Magnetism, Part 2

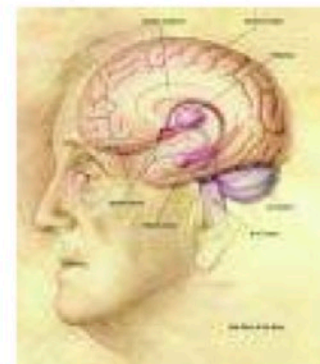
- ⚡ Examples/applications
- ⚡ Magnetic dipole & quadrupole
- ⚡ Force, torque and energy
- ⚡ Magnetic fields in matter
- ⚡ Boundary conditions
- ⚡ Magnetic materials

Magnetic fields in nature

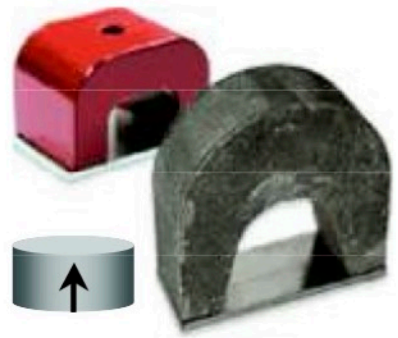
Typical values of *B*

Earth 50 μT

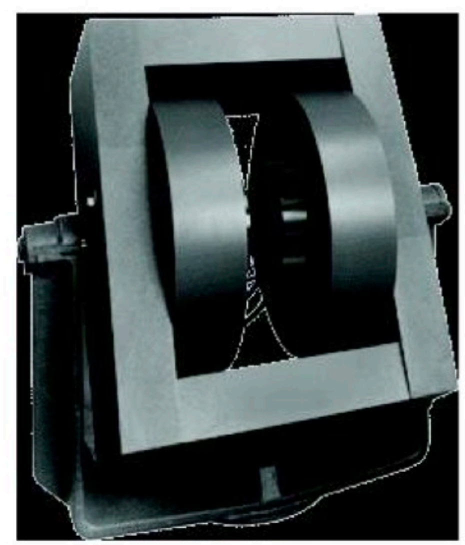
Helmholtz coils 0.01 T



Human brain 1 fT



Permanent magnets 0.5 T



Electromagnet 1 T

Superconducting magnet 10 T

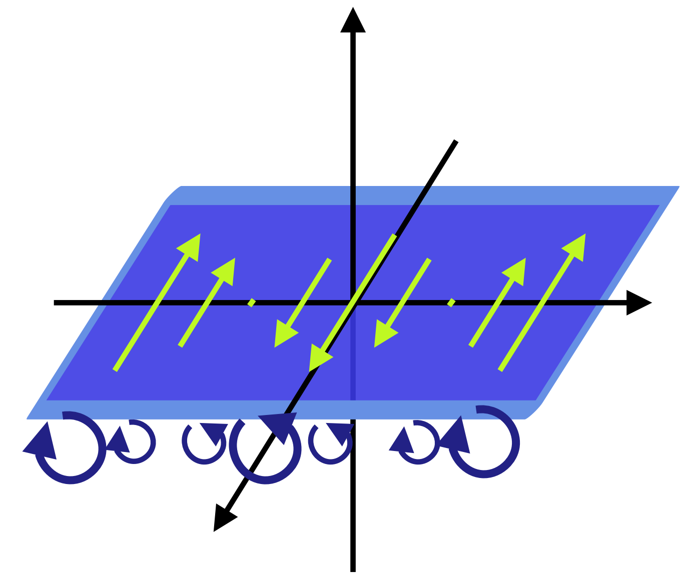
Magnetar 10^{12} T

Some simple examples

- We saw in the last class how an infinite plane with a simple current density generates a magnetic field that decays exponentially away from that plane:

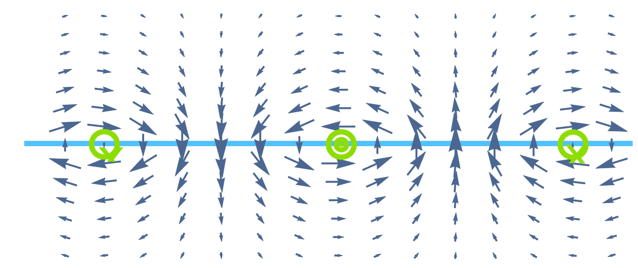
$$\vec{K} = K_0\,\hat{x}\,\cos ky \quad , \quad \text{or} \quad \vec{J} = K_0\,\hat{x}\,\cos ky\;\delta(z)$$

$$\Rightarrow \quad \vec{B} = \frac{\mu_0 K_0}{2}\left[\mp\hat{y}\,\cos ky\, e^{\mp kz} + \hat{z}\,\sin ky\, e^{\mp kz}\right]$$



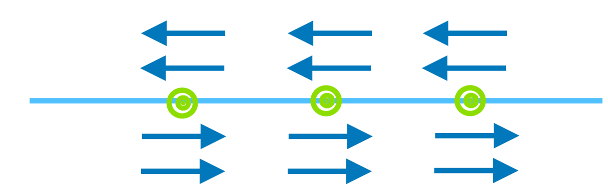
- We can take the limit of constant surface current density by making $k \to 0$. We then have:

$$\Rightarrow \quad \vec{B} = \mp\frac{\mu_0 K_0}{2}\,\hat{y} \quad , \quad \text{which is a familiar result.}$$

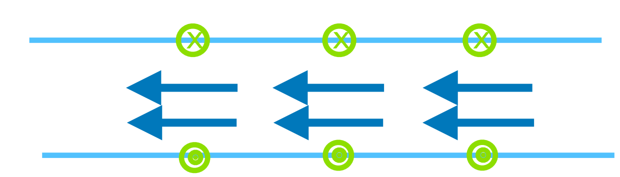


- On the basis of this result we can derive many more. For instance, two infinite planes with opposite currents will create a magnetic field only between those two planes:

$$\vec{B} = \mp\,\mu_0 K_0\,\hat{y}$$

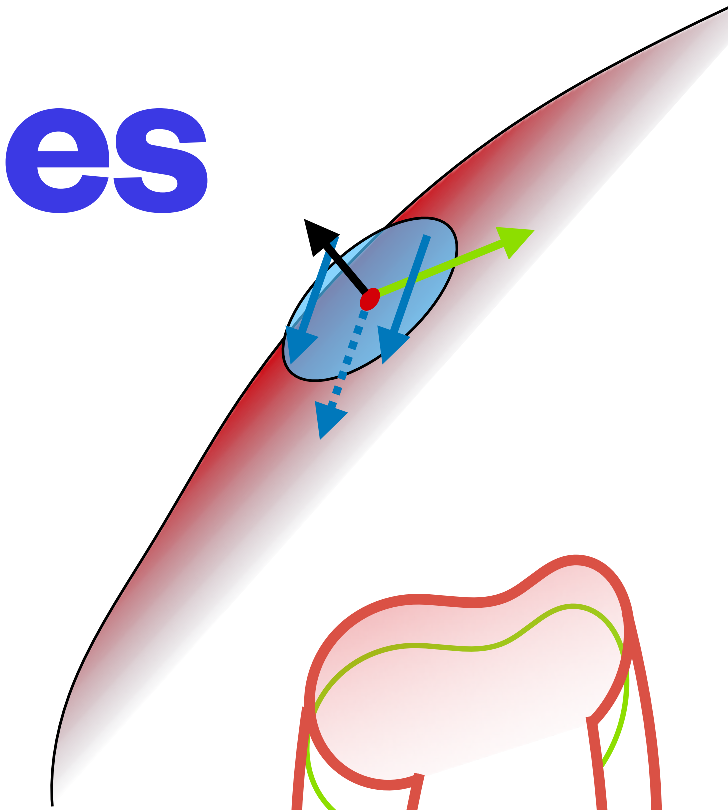


Outside the region between the two plates the magnetic field cancels out, and the field is therefore zero.



- This is the "building block" for many other results.

