

Nome: Sabato N° USP: \_\_\_\_\_  
 (Colocar nome em todas as folhas!)

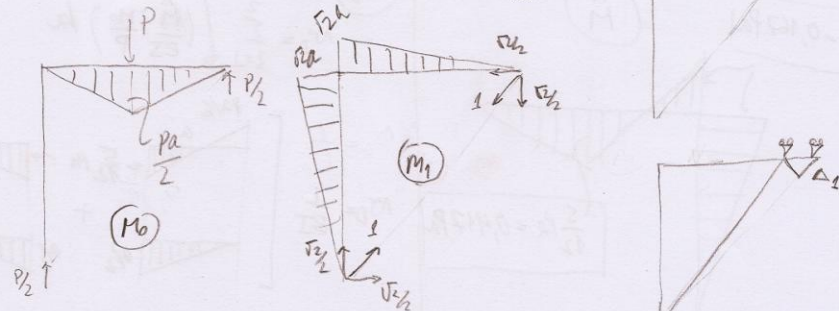
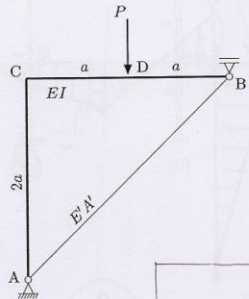
2ª Prova — 1º semestre de 2010

1ª Questão (3,5 pontos)

No pórtico esquematizado ao lado, as barras AC e CB têm módulo de rigidez à flexão  $EI$  e módulo de rigidez axial  $EA = \infty$ , enquanto que o tirante AB tem módulo de rigidez axial  $E'A' = 6\sqrt{2}EI/a^2$  e módulo de rigidez à flexão desprezível.

Determine o diagrama de momentos fletores atuando no pórtico sob a ação de uma carga vertical  $P$  agindo no ponto D, bem como o deslocamento vertical deste ponto.

2,5  
1,0



$$\Delta_0 = \int_0^2 \frac{M_0 m_1}{EI} dx = \frac{1}{EI} \left[ \int_0^a \frac{Pa/2}{\sqrt{2}a} dx + \int_a^{2a} \frac{Pa/2}{\sqrt{2}a} dx \right]$$

$$\Delta_0 = \frac{1}{EI} \left[ -\frac{a}{2} \times \frac{Pa}{2} \times (\sqrt{2}a + 2\frac{\sqrt{2}a}{2}) - \frac{a}{3} \times \frac{Pa}{2} \times \frac{\sqrt{2}a}{2} \right] = -\frac{\sqrt{2}Pa^3}{4EI}$$

$$\Delta_1 = \int_0^2 \frac{m_1 m_1}{EI} dx + \left( \frac{m_1 m_1}{EA} \right)_{b=3} = \frac{1}{EI} \left[ \int_0^a \frac{\sqrt{2}a}{2a} dx + \int_a^{2a} \frac{\sqrt{2}a}{2a} dx \right] + \frac{1^2}{E'A'}$$

$$\Delta_1 = \frac{8a^3}{3EI} + \frac{2\sqrt{2}a}{E'A'} = \frac{8a^3}{3EI} + \frac{2\sqrt{2}a}{(6\sqrt{2}EI/a^2)}$$

$$\Delta_1 = \frac{8a^3}{3EI} + \frac{a^3}{3EI} = \frac{3a^3}{EI}$$

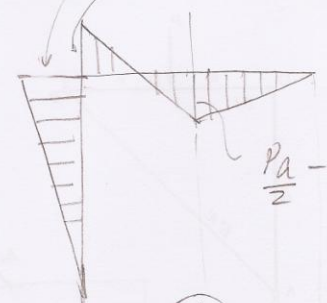
$$\Delta_0 + X_1 \Delta_1 = 0$$

$$-\frac{\sqrt{2}Pa^3}{4EI} + X_1 \frac{3a^3}{EI} = 0$$

$$X_1 = \frac{\sqrt{2}P}{12} = 0,1178P$$

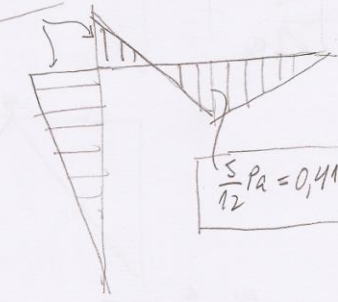
$$M = M_0 + X_1 m_1$$

$$\sqrt{2}a \times \left( \frac{\sqrt{2}P}{12} \right) = \frac{Pa}{6}$$



$$\frac{Pa}{2} - \frac{\sqrt{2}P}{12} \times \frac{\sqrt{2}a}{2} = \frac{Pa}{2} - \frac{Pa}{12} = \frac{5}{12}Pa$$

