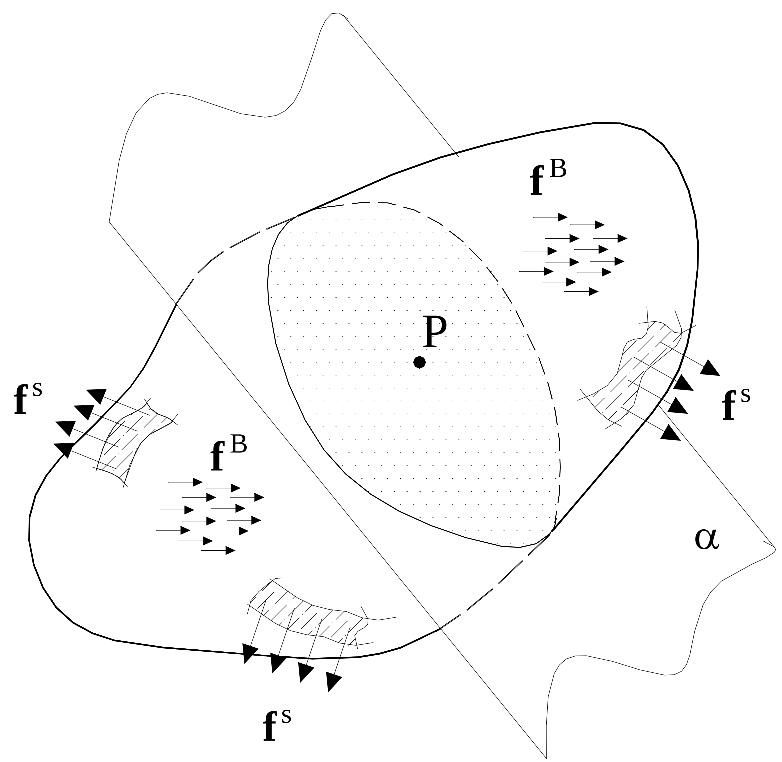
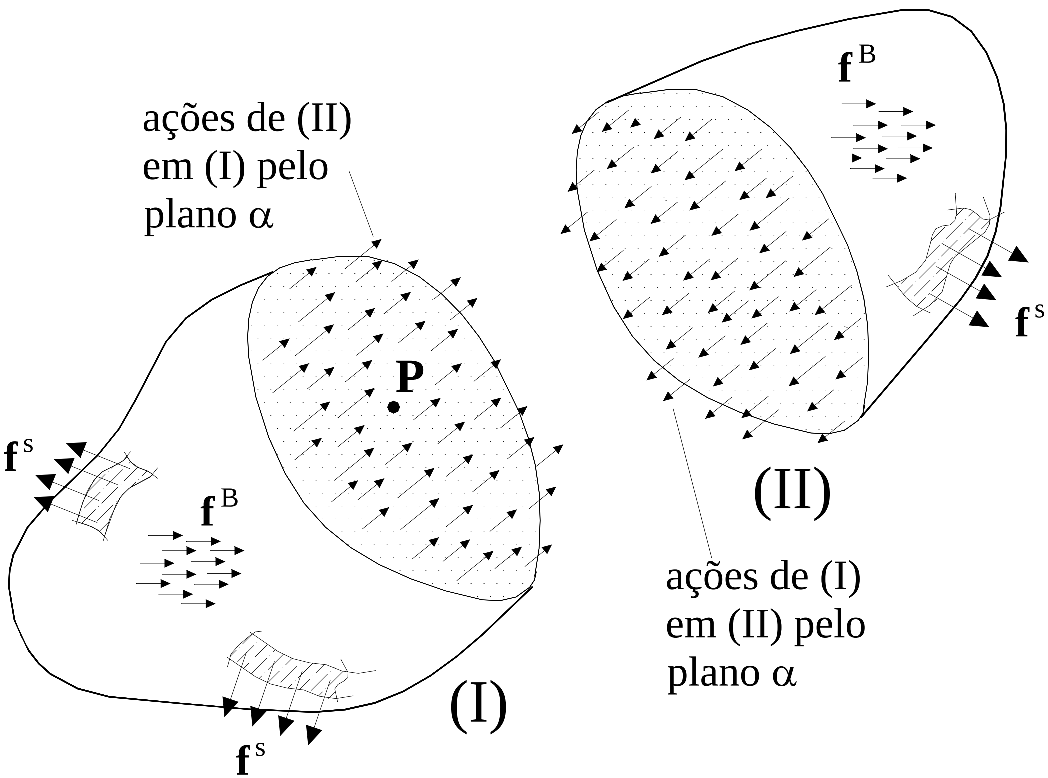
Conceituação Clássica de Tensões

