P2	W2	175	≤	200	$600
W1	RO1	100	≤	100	$470
W1	RO2	50	≤	150	$505
W1	RO3	100	≤	100	$490
W2	RO1	50	≤	125	$390
W2	RO2	150	≤	150	$410
W2	RO3	50	≤	75	$440

Total Cost = $ 488,125

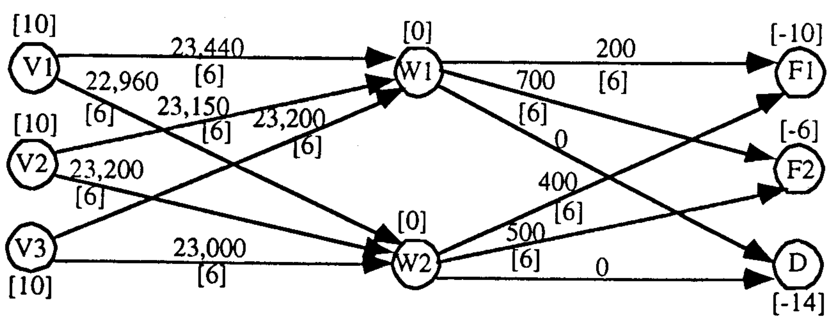
Nodes	Net Flow		Output/Demand
P1	200	=	200
P2	300	=	300
W1	0	=	0
W2	0	=	0
RO1	-150	=	-150
RO2	-200	=	-200
RO3	-150	=	-150

9.6-5

a)



b)



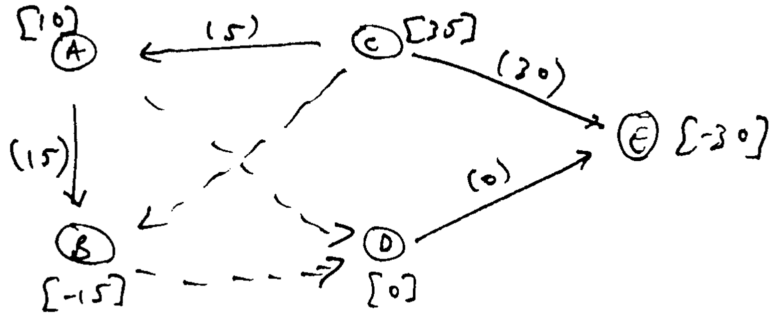
c)

From	To	Ship		Capacity	Unit Cost
V1	W1	4	≤	6	$23,440
V1	W2	6	≤	6	$22,960
V2	W1	6	≤	6	$23,150
V2	W2	4	≤	6	$23,200
V3	W1	4	≤	6	$23,200
V3	W2	6	≤	6	$23,000
W1	F1	6	≤	6	$200
W1	F2	0	≤	6	$700
W1	D	8		-	$0
W2	F1	4	≤	6	$400
W2	F2	6	≤	6	$500
W2	D	6		-	$0

Total Cost = $ 699,820

Nodes	Net Flow		Output/Demand
V1	10	=	10
V2	10	=	10
V3	10	=	10
W1	0	=	0
W2	0	=	0
F1	-10	=	-10
F2	-6	=	-6
D	-14	=	-14

9.7-1 a)



b) calculate Δ for nonbasic arcs:

$\Delta_{BD} = 5+4-3+(-6)+2 = 2$

$\Delta_{AD} = 5+4-3+(-6) = 0$

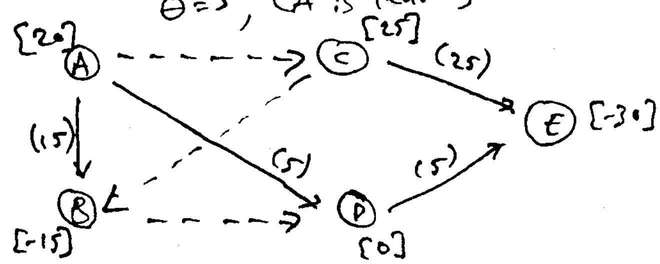
$\Delta_{CB} = (-3)-2-(-6) = 1$

all ≥ 0 ⇒ optimal

$\Delta_{AD} = 0$ ⇒ multiple optima exist

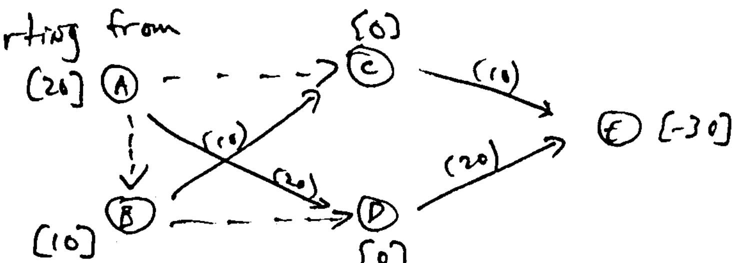
Network simplex:

$\theta = 5$, CA is leaving basic arc (reverses)



From this and part (a) we see that optimal nonbasic solutions have $x_{AB} = 15$, $x_{AC} = \theta$, $x_{AD} = 5-\theta$, $x_{CE} = 25+\theta$, $x_{DE} = 5-\theta$, where $0 \le \theta \le 5$ and C→B, B→D are 'nonbasic' arcs.

c) starting from



network simplex gives

$\Delta_{AC} = 6+3-4-5 = 0$

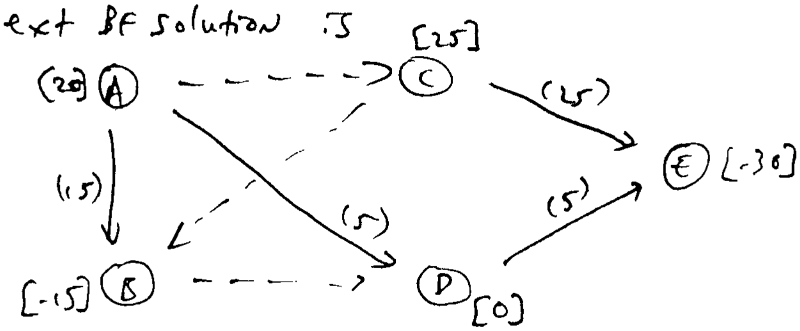
$\Delta_{AB} = 2+3+3-4-5 = -1 < 0$ ← entering arc

$\Delta_{BD} = 5+4-3-3 = 3$

$\theta = 15$, BC is leaving arc (reverses)

9.7-1 c) (CONT'D)

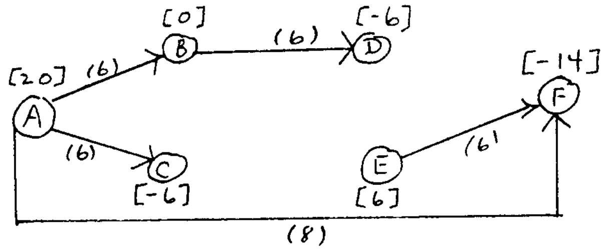
next BF solution is



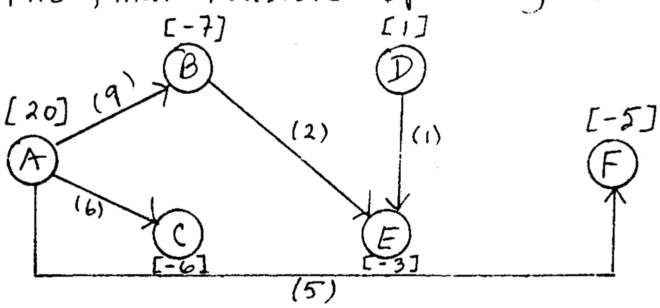
From (6) we recognize this as optimal.

9.7-2

a)



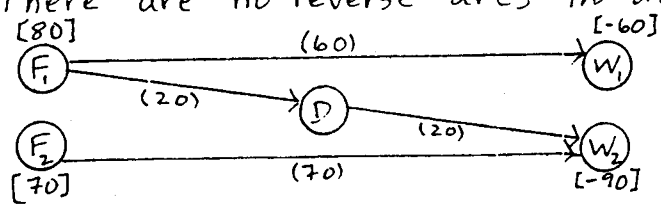
b) The final feasible spanning tree is



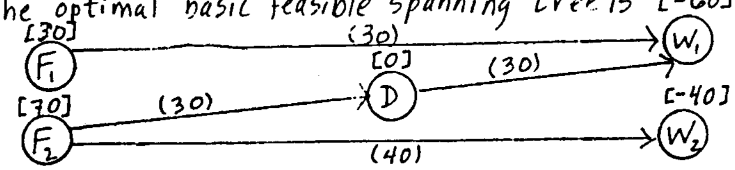
And the flow to which it corresponds is the same as in 9.5-6

9.7-3 There are no reverse arcs in this solution

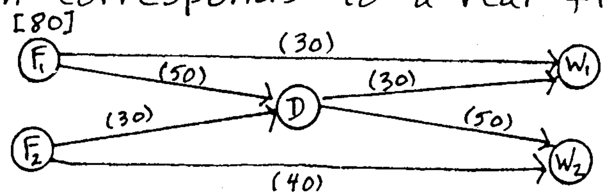
a)



b) The optimal basic feasible spanning tree is

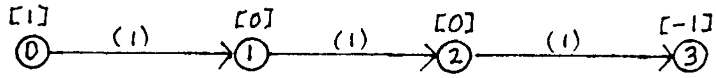


9.7-3 (CONT'D)
Which corresponds to a real flow of

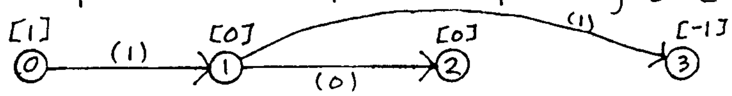


and a cost of 1,100

9.7-4 Initial basic feasible spanning tree is



The optimal basic feasible spanning tree is

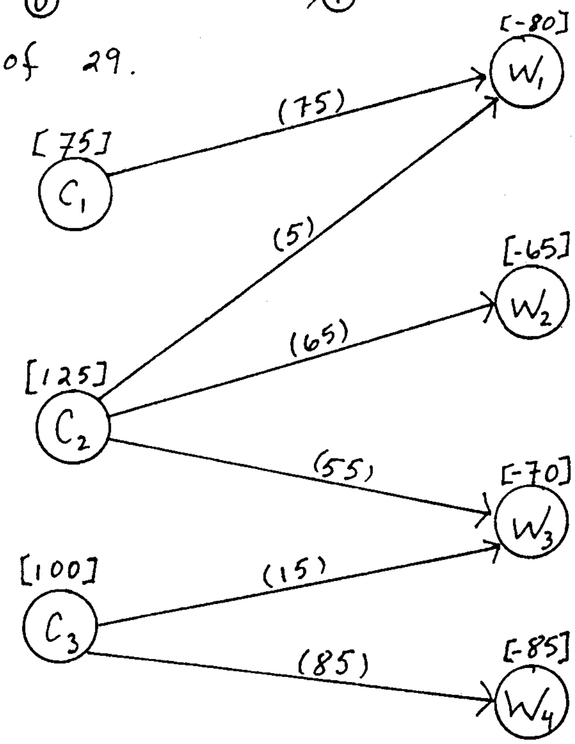


which has a real flow of

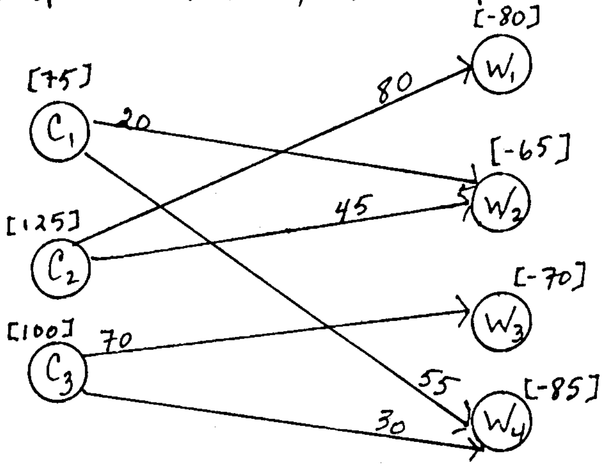
and a cost of 29.