$$\frac{Pa}{6} = 0,167Pa$$



$$\frac{5}{12}Pa = 0,417Pa$$

$$W_D = \int_0^2 \left( \frac{M}{EI} \right) \left( \frac{M_0}{P} \right) dx$$

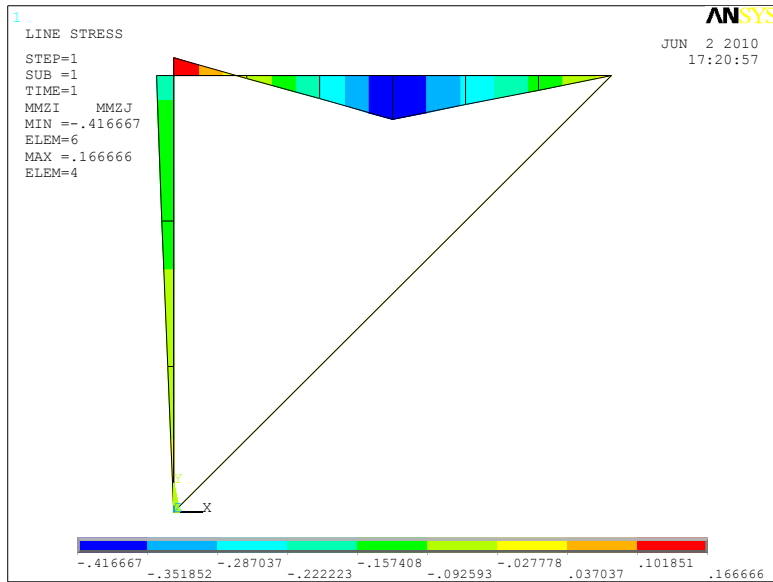
$$W_D = \frac{1}{EI} \left[ \int_0^a \frac{Pa/6}{\sqrt{2}a} dx + \int_a^{2a} \frac{5Pa/12}{\sqrt{2}a} dx \right]$$

$$W_D = \frac{1}{EI} \left[ \frac{a}{6} \times \left( \frac{a}{2} \right) \times \left( -\frac{Pa}{6} + 2 \times \frac{5}{12}Pa \right) + \frac{a}{3} \times \left( \frac{a}{2} \right) \times \left( \frac{5}{12}Pa \right) \right]$$

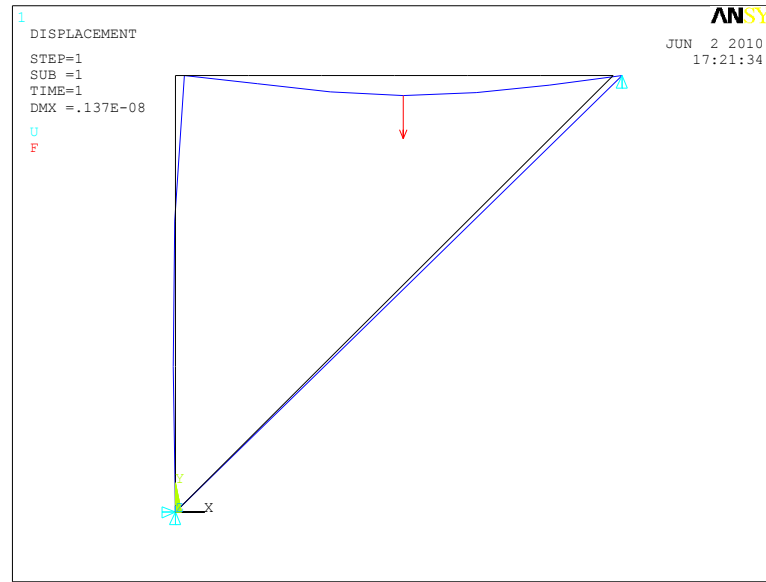
$$W_D = \frac{Pa^3}{8EI} = 0,125 \frac{Pa^3}{EI}$$

Resolução numérica, com Ansys, para os seguintes parâmetros:  $a = 1,0m$ ;  $E = 100GPa$ ;  $I = 10^{-3}m^4$ ;  $A' = 6\sqrt{2} \times 10^{-3}m^2$

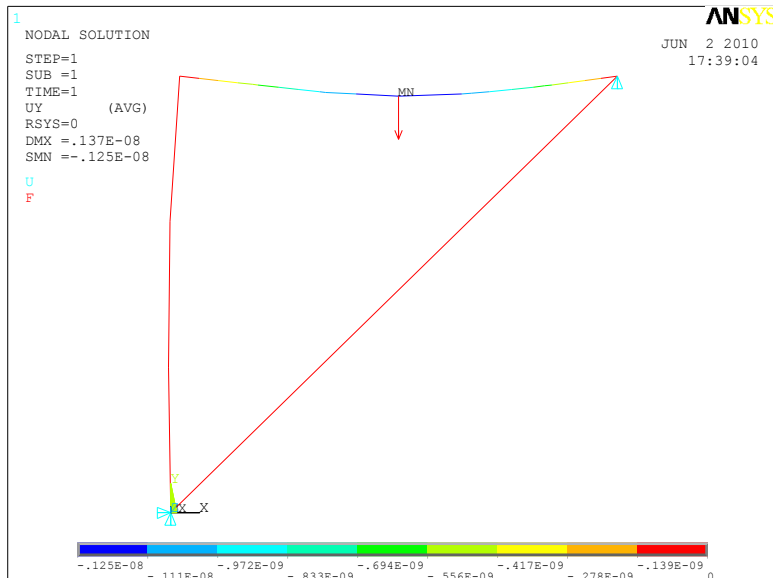
a) Momentos fletores



b) Deslocamentos



c) Deslocamentos verticais



d) forças normais