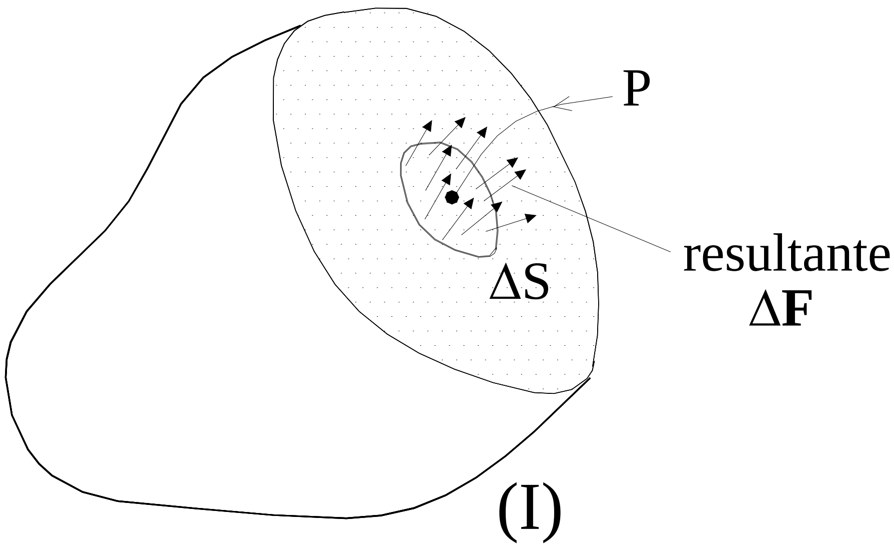
Considere um sólido em equilíbrio estático sob a ação de $\underline{f}^{B}$ e $\underline{f}^{S}$



Conceituação Clássica de Tensões



(I) e (II) também devem estar em equilíbrio



Define-se

$$\underline{\rho}_m = \frac{\underline{\Delta F}}{\Delta S} \qquad \underline{\rho}_m = \underline{\rho}_m(P, \alpha, \Delta S)$$

$\underline{\rho}_m$: Tensão média

$$\underline{\rho} = \lim_{\Delta S \to 0} \frac{\underline{\Delta F}}{\Delta S}$$

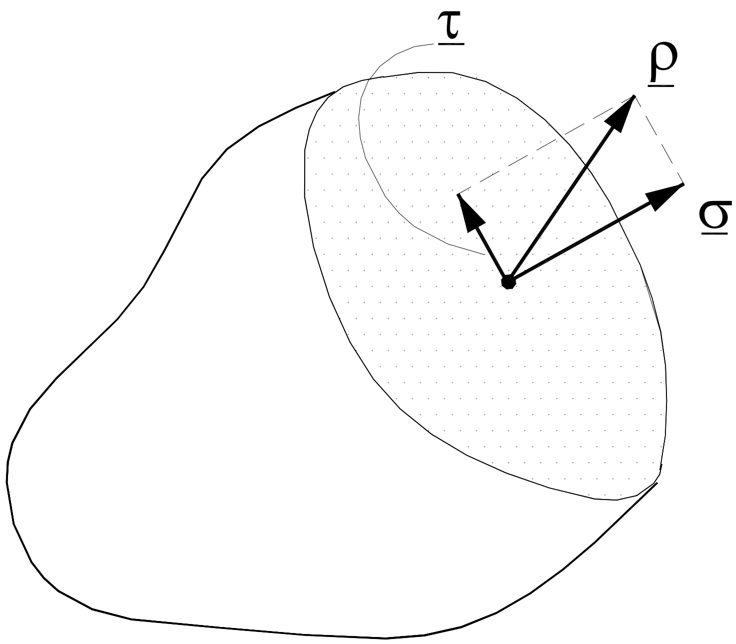
$$\underline{\rho} = \lim_{\Delta S \to 0} \underline{\rho}_m \qquad \underline{\rho} = \underline{\rho}(P, \alpha)$$

