

Linha de Transmissão

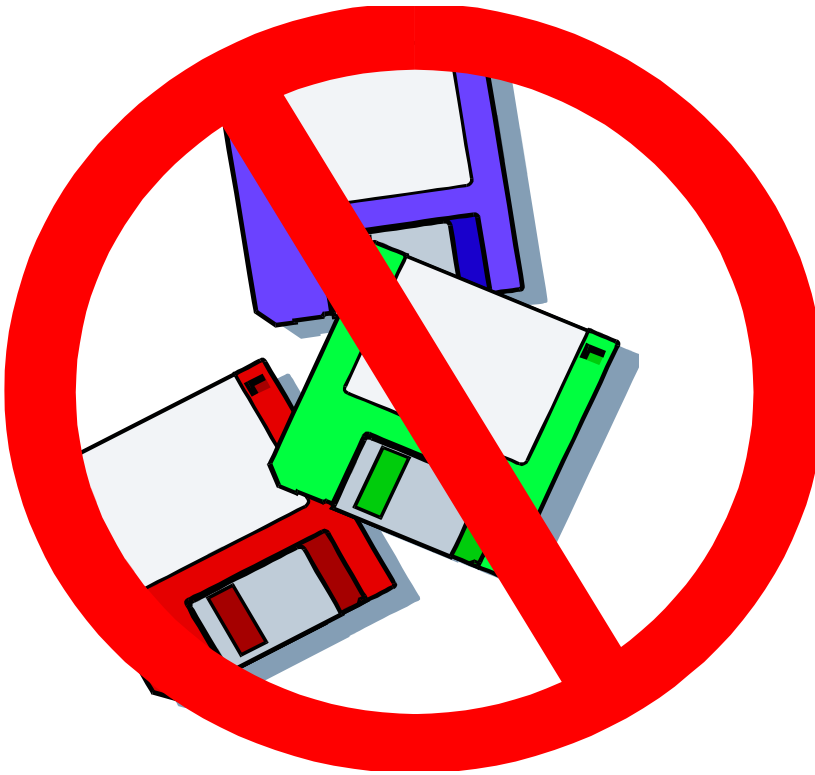
Parte 7

Onda Estacionária

SEL 310/612 Ondas Eletromagnéticas

Amílcar Careli César
Departamento de Engenharia Elétrica da EESC-USP

Atenção!



- ✓ Este material didático é planejado para servir de apoio às aulas de **SEL-310 E SEL-612: Ondas Eletromagnéticas**, oferecida aos alunos regularmente matriculados no curso de engenharia elétrica e engenharia de computação.
- ✓ Não são permitidas a reprodução e/ou comercialização do material.
- ✓ solicitar autorização ao docente para qualquer tipo de uso distinto daquele para o qual foi planejado.

Impedância ao longo da linha

$$Z(z) = Z_0 \frac{Z_L - jZ_0 \operatorname{tg}(k_I z)}{Z_0 - jZ_L \operatorname{tg}(k_I z)}, \quad z \leq 0$$

$$Z(z) = Z_0 \frac{Z_L - Z_0 \operatorname{tgh}(kz)}{Z_0 - Z_L \operatorname{tgh}(kz)}, \quad z \leq 0$$

com $k = k_R + jk_I$

Terminação: Curto-circuito

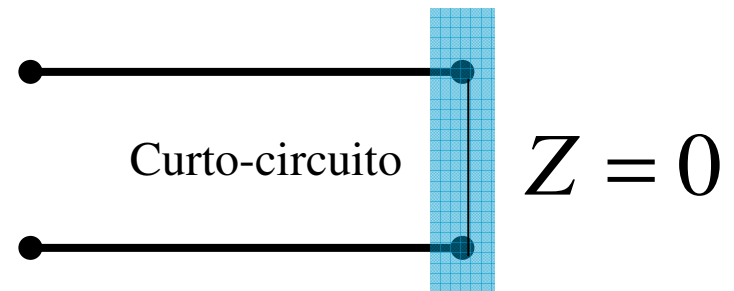
curto $Z_L = 0$

$$Z(z = -l) = Z_0 \frac{0 + jZ_0 \operatorname{tg} k_I l}{Z_0 + j(0) \operatorname{tg} k_I l} = jZ_0 \operatorname{tg}(k_I l)$$

