Linha de Transmissão
Parte 7
Onda Estacionária

SEL 310/612 Ondas Eletromagnéticas

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Atenção!

- Este material didático é planejado para servir de apoio às aulas de SEL-310 E SEL-612: Ondas Eletromagnéticas, oferecida aos alunos regularmente matriculados no curso de engenharia elétrica e engenharia de computação.
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Impedância ao longo da linha

$$Z(z) = Z_0 \frac{Z_L - jZ_0 \mathrm{tg}(k_I z)}{Z_0 - jZ_L \mathrm{tg}(k_I z)}, \; z \leq 0$$

$$Z(z) = Z_0 \frac{Z_L - Z_0 \mathrm{tgh}(kz)}{Z_0 - Z_L \mathrm{tgh}(kz)}, \; z \leq 0$$

$$\text{com } k = k_R + jk_I$$

Terminação: Curto-circuito

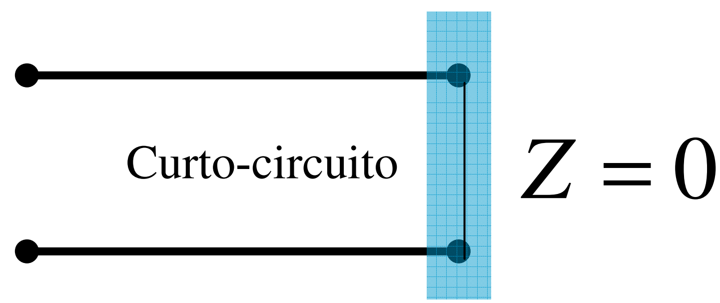
curto $Z_L = 0$

$$Z\left(z=-l\right) = Z_0 \frac{0 + jZ_0 \operatorname{tg} k_I l}{Z_0 + j\left(0\right) \operatorname{tg} k_I l} = jZ_0 \operatorname{tg}(k_I l)$$

$$Z\left(z=-l\right) = jZ_0 \operatorname{tg}(k_I l)$$

normalização

$$Z\left(z=-\ell\right) / \left(Z_0\right) = j\operatorname{tg}(k_I l) = jX$$



Terminação: Circuito aberto

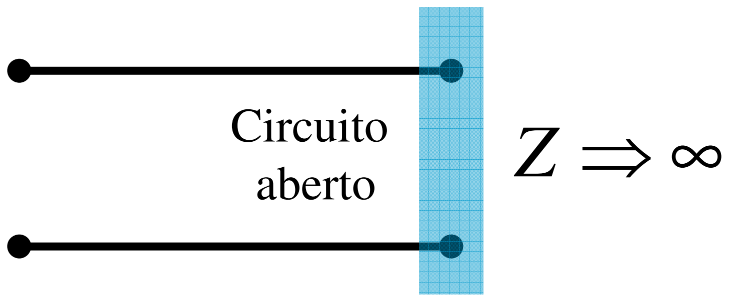
Circuito aberto $Z_L \rightarrow \infty$

$$Z\left(z=-l\right)=Z_0\frac{1+j\left(Z_0\,/\,Z_L\right)\mathrm{tg}(k_I l)}{\left(Z_0\,/\,Z_L\right)+j\mathrm{tg}(k_I l)}=-jZ_0\mathrm{cotg}(k_I l)$$