9.7-5

Initial basic spanning tree:

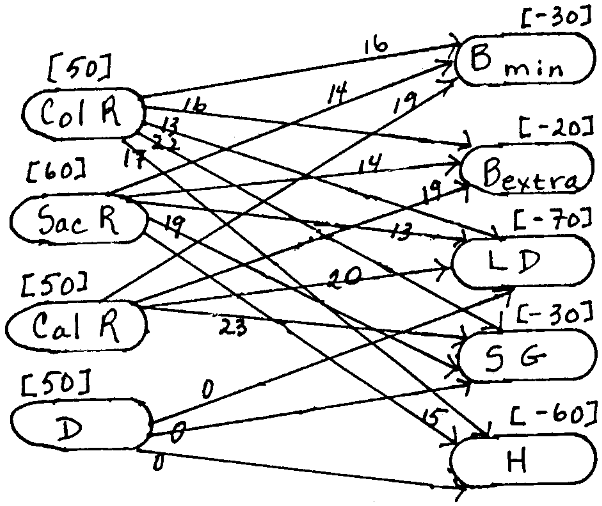


9.7-5 (continued)
The optimal basic feasible spanning tree is:

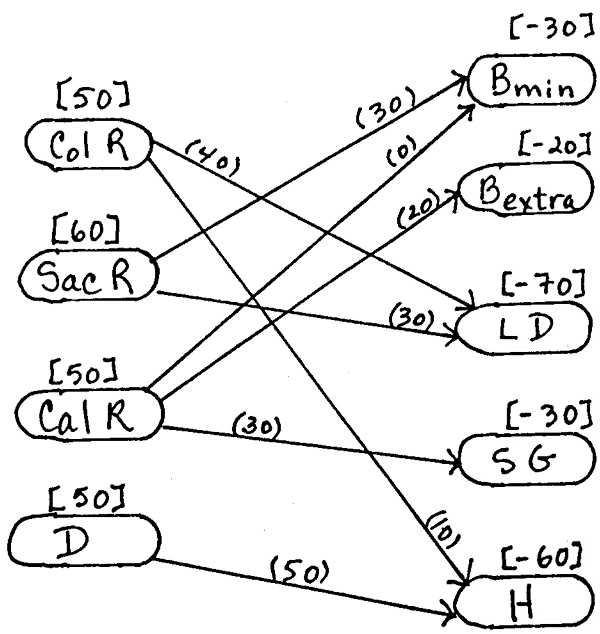


which corresponds to the optimal solution given in Sec. 8.1

9.7-6 a)

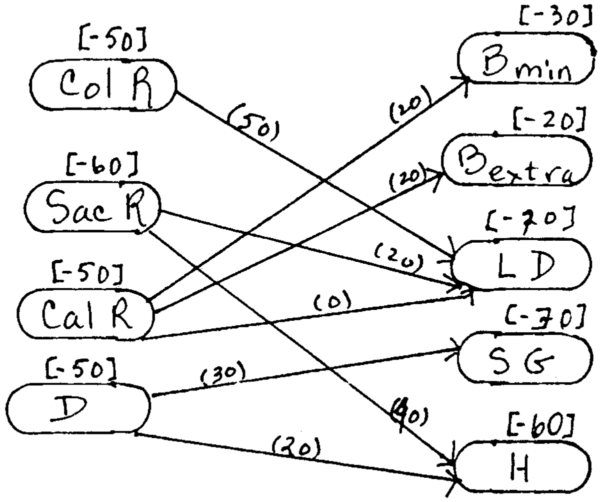


b) Initial basic feasible spanning tree



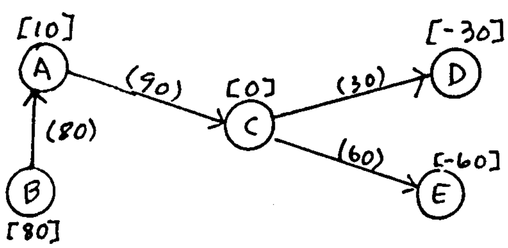
9.7-6 b) (continued)

The optimal basic feasible spanning tree is:

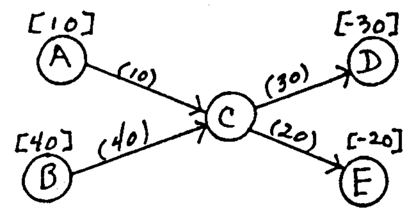


The sequence of basic feasible solutions is identical with the transportation simplex method.

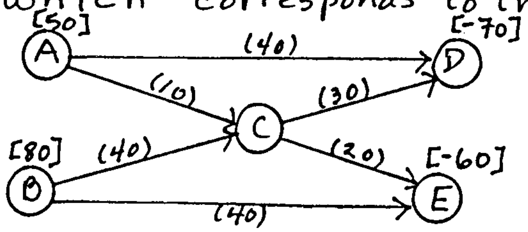
9.7-7



The optimal basic feasible spanning tree is →



which corresponds to the real flows



and a cost of 750.

9.8-1

Activity to Crash	Crash Cost	Length of Path A – C	B - D
		14	16
B	$5,000	14	15
B	$5,000	14	15
D	$6,000	14	14
C	$4,000	13	14
D	$6,000	13	13
C	$4,000	12	13
D	$6,000	12	12

9.8-2

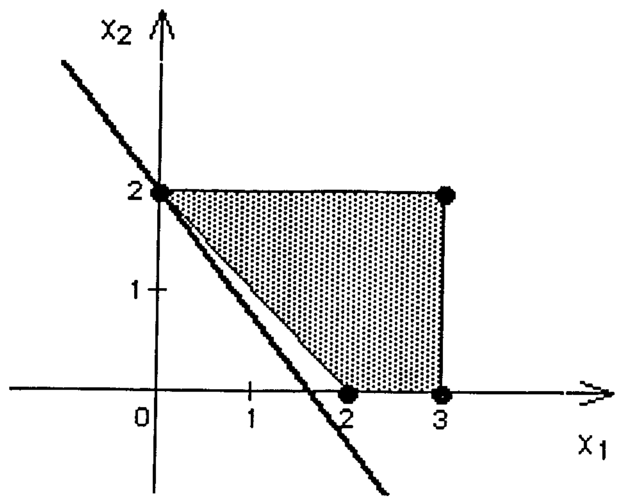
a)

Let x_A = reduction in A due to crashing
Let x_C = reduction in C due to crashing

Minimize $C = 5,000x_A + 4,000x_c$,
subject to $x_A \leq 3$
$x_C \leq 2$
$x_A + x_C \geq 2$
and $x_A \geq 0,\ x_C \geq 0$.

(CONT'D)

9.8-2 (CONT'D)

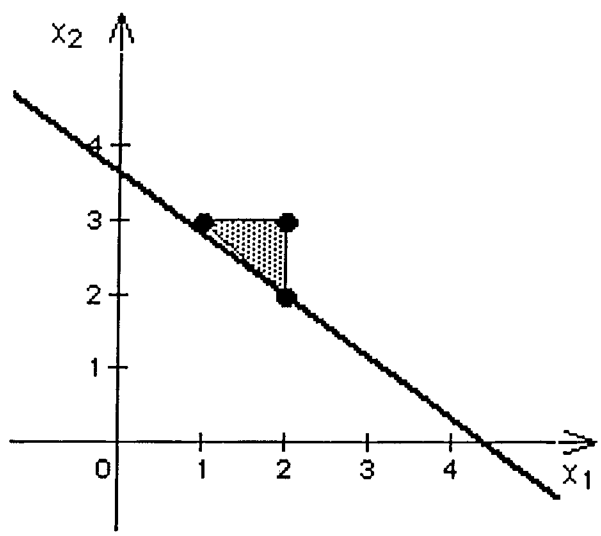


Optimal solution $(x_A, x_C) = (0, 2)$ and $C = 8{,}000$.

b)

Let $x_B =$ reduction in B due to crashing
Let $x_D =$ reduction in D due to crashing

Minimize $C = 5{,}000x_B + 6{,}000x_D$,
subject to $x_B \le 2$
$x_D \le 3$
$x_B + x_D \ge 4$
and $x_B \ge 0,\ x_D \ge 0$.



Optimal solution $(x_B, x_D) = (2, 2)$ and $C = 22{,}000$.

c)

Let x_A = reduction in A due to crashing
Let x_B = reduction in B due to crashing
Let x_C = reduction in C due to crashing
Let x_D = reduction in D due to crashing

Minimize $C = 5,000x_A + 5,000x_B + 4,000x_C + 6,000x_D$,

subject to
$$x_A \leq 3$$
$$x_B \leq 2$$
$$x_C \leq 2$$
$$x_D \leq 3$$
$$x_A + x_C \geq 2$$
$$x_B + x_D \geq 4$$
and
$$x_A \geq 0,\ x_B \geq 0,\ x_c \geq 0,\ x_D \geq 0.$$

Optimal solution $(x_A, x_B, x_C, x_D) = (0, 2, 2, 2)$ and $C = 30,000$.

d)

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	8	5	\$25000	\$40000	3	\$5000	0	0	8
B	9	7	\$20000	\$30000	2	\$5000	0	2	7
C	6	4	\$16000	\$24000	2	\$4000	8	2	12
D	7	4	\$27000	\$45000	3	\$6000	7	2	12

Finish Time = 12
Total Cost = \$118000

e) <u>Deadline of 11 months:</u>

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	8	5	\$25000	\$40000	3	\$5000	0	1	7
B	9	7	\$20000	\$30000	2	\$5000	0	2	7
C	6	4	\$16000	\$24000	2	\$4000	7	2	11
D	7	4	\$27000	\$45000	3	\$6000	7	3	11

Finish Time = 11
Total Cost = \$129000

g)

<u>Deadline of 13 months:</u>

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	8	5	\$25000	\$40000	3	\$5000	0	0	8
B	9	7	\$20000	\$30000	2	\$5000	0	2	7
C	6	4	\$16000	\$24000	2	\$4000	8	1	13
D	7	4	\$27000	\$45000	3	\$6000	7	1	13