$\underline{\rho}$: Tensão em P segundo o plano α

$$\underline{\rho} = \underline{\sigma} + \underline{\tau}$$

$\underline{\sigma}$: Tensão normal

$\underline{\tau}$: Tensão de cisalhamento



$$\underline{\sigma} = \sigma \, \underline{n}$$

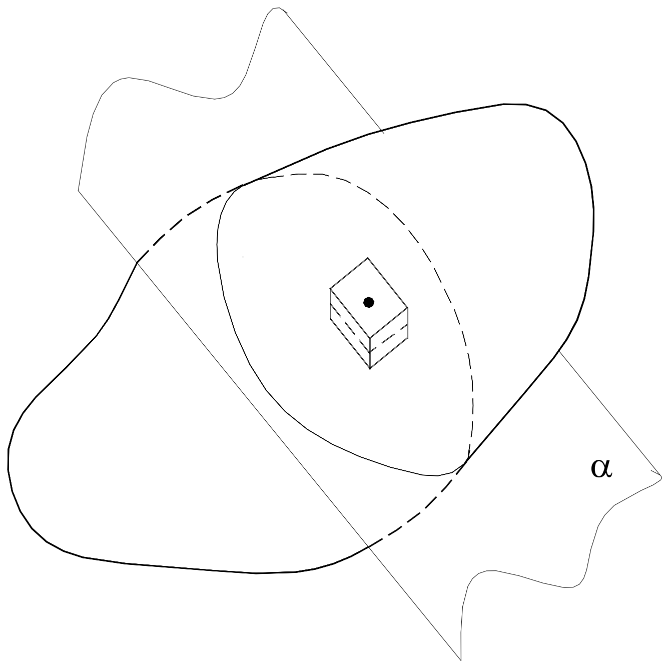
$\underline{n}$: Normal exterior

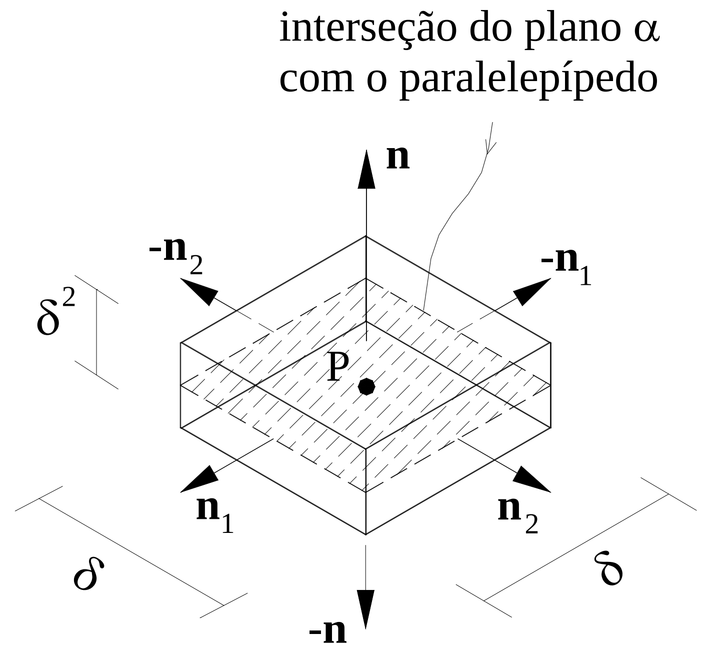
$$\underline{\sigma} = (\underline{\rho} \cdot \underline{n}) \, \underline{n} \qquad \left(\sigma = \underline{\rho} \cdot \underline{n}\right)$$