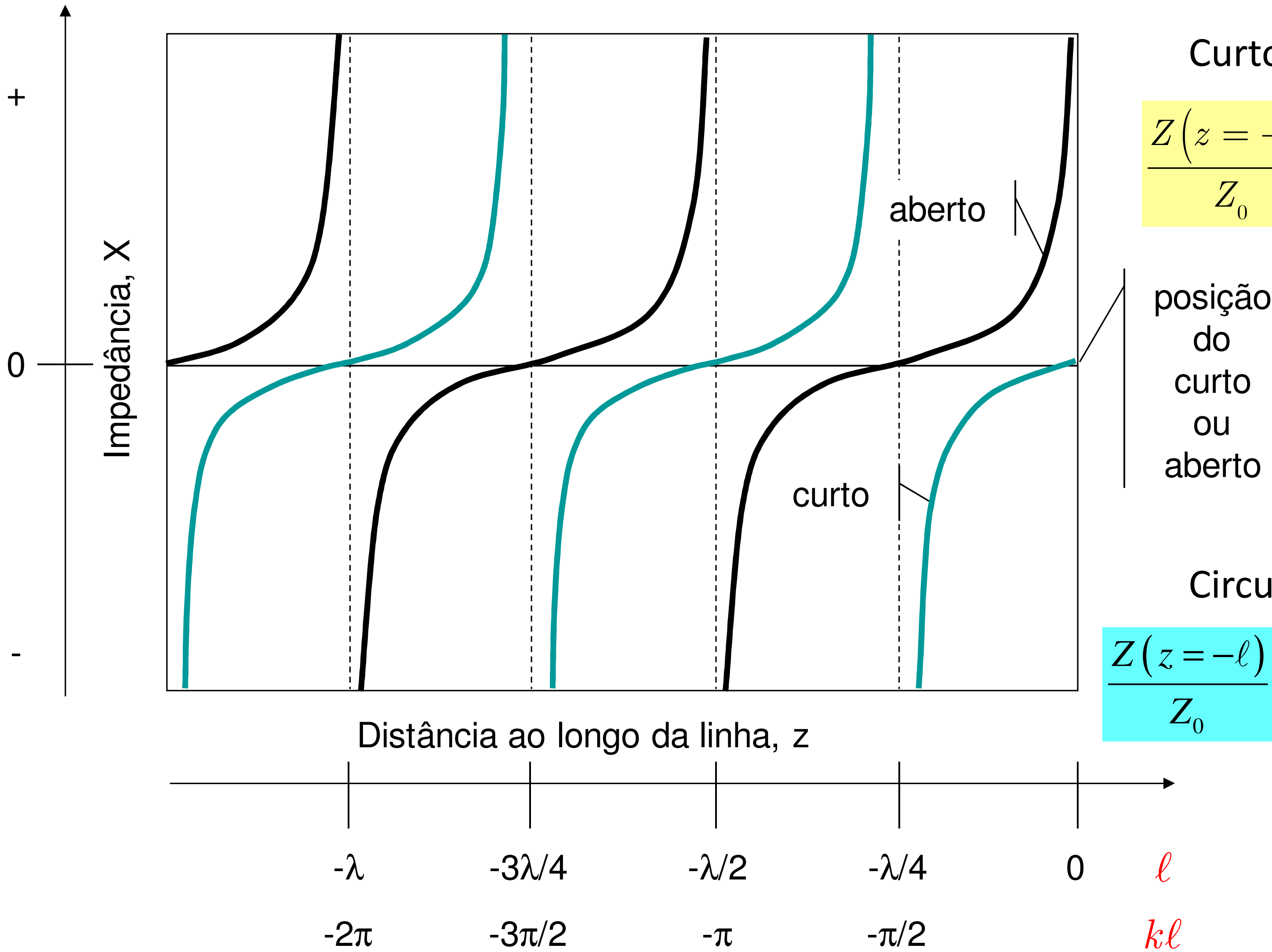
$$Z\left(z=-l\right)=-jZ_0\mathrm{cotg}(k_I l)$$