Finish Time = 13
Total Cost = \$108000

9.8-3

a)

Activity to Crash	Crash Cost	Length of Path A – B – D	A – B – E	A – C - E
		10	11	12
C	$1,333	10	11	11
E	$2,500	10	10	10
D & E	$4,000	9	9	9
B & C	$4,333	8	8	8

New Plan:

Activity	Duration	Cost
A	3 weeks	$54,000
B	3 weeks	$65,000
C	3 weeks	$68,666
D	2 weeks	$41,500
E	2 weeks	$80,000

$7834 is saved by this crashing schedule.

b)

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	0	3
B	4	3	$62000	$65000	1	$3000	4	0	8
C	5	2	$66000	$70000	3	$1333	3	0	8
D	3	1	$40000	$43000	2	$1500	9	0	12
E	4	2	$75000	$80000	2	$2500	8	0	12

Finish Time = 12
Total Cost = $297000

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	0	3
B	4	3	$62000	$65000	1	$3000	3	0	7
C	5	2	$66000	$70000	3	$1333	3	1	7
D	3	1	$40000	$43000	2	$1500	8	0	11
E	4	2	$75000	$80000	2	$2500	7	0	11

Finish Time = 11
Total Cost = $298333

(CONT'D)

9.8-3

(CONT'D)

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	0	3
B	4	3	$62000	$65000	1	$3000	3	0	7
C	5	2	$66000	$70000	3	$1333	3	1	7
D	3	1	$40000	$43000	2	$1500	7	1.22E-15	10
E	4	2	$75000	$80000	2	$2500	7	1	10

Finish Time = 10
Total Cost = $300833

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	0	3
B	4	3	$62000	$65000	1	$3000	3	4.66E-12	7
C	5	2	$66000	$70000	3	$1333	3	1	7
D	3	1	$40000	$43000	2	$1500	7	1	9
E	4	2	$75000	$80000	2	$2500	7	2	9

Finish Time = 9
Total Cost = $304833

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	3.66E-11	3
B	4	3	$62000	$65000	1	$3000	3	1	6
C	5	2	$66000	$70000	3	$1333	3	2	6
D	3	1	$40000	$43000	2	$1500	6	1	8
E	4	2	$75000	$80000	2	$2500	6	2	8

Finish Time = 8
Total Cost = $309167

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	3	2	$54000	$60000	1	$6000	0	1	2
B	4	3	$62000	$65000	1	$3000	2	1	5
C	5	2	$66000	$70000	3	$1333	2	2	5
D	3	1	$40000	$43000	2	$1500	5	1	7
E	4	2	$75000	$80000	2	$2500	5	2	7

Finish Time = 7
Total Cost = $315167

Crash to 8 weeks.

9.8-4

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	5	3	$20	$30	2	$5	0	2	3
B	3	2	$10	$20	1	$10	0	1	2
C	4	2	$16	$24	2	$4	3	0	7
D	6	3	$25	$43	3	$6	3	0	9
E	5	4	$22	$30	1	$8	2	0	7
F	7	4	$30	$48	3	$6	2	0	9
G	9	5	$25	$45	4	$5	7	1	15
H	8	6	$30	$44	2	$7	9	2	15

Finish Time = 15
Total Cost = $217

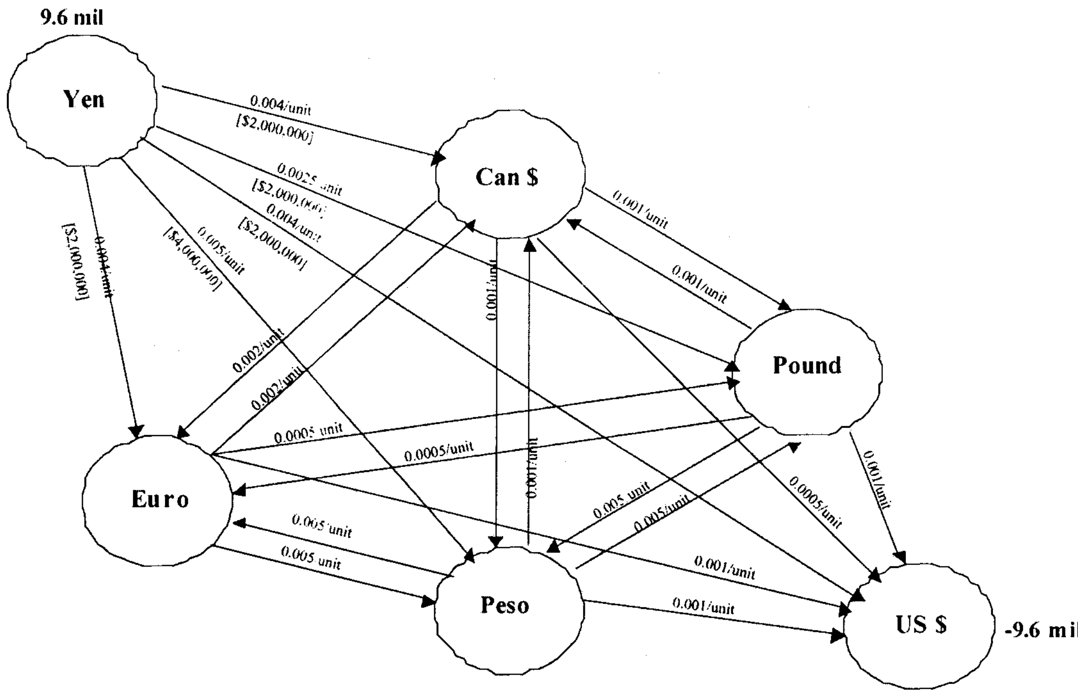
9.8-5

Activity	Time Normal	Time Crash	Cost Normal	Cost Crash	Maximum Time Reduction	Crash Cost per Week saved	Start Time	Time Reduction	Finish Time
A	32	28	$160	$180	4	$5	8	0	40
B	28	25	$125	$146	3	$7	0	3	25
C	36	31	$170	$210	5	$8	40	0	76
D	16	13	$60	$72	3	$4	25	0	41
E	32	27	$135	$160	5	$5	25	0	57
F	54	47	$215	$257	7	$6	25	3	76
G	17	15	$90	$96	2	$3	41	0	58
H	20	17	$120	$132	3	$4	58	0	78
I	34	30	$190	$226	4	$9	58	0	92
J	18	16	$80	$84	2	$2	76	2	92

Finish Time = 92
Total Cost = $1388

Cases

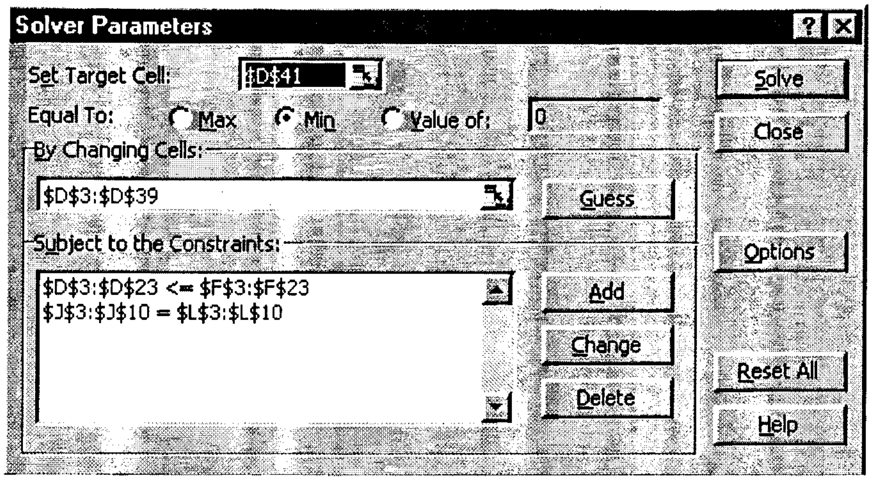
9.1 a) There are three supply nodes – the Yen node, the Rupiah node, and the Ringgit node. There is one demand node – the US$ node. Below, we draw the network originating from only the Yen supply node to illustrate the overall design of the network. In this network, we exclude both the Rupiah and Ringgit nodes for simplicity.

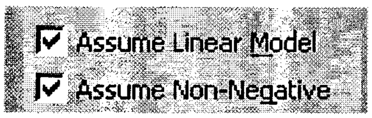


b) Since all transaction limits are given in the equivalent of 1000 dollars we define the flow variables as the amount in 1000's of dollars that Jake converts from one currency into another one. His total holdings in Yen, Rupiah, and Ringgit are equivalent to \$9.6 million, \$1.68 million, and \$5.6 million, respectively. So, the supplies at the supply nodes Yen, Rupiah, and Ringgit are -\$9.6 million, -\$1.68 million, and -\$5.6 million, respectively. The demand at the only demand node US\$ equals \$16.88 million. The transaction limits are capacity constraints for all arcs leaving from the nodes Yen, Rupiah, and Ringgit. The unit cost for every arc is given by the transaction cost for the currency conversion.

	A	B	C	D	E	F	G	H	I	J	K	L	M
1													
2		From	To	Ship		Capacity	Unit Cost		Nodes	Net Flow	Supply /Demand		
3		Yen	Rupiah	0	<=	5000	0.005		Yen	-9600	=	-9600	
4		Yen	Ringgit	0	<=	5000	0.005		Rupiah	-1680	=	-1680	
5		Yen	US$	2000	<=	2000	0.004		Ringgit	-5600	=	-5600	
6		Yen	Can$	2000	<=	2000	0.004		US$	16880	=	16880	
7		Yen	Euro	2000	<=	2000	0.004		Can$	0	=	0	
8		Yen	Pound	2000	<=	2000	0.0025		Euro	0	=	0	
9		Yen	Peso	1600	<=	4000	0.005		Pound	0	=	0	
10		Rupiah	Yen	0	<=	5000	0.005		Peso	0	=	0	
11		Rupiah	Ringgit	0	<=	2000	0.007						
12		Rupiah	US$	200	<=	200	0.005						
13		Rupiah	Can$	200	<=	200	0.003						
14		Rupiah	Euro	1000	<=	1000	0.003						
15		Rupiah	Pound	80	<=	500	0.0075						
16		Rupiah	Peso	200	<=	200	0.0075						
17		Ringgit	Yen	0	<=	3000	0.005						
18		Ringgit	Rupiah	0	<=	4500	0.007						
19		Ringgit	US$	1100	<=	1500	0.007						
20		Ringgit	Can$	0	<=	1500	0.007						
21		Ringgit	Euro	2500	<=	2500	0.004						
22		Ringgit	Pound	1000	<=	1000	0.0045						
23		Ringgit	Peso	1000	<=	1000	0.005						
24		Can$	US$	2200		-	0.0005						
25		Can$	Euro	0		-	0.002						
26		Can$	Pound	0		-	0.001						
27		Can$	Peso	0		-	0.001						
28		Euro	US$	5500		-	0.001						
29		Euro	Can$	0		-	0.002						
30		Euro	Pound	0		-	0.0005						
31		Euro	Peso	0		-	0.005						
32		Pound	US$	3080		-	0.001						
33		Pound	Can$	0		-	0.001						
34		Pound	Euro	0		-	0.0005						
35		Pound	Peso	0		-	0.005						
36		Peso	US$	2800		-	0.001						
37		Peso	Can$	0		-	0.001						
38		Peso	Euro	0		-	0.005						
39		Peso	Pound	0		-	0.005						
40													
41			Total Cost	$83,380.00									

