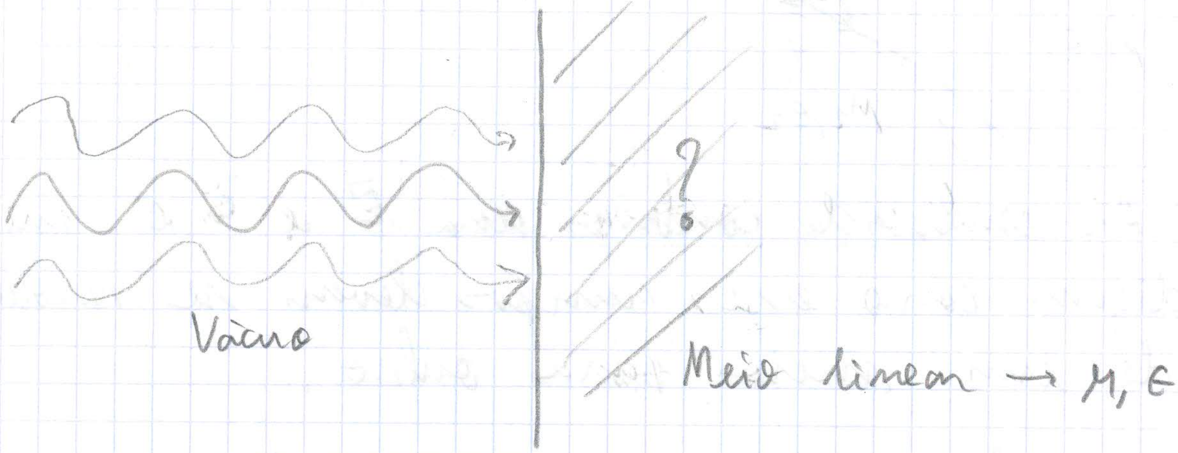
Hoje: * Reflexão e Transmissão de Ondas
* Lei de Snell
* Lei de Fresnel



Vamos sempre considerar que não há cargas nem correntes livres, de modo que as Eq. de Maxwell nesses meios lineares são:

$\vec{\nabla}\cdot\vec{E}=0$
$\vec{\nabla}\cdot\vec{B}=0$ $\Rightarrow$ Ondas transversais, $E_x \sim \vec{E}_0\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$, $\vec{k}\cdot\vec{E}_0=0$

$$\left.\begin{aligned}\vec{\nabla}\times\vec{E} &= -\frac{\partial \vec{B}}{\partial t}\\ \vec{\nabla}\times\vec{B} &= \mu\epsilon\frac{\partial \vec{E}}{\partial t}\end{aligned}\right\}\quad \square\begin{Bmatrix}\vec{E}\\ \vec{B}\end{Bmatrix}=0$$

$\square = \nabla^2 - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}$ $\qquad v^2 = \frac{1}{\mu\epsilon}$ $\quad \boxed{\omega = k\cdot v}$ $\quad \boxed{\vec{B}=\frac{\hat{k}\times\vec{E}}{v}}$

Velocidade de propagação: $v = \frac{1}{\sqrt{\mu\epsilon}} = \frac{c}{n} < c$

Índice de refração: $n = \sqrt{\frac{\epsilon\mu}{\epsilon_0\mu_0}} = \frac{c}{v}$; $n \simeq \sqrt{\frac{\epsilon}{\epsilon_0}} = \sqrt{\epsilon_r}$

Vetor de Poynting: $\vec{S} = \frac{\vec{E}\times\vec{B}}{\mu} = \frac{\epsilon}{\epsilon\mu}\, E\times B = \epsilon v \vec{E}^2\, \hat{k}$

Intensidade: $\langle|\vec{S}|\rangle_t = \epsilon v \langle \vec{E}^2\rangle_t = \epsilon v E_0^2 \langle \cos^2(\vec{k}\cdot\vec{r}-\omega t+\delta)\rangle_t$

