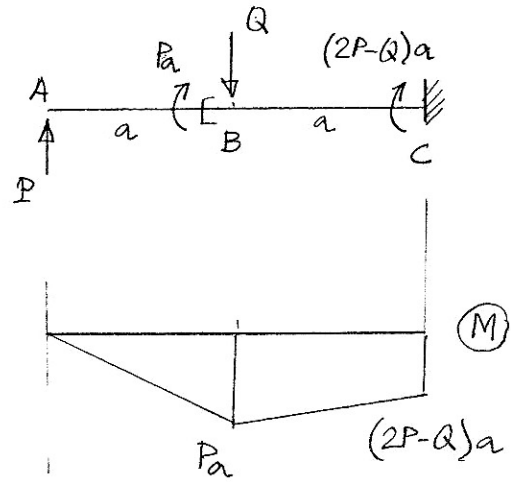


$$\begin{aligned}
 a) \quad U(P, Q) &= \int_0^{2a} \frac{M^2}{2EI} dx = \triangle + \square \\
 &= \frac{a}{3(2EI)} P^2 a^2 + \frac{a}{3(2EI)} [P^2 a^2 + P(2P-Q)a + (2P-Q)^2 a^2] \\
 &= \frac{P^2 a^3}{3EI} + \frac{P(2P-Q)a^3}{6EI} + \frac{(2P-Q)^2 a^3}{6EI}
 \end{aligned}$$



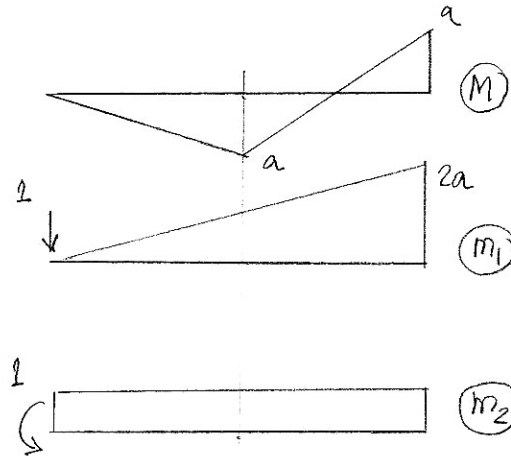
para $P = 1 \text{ kN}$ e $Q = 3 \text{ kN}$:

$$U = \frac{a^3}{EI} \left(\frac{1}{3} - \frac{1}{6} + \frac{1}{6} \right) = \frac{a^3}{3EI}$$

$$b) \quad v_B = \frac{\partial U}{\partial Q} \Big|_{P=1, Q=3} = -\frac{Pa^3}{6EI} - \frac{2(2P-Q)a^3}{6EI} = \frac{a^3}{6EI}$$

c) Carga unitária:

$$\begin{aligned}
 v_A &= \int_0^{2a} \frac{Mm}{EI} dx \\
 &= -\frac{a}{3EI} a^2 + \frac{a}{6EI} (-2a^2 + 4a^2 - 2a^2 + a^2) \\
 &= -\frac{a^3}{6EI} \quad (\uparrow)
 \end{aligned}$$



$$\varphi_A = \frac{a}{2EI} (-a) + \frac{a}{6EI} (-2a + 2a + a - a) = -\frac{a^2}{2EI} \quad (\uparrow)$$