	J
1	
2	Net Flow
3	=-SUM(D3:D9)+D10+D17
4	=-SUM(D10:D16)+D3+D18
5	=-SUM(D17:D23)+D4+D11
6	=D5+D12+D19+D24+D28+D32+D36
7	=D6+D13+D20-SUM(D24:D27)+D29+D33+D37
8	=D7+D14+D21+D25-SUM(D28:D31)+D34+D38
9	=D8+D15+D22+D26+D30-SUM(D32:D35)+D39
10	=D9+D16+D23+D27+D31+D35-SUM(D36:D39)
11	

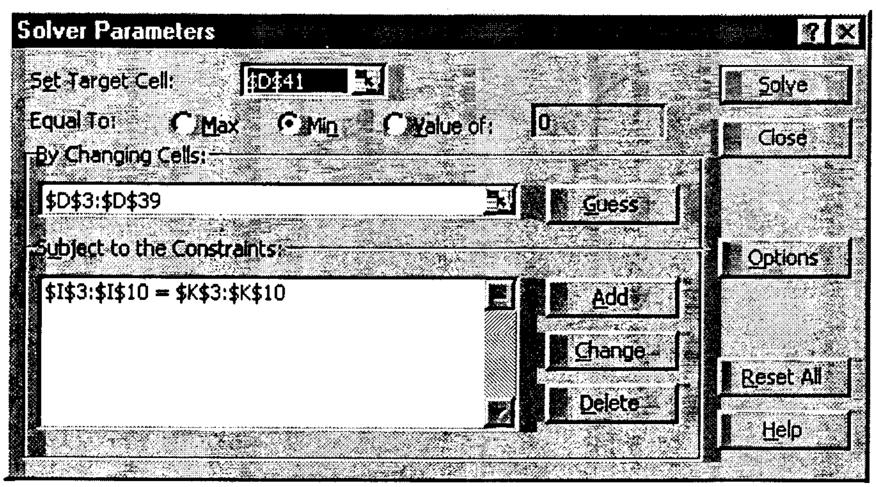




Jake should convert the equivalent of $2 million from Yen to each US$, Can$, Euro, and Pound. He should convert $1.6 million from Yen to Peso. Moreover, he should convert the equivalent of $200,000 from Rupiah to each US$, Can$, and Peso, $1 million from Rupiah to Euro, and $80,000 from Rupiah to Pound. Furthermore, Jake should convert the equivalent of $1.1 million from Ringgit to US$, $2.5 million from Ringgit to Euro, and $1 million from Ringgit to each Pound and Peso. Finally, he should convert all the money he converted into Can$, Euro, Pound, and Peso directly into US$. Specifically, he needs to convert into US$ the equivalent of $2.2 million, $5.5 million, $3.08 million, and $2.8 million Can$, Euro, Pound, and Peso, respectively. Assuming Jake pays for the total transaction costs of $83,380 directly from his American bank accounts he will have $16,880,000 dollars to invest in the US.

c) We eliminate all capacity restrictions on the arcs.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		From	To	Ship	Capacity	Unit Cost		Nodes	Net Flow	Supply /Demand		
3		Yen	Rupiah	0	-	0.005		Yen	-9600	=	-9600	
4		Yen	Ringgit	0	-	0.005		Rupiah	-1680	=	-1680	
5		Yen	US$	0	-	0.004		Ringgit	-5600	=	-5600	
6		Yen	Can$	0	-	0.004		US$	16880	=	16880	
7		Yen	Euro	0	-	0.004		Can$	0	=	0	
8		Yen	Pound	9600	-	0.0025		Euro	0	=	0	
9		Yen	Peso	0	-	0.005		Pound	0	=	0	
10		Rupiah	Yen	0	-	0.005		Peso	0	=	0	
11		Rupiah	Ringgit	0	-	0.007						
12		Rupiah	US$	0	-	0.005						
13		Rupiah	Can$	1680	-	0.003						
14		Rupiah	Euro	0	-	0.003						
15		Rupiah	Pound	0	-	0.0075						
16		Rupiah	Peso	0	-	0.0075						
17		Ringgit	Yen	0	-	0.005						
18		Ringgit	Rupiah	0	-	0.007						
19		Ringgit	US$	0	-	0.007						
20		Ringgit	Can$	0	-	0.007						
21		Ringgit	Euro	5600	-	0.004						
22		Ringgit	Pound	0	-	0.0045						
23		Ringgit	Peso	0	-	0.005						
24		Can$	US$	1680	-	0.0005						
25		Can$	Euro	0	-	0.002						
26		Can$	Pound	0	-	0.001						
27		Can$	Peso	0	-	0.001						
28		Euro	US$	5600	-	0.001						
29		Euro	Can$	0	-	0.002						
30		Euro	Pound	0	-	0.0005						
31		Euro	Peso	0	-	0.005						
32		Pound	US$	9600	-	0.001						
33		Pound	Can$	0	-	0.001						
34		Pound	Euro	0	-	0.0005						
35		Pound	Peso	0	-	0.005						
36		Peso	US$	0	-	0.001						
37		Peso	Can$	0	-	0.001						
38		Peso	Euro	0	-	0.005						
39		Peso	Pound	0	-	0.005						
40												
41			Total Cost	$67,480.00								
42												



Jake should convert the entire holdings in Japan from Yen into Pound and then into US$, the entire holdings in Indonesia from Rupiah into Can$ and then into US$, and the entire holdings in Malaysia from Ringgit into Euro and then into US$. Without the capacity limits the transaction costs are reduced to $67,480.00.

d) We multiply all unit cost for Rupiah by 6.

	A	B	C	D	E	F	G	H	I	J	K	L
1												
2		From	To	Ship	Capacity	Unit Cost		Nodes	Net Flow		Supply/ Demand	
3		Yen	Rupiah	0	-	0.005		Yen	-9600	=	-9600	
4		Yen	Ringgit	0	-	0.005		Rupiah	-1680	=	-1680	
5		Yen	US$	0	-	0.004		Ringgit	-5600	=	-5600	
6		Yen	Can$	0	-	0.004		US$	16880	=	16880	
7		Yen	Euro	0	-	0.004		Can$	0	=	0	
8		Yen	Pound	9600	-	0.0025		Euro	0	=	0	
9		Yen	Peso	0	-	0.005		Pound	0	=	0	
10		Rupiah	Yen	0	-	0.03		Peso	0	=	0	
11		Rupiah	Ringgit	0	-	0.042						
12		Rupiah	US$	0	-	0.03						
13		Rupiah	Can$	1680	-	0.018						
14		Rupiah	Euro	0	-	0.018						
15		Rupiah	Pound	0	-	0.045						
16		Rupiah	Peso	0	-	0.045						
17		Ringgit	Yen	0	-	0.005						
18		Ringgit	Rupiah	0	-	0.007						
19		Ringgit	US$	0	-	0.007						
20		Ringgit	Can$	0	-	0.007						
21		Ringgit	Euro	5600	-	0.004						
22		Ringgit	Pound	0	-	0.0045						
23		Ringgit	Peso	0	-	0.005						
24		Can$	US$	1680	-	0.0005						
25		Can$	Euro	0	-	0.002						
26		Can$	Pound	0	-	0.001						
27		Can$	Peso	0	-	0.001						
28		Euro	US$	5600	-	0.001						
29		Euro	Can$	0	-	0.002						
30		Euro	Pound	0	-	0.0005						
31		Euro	Peso	0	-	0.005						
32		Pound	US$	9600	-	0.001						
33		Pound	Can$	0	-	0.001						
34		Pound	Euro	0	-	0.0005						
35		Pound	Peso	0	-	0.005						
36		Peso	US$	0	-	0.001						
37		Peso	Can$	0	-	0.001						
38		Peso	Euro	0	-	0.005						
39		Peso	Pound	0	-	0.005						
40												
41			Total Cost =	$92,680.00								
42												

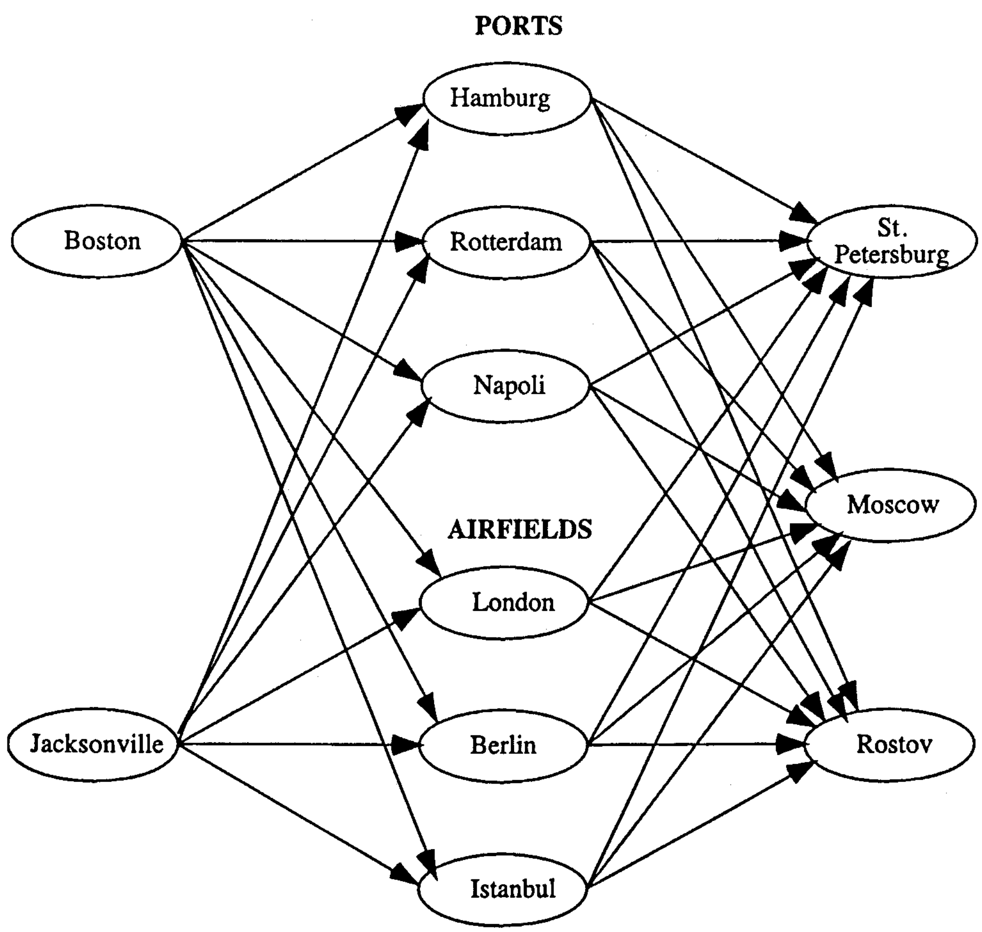
The optimal routing for the money doesn't change, but the total transaction costs are now increased to $92,680.

e) In the described crisis situation the currency exchange rates might change every minute. Jake should carefully check the exchange rates again when he performs the transactions.