$\sigma > 0$ Tração

$\sigma < 0$ Compressão

$$\underline{\tau} = \underline{\rho} - \underline{\sigma}$$



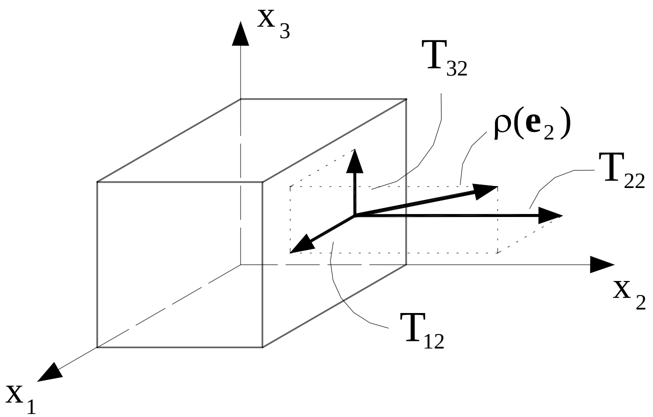


$$\underline{\rho}(P,\underline{n})\delta^2 + \underline{\rho}(P,-\underline{n})\delta^2 + \underline{\rho}(P,\underline{n}_1)\delta^3 + \underline{\rho}(P,-\underline{n}_1)\delta^3 +$$
$$\underline{\rho}(P,\underline{n}_2)\delta^3 + \underline{\rho}(P,-\underline{n}_2)\delta^3 + \underline{f}^B(P)\delta^4 = 0$$

Quando $\delta \to 0$

$$\underline{\rho}(P,\underline{n}) = -\underline{\rho}(P,-\underline{n})$$

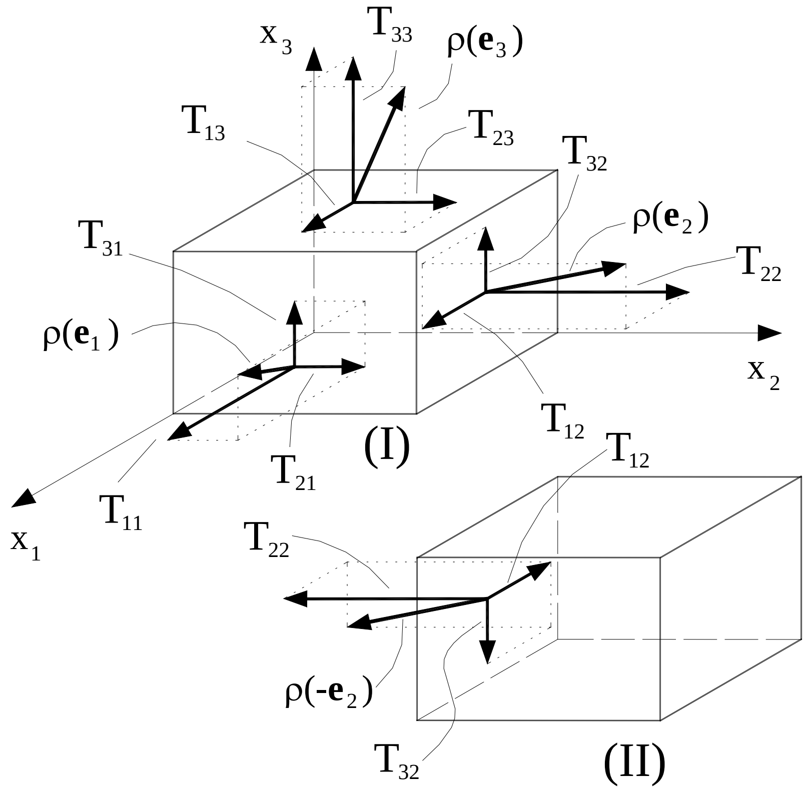
Componentes escalares de tensão



$$\underline{\rho}\left(\underline{e}_2\right) = T_{12}\,\underline{e}_1 + T_{22}\,\underline{e}_2 + T_{32}\,\underline{e}_3$$

