Instituto de Física

FAP 2292

Gabarito – Lista de Exercícios 7
Equações de Maxwell e condições de contorno

Exercícios Sugeridos (25/05/2010) — A numeração corresponde ao Livros Textos A e B.

B29.36 (a) $J_\mathrm{D} = \epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} = \epsilon_0 \dfrac{\partial}{\partial t}\left(\dfrac{Q}{\epsilon_0 \pi R^2}\right) = i_\mathrm{C}/\pi R^2 = 55{,}7\ \mathrm{A/m^2}$

(b) $\dfrac{\partial E}{\partial t} = \dfrac{1}{\epsilon_0} J_\mathrm{D} = 6{,}29\times10^{12}\ \mathrm{Vm^{-1}s^{-1}}$

$2\pi r B(r<R) = \mu_0 J_\mathrm{D} \pi r^2 = \mu_0 i_\mathrm{C} \dfrac{r^2}{R^2} \Rightarrow B(r) = \dfrac{\mu_0 i_\mathrm{C}}{2\pi R}\dfrac{r}{R} \quad (r \le R)$

(c) $r = 2{,}0$ cm $\Rightarrow B = 7{,}0\times10^{-7}$ T; (d) $r = 1{,}0$ cm $\Rightarrow B = 3{,}5\times10^{-7}$ T.

B29.39 (a) $E = \rho J$; $J = I/A = 7{,}62\times10^6\ \mathrm{A/m^2}$; $E = 0{,}152$ V/m; (b) $\dfrac{\mathrm{d}E}{\mathrm{d}t} = \dfrac{\rho}{A}\dfrac{\mathrm{d}I}{\mathrm{d}t} = 38{,}1\ \mathrm{Vm^{-1}s^{-1}}$;

(c) $J_\mathrm{D} \approx \epsilon_0 \dfrac{\mathrm{d}E}{\mathrm{d}t} = 3{,}37\times10^{-10}\ \mathrm{A/m^2}$; (d) $J_\mathrm{D}/J = 4\times10^{-17} \Rightarrow B(r) \approx \mu_0 I/2\pi r = 5{,}33\times10^{-5}$ T.

NA3.1 a) $\nabla\cdot\mathbf{F} = z e^{2y}\left(y + x^2 + 2x^2 y\right)$

$\nabla\times\mathbf{F} = e^{2y}\left[x^2(1+y)\,\hat{x} - xy\,\hat{y} - xz\,\hat{z}\right]$

b) $\nabla\cdot\mathbf{G} = yz^2\,(yz + 4xz + 9xy)$

$\nabla\times\mathbf{G} = 6xyz^2(z-y)\,\hat{x} + 3y^2z^2(x-z)\,\hat{y} + 2yz^3(y-x)\,\hat{z}$

NA3.2 a) $\rho(x,y,z) = \epsilon_0 \nabla\cdot\mathbf{E} = \dfrac{Q_0}{L^7}\left[(yz)^2 + (xz)^2 + (xy)^2\right]$

b) $Q = \displaystyle\int \rho(x,y,z)\mathrm{d}v = 3 \times \frac{Q_0}{L^7}\int_0^{2L}\mathrm{d}x \int_0^{2L} y^2\mathrm{d}y \int_0^{2L} z^2\mathrm{d}z = \frac{128}{3}Q_0.$

NA3.3 Para um campo magnético $\nabla\cdot\mathbf{B} = 0$ em todo o espaço, e $\nabla\times\mathbf{B}(\mathbf{r}) = \mu_0\mathbf{J}(\mathbf{r})$. Mas $\mathbf{J}(\mathbf{r})$ é a densidade de corrente no ponto $\mathbf{r}$. $\mathbf{B}$ pode ser gerado por correntes em outro lugar do espaço.

a) $\nabla\cdot\mathbf{A} = 0$, $\mu_0\mathbf{J}(x,y,z) = \nabla\times\mathbf{A} = A_1\left(x^2+z^2\right)\hat{y}$.