$$Z(z = -l) = jZ_0 \operatorname{tg}(k_I l)$$

normalização

$$Z(z = -l) / (Z_0) = j \operatorname{tg}(k_I l) = jX$$



Terminação: Circuito aberto

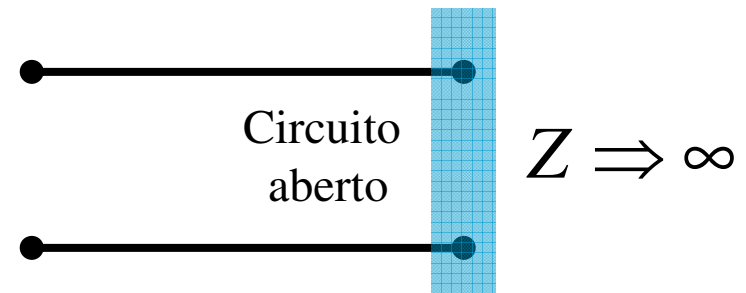
Circuito aberto $Z_L \rightarrow \infty$

$$Z(z = -l) = Z_0 \frac{1 + j(Z_0 / Z_L) \operatorname{tg}(k_I l)}{(Z_0 / Z_L) + j \operatorname{tg}(k_I l)} = -jZ_0 \operatorname{cotg}(k_I l)$$

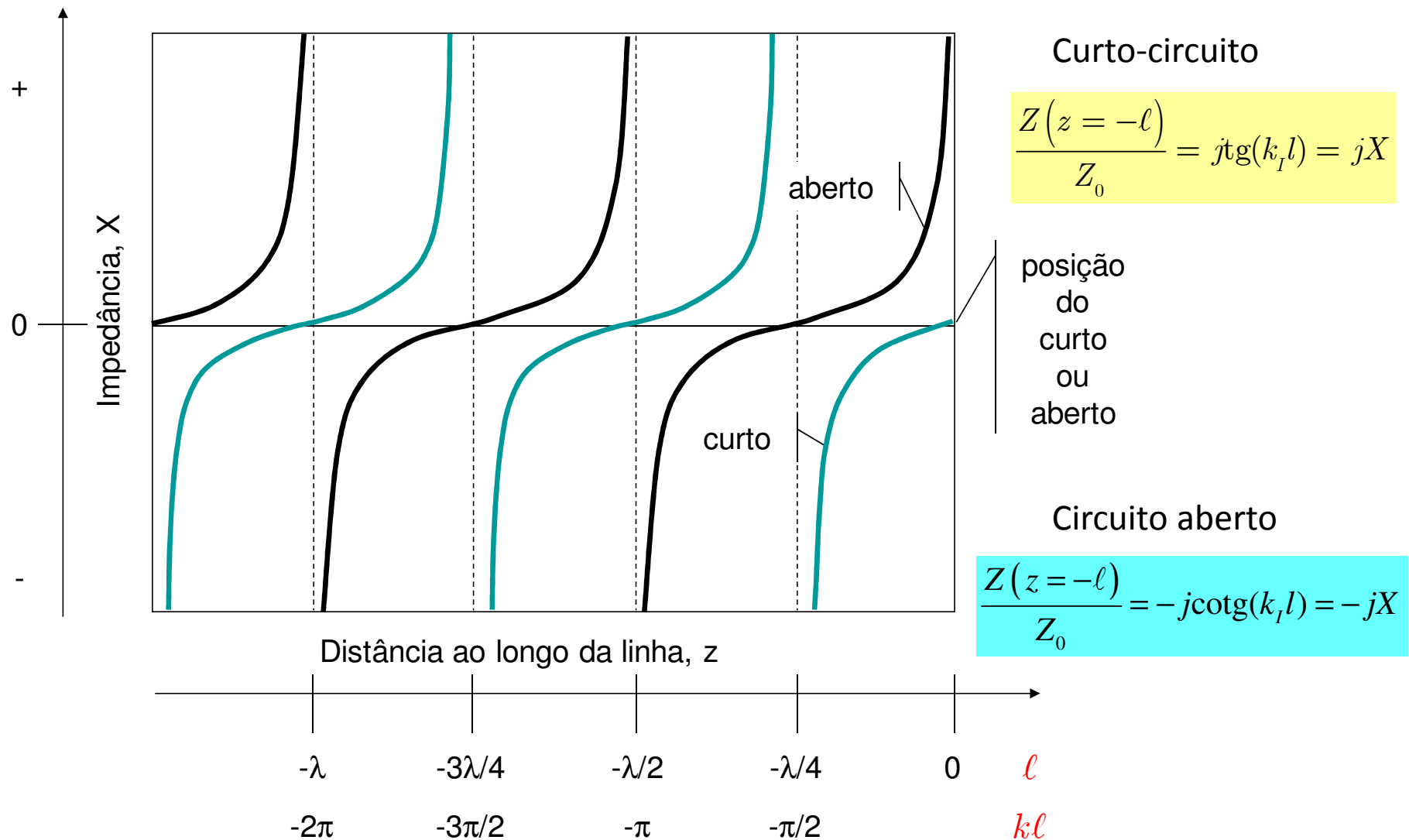
$$Z(z = -l) = -jZ_0 \operatorname{cotg}(k_I l)$$

