ENSAIO P3 - Pelicano FI-10 2013

(a) $A(t) = A_0 e^{-\frac{\gamma}{2}t}$ $\quad A_0 = 3{,}68\ cm$ $\quad t_1 = 20\ s$

$A(t_1) = A_0 e^{-\frac{\gamma}{2}t_1} = 3{,}68 \times e^{-\frac{\gamma}{2}\times 20} = 1$

$\frac{1}{3{,}68} = e^{-10\gamma}$ $\quad \ln\frac{1}{3{,}68} = -10\gamma = -1{,}3$

$\gamma = 0{,}13$ $\quad \omega^2 = \sqrt{\omega_0^2 - \left(\frac{\gamma}{2}\right)^2}$

$\omega = \frac{2\pi}{T} = \frac{6{,}28}{0{,}628} = 10\ rad/s$

$\omega_0 = \sqrt{\omega^2 + \left(\frac{\gamma}{2}\right)^2} = \sqrt{100 + \left(\frac{0{,}13}{2}\right)^2} \simeq 10\ rad/s$

(b) $\omega_0 = \sqrt{\frac{k_{ef}}{m}} = 10\ rad/s$ $\quad k_{ef} = 100 \times 14 = 1400\ N/m$

$k_{ef}\Delta x_{eq} = -mg = 140\ N$ $\quad \Delta x_{eq} = -0{,}1\ m$

$\Delta x_{eq} = -10\ cm$

(c) $E_{mec} = \frac{1}{2}mv_0^2 = \frac{1}{2}k_{ef}A_0^2$

$v_0 = \sqrt{\frac{k_{ef}}{m}}A_0 = 10 \times 0{,}0368 = 0{,}37\ m/s$

(d) $\omega_p = 2\pi f_p = 2\pi \times \frac{180}{60} = 6\pi = 18{,}8\ rad/s$

$A_e(\omega_p) = \frac{F_0/m}{\sqrt{(\omega_p^2 - \omega_0^2)^2 + \gamma^2\omega_p^2}}$ $\quad \omega_p^2 - \omega_0^2 = 255 \gg$

$A_e(\omega_p) \simeq \frac{F_0/m}{\omega_p^2 - \omega_0^2}$

$\frac{A_e(\omega_p)}{A_e(\omega_0)} \simeq A_e(\omega_p) = \frac{F_0/m}{255}$

$\omega_f = 2\pi \times \frac{90}{60} = 9,42 \text{ rad/s}$

$(\omega_f^2 - \omega_0^2)^2 = 126.9 \qquad \gamma^2\omega_f^2 = 1,5$

$$A_e(\omega_f) = \frac{F_0/m}{\sqrt{128}}$$

$$\frac{A_e(\omega_f)}{A_e(\omega_p)} = \frac{255}{\sqrt{128}} = 22,5 \qquad A_e(\omega_f) = 4,5 \text{ cm}$$

(e) $\omega_a \approx \omega_0 \qquad f_a = \frac{\omega_0}{2\pi} = \frac{10}{6,28} = 1,6 \text{ Hz}$

$F_0 = 140 N \qquad \omega_0 = 10 \text{ rad/s}$

$$A_a = \frac{10}{\sqrt{\gamma^2\omega_0^2}} = \frac{10}{0,13 \times 10} = \frac{1}{0,13} = 7,7 \text{ m} !$$

(f) $F = F_0 \cos(\omega t)$

$x = A_e \cos(\omega t + \varphi)$

$\varphi(\omega = \omega_0) = -\frac{\pi}{2}$