The European economies might be more insulated from the Asian financial collapse than the US economy. To impress his boss Jake might want to explore other investment opportunities in safer European economies that provide higher rates of return than US bonds.

Cases

9.2 a) The network showing the different routes troops and supplies may follow to reach the Russian Federation appears below.



b) The President is only concerned about how to most quickly move troops and supplies from the United States to the three strategic Russian cities. Obviously, the best way to achieve this goal is to find the fastest connection between the US and the three cities. We therefore need to find the shortest path between the US and each of the three cities.

The President only cares about the time it takes to get the troops and supplies to Russia. It does not matter how great a distance the troops and supplies cover. Therefore we define the arc length between two nodes in the network to be the time it takes to travel between the respective cities. For example, the distance between Boston and London equals 6,200 km. The mode of transportation between the cities is a Starfighter traveling at a speed of 400 miles per hour * 1.609 km per mile = 643.6 km per hour. The time is takes to bring troops and supplies from Boston to London equals 6,200 km / 643.6 km per hour = 9.6333 hours. Using this approach we can compute the time of travel along all arcs in the network.

By simple inspection and common sense it is apparent that the fastest transportation involves using only airplanes. We therefore can restrict ourselves to only those arcs in the network where the mode of transportation is air travel. We can omit the three port cities and all arcs entering and leaving these nodes.

Finally, we define a new node ("dummy" node) in the network called "US," and we introduce two new arcs: one going from the US to Boston and the other going from the US to Jacksonville. The arc length on both new arcs equals 0. The objective is now to find the shortest path from the US to each of the three Russian cities. We define the US node to be a supply node with supply 3, and we define each of the three nodes representing Russian cities as demand nodes with a demand of -1. The nodes representing the three European airfields – London, Berlin, and Istanbul – are all transshipment nodes.

The following spreadsheet shows the entire linear programming model, which identifies the three shortest paths.

	A	B	C	D	E	F	G	H	I	J	K
1		From	To	On Route	Distance	Time (hr)		Nodes	Net Flow		Supply/Demand
2		US	Boston	3	0	0		US	3	=	3
3		US	Jacksonville	0	0	0		Boston	0	=	0
4		Boston	London	2	6200	9.63331		Jacksonville	0	=	0
5		Boston	Berlin	1	7250	11.2648		London	0	=	0
6		Boston	Istanbul	0	8300	12.8962		Berlin	0	=	0
7		Jacksonville	London	0	7900	12.2747		Istanbul	0	=	0
8		Jacksonville	Berlin	0	9200	14.2946		St. Petersburg	-1	=	-1
9		Jacksonville	Istanbul	0	10100	15.693		Moscow	-1	=	-1
10		London	St. Petersburg	1	1980	3.07644		Rostov	-1	=	-1
11		London	Moscow	1	2300	3.57365					
12		London	Rostov	0	2860	4.44375					
13		Berlin	St. Petersburg	0	1280	1.98881					
14		Berlin	Moscow	0	1600	2.48602					
15		Berlin	Rostov	1	1730	2.688					
16		Istanbul	St. Petersburg	0	2040	3.16967					
17		Istanbul	Moscow	0	1700	2.64139					
18		Istanbul	Rostov	0	990	1.53822					
19											
20			Total Time =	39.86948415							

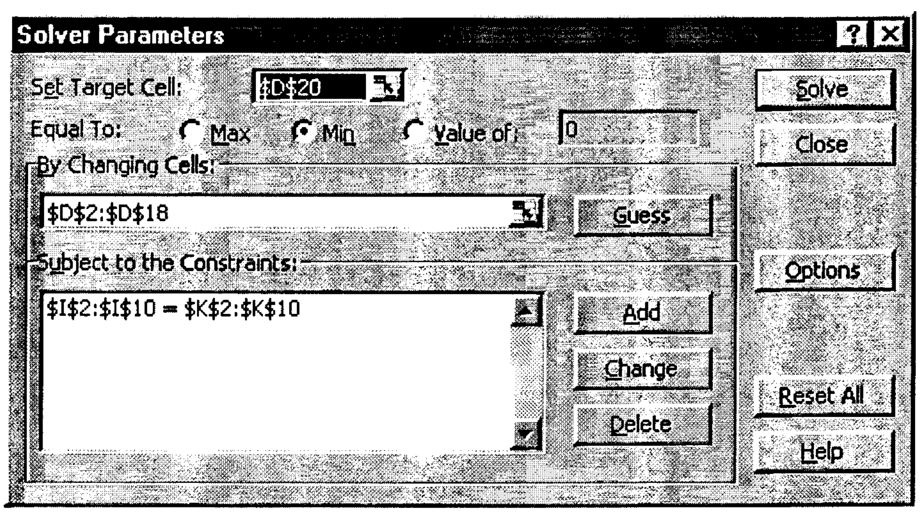
The spreadsheet contains the following formulas:

	F
1	Time (hr)
2	0
3	0
4	=E4/(400*1.609)
5	=E5/(400*1.609)
6	=E6/(400*1.609)
7	=E7/(400*1.609)
8	=E8/(400*1.609)
9	=E9/(400*1.609)
10	=E10/(400*1.609)
11	=E11/(400*1.609)
12	=E12/(400*1.609)
13	=E13/(400*1.609)
14	=E14/(400*1.609)
15	=E15/(400*1.609)
16	=E16/(400*1.609)
17	=E17/(400*1.609)
18	=E18/(400*1.609)
19	
20	

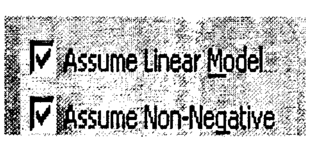
	I
1	Net Flow
2	=SUM(D2:D3)
3	=-D2+SUM(D4:D6)
4	=-D3+SUM(D7:D9)
5	=-D4-D7+D10+D11+D12
6	=-D5-D8+D13+D14+D15
7	=-D6-D9+D16+D17+D18
8	=-D10-D13-D16
9	=-D11-D14-D17
10	=-D12-D15-D18
11	

	C	D
20	Total Time =	=SUMPRODUCT(D2:D18,F2:F18)

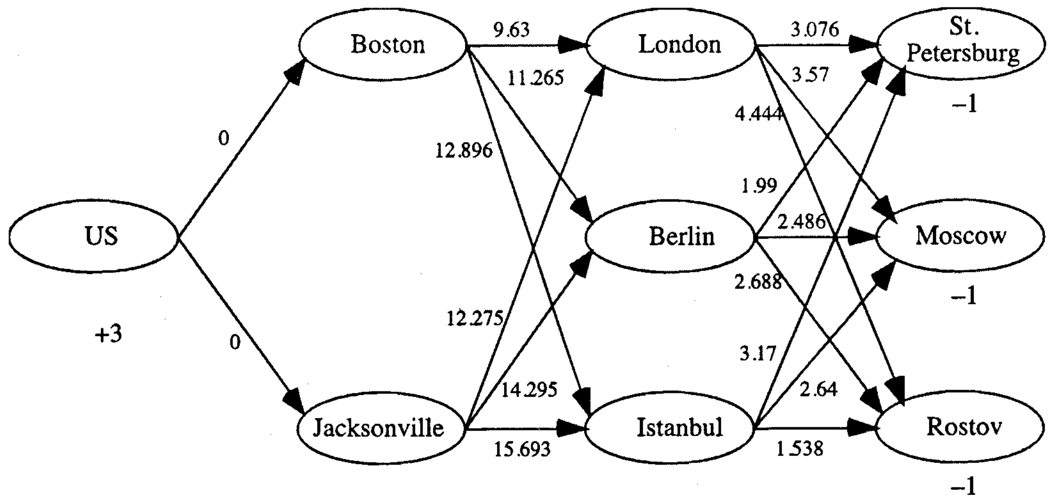
The solver dialogue box appears as follows.



Throughout the analysis of this case we use the following solver options.



From the optimal solution to the linear programming model we see that the shortest path from the US to Saint Petersburg is Boston → London → Saint Petersburg with a total travel time of 12.710 hours. The shortest path from the US to Moscow is Boston → London → Moscow with a total travel time of 13.207 hours. The shortest path from the US to Rostov is Boston → Berlin → Rostov with a total travel time of 13.953 hours. The following network diagram highlights these shortest paths.



c) The President must satisfy each Russian city's military requirements at minimum cost. Therefore, this problem can be solved as a minimum-cost network flow problem. The two nodes representing US cities are supply nodes with a supply of 500 each (we measure all weights in 1000 tons). The three nodes representing Saint Petersburg, Moscow, and Rostov are demand nodes with demands of –320. -440, and –240, respectively. All nodes representing European airfields and ports are transshipment nodes. We measure the flow along the arcs in 1000 tons. For some arcs, capacity constraints are given. All arcs from the European ports into Saint Petersburg have zero capacity. All truck routes from the European ports into Rostov have a transportation limit of 2,500*16 = 40,000 tons. Since we measure the arc flows in 1000 tons, the corresponding arc capacities equal 40. An analogous computation yields arc capacities of 30 for both the arcs connecting the nodes London and Berlin to Rostov. For all other nodes we determine natural arc capacities based on the supplies and demands at the nodes. We define the unit costs along the arcs in the network in S1000 per 1000 tons. For example, the cost of transporting 1 ton of material from Boston to Hamburg equals $30,000 / 240 = $125, so the costs of transporting 1000 tons from Boston to Hamburg equals $125,000.

The objective is to satisfy all demands in the network at minimum cost. The following spreadsheet shows the entire linear programming model.

