The preceding paragraph points out the similarity between four-phase systems in ternary alloys and three-phase systems in binary alloys. In the same manner, three-phase ternary systems are analogous to two-phase binary systems. Thus, at constant temperature and pressure, a three-phase system in a ternary alloy will have fixed compositions of the phases. This fact does not mean that the composition of the alloy as a whole has one value, but that the phases, which may be in any proportion, have fixed compositions.

Three-component alloys may also have structures containing two phases or even a single phase. It is important to notice that in a ternary alloy containing two phases, the compositions of the phases are not fixed. This statement is true of binary systems, but not of ternary systems.

PROBLEMS

10.1 List and identify the phases in Fig. 2.27. Explain whether the phase rule can be applied to this structure.

10.2 The three elements zirconium, titanium, and hafnium form an alloy system in which the components are able to form a single solid solution containing the elements mixed in any proportion. Assume that one has such a solid solution containing 0.50 kg titanium, 0.30 kg zirconium, and 1.00 kg hafnium, and determine the mole fraction of each of the components in the alloy. The gram-atomic weights of Ti, Zr, and Hf are 47.9, 91.2, and 178.6 g/mol, respectively.

10.3 Given an alloy containing 70 percent of copper and 30 percent nickel, by weight, determine the composition of this alloy in atomic percent. Note the atomic weights of copper and nickel are 63.546 and 58.71 grams per mole, respectively.

10.4 Consider the activity coefficient vs. composition curves for two alloys in Fig. 10.5. In Fig. 10.5A, a dashed line is drawn at the composition $N_A = 0.70$. As discussed in the text, the activity of the B component, a_B, equals 0.80. The activity a_A is 0.50. Assume the free energy of 1 mole of pure A is 50,000 J/mol and that of pure B is 70,000 J/mol and the temperature is 1000 K.

(a) Compute the free energy of 1 mole of the 0.70 N_A alloy.

(b) Determine the free-energy decrease when a mole of this solution forms from the pure components.

10.5 In Fig. 10.5B, at the composition $N_A = 0.70$ we have

$$a_A = 0.10$$

$$a_B = 0.44$$

$$T = 1000 \text{ K}$$

$$G_A^0 = 50{,}000 \text{ J/mol}$$

$$G_B^0 = 70{,}000 \text{ J/mol}$$

(a) Compute the free-energy per mole of the solution with $N_A = 0.30$.

(b) Determine the decrease in free energy if a mole of an ideal solution were to be formed from the pure components.

(c) What is the significance of the difference in the free energies of solution of the two $N_A = 0.70$ alloys in Figs. 10.5A and 10.5B?

10.6 (a) Compute the free-energy decrease associated with forming one mole of an ideal solution at 1000 K, if $G_A^0 = 50{,}000$ J/mol and $G_B^0 = 70{,}000$ J/mol.

(b) Compare your answer with those for the (b) parts in Probs. 10.4 and 10.5 and explain the significance of the differences.

10.7 Compute the activity coefficients corresponding to the four activities involved in Probs. 10.4 and 10.5.

10.8 Consider the hypothetical free-energy diagram of an ideal solution shown in Fig. 10.7. Now assume that a layer of pure metal A is bonded to a layer of pure metal B, that 40 percent by weight of the couple is in the A metal layer and that the atomic weights of metals A and B are 80 and 60 grams per mole, respectively. Further assume that the couple is held long enough at 500 K so that, as a result of diffusion, a homogeneous solid solution is obtained.

(a) What would be the free energy of the couple at the beginning of the diffusion anneal? Give the answer in Joules per mole.

(b) What would be the free energy of the homogeneous solid solution at the end of the diffusion anneal? Give the answer in Joules per mol.

10.9 Assume that the following data hold for an ideal binary solid solution; $T = 1000$ K, $G_A^0 = 7000$ J/mol, and $G_B^0 = 10{,}000$ J/mole. Draw the free energy versus mole fraction diagram for this solid solution (see Fig. 10.8) and determine the partial molal free energies per mole for the solid solution composition containing 0.4 N_A.

10.10 With reference to Fig. 10.10, take $\overline{G}_B^\alpha = 2100$ J/mol, $\overline{G}_A^\beta = 1400$ J/mol, $N_{A_2}^\beta = 0.22$, and $N_{A_2}^\alpha = 0.64$.

(a) Determine the free energy of 1.0 mole of the alloy whose mole fraction is 0.22.

(b) What fraction of this composition will be in the α phase?

(c) Determine the free energy of 1.0 mole of the composition $N_A = 0.50$.

(d) How much of this latter composition will be in the α phase?

10.11 (a) As explained in Sec. 10.10, a ternary alloy may have a ternary eutectic point at which four phases may coexist. Describe the nature of these four phases. How many degrees of freedom are there when four phases coexist in a ternary alloy? What is the meaning of this?

(b) A ternary alloy may also have a three-phase field. How many degrees of freedom will there be in the three-phase field? Explain the significance of this number of degrees of freedom.

(c) In a ternary alloy a single-phase field may also occur. How many degrees of freedom will it contain?

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