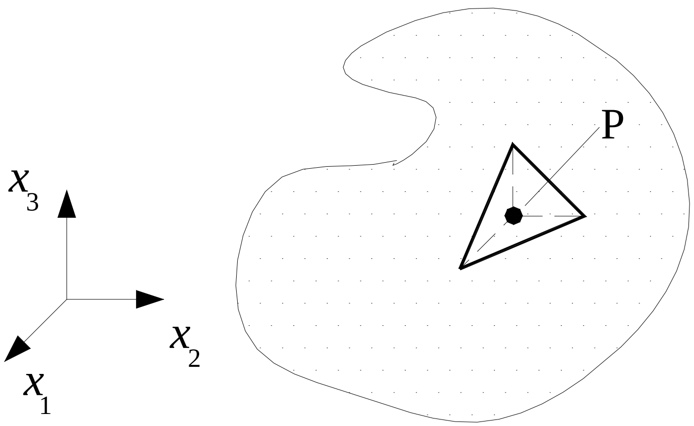
$$= \sum_{i=1}^{3} T_{i2}\,\underline{e}_i$$

$$\underline{\rho}\left(\underline{e}_j\right) = \sum_{i=1}^{3} T_{ij}\,\underline{e}_i$$



$$\underline{\rho}\left(-\underline{e}_2\right) = -\underline{\rho}\left(\underline{e}_2\right) = -T_{12}\,\underline{e}_1 - T_{22}\,\underline{e}_2 - T_{32}\,\underline{e}_3$$

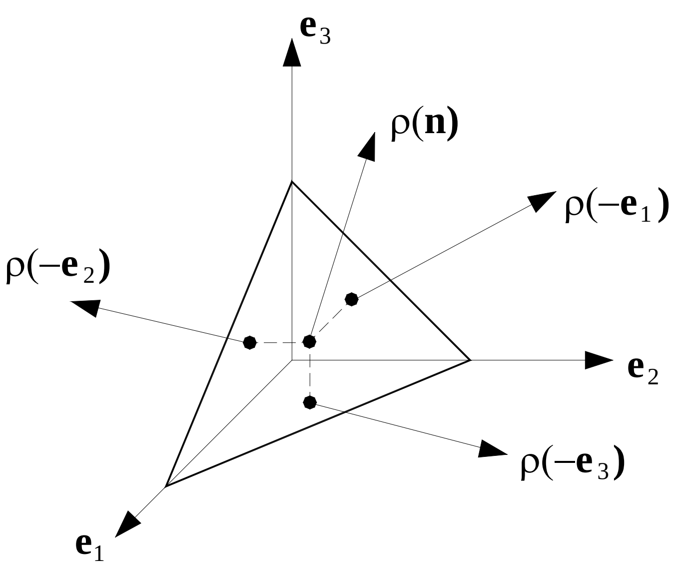
Estado de tensão



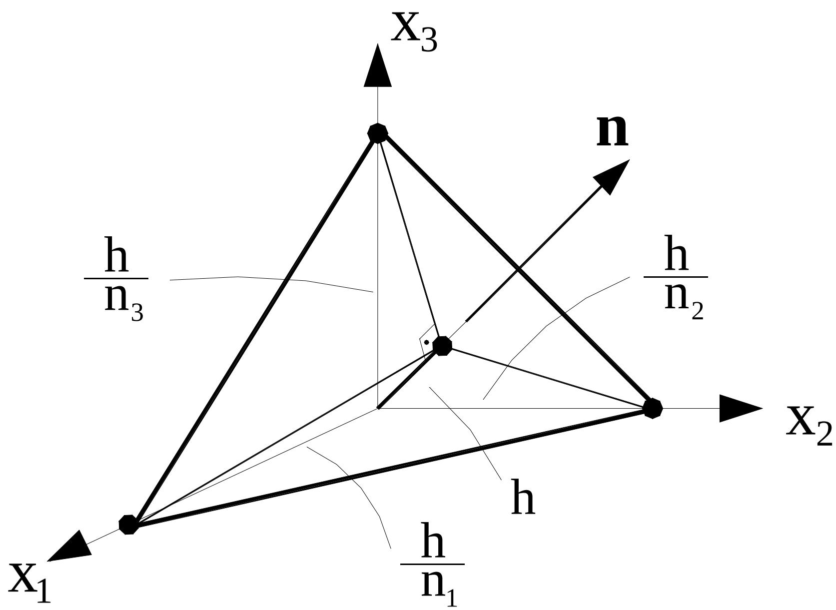
$\Delta V \equiv tetraedro$

Equilíbrio

Isolando o tetraedro



Propriedades geométricas



$$V = \frac{1}{3}Sh = \frac{1}{3}S_i\left(\frac{h}{n_i}\right)$$

$$S_i = n_i S$$

Equilíbrio do tetraedro $\Rightarrow$ $\underline{R} = \underline{0}$

$$\underline{R} = \underline{\rho}(\underline{n})S + \underline{\rho}(-\underline{e}_1)S_1 + \underline{\rho}(-\underline{e}_2)S_2 + \underline{\rho}(-\underline{e}_3)S_3 + \underline{f}^{B}V = 0$$