	A	B	C	D	E	F	G	H	I	J	K
1	From	To	Ship	Capacity (in 1000 tons)	Cost of Transport	Unit Cost (in $1000 per 1000 tons)		Nodes	Net Flow		Supply/Demand
2	Boston	Hamburg	440	500	30000	125		Boston	500	=	500
3	Boston	Rotterdam	0	500	30000	125		Jacksonville	500	=	500
4	Boston	Napoli	0	500	32000	133.3333333		Hamburg	0	=	0
5	Boston	London	0	500	45000	300		Rotterdam	0	=	0
6	Boston	Berlin	0	500	50000	333.3333333		Napoli	0	=	0
7	Boston	Istanbul	60	500	55000	366.6666667		London	0	=	0
8	Jacksonville	Hamburg	0	500	48000	200		Berlin	0	=	0
9	Jacksonville	Rotterdam	0	500	44000	183.3333333		Istanbul	0	=	0
10	Jacksonville	Napoli	0	500	56000	233.3333333		St. Petersburg	-320	=	-320
11	Jacksonville	London	350	500	49000	326.6666667		Moscow	-440	=	-440
12	Jacksonville	Berlin	0	500	57000	380		Rostov	-240	=	-240
13	Jacksonville	Istanbul	150	500	61000	406.6666667					
14	Hamburg	St. Petersburg	0	0	3000	187.5					
15	Rotterdam	St. Petersburg	0	0	3000	187.5					
16	Napoli	St. Petersburg	0	0	5000	312.5					
17	London	St. Petersburg	320	320	22000	146.6666667					
18	Berlin	St. Petersburg	0	320	24000	160					
19	Istanbul	St. Petersburg	0	320	28000	186.6666667					
20	Hamburg	Moscow	440	440	4000	250					
21	Rotterdam	Moscow	0	440	5000	312.5					
22	Napoli	Moscow	0	440	5000	312.5					
23	London	Moscow	0	440	19000	126.6666667					
24	Berlin	Moscow	0	440	22000	146.6666667					
25	Istanbul	Moscow	0	440	25000	166.6666667					
26	Hamburg	Rostov	0	40	7000	437.5					
27	Rotterdam	Rostov	0	40	8000	500					
28	Napoli	Rostov	0	40	9000	562.5					
29	London	Rostov	30	30	4000	26.66666667					
30	Berlin	Rostov	0	30	23000	153.3333333					
31	Istanbul	Rostov	210	240	2000	13.33333333					
32											
33		Total Cost =	412866.6667								

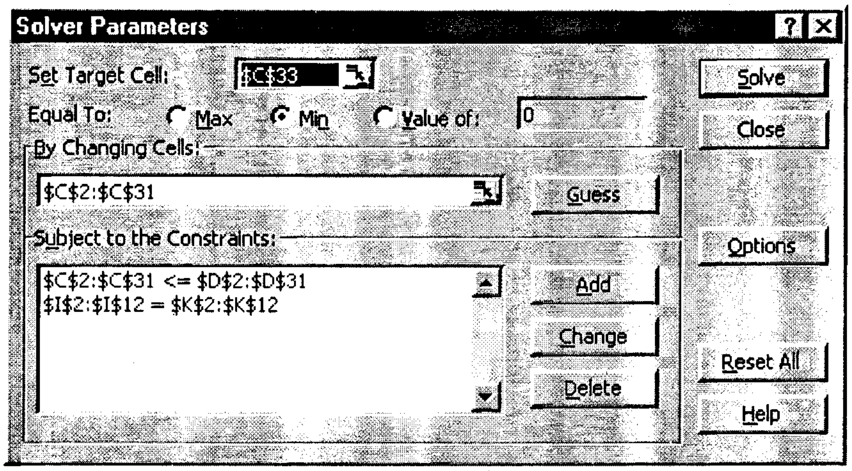
The following formulas appear in the spreadsheet.

	F
1	Unit Cost (in $1000 per 1000 tons)
2	=E2/240
3	=E3/240
4	=E4/240
5	=E5/150
6	=E6/150
7	=E7/150
8	=E8/240
9	=E9/240
10	=E10/240
11	=E11/150
12	=E12/150
13	=E13/150
14	=E14/16
15	=E15/16
16	=E16/16
17	=E17/150
18	=E18/150
19	=E19/150
20	=E20/16
21	=E21/16
22	=E22/16
23	=E23/150
24	=E24/150
25	=E25/150
26	=E26/16
27	=E27/16
28	=E28/16
29	=E29/150
30	=E30/150
31	=E31/150
32	

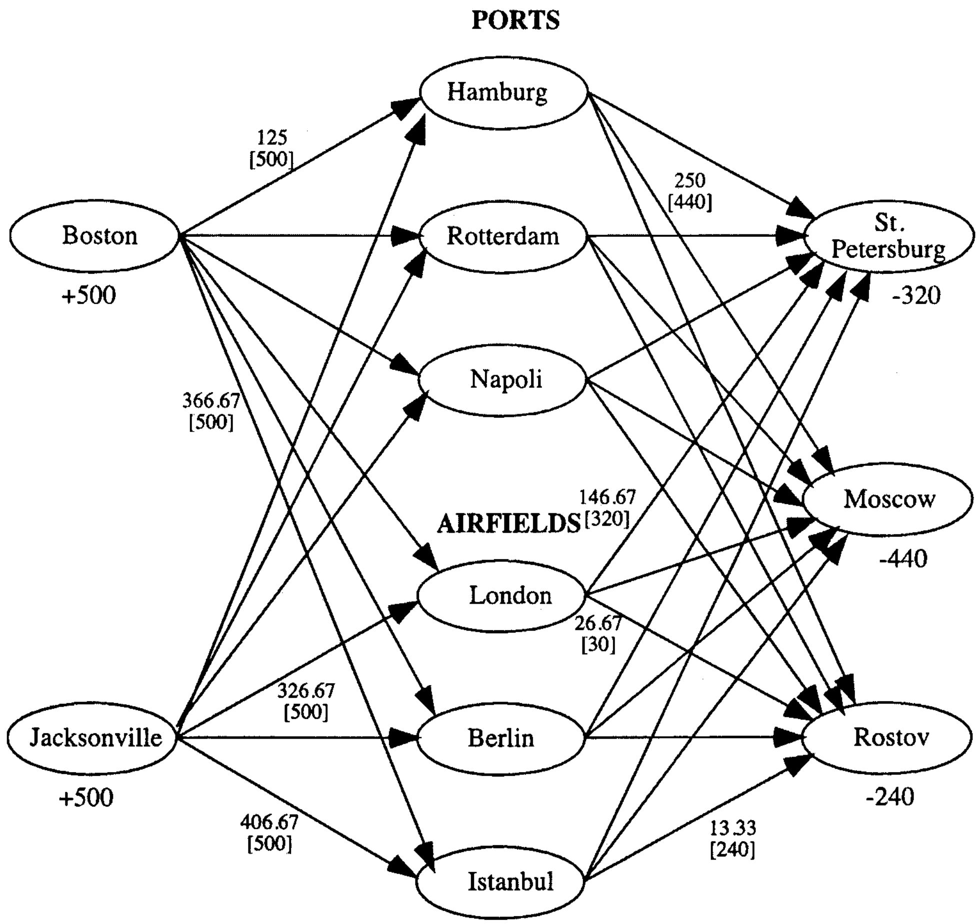
	I
1	Net Flow
2	=SUM(C2:C7)
3	=SUM(C8:C13)
4	=-C2-C8+C14+C20+C26
5	=-C3-C9+C15+C21+C27
6	=-C4-C10+C16+C22+C28
7	=-C5-C11+C17+C23+C29
8	=-C6-C12+C18+C24+C30
9	=-C7-C13+C19+C25+C31
10	=-SUM(C14:C19)
11	=-SUM(C20:C25)
12	=-SUM(C26:C31)
13	

	B	C
33	Total Cost =	=SUMPRODUCT(C2:C31,F2:F31)

We use the following solver dialogue box for this model.



The total cost of the operation equals $412,866,666.67. The entire supply for Saint Petersburg is supplied from Jacksonville via London. The entire supply for Moscow is supplied from Boston via Hamburg. Of the 240 (= 240,000 tons) demanded by Rostov, 60 are shipped from Boston via Istanbul, 150 are shipped from Jacksonville via Istanbul, and 30 are shipped from Jacksonville via London. The paths used to ship supplies to Saint Petersburg, Moscow, and Rostov are highlighted on the following network diagram.


PORTS
Hamburg
Rotterdam
Napoli
AIRFIELDS
London
Berlin
Istanbul
Boston
+500
Jacksonville
+500
St. Petersburg
-320
Moscow
-440
Rostov
-240
125
[500]
366.67
[500]
326.67
[500]
406.67
[500]
250
[440]
146.67
[320]
26.67
[30]
13.33
[240]

d) Now the President wants to maximize the amount of cargo transported from the US to the Russian cities. In other words, the President wants to maximize the flow from the two US cities to the three Russian cities. All the nodes representing the European ports and airfields are once again transshipment nodes. The flow along an arc is again measured in thousands of tons. The new restrictions can be transformed into arc capacities using the same approach that was used in part (c). The objective is now to maximize the combined flow into the three Russian cities.

The linear programming model describing the maximum flow problem appears as follows.

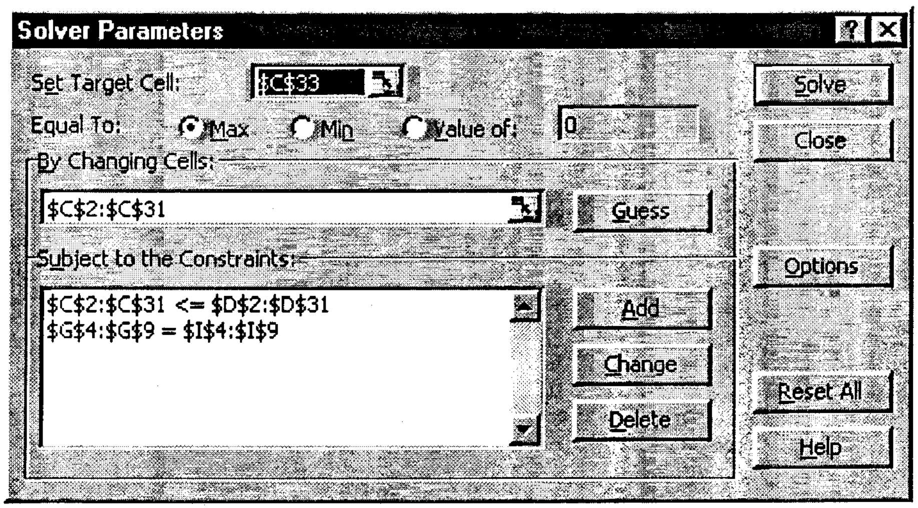
	A	B	C	D	E	F	G	H	I
1	From	To	Ship	Capacity (in 1000 tons)		Nodes	Net Flow		Supply/Demand
2	Boston	Hamburg	19.2	500		Boston	282.2		
3	Boston	Rotterdam	21.6	500		Jacksonville	240		
4	Boston	Napoli	46.4	500		Hamburg	0	=	0
5	Boston	London	75	75		Rotterdam	0	=	0
6	Boston	Berlin	45	45		Napoli	0	=	0
7	Boston	Istanbul	75	75		London	0	=	0
8	Jacksonville	Hamburg	0	500		Berlin	0	=	0
9	Jacksonville	Rotterdam	0	500		Istanbul	0	=	0
10	Jacksonville	Napoli	0	500		St. Petersburg	-225		
11	Jacksonville	London	90	90		Moscow	-104.8		
12	Jacksonville	Berlin	75	75		Rostov	-192.4		
13	Jacksonville	Istanbul	75	105					
14	Hamburg	St. Petersburg	0	0					
15	Rotterdam	St. Petersburg	0	0					
16	Napoli	St. Petersburg	0	0					
17	London	St. Petersburg	150	150					
18	Berlin	St. Petersburg	75	75					
19	Istanbul	St. Petersburg	0	0					
20	Hamburg	Moscow	11.2	11.2					
21	Rotterdam	Moscow	9.6	9.6					
22	Napoli	Moscow	24	24					
23	London	Moscow	0	30					
24	Berlin	Moscow	45	45					
25	Istanbul	Moscow	15	15					
26	Hamburg	Rostov	8	8					
27	Rotterdam	Rostov	12	12					
28	Napoli	Rostov	22.4	22.4					
29	London	Rostov	15	15					
30	Berlin	Rostov	0	0					
31	Istanbul	Rostov	135	135					
32									
33		Total Cost =	522.2						

The following formulas appear in the spreadsheet.

