Instituto de Física

FAP 2292

Gabarito – Lista de Exercícios 8
Ondas Eletromagnéticas

Exercícios Sugeridos (01/06/2010) — A numeração corresponde ao Livros Textos A e B.

A24.0 $f{=}c/\lambda$: a) $3{\times}10^{10}$ Hz=30 GHz; b) $3{\times}10^{14}$ Hz; c) $5{,}17{\times}10^{14}$ Hz; d) $3{\times}10^{15}$ Hz; e) $3{\times}10^{20}$ Hz.

A24.7 $\mathbf{E}{=}E_0\,\mathrm{sen}(kx-\omega t)\,\hat{y}$: $E_0{=}100$ V/m, $k{=}1{,}00{\times}10^7\ \mathrm{m}^{-1}$.
a) $\lambda{=}2\pi/k{=}6{,}28{\times}10^{-7}$ m=0,628 μm, $f{=}4{,}77{\times}10^{14}$ Hz;
b) $\mathbf{B}{=}B_0\,\mathrm{sen}(kx-\omega t)\,\hat{z}$; $(B_0{=}3{,}34{\times}10^{-7}\ \mathrm{T})$;
c) $\mathbf{S}{=}(1/\mu_0)\mathbf{E}\times\mathbf{B}{=}S_0\,\mathrm{sen}^2(kx-\omega t)\,\hat{x}$, $S_0{=}26{,}5\ \mathrm{W/m^2}$.

A24.17 $I=\dfrac{P}{4\pi r^2}=\dfrac{1}{2}\epsilon_0 c E_0^2; \mathcal{E}_0=E_0\ell=27{,}1\ \mathrm{mV}$

A24.19 $P=0{,}3IA \Rightarrow A=3{,}33{\times}10^3\ \mathrm{m}^2$.

A24.25 $P{=}I/c{=}8{,}3{\times}10^{-8}$ Pa.

A24.27 $P{=}15$ mW, $A{=}\pi d^2/4{=}3{,}1\ \mathrm{mm}^2$, $I{=}P/A{=}4{,}8\ \mathrm{kW/m^2}$.
a) $E_0{=}\sqrt{2\mu_0 c I}{=}1{,}9$ kV/m; b) $U{=}(I/c)LA{=}5{,}0{\times}10^{-11}$ J; c) $P{=}U/c{=}1{,}7{\times}10^{-19}$ kg m/s.

A24.28 a) $F{=}(I_S/c)\pi R_T^2{=}5{,}70{\times}10^8$ N; b) $F_G=GM_TM_S/R^2=3{,}55{\times}10^{22}$ N.

A24.52 a) $B_0{=}E_0/c{=}6{,}7{\times}10^{-16}$ T; b) $I{=}\frac{1}{2}c\epsilon_0E_0^2{=}5{,}3{\times}10^{-17}\ \mathrm{W/m^2}$;
c) $P{=}IA{=}6{,}7{\times}10^{-12}$ W; d) $F{=}IA/c{=}P/c{=}2{,}2{\times}10^{-20}$ N.

A24.55 a) $\mathbf{E}(x,t){=}E_0\,\mathrm{sen}(kx-\omega t)\,\hat{y}$, $\mathbf{B}(x,t){=}B_0\,\mathrm{sen}(kx-\omega t)\,\hat{z}$:
$k{=}2\pi/\lambda{=}4{,}19\ \mathrm{cm}^{-1}$, $\omega{=}kc{=}1{,}26{\times}10^9$ rad/s, $B_0{=}E_0/c=5{,}84{\times}10^{-7}$ T;
b) $\mathbf{S}{=}S_0\,\mathrm{sen}^2(kx-\omega t)\,\hat{k}$, $S_0{=}c\epsilon_0E_0^2{=}81{,}3\ \mathrm{A/m^2}$;
c) (reflexão) $P_{\mathrm{rad}}=2I/c=S_0/c=2{,}7{\times}10^{-7}$ Pa; d) $a=P_{\mathrm{rad}}A/m=4{,}1{\times}10^{-7}\ \mathrm{m/s^2}$.