normalização

$$\frac{Z\left(z=-\ell\right)}{Z_0}=-j\mathrm{cotg}(k_I l)=-jX$$



Terminações: Curto e aberto



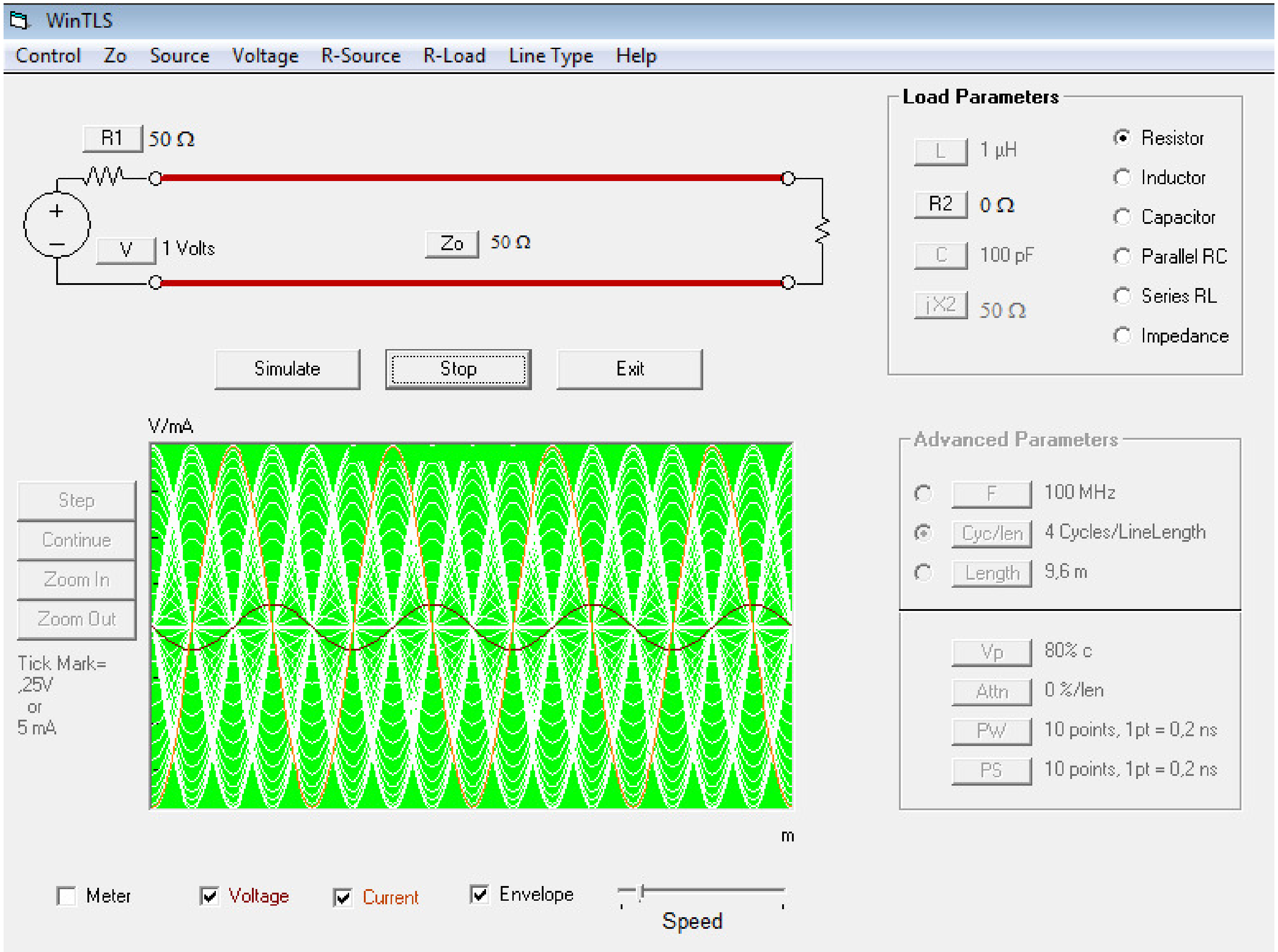
Curto-circuito

$$\frac{Z(z=-\ell)}{Z_0} = j\mathrm{tg}(k_I l) = jX$$

Circuito aberto

$$\frac{Z(z=-\ell)}{Z_0} = -j\mathrm{cotg}(k_I l) = -jX$$

Simulação no WinTls



Onda Estacionária: Módulo-1

$$V\left(z\right) = V^{+}e^{-jk_I z} + V^{-}e^{jk_I z}$$

$$V^{-}e^{+jk_I z} = \Gamma_0 V^{+}e^{+jk_I z}$$

$$\Gamma_0 = \left|\Gamma_0\right|e^{j\phi_r}$$

$$V\left(z\right) = V^{+}\left(e^{-jk_I z} + \Gamma_0 e^{jk_I z}\right)$$

Onda Estacionária: Módulo-2

$$V(z) = V^+\left(e^{-jk_I z} + \Gamma_0 e^{jk_I z}\right)$$

$$\left|V(z)\right|^2 = V(z)V(z)^* =$$

$$\left[V^+\left(e^{-jk_I z} + \Gamma_0 e^{jk_I z}\right)\right]\left[\left(V^+\right)^*\left(e^{jk_I z} + \Gamma_0^* e^{-jk_I z}\right)\right] =$$

$$V^+\left(V^+\right)^*\left(e^{-jk_I z} + \Gamma_0 e^{jk_I z}\right)\left(e^{jk_I z} + \Gamma_0^* e^{-jk_I z}\right) =$$

$$\left|V^+\right|^2\left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + \Gamma_0 \Gamma_0^*\right) =$$

$$\left|V^+\right|^2\left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + \left|\Gamma_0\right|^2\right)$$