Some simple examples



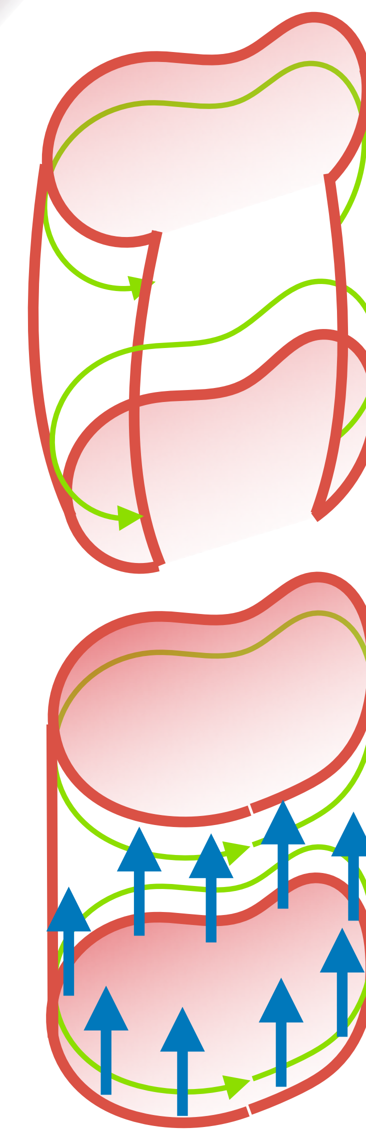
- The previous results indicate that ***just above*** a surface (with normal $\hat{n}$) that carries a surface current density $\vec{K}$, the field is approximately:

  $$\vec{B} \to \mu_0 \vec{K} \times \hat{n}$$

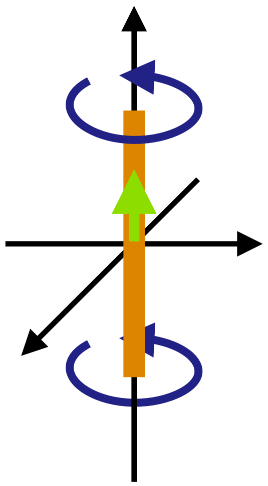
- Now, imagine that we curve that surface in such a way that we close it in on itself — i.e., we wrap the surface such that the current density connects with itself at the other end, as indicated in the figure.
- Suppose that we are careful enough that we make the normal to the surface always point perpendicular to the z direction — i.e., the normal is always in the x, y plane. In this way we form a **tube**, with a section shown in the figure.
- Therefore, at any given section of this tube, the field right near the surface of the tube is always $\vec{B} \to \mu_0 K_0 \, \hat{z}$
- OK, now consider that this tube is infinite, therefore the magnetic field in **all of the boundary** (the inner surface of the tube) is given by $\vec{B} \to \mu_0 K_0 \, \hat{z}$. What does that mean for the field inside the tube?
- Of course, it means that the field has to be $\vec{B} = \mu_0 K_0 \, \hat{z}$ everywhere!! Remember: the magnetic field inside the tube obeys $\nabla^2 \vec{B} = 0$, and we know that such a field, if subject to constant boundary conditions, will take the same value inside the volume as it has on the boundary! (E.g., the relaxation method.)
- Therefore, we conclude that, for any tube of any section, if the surface current density is constant, then the magnetic field is $\vec{B} = \mu_0 K_0 \, \hat{z}$, where $\hat{z}$ defines the direction along the length of the tube.
- A corollary of this result is that the field inside a solenoid is given by:

  $$\vec{B} = \mu_0 K_0 \hat{z} \quad , \qquad \text{with } K_0 = \frac{N I}{L}$$

  , where N are the number of loops, I is the current, and L is the length.



Another simple example



- Consider now another "building block": a segment of a wire of length L, carrying a current I, oriented in the z direction.
- Ok, this cannot be entirely correct, right? Where does the current "go"? It's being created at the lower ($z = -L/2$) end, and destroyed at the upper $z = L/2$ end. So, it cannot be that $\vec{\nabla} \cdot \vec{J} = 0$ everywhere, right?…
- Nevertheless, we can try to solve the problem defined by the current density:

$$\vec{J}\, dV = I\, dz\, \hat{z} \qquad \text{for } -L/2 \leq z \leq L/2 \,.$$

- We then have the vector potential:

$$\vec{A}(\rho, \varphi, z) = \frac{\mu_0}{4\pi} \int_{-L/2}^{L/2} dz' \frac{I\, \hat{z}}{\sqrt{\rho^2 + (z - z')^2}}$$

- But the integral above is basically a log: $\int dx\, (a + x^2)^{-1/2} = \log\left(x + \sqrt{a + x^2}\right)$. Therefore:

$$\vec{A} = \frac{\mu_0 I}{4\pi} \ln\left[\frac{z + L/2 + \sqrt{\rho^2 + (z + L/2)^2}}{z - L/2 + \sqrt{\rho^2 + (z - L/2)^2}}\right] \hat{z} \quad , \qquad \text{which leads to:}$$

$$\vec{B} = \frac{\mu_0 I}{4\pi} \frac{1}{\rho} \left[\frac{z + L/2}{\sqrt{\rho^2 + (z + L/2)^2}} - \frac{z - L/2}{\sqrt{\rho^2 + (z - L/2)^2}}\right] \hat{\varphi}$$

- Notice that in the limit $L \to \infty$ we recover the result for an infinite wire, $\vec{B} \to \dfrac{\mu_0 I}{2\pi} \dfrac{1}{\rho} \hat{\varphi}$.

Another simple example

- This simple "building block" can be used to construct many other well-know results. Take, for instance, the magnetic field along the axis of a current loop.
- A simple application of the Biot-Savart law gives us that, along the z axis, we have:

$$\vec{B}(\rho = 0, z) = \frac{\mu_0 I}{2} \frac{R^2}{(R^2 + z^2)^{3/2}} \quad , \quad \text{and therefore} \quad \vec{B}(\rho = 0, z = 0) = \frac{\mu_0 I}{2} \frac{1}{R}$$

- Let's consider that this loop is made up of a large number of connected wire segments of length ΔL, subject to the condition that $\sum \Delta L = 2\pi R$.
- For simplicity, let's take the plane $\{x, y\}$, and place our little wire segment somewhere along the circle, with the current in the anti-clockwise direction. This piece of wire will create a field, right at the center of the loop, which points in the $+z$ direction, with strength:

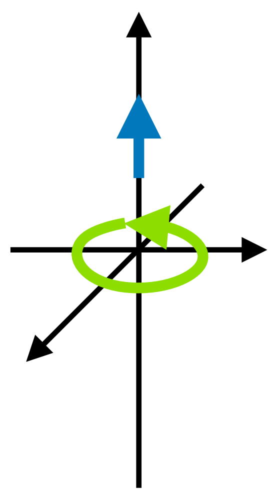
$$\Delta B = \frac{\mu_0 I}{4\pi} \frac{1}{R} \frac{\Delta L}{\sqrt{R^2 + (\Delta L/2)^2}}$$

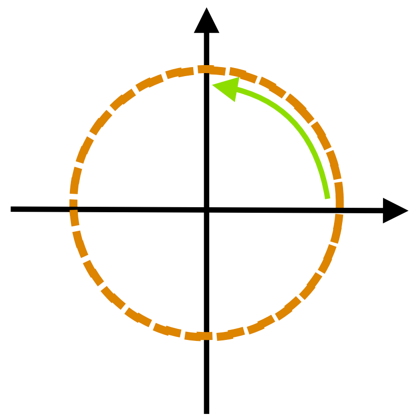
- Now, we take $\Delta L \to 0$ and sum over all the contributions of all those little pieces. We get:

$$B = \sum \Delta B = \frac{\mu_0 I}{4\pi} \frac{1}{R} \sum \frac{\Delta L}{\sqrt{R^2 + (\Delta L/2)^2}} = \frac{\mu_0 I}{4\pi} \frac{1}{R} \frac{\sum \Delta L}{\sqrt{R^2}} = \frac{\mu_0 I}{4\pi} \frac{2\pi R}{R^2} = \frac{\mu_0 I}{2} \frac{1}{R}$$

which is the expected result!

- I will leave it to you to show that the result for any z can be obtained from the result of the segments (you can check it against the solution in, e.g., Jackson). And, as a bonus, we can now generate the field of **any closed loop** made of line segments!!