A24.57 (a) $F_{\mathrm{rad}}=P/c=Ma; \ell=\dfrac{1}{2}at^2 \Rightarrow t=\sqrt{2\ell Mc/P}=1{,}28{\times}10^4\ \mathrm{s}=3{,}57$ hora.
(b) $(M-m)V=mv, \quad v+V=v_0 \Rightarrow V=\dfrac{m}{M}v_0 \Rightarrow t=\dfrac{\ell}{V}=30{,}6$ s.

B32.10 (a) No sentido de $-\hat{\imath}$ (b) $2\pi f=\omega=kc \Rightarrow f=6{,}58{\times}10^{11}$ Hz
(c) $\mathbf{E}=c\mathbf{B}\times(-\hat{\imath}) \Rightarrow \mathbf{E}(x,t)=(2{,}47\ \mathrm{V/m})\ \hat{k}\,\mathrm{sen}\left[\left(1{,}38{\times}10^4\ \mathrm{rad/m}\right)x+\omega t\right]$

B32.45 $2mr^2\left.\dfrac{\mathrm{d}\omega}{\mathrm{d}t}\right|_{\mathrm{max}}=\dfrac{IA}{c}(2r-r) \Rightarrow \left.\dfrac{\mathrm{d}\omega}{\mathrm{d}t}\right|_{\mathrm{max}}=\dfrac{\epsilon_0E_0^2}{4mr}=3{,}9\ \mathrm{rad/s^2}$.

B32.47 (a) $\mathbf{E}=\rho\mathbf{J}=\dfrac{\rho I}{\pi a^2}\,\hat{z}$; (b) $\mathbf{B}(r{<}a)=\dfrac{\mu_0 I}{2\pi a^2}r\,\hat{\varphi}$; (c) $\mathbf{S}(r{<}a)=\dfrac{1}{\mu_0}\mathbf{E}\times\mathbf{B}(r)=-\dfrac{\rho I^2 r}{2\pi^2a^4}\,\hat{\rho}$;
(d) $\left.\dfrac{\mathrm{d}U}{\mathrm{d}t}\right|_{\text{para dentro}}=-\oiint \mathbf{S}\cdot\mathrm{d}\mathbf{A}=S2\pi r\ell=\left(\dfrac{\rho\ell}{\pi r^2}\right)\left(I\dfrac{r^2}{a^2}\right)^2=R(r)I(r)^2$.
Note que o fluxo do vetor de Poynting para dentro de uma porção do cilindro de raio r é igual à potência dissipada nesta porção do resistor que tem resistência $R(r)=\rho\ell/\pi r^2$ pela qual passa uma corrente $I(r)=Ir^2/a^2$.

B32.49 $I = \dfrac{P}{4\pi r^2} = \dfrac{c}{2\mu_0}B_0^2; \quad \lambda = c/f = 3{,}16\text{ m} \gg D : \mathcal{E}_{\max} = \left.\dfrac{\mathrm{d}\Phi}{\mathrm{d}t}\right|_{\max} \approx \omega B_0 \dfrac{\pi D^2}{4} = 36{,}8\text{ mV}$

B32.57 (b) $\dfrac{1}{2}m_\mathrm{p}v^2 = K \Rightarrow v = 3{,}39\times10^7\text{ m/s}; a = v^2/r = 1{,}53\times10^{15}\text{ m/s}^2$

$P = 1{,}34\times10^{-23}\text{ W} = 8{,}37\times10^{-11}\text{ MeV/s}; \quad \dfrac{P}{K} = 1{,}4\times10^{-11}\text{ s}^{-1}$

(c) $K = \dfrac{1}{2}m_\mathrm{e}v^2 = 5{,}24\times10^{-16}\text{ J} = 3{,}27\text{ keV}; \quad \dfrac{P}{K} = 2{,}6\times10^{-8}\text{ s}^{-1}$

B32.58 (c) $k_\mathrm{C} = 1{,}5\times10^4\text{ m}^{-1} \Rightarrow \dfrac{1}{k_\mathrm{C}} = 66\ \mu\text{m}$