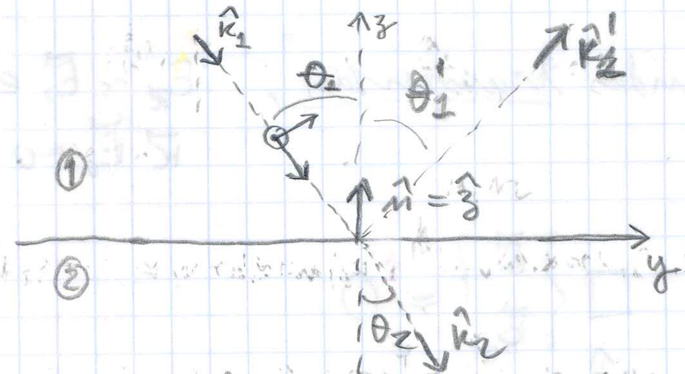
$$\rightarrow \boxed{I = \frac{1}{2}\epsilon v E_0^2} \quad \propto E^2 \,!$$

As condições de contorno para $\vec{E}$ e $\vec{B}$ é que nos dizem como esses campos devem se propagar de um meio para outro.

Já sabemos que, na ausência de cargas livres:

$$\Delta D^{\perp} = 0 \Rightarrow \epsilon_1 E_1^{\perp} = \epsilon_2 E_2^{\perp} \qquad \Delta \vec{E}_{\parallel} = 0 \Rightarrow \vec{E}_1^{\parallel} = \vec{E}_2^{\parallel}$$

$$\Delta B^{\perp} = 0 \Rightarrow B_1^{\perp} = B_2^{\perp} \qquad \Delta \vec{H}_{\parallel} = 0 \Rightarrow \frac{1}{\mu_1}\vec{B}_1^{\parallel} = \frac{1}{\mu_2}\vec{B}_2^{\parallel}$$



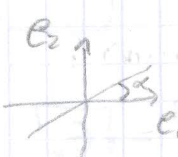
$$\boxed{\hat{k}_1 = -\hat{z}\cos\theta_1 + \hat{y}\,\mathrm{sen}\,\theta_1}$$

$$\begin{cases} \hat{x}, \hat{y} : \text{"}\parallel\text{"} \\ \hat{z} : \text{"}\perp\text{"} \end{cases}$$

Vamos considerar uma onda plana monocromática (ω) unidirecional ($\hat{k}$) e com polarização linear:

QQR POLARIZAÇÃO PODE SER ESCRITA COMO COMBIN. DE POLAR. LINEARES!

$$\vec{E}_1 = \vec{E}_{01} e^{i(\vec{k}_1\cdot\vec{r} - \omega_1 t)} \qquad \omega_1 = |\vec{k}_1|\, v_1$$



$$\vec{E}_{01} = E_{01}\left[\hat{x}\,\mathrm{sen}\,\alpha_1 + \cos\alpha_1\left(\hat{y}\cos\theta_1 + \hat{z}\,\mathrm{sen}\,\theta_1\right)\right]$$

$$\vec{E}_{01}^2 = E_{01}^2 \quad , \quad \vec{E}_{01}\cdot\hat{k}_1 = 0 \quad \checkmark$$

$$\vec{B}_1 = \frac{1}{v_1}\hat{k}_1 \times \vec{E}_1 = \frac{E_{01}}{v_1}\left(\hat{y}\,\mathrm{sen}\,\theta_1 - \hat{z}\cos\theta_1\right) \times \Big(\hat{x}\,\mathrm{sen}\,\alpha_1 + \cos\alpha_1\left(\hat{y}\cos\theta_1 + \hat{z}\,\mathrm{sen}\,\theta_1\right)\Big)$$

α_1: GRAU DE POLARIZAÇÃO NO PLANO DE INCIDÊNCIA (y, z)

$$\Rightarrow \vec{B}_1 = \frac{E_{01}}{v_1}\left[\hat{x}\cos\alpha_1 - \operatorname{sen}\alpha_1(\hat{y}\cos\theta_1 + \hat{z}\operatorname{sen}\theta_1)\right]$$

$$\vec{B}_1 \cdot \hat{k}_1 = \vec{B}_1 \cdot \vec{E}_1 = 0$$

$\hat{k}_1' = +\hat{z}\cos\theta_1' + \hat{y}\operatorname{sen}\theta_1'$

E o mesmo para a onda "1'" ("refletida", no meio 1) e a onda (2) (transmitida, no meio 2).