Onda Estacionária: Módulo-2

$$\left|V(z)\right|^2 = V(z)V(z)^* = \left|V^+\right|^2\left(1 + \Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} + \left|\Gamma_0\right|^2\right)$$

$$\Gamma_0^* e^{-j2k_I z} + \Gamma_0 e^{j2k_I z} = \left|\Gamma_0\right| e^{-j\phi_r} e^{-j2k_I z} + \left|\Gamma_0\right| e^{j\phi_r} e^{jk_I z} =$$

$$\left|\Gamma_0\right|\left[e^{-j\left(2k_I z+\phi_r\right)} + e^{j\left(2k_I z+\phi_r\right)}\right] = 2\left|\Gamma_0\right|\cos\left(2k_I z + \phi_r\right)$$

$$\left|V(z)\right|^2 = \left|V^+\right|^2\left[1 + \left|\Gamma_0\right|^2 + 2\left|\Gamma_0\right|\cos\left(2k_I z + \phi_r\right)\right]$$

Onda Estacionária: Valor Máximo

$$\left|V(z)\right|^2 = \left|V^+\right|^2\left[1 + \left|\Gamma_0\right|^2 + 2\left|\Gamma_0\right|\cos\left(2k_I z + \phi_r\right)\right]$$

Máximo ocorre para $\cos(2k_I z + \phi_r) = 1$ $\quad z = z_{\max}$

$$2k_I z_{\max} + \phi_r = 2n\pi \ , \ n = 0, 1, 2, 3 \ldots$$

$$\left|V(z)\right|^2_{\max} = \left|V^+\right|^2\left(1 + \left|\Gamma_0\right|^2 + 2\left|\Gamma_0\right|\right) = \left|V^+\right|^2\left(1 + \left|\Gamma_0\right|\right)^2$$

$$\left|V(z)\right|_{\max} \equiv V_{\max} = \left|V^+\right|\left(1 + \left|\Gamma_0\right|\right)$$

Onda Estacionária: Valor Mínimo

$$\left|V(z)\right|^2 = \left|V^+\right|^2\left[1+\left|\Gamma_0\right|^2 + 2\left|\Gamma_0\right|\cos\left(2k_I z + \phi_r\right)\right]$$

Mínimo ocorre para $\cos(2k_I z + \phi_r) = -1$ $\quad z = z_{\min}$

$$2k_I z_{\min} + \phi_r = (2n+1)\pi \ , \ n = 0,1,2,3....$$

$$\left|V(z)\right|^2_{\min} = \left|V^+\right|^2\left(1+\left|\Gamma_0\right|^2 - 2\left|\Gamma_0\right|\right) = \left|V^+\right|^2\left(1-\left|\Gamma_0\right|\right)^2$$

$$\left|V(z)\right|_{\min} \equiv V_{\min} = \left|V^+\right|\left(1-\left|\Gamma_0\right|\right)$$

Relação de Onda Estacionária (ROE)

$$R0E = \frac{\left|V(z)\right|_{\max}}{\left|V(z)\right|_{\min}} = \frac{\left|V^+\right|\left(1+\left|\Gamma_0\right|\right)}{\left|V^+\right|\left(1-\left|\Gamma_0\right|\right)} = \frac{\left(1+\left|\Gamma_0\right|\right)}{\left(1-\left|\Gamma_0\right|\right)}$$

$$R0E = \frac{\left|I(z)\right|_{\max}}{\left|I(z)\right|_{\min}} = \frac{\left|I^+\right|\left(1+\left|\Gamma_0\right|\right)}{\left|I^+\right|\left(1-\left|\Gamma_0\right|\right)} = \frac{\left(1+\left|\Gamma_0\right|\right)}{\left(1-\left|\Gamma_0\right|\right)}$$

$|\Gamma_0| = 0$ $\quad R0E = 1$ $\quad$ Casamento de impedância

$|\Gamma_0| = 1$ $\quad R0E \rightarrow \infty$ $\quad$ Reflexão total

$$R0E \geq 1$$