P3.1 a) $\rho(\mathbf{r},t) = \dfrac{\epsilon_0 E_0}{a}\,\mathrm{sen}(\omega t).$

b) $\mathbf{J}(\mathbf{r},t) = -\omega\epsilon_0 E_0 \dfrac{x}{a}\cos(\omega t)\,\hat{x}.$

c) $\boldsymbol{\nabla}\times\mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0\dfrac{\partial\mathbf{E}}{\partial t}\right) = 0, \quad -\dfrac{\partial\mathbf{B}}{\partial t} = \boldsymbol{\nabla}\times\mathbf{E} = 0, \Rightarrow \mathbf{B} = \vec{0}.$

P3.2 a) $\mathbf{J}_d(\rho,\varphi,z,t) = -\dfrac{\epsilon_0 E_0}{t_0}\,\hat{z}$ na região $0 \le \rho < R$, e $\mathbf{J}_d(\rho,\varphi,z,t) = \mathbf{0}$ para $\rho > R$.

b) $\mathbf{B}(\rho,\varphi,z,t) = -\mu_0\dfrac{\epsilon_0 E_0}{2t_0}\rho\,\hat{\varphi}$, para $\rho < R$

$\mathbf{B}(\rho,\varphi,z,t) = -\mu_0\dfrac{\epsilon_0 E_0}{2t_0}\dfrac{R^2}{\rho}\,\hat{\varphi}$, para $\rho > R$.

c) $U = \frac{1}{2}\epsilon_0 E_0^2\pi R^2 d\left[\left(1 - \dfrac{t}{t_0}\right)^2 + \dfrac{\mu_0\epsilon_0 R^2}{8t_0^2}\right].$

P3.3 a) $\mathbf{k} = k\,\hat{y} = \dfrac{2\pi}{y_0}\,\hat{y}, \quad \lambda = \dfrac{2\pi}{k} = y_0, \quad f = \dfrac{c}{\lambda} = 7{,}5\times10^{17}\text{ Hz}..$

b) $\mathbf{E} = cB_0\dfrac{\hat{z} - \sqrt{3}\,\hat{y}}{2}\,\mathrm{sen}\left[2\pi\left(\dfrac{y}{y_0} - \dfrac{t}{t_0}\right)\right].$

c) $I = \frac{1}{2}\epsilon_0 c E_0^2 = \frac{1}{2}\dfrac{c}{\mu_0}B_0^2.$

NA5.1 (a) $r = -0{,}20$; (b) $T = 0{,}96$.

NA5.2 $\dfrac{E_r}{E_i} = r = \dfrac{1 - n_v}{1 + n_v} = -0{,}09; \quad \dfrac{E_t}{E_i} = t = \dfrac{2}{1 + n_v} = 0{,}91; \quad T = \dfrac{S_t}{S_i} = \dfrac{4n_v}{(1 + n_v)^2} = 0{,}99$

$$\begin{aligned}\text{(a)} \quad \mathbf{E}_r(x,t) &= 2rE_0\cos\left(-k_0x - \omega t\right)\,\hat{y} + 3rE_0\,\mathrm{sen}\left(-k_0x - \omega t\right)\,\hat{z} \\ &= 2rE_0\cos\left(k_0x + \omega t\right)\,\hat{y} - 3rE_0\,\mathrm{sen}\left(k_0x + \omega t\right)\,\hat{z}\end{aligned}$$

$$\begin{aligned}\text{(b)} \quad \mathbf{S}_t(x,t) &= \frac{n_v}{\mu_0 c}\left|\mathbf{E}_t(x,t)\right|^2\,\hat{x} = \frac{n_v t^2 E_0^2}{\mu_0 c}\left[4\cos^2\left(k_0x - \omega t\right) + 9\,\mathrm{sen}^2\left(k_0x - \omega t\right)\right]\,\hat{x} \\ &= \frac{TE_0^2}{\mu_0 c}\left[4 + 5\,\mathrm{sen}^2\left(k_0x - \omega t\right)\right]\,\hat{x}\end{aligned}$$