Lembrando que: $\hat{x}, \hat{y} \rightarrow //$, $\hat{z} \rightarrow \perp$

Interface: $z = 0$

$$\begin{cases} \phi_1 \equiv (\vec{k}_1 \cdot \vec{r} - \omega_1 t)\big|_{z=0} = k_1 y \operatorname{sen}\theta_1 - \omega_1 t \\ \phi_2 \equiv k_2 y \operatorname{sen}\theta_2 - \omega_2 t \\ \phi_1' \equiv k_1' y \operatorname{sen}\theta_1' - \omega_1' t \end{cases}$$

Temos então 6 equações:

i) $E_1^x + E_{1'}^x = E_2^x \Rightarrow E_{01}\operatorname{sen}\alpha_1 e^{i\phi_1} + E_{01}'\operatorname{sen}\alpha_1' e^{i\phi_1'} = E_{02}\operatorname{sen}\alpha_2 e^{i\phi_2}$

ii) $E_1^y + E_{1'}^y = E_2^y \Rightarrow E_{01}\cos\alpha_1\cos\theta_1 e^{i\phi_1} \ominus E_{01}'\cos\alpha_1'\cos\theta_1' e^{i\phi_1'} = E_{02}\cos\alpha_2\cos\theta_2 e^{i\phi_2}$

iii) $\epsilon_1(E_1^z + E_{1'}^z) = \epsilon_2 E_2 \Rightarrow \epsilon_1(E_{01}\cos\alpha_1\operatorname{sen}\theta_1 e^{i\phi_1} \oplus E_{01}'\cos\alpha_1'\operatorname{sen}\theta_1' e^{i\phi_1'}) = \epsilon_2 E_{02}\cos\alpha_2\operatorname{sen}\theta_2 e^{i\phi_2}$

$\frac{1}{\mu}(\vec{B}_1^{//} + \vec{B}_{1'}^{//}) = \frac{1}{\mu_2}\vec{B}_2^{//}$

iv) $\frac{1}{\mu_1 v_1}\left(E_{01}\cos\alpha_1 e^{i\phi_1} + E_{01}'\cos\alpha_1' e^{i\phi_1'}\right) = \frac{1}{\mu_2 v_2} E_{02}\cos\alpha_2 e^{i\phi_2}$

v) $-\frac{1}{\mu_1 v_1}\left(E_{01}\operatorname{sen}\alpha_1\cos\theta_1 e^{i\phi_1} \ominus E_{01}'\operatorname{sen}\alpha_1'\cos\theta_1' e^{i\phi_1'}\right) = -\frac{1}{\mu_2 v_2} E_{02}\operatorname{sen}\alpha_2\cos\theta_2 e^{i\phi_2}$

vi) $-\frac{1}{v_1}\left(E_{01}\operatorname{sen}\alpha_1\operatorname{sen}\theta_1 e^{i\phi_1} + E_{01}'\operatorname{sen}\alpha_1'\operatorname{sen}\theta_1' e^{i\phi_1'}\right) = -\frac{1}{v_2} E_{02}\operatorname{sen}\alpha_2\cos\theta_2 e^{i\phi_2}$

Todas as fases dependem de y e t, e portanto a única maneira de satisfazermos todas essas equações simultaneamente p/ qq y e qq t é:

$$k_1\operatorname{sen}\theta_1 = k_2\operatorname{sen}\theta_2 = k_1'\operatorname{sen}\theta_1', \quad \text{e} \quad \omega_1 = \omega_2 = \omega_1'$$

Mas $k_1 = \frac{w_1}{v_1}$, $\boxed{k_1' = \frac{w_1'}{v_1} = \frac{w_1}{v_1} = k_1}$

$$k_2 = \frac{w_2}{v_2} = \frac{w_1}{v_2} = k_1 \frac{v_1}{v_2} = k_1 \cdot \frac{n_2}{n_1}$$

Além disso, $\cancel{k_1} \operatorname{sen}\theta_1 = \cancel{k_1'} \operatorname{sen}\theta_1' \Rightarrow \boxed{\theta_1 = \theta_1'}$

$$k_1 \operatorname{sen}\theta_1 = k_2 \operatorname{sen}\theta_2 = k_1 \frac{n_2}{n_1} \operatorname{sen}\theta_2$$

$$\Rightarrow \boxed{n_1 \operatorname{sen}\theta_1 = n_2 \operatorname{sen}\theta_2}$$

