Gabarito de Rec - J.R.B. Olivaro

Física I - IO - 2013 (2014)

1. (a) $T_a - \mu_c m_1 g = m_1 a$

$$(T_b - T_a)R = \frac{1}{2} m_2 R^2 \left(\frac{a}{R}\right)$$

$$m_3 g \,\text{sen}\,\theta - T_b = m_3 a$$

$$T_a = m_1 a + \mu_c m_1 g$$

$$T_b - T_a = \frac{1}{2} m_2 a$$

$$T_b = \frac{1}{2} m_2 a + T_a = \frac{1}{2} m_2 a + m_1 a + \mu_c m_1 g$$

$$m_3 g \,\text{sen}\,\theta = m_3 a + T_b = m_3 a + \frac{1}{2} m_2 a + m_1 a + \mu_c m_1 g$$

$$m_3 g \,\text{sen}\,\theta - \mu_c m_1 g = \left(m_3 + \frac{1}{2} m_2 + m_1\right) a$$

$$a = \frac{(m_3 \,\text{sen}\,\theta - m_1 \mu_c)\, g}{m_3 + \frac{1}{2} m_2 + m_1}$$

$$a = \frac{6 \times 0{,}4 - 3{,}5 \times 0{,}8}{6 + 0{,}5 + 3{,}5} \times 10 = -0{,}40 \text{ m/s}^2$$

Acel. p/ esquerda. v diminui.

(b) $E_0 = \frac{1}{2}(m_1 + m_3)\, v_0^2 + \frac{1}{2}\left(\frac{1}{2} m R^2\right)\left(\frac{v_0}{R}\right)^2 \left(= K_0\right)$

$$E_0 = K_0 = \frac{1}{2}\left(m_1 + m_3 + \frac{1}{2} m_2\right) v_0^2 = 5 \times (1{,}672)^2$$

$$E_0 = 14{,}0\ \mathrm{J} \qquad K_f = 0$$

$$U_f = \frac{1}{2} k x^2 - m_3 g x \operatorname{sen}\theta \quad \left(= U_f = U_e + U_g\right)$$

$$E_0 = E_f = U_f + \Delta E_{Int} = U_f + \mu_c m_1 g x$$

$$\frac{1}{2} k x^2 - m_3 g \operatorname{sen}\theta\, x + \mu_c m_1 g x - E_0 = 0$$

$$\frac{1}{2} k x^2 - (m_3 \operatorname{sen}\theta - m_1 \mu_c) g x - E_0 = 0$$

$$10 x^2 + 4x - 14 = 0$$

$$x = \frac{-4 \pm \sqrt{16 + 40 \times 14}}{20} = 1{,}0\ \mathrm{m}$$

2. $\vec{v}(t) = At\hat{x} + B\cos(Ct)\hat{y}$

(a) $\vec{a}(t) = \frac{d\vec{v}}{dt} = A\hat{x} - BC\,\mathrm{sen}(Ct)\hat{y}$

$\vec{F}(t) = m\vec{a} = mA\hat{x} - mBC\,\mathrm{sen}(Ct)\hat{y}$

(b) $\vec{r}(t) = \int_0^t \vec{v}(t')dt' + \vec{r}_0$

$\vec{r}(t) = \frac{At^2}{2}\hat{x} + \frac{B}{C}\mathrm{sen}(Ct)\hat{y}$

(c) $\vec{\ell}(t) = \vec{r}(t) \times m\vec{v}(t)$

$\vec{\ell}(t) = m\left(\frac{At^2}{2}\hat{x} + \frac{B}{C}\mathrm{sen}(Ct)\hat{y}\right) \times \left(At\hat{x} + B\cos(Ct)\hat{y}\right)$

$\vec{\ell}(t) = m\left(\frac{ABt^2}{2}\cos(Ct) - \frac{ABt}{C}\mathrm{sen}(Ct)\right)\hat{z}$ $\quad (\hat{x} \times \hat{y} = \hat{z})$

$\vec{\tau}(t) = \vec{r} \times \vec{F} = \left(\frac{At^2}{2}\hat{x} + \frac{B}{C}\mathrm{sen}(Ct)\hat{y}\right) \times \left(A\hat{x} - BC\,\mathrm{sen}(Ct)\hat{y}\right)m$

$\vec{\tau}(t) = -m\left(\frac{ABC}{2}t^2\mathrm{sen}(Ct) + \frac{AB}{C}\mathrm{sen}(Ct)\right)\hat{z}$

(a) $\vec{F}(t) = mA\hat{x} - mBC\,\text{sen}(Ct)\,\hat{y}$

$$\vec{r}(t) = \frac{At^2}{2}\hat{x} + \frac{B}{C}\,\text{sen}(Ct)\,\hat{y}$$

$$x(t) = \frac{At^2}{2} \qquad y(t) = \frac{B}{C}\,\text{sen}(Ct)$$

$$\vec{F}(x,y) = mA\hat{x} - mBC\frac{C}{B}y\hat{y}$$

$$\vec{F}(x,y) = mA\hat{x} - mC^2 y\hat{y}$$

$$F_x = mA \qquad F_y = -mC^2 y$$

$F_x =$ cte análoga à gravitacional

$F_y = -ky$ análoga à elástica:

$$\begin{cases} U_g = +mgx & -\frac{\partial U}{\partial x} = -mg \\ U_e = \frac{1}{2}ky^2 & -\frac{\partial U}{\partial y} = -ky \end{cases}$$

$$U(x,y) = -mAx + \frac{1}{2}mC^2y^2$$

$$-\nabla U = -\left(\frac{\partial U}{\partial x}\hat{x} + \frac{\partial y}{\partial y}\hat{y}\right) = mA\hat{x} - mC^2y\hat{y} = \vec{F}(x,y)$$

Ok.