Magnetic dipoles and quadrupoles

- In the last class we saw that the current density, the vector potential and the magnetic field can be decomposed into multipoles.
- We also saw that the monopole of the magnetic field vanishes — a manifestation of the fact that in nature there are no magnetic monopoles.
- This means that the leading order contribution to the magnetic field comes from the magnetic dipole moment. We derived this moment to be:

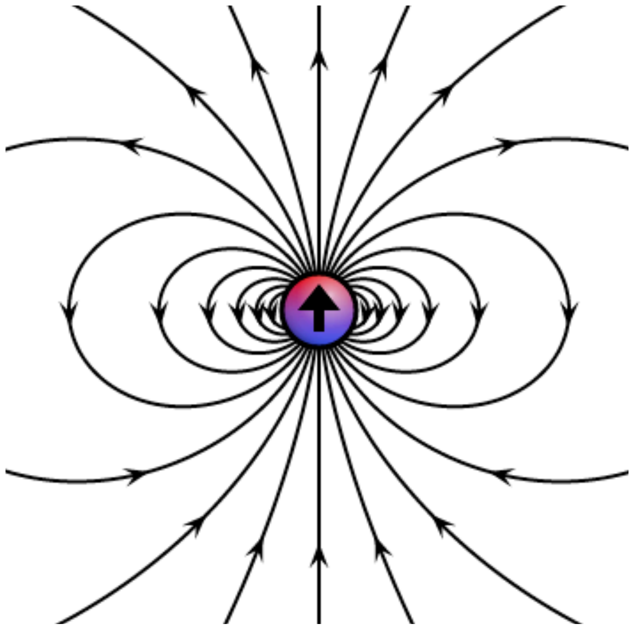
$$\vec{m} = \frac{1}{2}\int dV' \, \vec{r}' \times \vec{J}(\vec{r}') \quad , \quad \text{and the vector potential is:} \quad \vec{A}_1(\vec{r}) = \frac{\mu_0}{4\pi}\frac{\vec{m}\times\vec{r}}{r^3}$$

- The magnetic field of a magnetic dipole is:

$$\vec{B}_1 = \nabla \times \vec{A}_1 = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3}$$

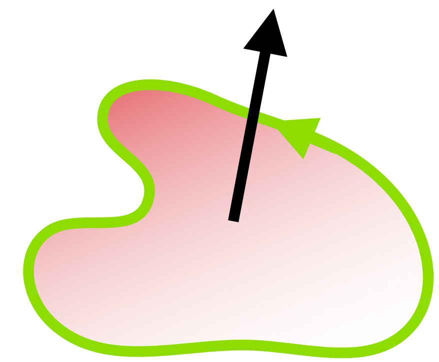
which you can compare with the electric field of an electric dipole:

$$\vec{E}_1 = \frac{1}{4\pi\,\epsilon_0}\frac{3\hat{r}(\vec{p}\cdot\hat{r}) - \vec{p}}{r^3}$$



- The two fields have exactly the same shape, but their origin is quite different

Magnetic dipoles and quadrupoles



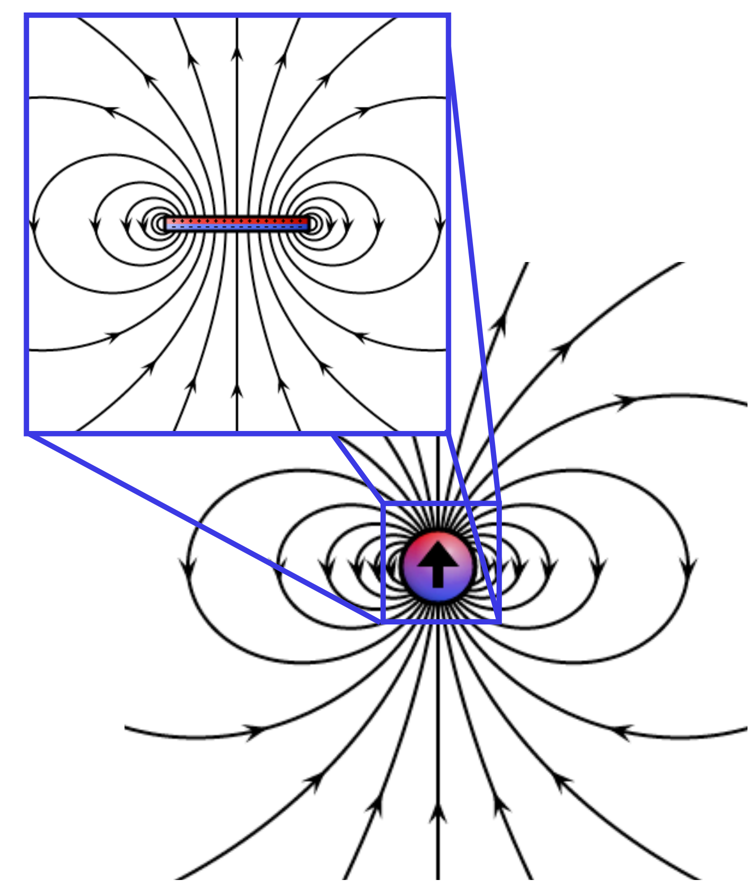
- The magnetic dipole moment of a current-carrying loop has a simple expression in terms of the area circumscribed by the loop:

$$\vec{m} = \frac{1}{2}\int dV'\, \vec{r}\,' \times \vec{J}(\vec{r}\,') = \frac{1}{2}\int \vec{r}\,' \times (I d\vec{l}\,') = I\,\vec{S} \quad ,$$

  where $\vec{S}$ is the area of the loop. Another equivalent way to write the magnetic moment is in terms of the angular momentum of the charges circulating in the loop:

$$\vec{m} = \frac{q\,\vec{L}}{2\,M} \quad ,$$ where $\vec{L}$ is the orbital angular momentum of the charges q (of total mass M).

  [In some cases we associate the magnetic dipole momentum of a particle with its spin. You may have heard of the famous "anomalous magnetic dipole of the muon", which at present indicates at almost 4σ that the Standard Model may be incomplete — or even wrong altogether!]



- The field for a dipole is, formally:

$$\vec{B}_1 = \nabla \times \vec{A}_1 = \frac{\mu_0}{4\pi}\frac{3\hat{r}(\vec{m}\cdot\hat{r}) - \vec{m}}{r^3}$$

  However, obviously the field inside a real loop is well behaved — there is nothing strange going on at $r \to 0$, as long as the loop is finite. ***[See Mathematica Notebook.]***

- OK, so this (the dipole, $\ell = 1$) is the leading order contribution to the magnetic field. Now, what is the next-to-leading order term?

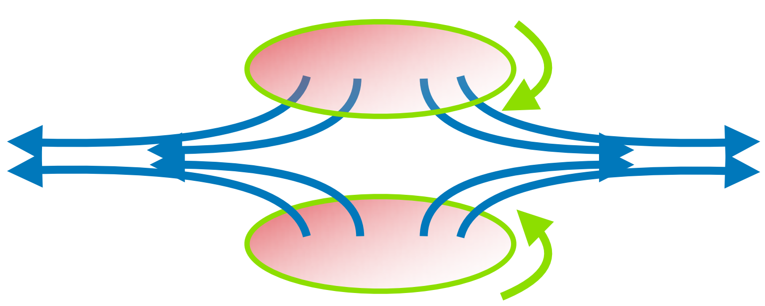
- Let's look at the magnetic quadrupole ($\ell = 2$) in some cases.

Magnetic quadrupole

- Let's write the magnetic quadrupole using the simple expression in terms of the Legendre polynomials:

$$\vec{A}_2 = \frac{\mu_0}{4\pi}\frac{1}{r^3}\int dV' \, (r')^2 \, P_2(\hat{n}\cdot\hat{n}') \; \vec{J}(\vec{r}\,')$$

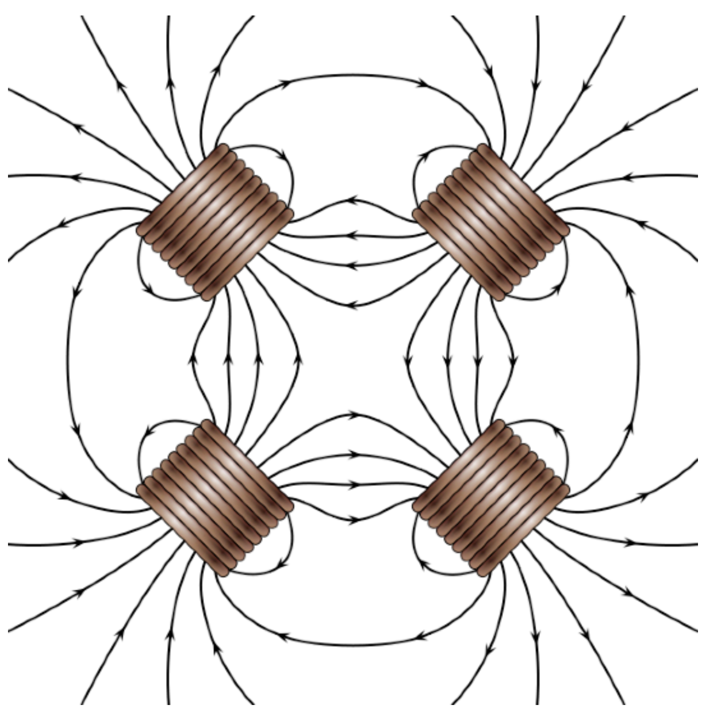
$$= \frac{\mu_0}{4\pi}\frac{1}{r^3}\int dV' \, (r')^2 \, \frac{1}{2}\left[3(\hat{n}\cdot\hat{n}')^2 - 1\right] \; \vec{J}(\vec{r}\,')$$



- Just as we did for the electric quadrupole, we can write the expression above as:

$$\vec{A}_2 = \frac{\mu_0}{4\pi}\frac{1}{r^5}\sum_{i,j=3}^{3} r_i \, r_j \, \vec{Q}_{ij} \quad , \quad \text{where}$$