Dividindo-se por *S*

$$\underline{\rho}(\underline{n})S + \underline{\rho}(-\underline{e}_1)n_1 + \underline{\rho}(-\underline{e}_2)n_2 + \underline{\rho}(-\underline{e}_3)n_3 + \frac{1}{3}\underline{f}^{B}h = 0$$

Arestas infinitesimais $\Rightarrow$ *h* é infinitesimal

Despreza-se $\frac{1}{3}\underline{f}^{B}h$ em relação aos demais

$$\underline{\rho}(\underline{n}) = \underline{\rho}(\underline{e}_1)n_1 + \underline{\rho}(\underline{e}_2)n_2 + \underline{\rho}(\underline{e}_3)n_3$$

ou

$$\underline{\rho}(\underline{n}) = \underline{\rho}\left(\sum_{i=1}^{3} T_{i1}e_i\right)n_1 + \underline{\rho}\left(\sum_{i=1}^{3} T_{i2}e_i\right)n_2 + \underline{\rho}\left(\sum_{i=1}^{3} T_{i3}e_i\right)n_3$$

Igualando as componentes

$$\rho_1 = T_{11}n_1 + T_{12}n_2 + T_{13}n_3$$
$$\rho_2 = T_{21}n_1 + T_{22}n_2 + T_{23}n_3$$
$$\rho_3 = T_{31}n_1 + T_{32}n_2 + T_{33}n_3$$

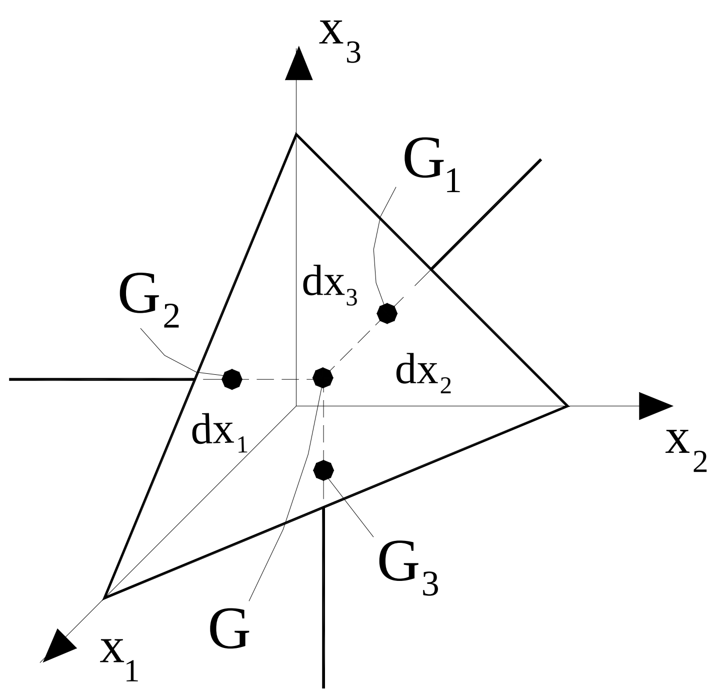
Matricialmente

$$\begin{bmatrix} \rho_1 \\ \rho_2 \\ \rho_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} n_1 \\ n_2 \\ n_3 \end{bmatrix}$$

$$\underline{\rho}(\underline{n}) = \underline{T}\,\underline{n}$$

Equilíbrio do tetraedro $\Rightarrow$ Momento em relação a qualquer ponto deve ser nulo

Propriedades geométricas relacionando os centros de gravidade das faces do tetraedro