Leis de Snell

Agora as nossas 6 equações se resumem a:

i) $E_{01}\operatorname{sen}\alpha_1 + E_{01}'\operatorname{sen}\alpha_1' = E_{02}\operatorname{sen}\alpha_2$

ii) $\cos\theta_1 (E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1') = \cos\theta_2\, E_{02}\cos\alpha_2$

iii) $\epsilon_1 \operatorname{sen}\theta_1 (E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1') = \epsilon_2 \operatorname{sen}\theta_2\, E_{02}\cos\alpha_2$

iv) $E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1' = \frac{\mu_1 v_1}{\mu_2 v_2} E_{02}\cos\alpha_2$

v) $\cos\theta_1 (E_{01}\operatorname{sen}\alpha_1 + E_{01}'\operatorname{sen}\alpha_1') = \cos\theta_2 \frac{\mu_1 v_1}{\mu_2 v_2} E_{02}\operatorname{sen}\alpha_2$

vi) $\cancel{\operatorname{sen}\theta_1} (E_{01}\operatorname{sen}\alpha_1 + E_{01}'\operatorname{sen}\alpha_1') = \cancel{\operatorname{sen}\theta_2} \frac{v_1}{v_2} E_{02}\operatorname{sen}\alpha_2$

=

Vamos reescrevê-las:

i) $E_{01}\operatorname{sen}\alpha_1 + E_{01}'\operatorname{sen}\alpha_1' = E_{02}\operatorname{sen}\alpha_2$

iii) $E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1' = \frac{\epsilon_2 \operatorname{sen}\theta_2}{\epsilon_1 \operatorname{sen}\theta_1} E_{02}\cos\alpha_2$

v) $E_{01}\operatorname{sen}\alpha_1 + E_{01}'\operatorname{sen}\alpha_1' = \frac{\mu_1 v_1 \cos\theta_2}{\mu_2 v_2 \cos\theta_1} E_{02}\operatorname{sen}\alpha_2$

ii) $E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1' = \frac{\cos\theta_2}{\cos\theta_1} E_{02}\cos\alpha_2$

iv) $E_{01}\cos\alpha_1 + E_{01}'\cos\alpha_1' = \frac{\mu_1 v_1}{\mu_2 v_2} E_{02}\cos\alpha_2$

= !

Ficamos, então, com 4 equações:

i) $E_I \operatorname{sen}\alpha_I + E_R \operatorname{sen}\alpha_R = E_T \operatorname{sen}\alpha_T$

ii) $E_I \cos\alpha_I - E_R \cos\alpha_R = \frac{\cos\theta_T}{\cos\theta_I} \cdot E_T \cos\alpha_T$ a

iii) $E_I \cos\alpha_I + E_R \cos\alpha_R = \frac{\mu_1 v_1}{\mu_2 v_2} E_T \cos\alpha_T$ b

iv) $E_I \operatorname{sen}\alpha_I - E_R \operatorname{sen}\alpha_R = \frac{\mu_1 v_1 \cos\theta_T}{\mu_2 v_2 \cos\theta_R} E_T \operatorname{sen}\alpha_T$

$c = a \cdot b = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}$

Dados: $E_I, \alpha_I, \theta_I \longrightarrow \operatorname{sen}\theta_T = \frac{n_2}{n_1} \operatorname{sen}\theta_I$

Incógnitas: $E_R, \alpha_R, E_T, \alpha_T$

A) Combinando i) e v) temos:

$$2E_I \operatorname{sen}\alpha_I = (1+ab) E_T \operatorname{sen}\alpha_T$$

B) Combinando ii e iii) temos:

$$2E_I \cos\alpha_I = (a+b) E_T \cos\alpha_T$$

$E_T, \operatorname{sen}\alpha_T$

$$\Rightarrow \left(\frac{2}{1+ab}\right) E_I \operatorname{sen}\alpha_I = E_T \operatorname{sen}\alpha_T$$

$$\frac{2}{a+b} E_I \cos\alpha_I = E_T \cos\alpha_T$$

$$\Rightarrow \boxed{\operatorname{tg}\alpha_T = \frac{a+b}{1+ab} \operatorname{tg}\alpha_I}$$

$$\Rightarrow \boxed{E_T^2 = 4E_I^2 \left(\frac{\operatorname{sen}^2\alpha_I}{(1+ab)^2} + \frac{\cos^2\alpha_I}{(a+b)^2} \right)}$$