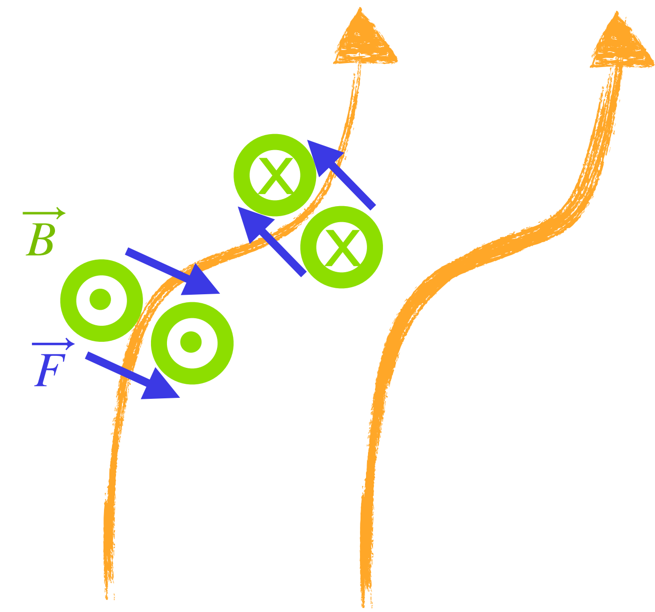
$$\vec{Q}_{ij} = \frac{1}{2}\int dV' \left(3r_i' r_j' - r'^2 \delta_i j\right) \vec{J}(\vec{r}\,')$$

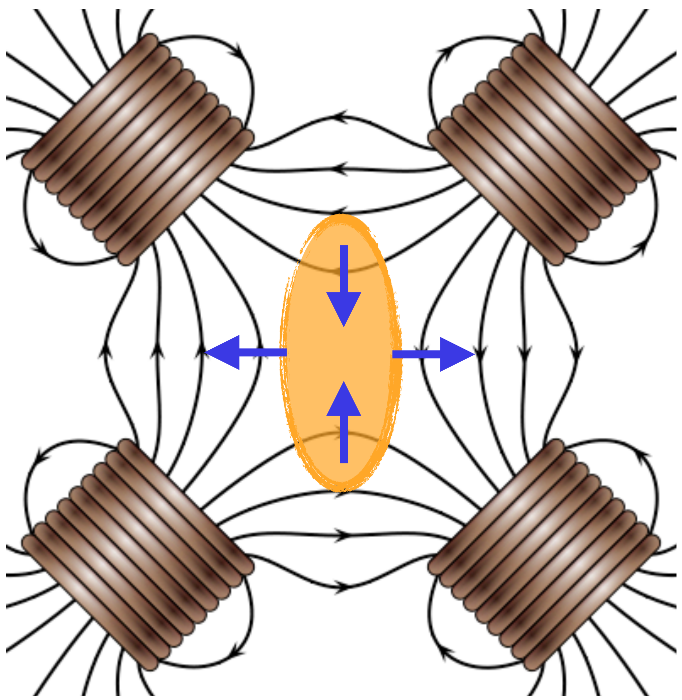


- A simple example of a quadrupole can be found by combining two identical loops with opposite magnetic dipole moments that are aligned, but placed at a distance.
- A more neat quadrupole is composed by combining four dipoles in a "cross", as shown in the figure.

Applications

- A very nice application of magnetic dipoles and quadrupoles is in collimating beams inside accelerators or tokamaks.
- In particle accelerators, one uses magnetic dipoles to change the paths of charge particles such as electrons or protons.
- However, with time the beam starts to lose focus: it gets wider and wider…
- One of the ways to restore the focus is to employ a quadrupole.
- If in a particular region the beam's cross section starts to get too "flattened", the quadrupole helps restore it to a more circular section.
- The same type of tricks are used in Tokamaks in order to keep the beams of electrons and protons well behaved.





Magnetic force and torque

- Now let's go back to the very beginning of our discussion about the magnetic field: the mutual force of two current-carrying pieces of wire. In terms of the current and of a line element we have:

$$d\vec{F}_{1\leftarrow 2} = (I_1\, d\vec{l}_1) \times \vec{B}_2 = (\vec{J}_1\, dV_1) \times \vec{B}_2 \quad ,$$

for the force on the wire 1 due to the wire 2, and vice-versa:

$$d\vec{F}_{2\leftarrow 1} = (I_2\, d\vec{l}_2) \times \vec{B}_1 = (\vec{J}_2\, dV_2) \times \vec{B}_1 \quad .$$

- On the other hand, the field due to the entire loop 1 on a position $\vec{r}_2$ on wire 2 is:

$$\vec{B}_1(\vec{r}_2) \;=\; \frac{\mu_0}{4\pi} \oint I_1(\vec{r}_1)\, d\vec{l}_1 \times \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

- Therefore, the total force of the wire 1 on the wire 2 is given by:

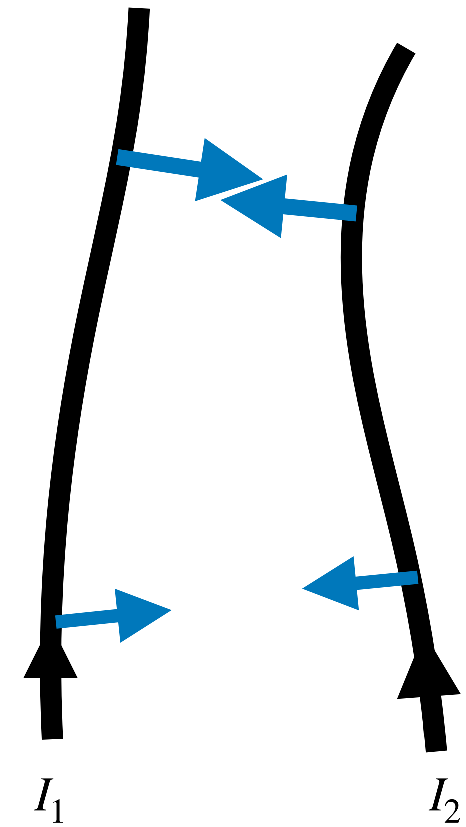
$$\vec{F}_{2\leftarrow 1} = \oint (I_2\, d\vec{l}_2) \times \vec{B}_1 = \oint (I_2\, d\vec{l}_2) \times \frac{\mu_0}{4\pi} \oint (I_1\, d\vec{l}_1) \times \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \oint \oint \frac{d\vec{l}_2 \times \left[d\vec{l}_1 \times (\vec{r}_2 - \vec{r}_1) \right]}{|\vec{r}_2 - \vec{r}_1|^3}$$

$$= \frac{\mu_0 I_1 I_2}{4\pi} \oint \oint \frac{-(d\vec{l}_2 \cdot d\vec{l}_1)(\vec{r}_2 - \vec{r}_1) + d\vec{l}_1 [d\vec{l}_2 \cdot (\vec{r}_2 - \vec{r}_1)]}{|\vec{r}_2 - \vec{r}_1|^3}$$

- The last term is zero for a closed loop, since:

$$\oint d\vec{l}_1 \cdot (\vec{r}_2 - \vec{r}_1)/|\vec{r}_2 - \vec{r}_2|^3 = \oint d\vec{l}_1 \cdot \vec{\nabla}(1/|\vec{r}_2 - \vec{r}_2|) = \int d\vec{S} \cdot (\vec{\nabla} \times \vec{\nabla}(1/|\vec{r}_2 - \vec{r}_2|) = 0$$



Magnetic force and torque

- Therefore, we get the expression for the total force that loop 1 exerts on loop 2:

$$\vec{F}_{2\leftarrow 1} = -\frac{\mu_0 I_1 I_2}{4\pi}\oint\oint (d\vec{l}_2 \cdot d\vec{l}_1)\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3}$$

, from which it is clear that:

$$\vec{F}_{1\leftarrow 2} = -\vec{F}_{2\leftarrow 1} \quad .$$

- As an example, two infinite straight wires have a mutual force per unit length which is:

$$F_{12} = \frac{\mu_0 I_1 I_2}{2\pi}\frac{1}{d}$$

, where d is the distance between the wires.