normalização

$$\frac{Z(z = -l)}{Z_0} = -j \operatorname{cotg}(k_I l) = -jX$$



Terminações: Curto e aberto



Simulação no WinTls

The screenshot displays the WinTls simulation software interface. At the top, the menu bar includes "Control", "Zo", "Source", "Voltage", "R-Source", "R-Load", "Line Type", and "Help". The main workspace shows a circuit diagram with a voltage source labeled "V 1 Volts" in series with a resistor "R1 50 Ω". This is connected to a transmission line with characteristic impedance "Zo 50 Ω", which is terminated at the other end by a resistor "R2 0 Ω". Below the circuit are "Simulate", "Stop", and "Exit" buttons.

On the right side, there are two parameter panels:

- Load Parameters:** Includes fields for inductance "L 1 μH", resistance "R2 0 Ω", capacitance "C 100 pF", and impedance multiplier "iX2 50 Ω". Radio buttons are present for "Resistor" (selected), "Inductor", "Capacitor", "Parallel RC", "Series RL", and "Impedance".
- Advanced Parameters:** Includes fields for frequency "F 100 MHz", cycles per line length "Cyc/len 4 Cycles/LineLength", and line length "Length 9,6 m". Below these are fields for propagation velocity "Vp 80% c", attenuation "Attn 0 %/len", pulse width "PW 10 points, 1pt = 0,2 ns", and pulse shape "PS 10 points, 1pt = 0,2 ns".

At the bottom left, there are control buttons: "Step", "Continue", "Zoom In", and "Zoom Out". Below these, it states "Tick Mark= .25V or 5 mA".

The central plot area shows a waveform with a green background. The vertical axis is labeled "V/mA" and the horizontal axis is labeled "m". The plot displays multiple overlapping waveforms, including a red envelope and several green and white waveforms. At the bottom of the plot area, there are checkboxes for "Meter", "Voltage" (checked), "Current" (checked), and "Envelope" (checked). To the right of these checkboxes is a "Speed" slider.