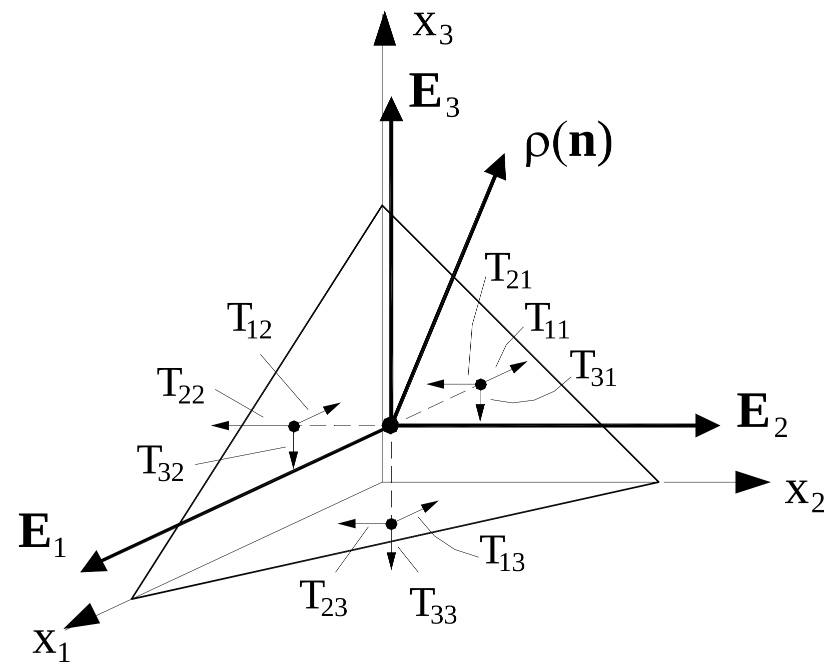


$$\sum \underline{M}_G = \underline{0} \quad \Longleftrightarrow \quad \sum M_{E_1} = \sum M_{E_2} = \sum M_{E_3} = 0$$

$$\sum M_{E_1} = 0 = T_{32} S_2 \frac{dx_2}{3} - T_{23} S_3 \frac{dx_3}{3}$$

Então

$$T_{32}\left(\frac{dx_1 dx_3}{2}\right)\frac{dx_2}{3} = T_{23}\left(\frac{dx_1 dx_2}{2}\right)\frac{dx_3}{3}$$



que leva a

$$T_{32} = T_{23}$$

Tem-se ainda

$$\sum M_{E_2} = 0 = -T_{31} S_1 \frac{dx_1}{3} + T_{13} S_3 \frac{dx_3}{3}$$

$$T_{31}\left(\frac{dx_2 dx_3}{2}\right)\frac{dx_1}{3} = T_{13}\left(\frac{dx_1 dx_2}{2}\right)\frac{dx_3}{3}$$

ou

$$T_{31} = T_{13}$$

e

$$\sum M_{E_3} = 0 = T_{21} S_1 \frac{dx_1}{3} - T_{12} S_2 \frac{dx_2}{3}$$

$$T_{21}\left(\frac{dx_2 dx_3}{2}\right)\frac{dx_1}{3} = T_{12}\left(\frac{dx_1 dx_3}{2}\right)\frac{dx_2}{3}$$