b) $\nabla\cdot\mathbf{B} = B_1 y^2 - B_2\dfrac{x^2}{y^2} \neq 0$.

c) $\nabla\cdot\mathbf{C} = C_0\left(\dfrac{e^{x/y}}{y} + \dfrac{e^{y/z}}{z} + \dfrac{e^{z/x}}{x}\right) \neq 0$.

d) $\nabla\cdot\mathbf{D} = 0$, $\mu_0\mathbf{J}(x,y,z) = \nabla\times\mathbf{D} = D_1\left(\dfrac{y}{x^2} + \dfrac{1}{y}\right)\hat{z}$.

e) $\nabla\cdot\mathbf{E} = 2E_0 y \neq 0$.

NA3.4 $\mathbf{B}(x,y,z) = \mu_0 k\rho\,\hat{\theta}$, $\quad \mathrm{d}\mathbf{s} = a\,\mathrm{d}\theta\,\hat{\theta} \to \displaystyle\oint_\circlearrowleft \mathbf{B}\cdot\mathrm{d}\mathbf{s} = \mu_0 k a^2 \int_0^{2\pi}\mathrm{d}\theta = 2\pi\mu_0 k a^2.$

NA3.5 $J = (E_0/\rho)\cos\omega t$; $\quad J_d = \epsilon_0\dfrac{\partial E}{\partial t} = -\omega\epsilon_0 E_0 \operatorname{sen}\omega t \to A_J/A_{J_d} = 1/\rho\omega\epsilon_0 = 9\times10^{14}.$

NA3.6 a) $\epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} = \mathbf{J}(z,t) \rightarrow \mathbf{E}(z,t) = (J_0/\epsilon_0\omega)\,\mathrm{sen}(\omega t - kz)\,\hat{x}.$

b) $\nabla\times\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{B}(z,t) = (kJ_0/\epsilon_0\omega^2)\,\mathrm{sen}(\omega t - kz)\,\hat{y}.$

c) $\nabla\times\mathbf{B} = \mu_0\epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \rightarrow k^2/\omega^2 = \mu_0\epsilon_0.$

NA3.7 $\nabla\times\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{B} = (E_0/kx_0)\mathrm{e}^{x/x_0 - kt}\,\hat{z}$

$\nabla\times\mathbf{B} = \mu_0\epsilon_0 \dfrac{\partial \mathbf{E}}{\partial t} \rightarrow k^2 x_0^2 = 1/\mu_0\epsilon_0.$

NA3.8 a) $\nabla\cdot\mathbf{B} = 0, \quad \nabla\times\mathbf{B} = 0$

b) $\nabla\times\mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \rightarrow \mathbf{E}(\rho,t) = -\dfrac{1}{2}kB_0\mathrm{e}^{kt}\rho\,\hat{\varphi}$

c) $\nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \epsilon_0\frac{\partial \mathbf{E}}{\partial t}\right) = 0 \rightarrow \mathbf{J} = \frac{1}{2}\epsilon_0 k^2 B_0 \mathrm{e}^{kt}\rho\,\hat{\varphi}$

NA3.9 $0 \le \rho \le a \rightarrow \mathbf{E}(\rho,\varphi,z,t) = -\dfrac{1}{2}\beta\rho\,\hat{\varphi}; \quad \rho > a \rightarrow \mathbf{E}(\rho,\varphi,z,t) = -\dfrac{1}{2}\dfrac{\beta a^2}{\rho}\,\hat{\varphi}$