- A more general way to write the total force over a system with a current density is:

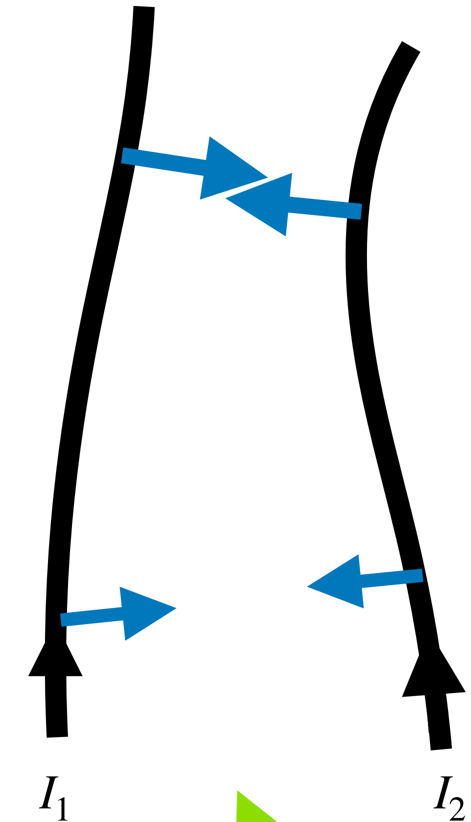
$$\vec{F} = \int dV\, \vec{J} \times \vec{B}$$

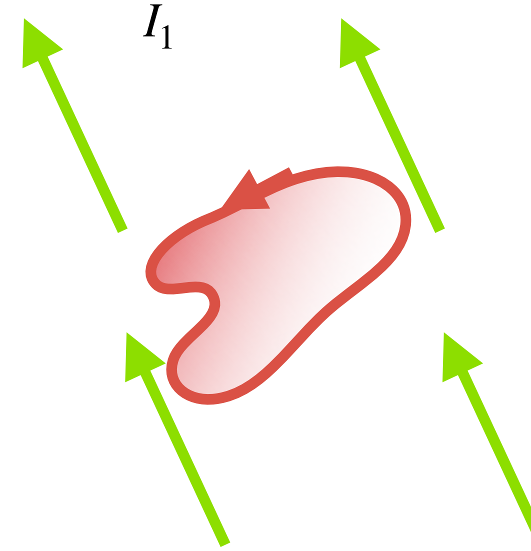
- Sometimes the total force on a system vanishes. As an example, consider any closed loop immersed in a constant magnetic field. We have:

$$\vec{F} = \oint I d\vec{l} \times \vec{B} = I\left[\oint d\vec{l}\right] \times \vec{B} = 0$$

- However, it is **not** true that nothing will happen here: there is a torque on this loop! The torque is:

$$\vec{\tau} = \oint \vec{r} \times d\vec{F} = \oint \vec{r} \times \left(I d\vec{l} \times \vec{B}\right) = \int dV\, \vec{r} \times \left(\vec{J} \times \vec{B}\right)$$





Magnetic force and torque

- So, let's compute the torque on that loop due to a constant magnetic field. We have:

$$\vec{\tau} = \oint \vec{r} \times d\vec{F} = \oint \vec{r} \times \left(I d\vec{l} \times \vec{B} \right) = I \left[\oint \vec{r} \times d\vec{l} \right] \times \vec{B}$$

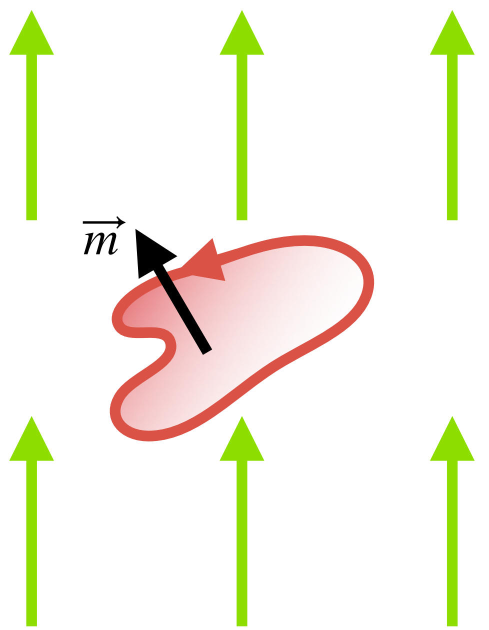
- But the term inside square brackets is exactly the expression for the area of the loop! In fact, if we multiply that area by the current, we have exactly the magnetic dipole moment of that loop! Hence,

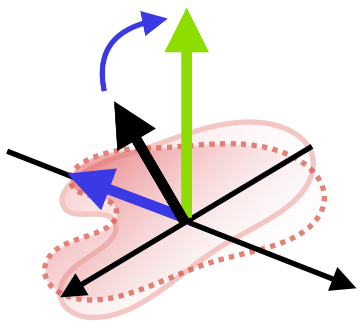
$$\vec{\tau} = \vec{m} \times \vec{B}$$

- Therefore, the magnetic field wants to align the magnetic moment with itself.
- Given this torque, we can in fact associate a potential energy with the orientation of the magnetic field:

$$U_B = - \vec{m} \cdot \vec{B} \quad .$$

- It is important to remark here that this is *not* the total energy of the magnetic dipole in the external field. As we align the dipole with the field, we will create an *induction* on the loop, and therefore there will be some work done by an electric field in order to maintain the strength of that magnetic moment.
- With that in mind, we delay the discussion of energy until we review Faraday's law.





Magnetic fields in matter

- As we have just seen, any existing magnetic dipoles will tend to align with an external magnetic field, in a way that is proportional to the strength of that dipole:

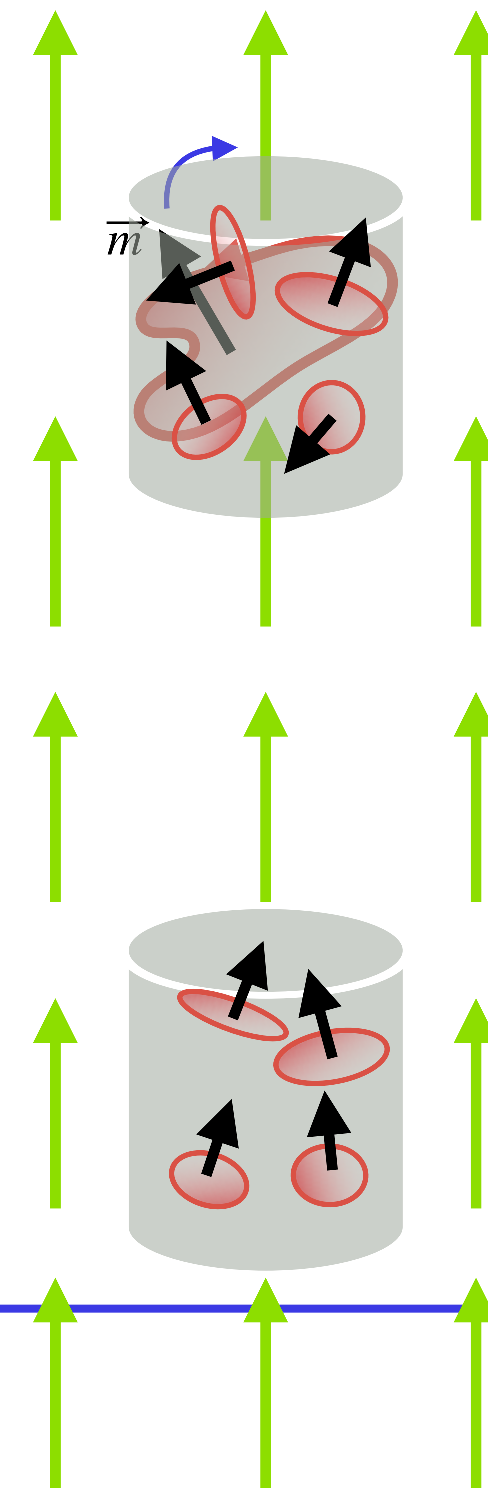
$$U_B = -\,\vec{m} \cdot \vec{B} \quad .$$

- Now, consider that most atoms have magnetic dipoles, since they have **electron orbitals** with a net, non-vanishing total orbital angular momentum — as we saw before, $\vec{m} = q\,\vec{L}/(2M)$. Although atomic nuclei also possess magnetic dipoles, they are very small, and can be neglected.
- In some materials, the orbital momentum is sub-dominant compared with the **spin** of the electrons.
- As the dipoles align with the magnetic field, they create a magnetic field of their own, changing the field configuration in and near that magnetic material.
- Remember that for a single magnetic dipole moment at the position $\vec{r}\,'$ we have:

$$\vec{A}_m(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times (\vec{r} - \vec{r}\,')}{|\,\vec{r} - \vec{r}\,'|^3}$$

- Now, consider that we have a distribution of magnetic moments in our material:

$$\vec{M} = \frac{d\vec{m}}{dV} \quad , \quad \vec{A}_M(\vec{r}) = \frac{\mu_0}{4\pi} \int dV' \frac{\vec{M}(\vec{r}\,') \times (\vec{r} - \vec{r}\,')}{|\,\vec{r} - \vec{r}\,'|^3}$$