Onda Estacionária: Módulo-1

$$V(z) = V^+ e^{-jk_I z} + V^- e^{jk_I z}$$

$$V^- e^{+jk_I z} = \Gamma_0 V^+ e^{+jk_I z}$$

$$\Gamma_0 = |\Gamma_0| e^{j\phi_r}$$

$$V(z) = V^+ \left(e^{-jk_I z} + \Gamma_0 e^{jk_I z} \right)$$

Onda Estacionária: Módulo-2

$$V(z) = V^+ \left(e^{-jk_I z} + \Gamma_0 e^{jk_I z} \right)$$

$$\begin{aligned} |V(z)|^2 &= V(z)V(z)^* = \\ & \left[V^+ \left(e^{-jk_I z} + \Gamma_0 e^{jk_I z} \right) \right] \left[\left(V^+ \right)^* \left(e^{jk_I z} + \Gamma_0^* e^{-jk_I z} \right) \right] = \\ & V^+ \left(V^+ \right)^* \left(e^{-jk_I z} + \Gamma_0 e^{jk_I z} \right) \left(e^{jk_I z} + \Gamma_0^* e^{-jk_I z} \right) = \\ & |V^+|^2 \left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + \Gamma_0 \Gamma_0^* \right) = \\ & |V^+|^2 \left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + |\Gamma_0|^2 \right) \end{aligned}$$

Onda Estacionária: Módulo-2

$$|V(z)|^2 = V(z)V(z)^* = |V^+|^2 \left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + |\Gamma_0|^2 \right)$$

$$\begin{aligned} \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} &= |\Gamma_0| e^{-j\phi_r} e^{-j2k_I z} + |\Gamma_0| e^{j\phi_r} e^{jk_I z} = \\ |\Gamma_0| \left[e^{-j(2k_I z + \phi_r)} + e^{j(2k_I z + \phi_r)} \right] &= 2|\Gamma_0| \cos(2k_I z + \phi_r) \end{aligned}$$

$$|V(z)|^2 = |V^+|^2 \left[1 + |\Gamma_0|^2 + 2|\Gamma_0| \cos(2k_I z + \phi_r) \right]$$

Onda Estacionária: Valor Máximo

$$|V(z)|^2 = |V^+|^2 \left[1 + |\Gamma_0|^2 + 2|\Gamma_0| \cos(2k_I z + \phi_r) \right]$$

Máximo ocorre para $\cos(2k_I z + \phi_r) = 1$ $z = z_{\max}$

$$2k_I z_{\max} + \phi_r = 2n\pi, \quad n = 0, 1, 2, 3, \dots$$

$$|V(z)|_{\max}^2 = |V^+|^2 (1 + |\Gamma_0|^2 + 2|\Gamma_0|) = |V^+|^2 (1 + |\Gamma_0|)^2$$

$$|V(z)|_{\max} \equiv V_{\max} = |V^+| (1 + |\Gamma_0|)$$

Onda Estacionária: Valor Mínimo

$$|V(z)|^2 = |V^+|^2 \left[1 + |\Gamma_0|^2 + 2|\Gamma_0| \cos(2k_I z + \phi_r) \right]$$

Mínimo ocorre para $\cos(2k_I z + \phi_r) = -1$ $z = z_{\min}$

$$2k_I z_{\min} + \phi_r = (2n + 1)\pi, \quad n = 0, 1, 2, 3, \dots$$

$$|V(z)|_{\min}^2 = |V^+|^2 (1 + |\Gamma_0|^2 - 2|\Gamma_0|) = |V^+|^2 (1 - |\Gamma_0|)^2$$

$$|V(z)|_{\min} \equiv V_{\min} = |V^+| (1 - |\Gamma_0|)$$

Relação de Onda Estacionária (ROE)

$$ROE = \frac{|V(z)|_{\max}}{|V(z)|_{\min}} = \frac{|V^+|(1 + |\Gamma_0|)}{|V^+|(1 - |\Gamma_0|)} = \frac{(1 + |\Gamma_0|)}{(1 - |\Gamma_0|)}$$

$$ROE = \frac{|I(z)|_{\max}}{|I(z)|_{\min}} = \frac{|I^+|(1 + |\Gamma_0|)}{|I^+|(1 - |\Gamma_0|)} = \frac{(1 + |\Gamma_0|)}{(1 - |\Gamma_0|)}$$

$$|\Gamma_0| = 0 \quad ROE = 1 \quad \text{Casamento de impedância}$$

$$|\Gamma_0| = 1 \quad ROE \rightarrow \infty \quad \text{Reflexão total}$$

$$ROE \geq 1$$