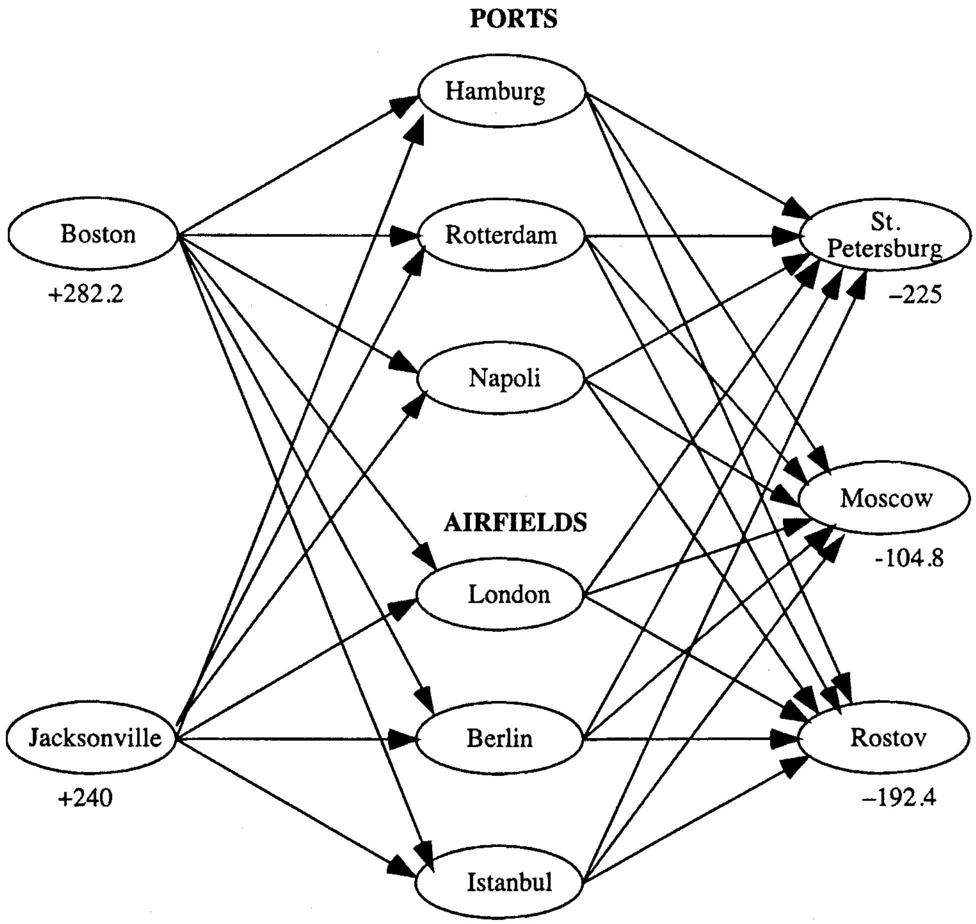
	G
1	Net Flow
2	=SUM(C2:C7)
3	=SUM(C8:C13)
4	=-C2-C8+C14+C20+C26
5	=-C3-C9+C15+C21+C27
6	=-C4-C10+C16+C22+C28
7	=-C5-C11+C17+C23+C29
8	=-C6-C12+C18+C24+C30
9	=-C7-C13+C19+C25+C31
10	=-SUM(C14:C19)
11	=-SUM(C20:C25)
12	=-SUM(C26:C31)
13	

	B	C
33	Total Cost =	=SUM(G2:G3)

We use the following solver dialogue box.



The worksheet shows all the amounts that are shipped between the various cities. The total supply for Saint Petersburg, Moscow, and Rostov equals 225,000 tons, 104,800 tons, and 192,400 tons, respectively. The following network diagram highlights the paths used to ship supplies between the US and the Russian Federation.



e) The creation of the new communications network is a minimum spanning tree problem. As usual, a greedy algorithm solves this type of problem.

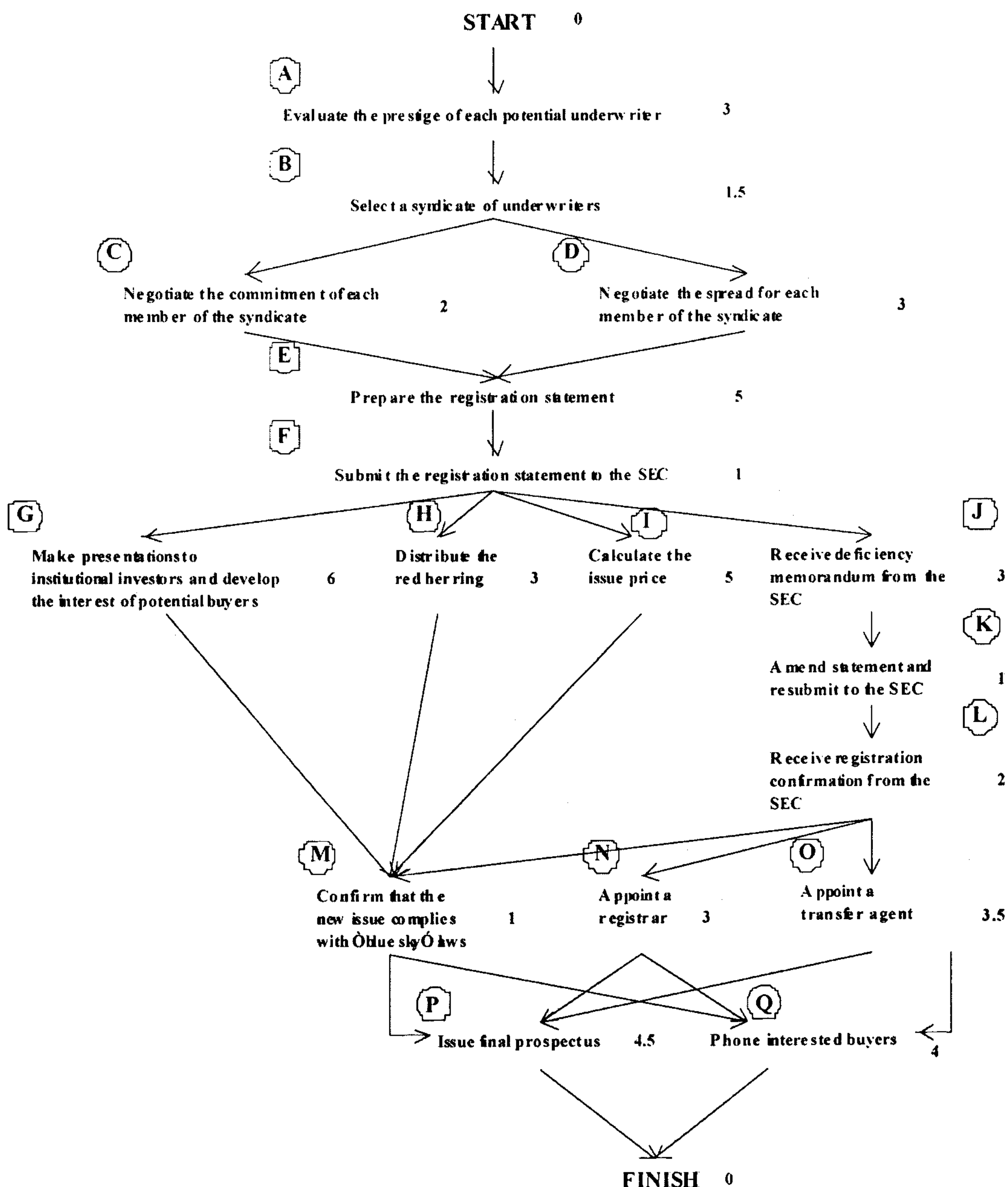
Arcs are added to the network in the following order (one of several optimal solutions):

Rostov - Orenburg	120
Ufa - Orenburg	75
Saratov - Orenburg	95
Saratov - Samara	100
Samara - Kazan	95
Ufa – Yekaterinburg	125
Perm – Yekaterinburg	85

The minimum cost of reestablishing the communication lines is $695,000.

Cases

9.3 a) A diagram of the project network appears below.



By inspection, the longest path and so the critical path is START → A → B → C → D → E → F → J → K → L → O → P → Finish. The length of this path and so the duration of the initial public offering process is 27.5 weeks.

	A	B	C	D	E	F	G	H	I	J
3		Activity	Description	Time	ES	EF	LS	LF	Slack	Critical?
4		A	Evaluate prestige	3	0	=E4+D4	=H4-D4	=G5	=H4-F4	=IF(I4=0,"Yes","No")
5		B	Select syndicate	1.5	=MAX(F4)	=E5+D5	=H5-D5	=MIN(G6,G7)	=H5-F5	=IF(I5=0,"Yes","No")
6		C	Negotiate commitment	2	=MAX(F5)	=E6+D6	=H6-D6	=G8	=H6-F6	=IF(I6=0,"Yes","No")
7		D	Negotiate spread	3	=MAX(F5)	=E7+D7	=H7-D7	=G8	=H7-F7	=IF(I7=0,"Yes","No")
8		E	Prepare registration	5	=MAX(F6,F7)	=E8+D8	=H8-D8	=G9	=H8-F8	=IF(I8=0,"Yes","No")
9		F	Submit registration	1	=MAX(F8)	=E9+D9	=H9-D9	=MIN(G10,G11,G12,G13)	=H9-F9	=IF(I9=0,"Yes","No")
10		G	Present	6	=MAX(F9)	=E10+D10	=H10-D10	=G16	=H10-F10	=IF(I10=0,"Yes","No
11		H	Distribute red herring	3	=MAX(F9)	=E11+D11	=H11-D11	=G16	=H11-F11	=IF(I11=0,"Yes","No
12		I	Calculate price	5	=MAX(F9)	=E12+D12	=H12-D12	=G16	=H12-F12	=IF(I12=0,"Yes","No
13		J	Receive deficiency	3	=MAX(F9)	=E13+D13	=H13-D13	=G14	=H13-F13	=IF(I13=0,"Yes","No
14		K	Amend statement	1	=MAX(F13)	=E14+D14	=H14-D14	=G15	=H14-F14	=IF(I14=0,"Yes","No
15		L	Receive registration	2	=MAX(F14)	=E15+D15	=H15-D15	=MIN(G18,G17,G16)	=H15-F15	=IF(I15=0,"Yes","No
16		M	Confirm blue sky	1	=MAX(F10,F11,F12,F15)	=E16+D16	=H16-D16	=MIN(G19,G20)	=H16-F16	=IF(I16=0,"Yes","No
17		N	Appoint registrar	3	=MAX(F15)	=E17+D17	=H17-D17	=MIN(G19,G20)	=H17-F17	=IF(I17=0,"Yes","No
18		O	Appoint transfer	3.5	=MAX(F15)	=E18+D18	=H18-D18	=MIN(G19,G20)	=H18-F18	=IF(I18=0,"Yes","No
19		P	Issue prospectus	4.5	=MAX(F16,F17,F18)	=E19+D19	=H19-D19	=F22	=H19-F19	=IF(I19=0,"Yes","No
20		Q	Phone buyers	4	=MAX(F16,F17,F18)	=E20+D20	=H20-D20	=F22	=H20-F20	=IF(I20=0,"Yes","No
21										
22					Project Duration	=MAX(F19,F20)				

The values in the new spreadsheet appear below.