Magnetic fields in matter

- Just as we did in the case of dielectric materials, we can express the contributions from the magnetic dipole moments in terms of a surface and a volume term. To see that, remember once again that:

$$\vec{\nabla}\frac{1}{|\vec{r}-\vec{r}'|} = -\frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3} \quad , \quad \text{and conversely} \quad \vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}'|} = \frac{\vec{r}-\vec{r}'}{|\vec{r}-\vec{r}'|^3}$$

- Now, using the fact that $\vec{\nabla}\times(f\vec{F}) = f\vec{\nabla}\times\vec{F} + (\vec{\nabla}f)\times\vec{F}$ we obtain that:

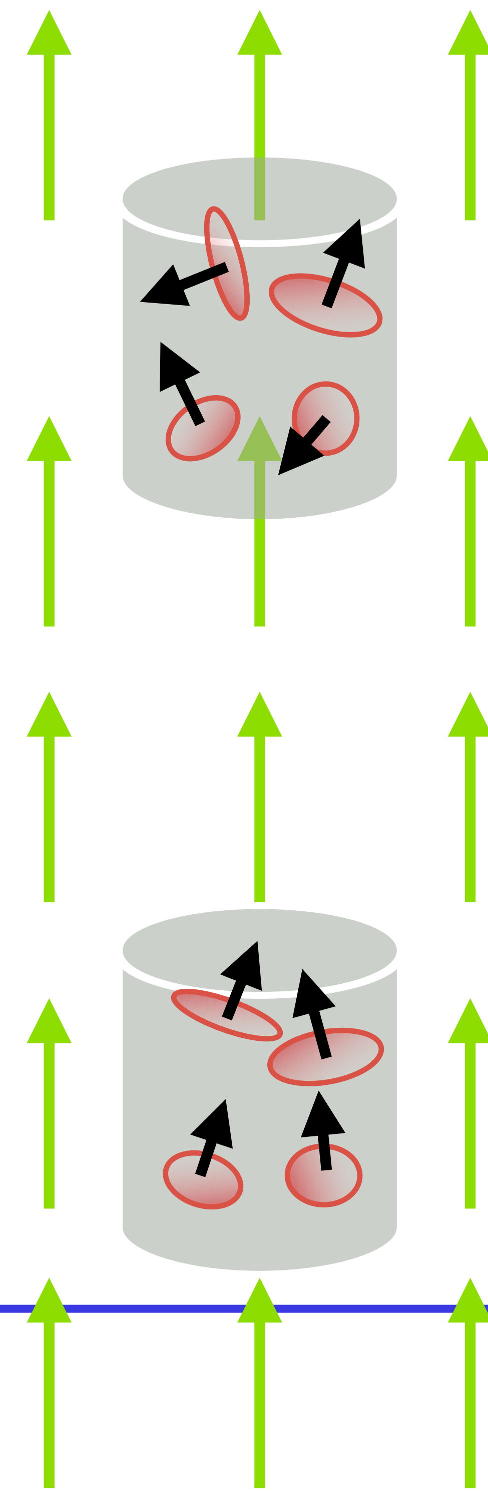
$$\vec{A}_M(\vec{r}) = \frac{\mu_0}{4\pi}\int dV'\vec{M}(\vec{r}')\times\left[\vec{\nabla}'\frac{1}{|\vec{r}-\vec{r}'|}\right]$$

$$= \frac{\mu_0}{4\pi}\int dV'\frac{\vec{\nabla}'\times\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} - \frac{\mu_0}{4\pi}\int dV'\,\vec{\nabla}'\times\left[\frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right]$$

$$= \frac{\mu_0}{4\pi}\int dV'\frac{\vec{\nabla}'\times\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|} - \frac{\mu_0}{4\pi}\int d\vec{S}'\times\left[\frac{\vec{M}(\vec{r}')}{|\vec{r}-\vec{r}'|}\right]$$

- Again, in full analogy with what we did in the case of dielectric media, we now introduce magnetization current densities and surface currents as:

$$\vec{J}_M = \vec{\nabla}\times\vec{M} \quad , \quad \text{and} \quad \vec{K}_M = \vec{M}\times\hat{n} \quad ,$$ where $\hat{n}$ is the normal to the surface.



Magnetic fields in matter

- The interpretation of these magnetization currents are very similar to what we had in the case of electric polarization:

Polarization charges:	Magnetization currents:
$\rho_P = -\vec{\nabla} \cdot \vec{P}$	$\vec{J}_M = \vec{\nabla} \times \vec{M}$
$\sigma_P = \vec{P} \cdot \hat{n}$	$\vec{K}_M = \vec{M} \times \hat{n}$

- It is now useful to introduce an effective magnetic field given the changes caused by the magnetization currents. Now, in addition to the "free" currents we have these "**magnetization currents**" that are another source for the magnetic field:

$$\vec{J} = \vec{J}_f + \vec{J}_M$$

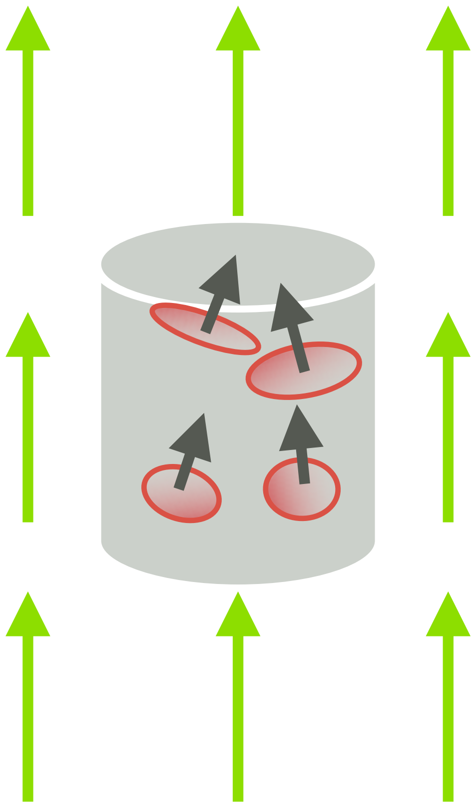
- Since $\vec{J}_M = \vec{\nabla} \times \vec{M}$, Ampère's law now states that:

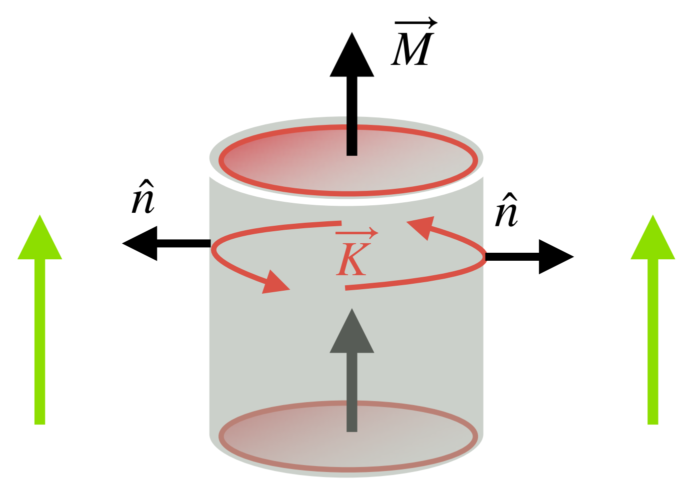
$$\frac{1}{\mu_0} \vec{\nabla} \times \vec{B} = \vec{J} = \vec{J}_f + \vec{J}_M = \vec{J}_f + \vec{\nabla} \times \vec{M}$$

- Passing the magnetization to the left-hand side we can redefine the magnetic field as a **"macroscopic" magnetic field**:

$$\vec{\nabla} \times \vec{H} = \vec{\nabla} \times \left(\frac{1}{\mu_0} \vec{B} - \vec{M} \right) = \vec{J}_f \quad .$$

- Notice, however, that $\vec{\nabla} \cdot \vec{H} = \frac{1}{\mu_0} \vec{\nabla} \cdot \vec{B} - \vec{\nabla} \cdot \vec{M} = - \vec{\nabla} \cdot \vec{M}$, but It is not true in general that $\vec{\nabla} \cdot \vec{M} \to 0$!





Boundary conditions

- The exact same calculations that led to the boundary conditions for magnetic fields can now be applied, in the presence of magnetized media. We obtain that:

  $\Delta B_\perp = 0$, and $\Delta \vec{H}_{||} = \vec{K}_f \times \hat{n}$

  Often times instead of the first condition it is more useful to use:

  $\Delta H_\perp = - \Delta M_\perp$

- A simple case of magnetized media happens when the material has a linear magnetization, i.e.:

  $\vec{M} \sim \vec{B}$

- In fact, it is more convenient to write the magnetization in terms of the effective (or "macroscopic") magnetic field $\vec{H}$ (since they have the same dimensions!):

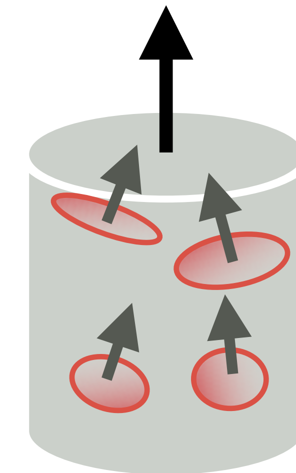
  $\vec{M} = \chi_M \vec{H}$

- In this way we can now write the magnetic field (now called "flux density") in terms of the effective magnetic field as:

  $\vec{B} = \mu \vec{H}$, with $\mu = \mu_0 (1 + \chi_M)$ expressing the **magnetic permeability** of the material.