Leis de Fresnel

Para encontrar E_R, α_R :

c) Combinando i) e v) :

$$2 E_R \operatorname{sen}\alpha_R = (1-ab) E_T \operatorname{sen}\alpha_T$$

$$= (1-ab) \cdot \frac{2}{1+ab} E_I \operatorname{sen}\alpha_I$$

$$\Rightarrow \boxed{E_R \operatorname{sen}\alpha_R = \frac{1-ab}{1+ab} E_I \operatorname{sen}\alpha_I}$$

D) Combinando ii) e iii) :

$$2E_R \cos\alpha_R = (b-a) \cdot E_T \cos\alpha_T$$

$$= (b-a) \cdot \frac{2}{c_1+c_2} E_I \cos\alpha_I$$

$$\Rightarrow \boxed{E_R \cos\alpha_R = \frac{b-a}{b+a} E_I \cos\alpha_I}$$

$$\Rightarrow \operatorname{tg}\alpha_R = \frac{1-ab}{1+ab} * \frac{b+a}{b-a} * \operatorname{tg}\alpha_I$$

$$\boxed{E_R^2 = E_I^2 \left[\left(\frac{1-ab}{1+ab}\right)^2 \operatorname{sen}^2\alpha_I + \left(\frac{b-a}{b+a}\right)^2 \cos^2\alpha_I\right]}$$

Intensidades

Incidente : $I_I = \frac{1}{2} \epsilon_1 v_1 E_I^2$

Refletida : $I_R = \frac{1}{2} \epsilon_1 v_1 E_R^2$

Transmitida : $I_T = \frac{1}{2} \epsilon_2 v_2 E_T^2$

A potência total da onda incidente deve ser conservada ⟶ uma parte vai p/ a onda refletida, outra parte vai para a onda transmitida!

$$R = \frac{I_R}{I_I} = \frac{E_R^2}{E_I^2} = \left(\frac{1-ab}{1+ab}\right)^2 \text{sen}^2\alpha_I + \left(\frac{b-a}{b+a}\right)^2 \cos^2\alpha_I$$

$$T = \frac{I_T}{I_I} = \frac{\epsilon_2 v_2}{\epsilon_1 v_1}\frac{E_T^2}{E_I^2} = ab \times \left(\frac{4}{(1+ab)^2}\,\text{sen}^2\alpha_I + \frac{4}{(a+b)^2}\cos^2\alpha_I\right)$$

De fato:

$$R+T = \overbrace{\frac{(1-ab)^2+4ab}{(1+ab)^2}}^{1}\,\text{sen}^2\alpha_I + \overbrace{\frac{(b-a)^2+4ab}{(a+b)^2}}^{1}\cos^2\alpha_I$$

$$= 1 \quad !!$$

Casos particulares

1) Polarização paralela ao plano de incidência:

$$\alpha_I = 0 \Rightarrow \text{sen}\,\alpha_I = 0, \quad \cos\alpha_I = 1, \quad \text{tg}\,\alpha_I = 0$$

$$\Rightarrow \alpha_T = \alpha_R = 0$$

$$E_T = \frac{2}{a+b}E_I \qquad a = \frac{\cos\theta_T}{\cos\theta_I}$$

$$E_R = \frac{b-a}{b+a}E_I \qquad b = \frac{\mu_1 v_1}{\mu_2 v_2} \approx \frac{v_1}{v_2} \approx \frac{\text{sen}\,\theta_T}{\text{sen}\,\theta_I}$$

2) Polarização ⊥ ao plano de incidência:

$$\alpha_I = \pi/2 \Rightarrow \text{sen}\,\alpha_I = 1, \quad \cos\alpha_I = 0, \quad \text{tg}\,\alpha_I = \infty$$

$$\Rightarrow \alpha_T = \alpha_R = \pm\frac{\pi}{2}$$

$$\Rightarrow E_t = \frac{2}{1+ab} E_I$$

$$E_R = \frac{1-ab}{1+ab} E_I$$

OBS: $\frac{tg\, \alpha_R}{tg\, \alpha_T} = \frac{1-ab}{b-a}$ NÃO DEP. DE α_I!