	A	B	C	D	E	F	G	H	I	J
3		Activity	Description	Time	ES	EF	LS	LF	Slack	Critical?
4		A	Evaluate prestige	3	0	3	0	3	0	Yes
5		B	Select syndicate	1.5	3	4.5	3	4.5	0	Yes
6		C	Negotiate commitment	2	4.5	6.5	5.5	7.5	1	No
7		D	Negotiate spread	3	4.5	7.5	4.5	7.5	0	Yes
8		E	Prepare registration	5	7.5	12.5	7.5	12.5	0	Yes
9		F	Submit registration	1	12.5	13.5	12.5	13.5	0	Yes
10		G	Present	6	13.5	19.5	16	22	2.5	No
11		H	Distribute red herring	3	13.5	16.5	19	22	5.5	No
12		I	Calculate price	5	13.5	18.5	17	22	3.5	No
13		J	Receive deficiency	3	13.5	16.5	13.5	16.5	0	Yes
14		K	Amend statement	1	16.5	17.5	16.5	17.5	0	Yes
15		L	Receive registration	2	17.5	19.5	17.5	19.5	0	Yes
16		M	Confirm blue sky	1	19.5	20.5	22	23	2.5	No
17		N	Appoint registrar	3	19.5	22.5	20	23	0.5	No
18		O	Appoint transfer	3.5	19.5	23	19.5	23	0	Yes
19		P	Issue prospectus	4.5	23	27.5	23	27.5	0	Yes
20		Q	Phone buyers	4	23	27	23.5	27.5	0.5	No
21										
22					Project Duration	27.5				

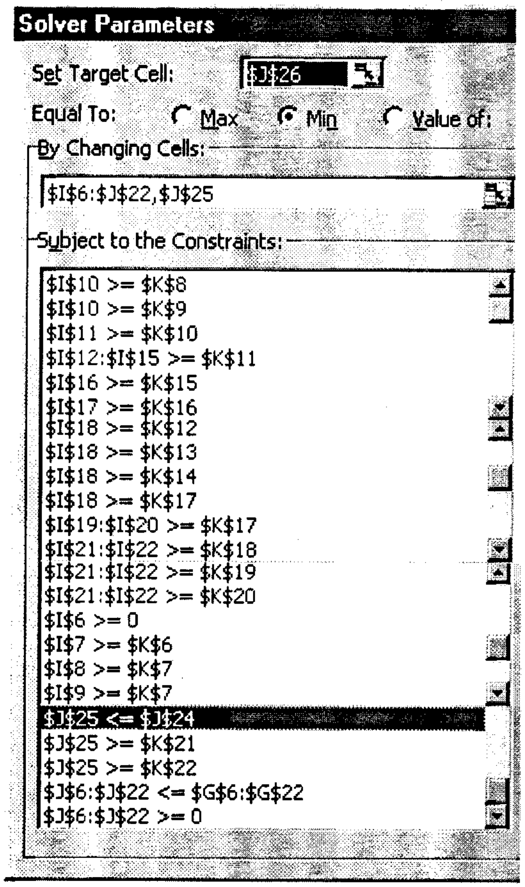
b) We formulate a linear programming problem to make the crashing decisions.

	A	B	C	D	E	F	G	H	I	J	K
3							Maximum	Crash Cost			
4			Time		Cost		Time	per Week	Start	Time	Finish
5		Activity	Normal	Crash	Normal	Crash	Reduction	saved	Time	Reduction	Time
6		A	3	1.5	8000	14000	=C6-D6	=(F6-E6)/G6	0	1.5	=I6+C6-J6
7		B	1.5	0.5	4500	8000	=C7-D7	=(F7-E7)/G7	1.5	1	=I7+C7-J7
8		C	2	2	9000	0	=C8-D8	0	2	0	=I8+C8-J8
9		D	3	3	12000	0	=C9-D9	0	2	0	=I9+C9-J9
10		E	5	4	50000	95000	=C10-D10	=(F10-E10)/G10	5	0	=I10+C10-J10
11		F	1	1	1000	0	=C11-D11	0	10	0	=I11+C11-J11
12		G	6	4	25000	60000	=C12-D12	=(F12-E12)/G12	11	0	=I12+C12-J12
13		H	3	2	15000	22000	=C13-D13	=(F13-E13)/G13	14	0	=I13+C13-J13
14		I	5	3.5	12000	31000	=C14-D14	=(F14-E14)/G14	11	0	=I14+C14-J14
15		J	3	3	0	0	=C15-D15	0	11	0	=I15+C15-J15
16		K	1	0.5	6000	9000	=C16-D16	=(F16-E16)/G16	14	0.5	=I16+C16-J16
17		L	2	2	0	0	=C17-D17	0	14.5	0	=I17+C17-J17
18		M	1	0.5	5000	8300	=C18-D18	=(F18-E18)/G18	17	0	=I18+C18-J18
19		N	3	1.5	12000	19000	=C19-D19	=(F19-E19)/G19	16.5	1.5	=I19+C19-J19
20		O	3.5	1.5	13000	21000	=C20-D20	=(F20-E20)/G20	16.5	2	=I20+C20-J20
21		P	4.5	2	40000	99000	=C21-D21	=(F21-E21)/G21	18	0.5	=I21+C21-J21
22		Q	4	1.5	9000	20000	=C22-D22	=(F22-E22)/G22	18	0	=I22+C22-J22
23											
24									Desired Finish	22	
25									Finish Time =	22	
26									Total Cost	==SUM(E6:E22)+SUMPRODUCT(H6:H22,J6:J22)	

The values used in the spreadsheet appear below.

	A	B	C	D	E	F	G	H	I	J	K
3							Maximum	Crash Cost			
4			Time		Cost		Time	per Week	Start	Time	Finish
5		Activity	Normal	Crash	Normal	Crash	Reduction	saved	Time	Reduction	Time
6		A	3	1.5	$8000	$14000	1.5	$4000	0.0	1.5	1.5
7		B	1.5	0.5	$4500	$8000	1	$3500	1.5	1	2
8		C	2	2	$9000	$0	0	$0	2.0	0	4
9		D	3	3	$12000	$0	0	$0	2.0	0	5
10		E	5	4	$50000	$95000	1	$45000	5.0	0	10
11		F	1	1	$1000	$0	0	$0	10.0	0	11
12		G	6	4	$25000	$60000	2	$17500	11.0	0	17
13		H	3	2	$15000	$22000	1	$7000	14.0	0	17
14		I	5	3.5	$12000	$31000	1.5	$12667	11.0	0	16
15		J	3	3	$0	$0	0	$0	11.0	0	14
16		K	1	0.5	$6000	$9000	0.5	$6000	14.0	0.5	14.5
17		L	2	2	$0	$0	0	$0	14.5	0	16.5
18		M	1	0.5	$5000	$8300	0.5	$6600	17.0	0	18
19		N	3	1.5	$12000	$19000	1.5	$4667	16.5	1.5	18
20		O	3.5	1.5	$13000	$21000	2	$4000	16.5	2.0	18
21		P	4.5	2	$40000	$99000	2.5	$23600	18.0	0.5	22
22		Q	4	1.5	$9000	$20000	2.5	$4400	18.0	0	22
23											
24									Desired Finish	22	
25									Finish Time =	22	
26									Total Cost =	$260800	

The Solver settings for the linear programming appear below.



Janet and Gilbert should reduce the time for step A (evaluating the prestige of each potential underwriter) by 1.5 weeks, the time for step B (selecting a syndicate of underwriters) by one week, the time for step K (amending statement and resubmitting it to the SEC) by 0.5 weeks, the time for step N (appointing a registrar) by 1.5 weeks, the time for step O (appointing a transfer agent) by two weeks, and the time for step P (issuing final prospectus) by 0.5 weeks. Janet and Gilbert can now meet the new deadline of 22 weeks at a total cost of $260,800.

C) We use the same model formulation that was used in part (c). We change one constraint, however. The project duration now has to be greater than or equal to 24 weeks instead of 22 weeks. We obtain the following solution in Excel.

	A	B	C	D	E	F	G	H	I	J	K
3							Maximum	Crash Cost			
4			Time		Cost		Time	per Week	Start	Time	Finish
5		Activity	Normal	Crash	Normal	Crash	Reduction	saved	Time	Reduction	Time
6		A	3	1.5	$8000	$14000	1.5	$4000	0.0	1.5	1.5
7		B	1.5	0.5	$4500	$8000	1	$3500	1.5	1	2
8		C	2	2	$9000	$0	0	$0	2.0	0	4
9		D	3	3	$12000	$0	0	$0	2.0	0	5
10		E	5	4	$50000	$95000	1	$45000	5.0	0	10
11		F	1	1	$1000	$0	0	$0	10.0	0	11
12		G	6	4	$25000	$60000	2	$17500	12.5	0	18.5
13		H	3	2	$15000	$22000	1	$7000	15.5	0	18.5
14		I	5	3.5	$12000	$31000	1.5	$12667	11.0	0	16
15		J	3	3	$0	$0	0	$0	11.0	0	14
16		K	1	0.5	$6000	$9000	0.5	$6000	14.0	0.5	14.5
17		L	2	2	$0	$0	0	$0	14.5	0	16.5
18		M	1	0.5	$5000	$8300	0.5	$6600	18.5	0	19.5
19		N	3	1.5	$12000	$19000	1.5	$4667	16.5	0	19.5
20		O	3.5	1.5	$13000	$21000	2	$4000	16.5	0.5	19.5
21		P	4.5	2	$40000	$99000	2.5	$23600	19.5	0	24
22		Q	4	1.5	$9000	$20000	2.5	$4400	19.5	0	23.5
23											
24									Desired Finish	24	
25									Finish Time =	24	
26									Total Cost =	$236000	

Janet and Gilbert should reduce the time for step A (evaluating the prestige of each potential underwriter) by 1.5 weeks, the time for step B (selecting a syndicate of underwriters) by one week, the time for step K (amending statement and resubmitting it to the SEC) by 0.5 weeks, and the time for step O (appointing a transfer agent) by 0.5 weeks. Janet and Gilbert can now meet the new deadline of 24 weeks at a total cost of $236,000.