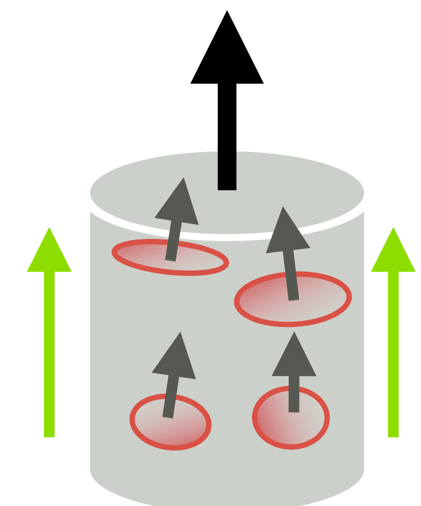
  [See examples in Mathematica Notebooks.]

Constitutive relations

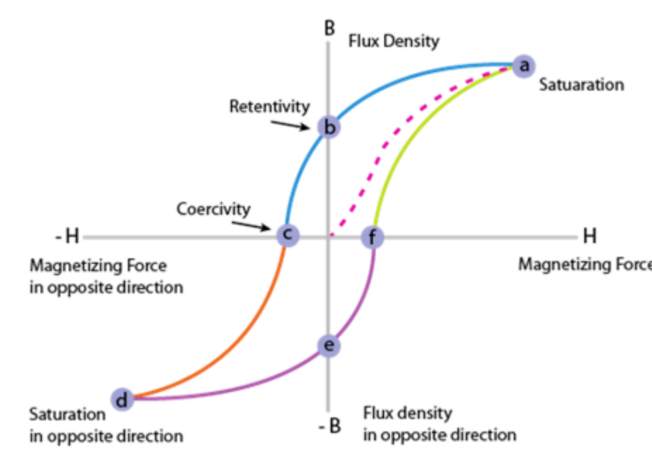


- For very simple materials without a permanent magnetization we can write something simple such as:

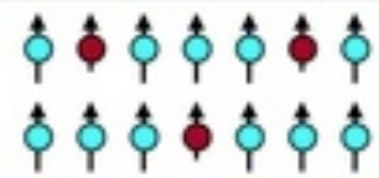
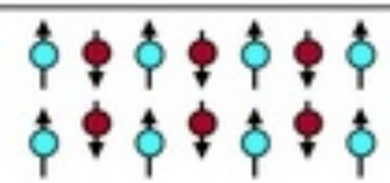
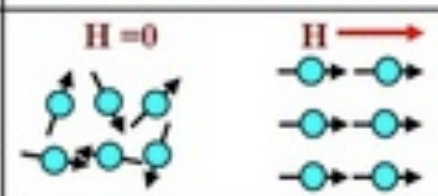
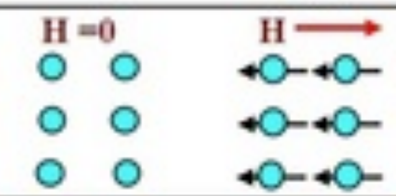
$$\vec{B} = \mu \, \vec{H} \quad , \qquad \text{where } \mu \text{ is very close to } \mu_0 \text{ (to one part in } 10^4 \text{ or } 10^5\text{).}$$



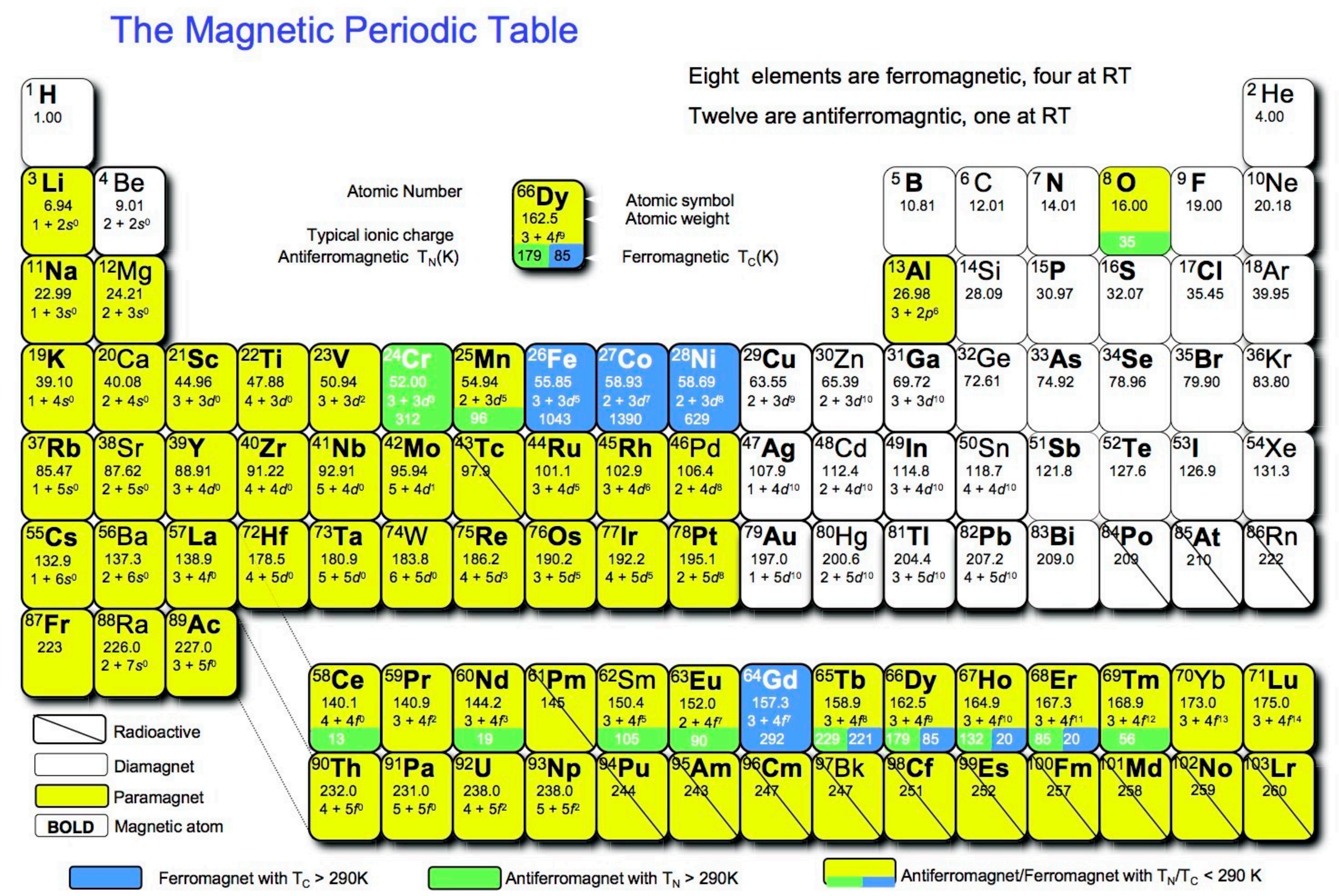
- However, many materials (e.g., ferromagnetic) present a much more interesting behavior: they have a **permanent magnetization**. This can be interpreted as a tendency of the magnetic dipoles of its atoms to align, which creates a permanent dipole moment to a piece of that material. Those materials can have $\mu \gg \mu_0$!
- We can also "nudge" those materials into acquiring even greater magnetization, by applying a magnetic field.
- That magnetization can persist, even after we remove the magnetic field.
- We can then remove that magnetization by applying a reverse magnetic field (opposite to the magnetic dipole moment).
- What this all means is that the magnetic field can be an interesting function of the macroscopic (effective) magnetic field that has a "memory", as show in the "hysteresis" loop (for a nice little demonstration, see https://demonstrations.wolfram.com/MagneticHysteresis/ .)