NA3.10

$$(\nabla\times\mathbf{E})_y = \frac{\partial E_x}{\partial z} = E_0\mathrm{e}^{-\beta z}\left[-\beta\,\mathrm{sen}(kz-\omega t) + k\cos(kz-\omega t)\right] = -\frac{\partial B_y}{\partial t}$$

$$B_y = E_0\mathrm{e}^{-\beta z}\left[\frac{\beta}{\omega}\cos(kz-\omega t) + \frac{k}{\omega}\,\mathrm{sen}(kz-\omega t)\right]$$

$$(\nabla\times\mathbf{B})_x = -\frac{\partial B_y}{\partial z} = E_0\mathrm{e}^{-\beta z}\left[\frac{\beta^2-k^2}{\omega}\cos(kz-\omega t) + \frac{2k\beta}{\omega}\,\mathrm{sen}(kz-\omega t)\right]$$

$$= \mu_0 J_x + \mu_0\epsilon_0\frac{\partial E_x}{\partial t} = \mu_0 J_x - \mu_0\epsilon_0\omega E_0\mathrm{e}^{-\beta z}\cos(kz-\omega t)$$

a) $k^2 - \beta^2 = \mu_0\epsilon_0\omega^2.$

b) $B_y = E_0\mathrm{e}^{-\beta z}\left[\dfrac{\beta}{\omega}\cos(kz-\omega t) + \dfrac{k}{\omega}\,\mathrm{sen}(kz-\omega t)\right].$

c) $J_x = \dfrac{2k\beta}{\mu_0\omega}E_0\mathrm{e}^{-\beta z}\,\mathrm{sen}(kz-\omega t).$

NA3.11 $(\nabla\times\mathbf{E})_\varphi = \dfrac{1}{r}\dfrac{\partial}{\partial r}(rE_\theta) = -\mu_0\dfrac{\partial H_\varphi}{\partial t} \Rightarrow \mathbf{H}(r,\theta,\varphi,t) = \dfrac{k}{\mu_0\omega}\dfrac{A\,\mathrm{sen}\,\theta}{r}\cos(\omega t - kr)\,\hat{\varphi} = \dfrac{k}{\mu_0\omega}\,\hat{r}\times\mathbf{E}$

$(\nabla\times\mathbf{H})_\theta = -\dfrac{1}{r}\dfrac{\partial}{\partial r}(rH_\varphi) = \epsilon_0\dfrac{\partial E_0}{\partial t} \Rightarrow \dfrac{k^2}{\omega^2} = \mu_0\epsilon_0 \Rightarrow \dfrac{\mu_0\omega}{k} = \sqrt{\dfrac{\mu_0}{\epsilon_0}} = Z_0.$

NA4.1 (a) $D_{1n} = D_{2n} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\epsilon_1\ell^2} + \dfrac{d_2}{\epsilon_2\ell^2} = \dfrac{1}{C_1} + \dfrac{1}{C_2}$

(b) $E_{1t} = E_{2t} \Rightarrow C = \dfrac{\epsilon_1\ell\ell_1}{d} + \dfrac{\epsilon_2\ell\ell_2}{d} = C_1 + C_2$

NA4.2 $E_{1t} = E_{2t} \Rightarrow \mathbf{E} = \dfrac{q}{2\pi(\epsilon_1+\epsilon_2)}\dfrac{1}{r^2}\,\hat{r}$

NA4.3 $\mathbf{E}_{1t} = \mathbf{E}_{0t} = -E_0\,\mathrm{sen}\,\alpha\,\hat{x}$

$\mathbf{E}_1 = \mathbf{E}_{1t} + \mathbf{E}_{1n} = \mathbf{E}_{0t} + \dfrac{1}{\epsilon_1}\mathbf{D}_{0n}$

$B_{1n} = B_{0n} = 0$

$D_{1n} = D_{0n} = \epsilon_0 E_0\cos\alpha$

$\mathbf{E}_1 = -E_0\,\mathrm{sen}\,\alpha\,\hat{x} + \frac{\epsilon_0}{\epsilon_1}E_0\cos\alpha\,\hat{z}$

$\mathbf{D}_1 = -\epsilon_1 E_0\,\mathrm{sen}\,\alpha\,\hat{x} + \epsilon_0 E_0\cos\alpha\,\hat{z}$

$\mathbf{H}_{1t} = \mathbf{H}_{0t} = \dfrac{E_0}{\mu_0 c}\,\hat{y}$