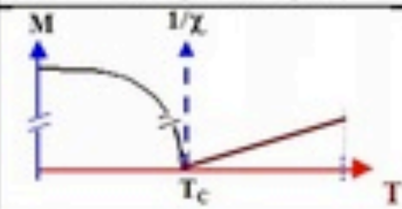
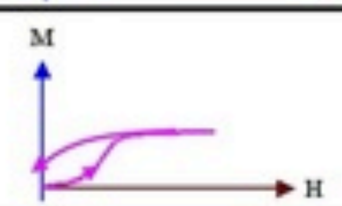
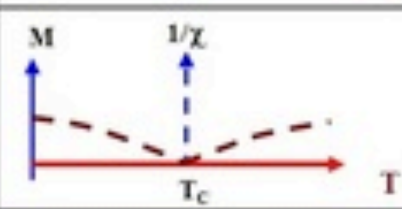
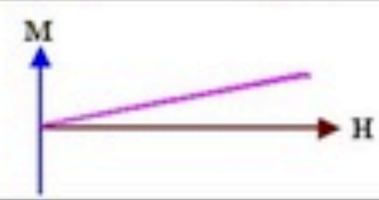
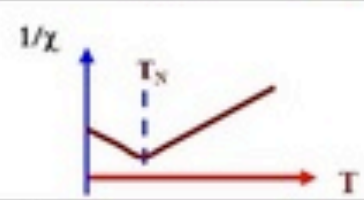
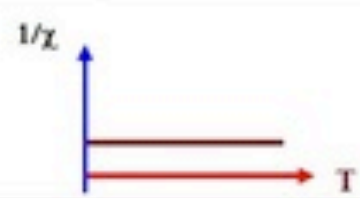
Magnetic materials

type	spin alignment	spin in simplified plot	examples
ferromagnetic	all spins align parallel to one another: spontaneous magnetization- $M = a + b$		Fe, Co, Ni, Gd, Dy, $SmCo_5$, Sm_2Co_{17}, $Nd_2Fe_{14}B$
ferrimagnetic	most spins parallel to one another, some spins antiparallel: spontaneous magnetization- $M = a - b > 0$		magnetite (Fe_3O_4), yttrium iron garnet (YIG), $GdCo_5$
antiferromagnetic	periodic parallel-antiparallel spin distribution: $M = a - b = 0$		chromium, FeMn, NiO
paramagnetic	spins tend to align parallel to an external magnetic field: $M = 0$ @ $H = 0$, $M > 0$ @ $H > 0$		oxygen, sodium, aluminum, calcium, uranium
diamagnetic	spins tend to align antiparallel to an external magnetic field $M = 0$ @ $H = 0$, $M < 0$ @ $H > 0$		superconductors, nitrogen, copper, silver, gold, water, organic compounds

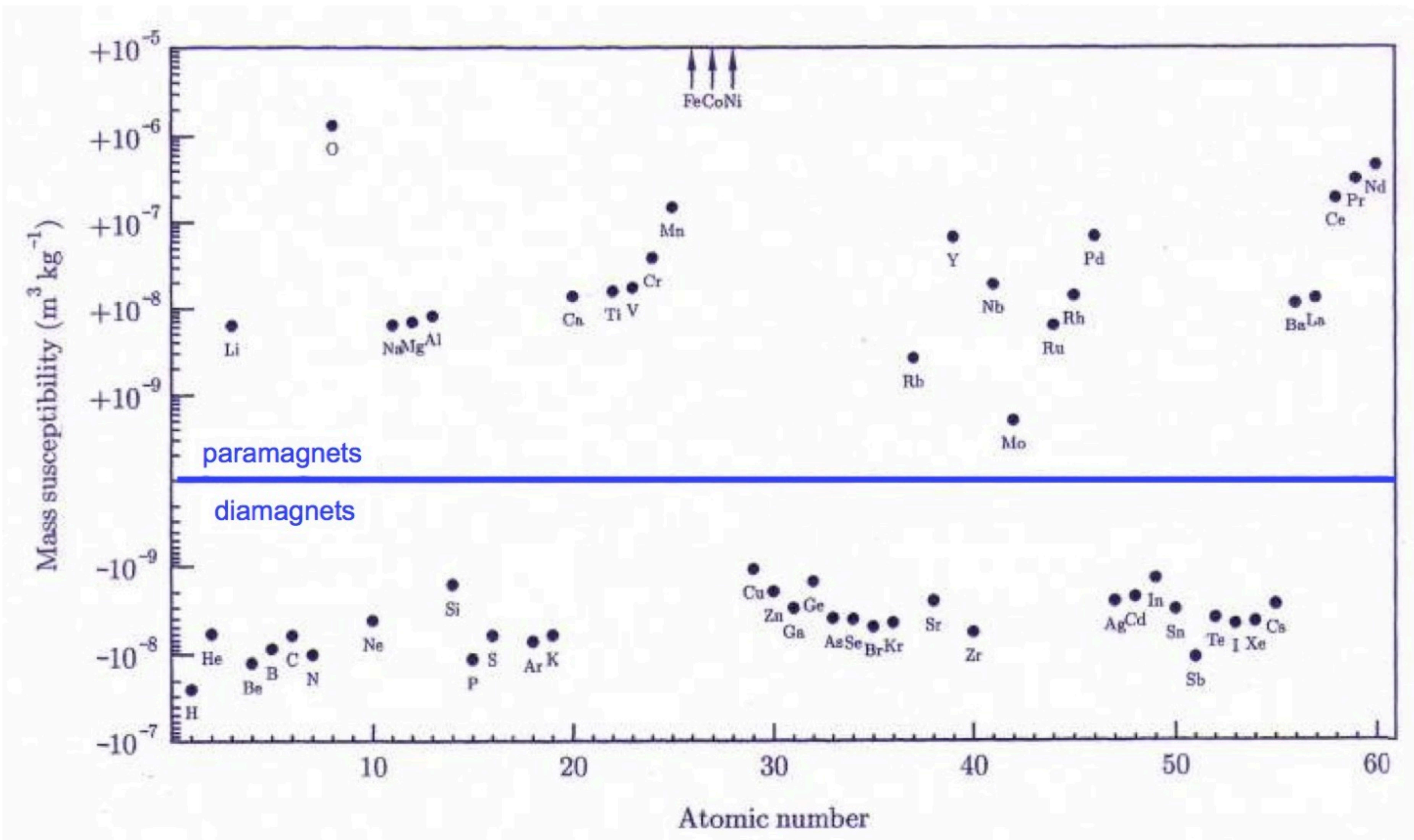
Magnetic materials

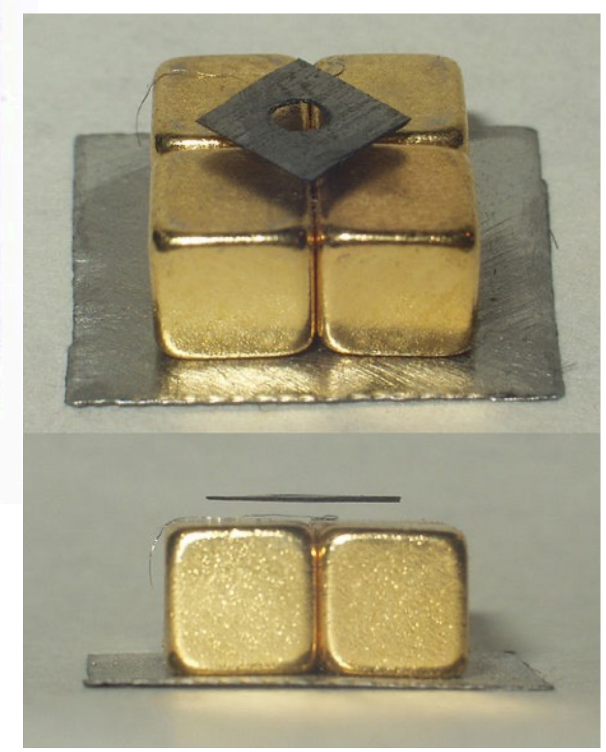


Magnetic materials

Magnetization $M = M(H)$	Magnetic susceptibility $\chi_{mag} = \chi_{mag}(T)$	Remakes
		**ferromagnets:** the susceptibility is large; the magnetization increases massively with h. above the *curie temperature*, t_c, paramagnetic behavior is observed.
		**ferrimagnets** behave like *ferro*magnets, except that the effect tends to be smaller. the $1/\chi$ is not linear at $t > t_c$
		**antiferromagnets:** like paramagnets at $t > t_{ne}$ (neél temperature). at $t < t_{ne}$, the χ is small, but with a *t*-dependence quite different from paramagnets.
		**paramagnets:** the χ is slightly larger than zero and decreases with t, plotted as $1/\chi(t)$ - a linear relationship
		**diamagnets:** the susceptibility, χ , is negative and close to zero; and there is no temperature dependence

Magnetic materials





A more complete explanation of diamagnetism (and superconductors) will have to wait until we consider Faraday's law of induction

Next class:

- Induction: Faraday’s law
- Induction and coordinate transformations
- Magnetic diffusion
- Energy of the magnetic field
- Jackson, Ch. 5