ou

$$T_{21} = T_{12}$$

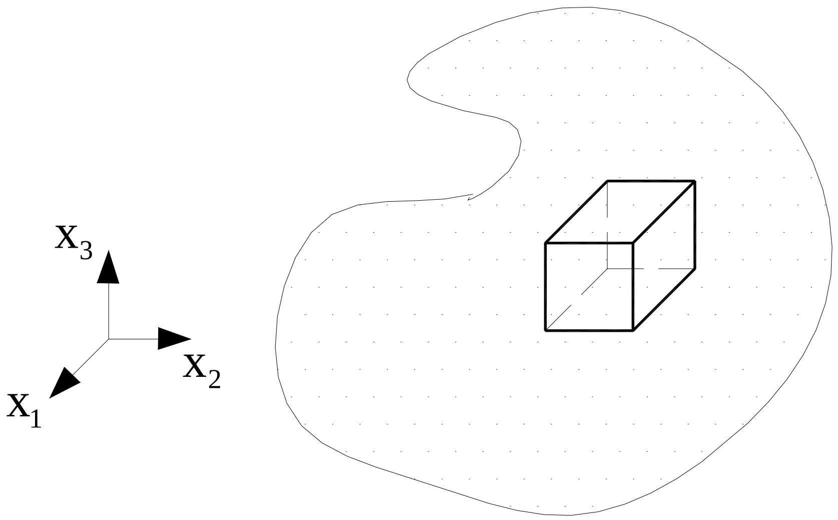
Portanto

$$[T] = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix}$$

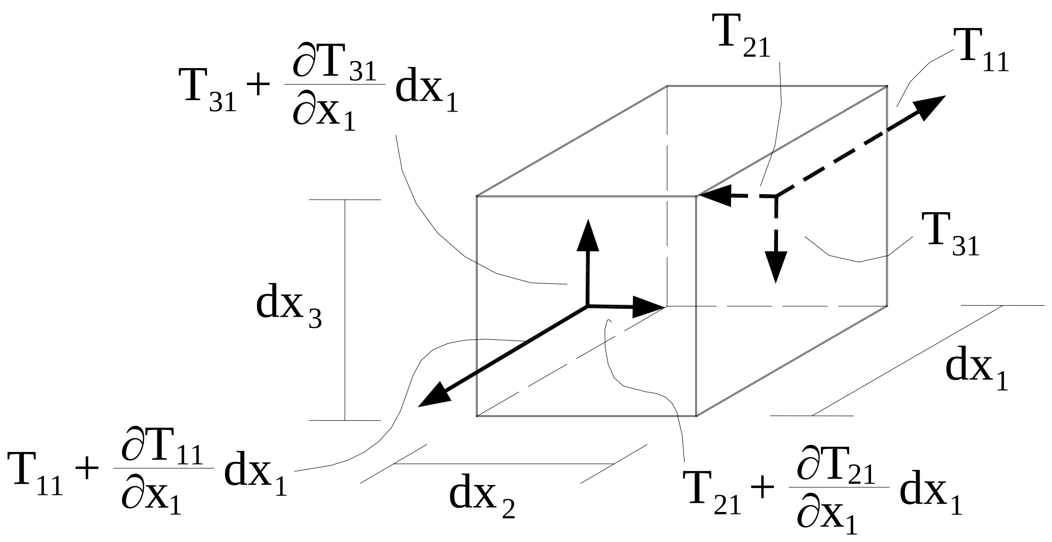
é simétrico e

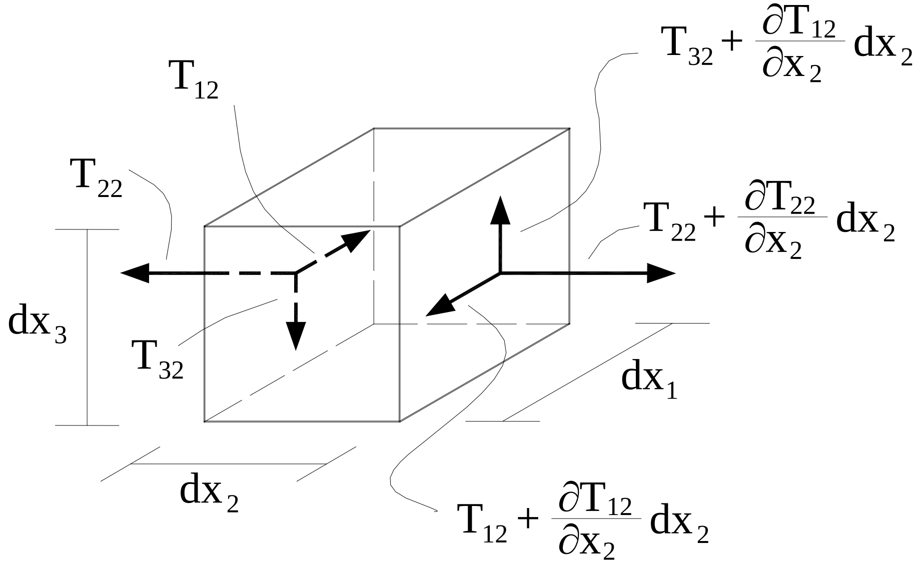
$$\underline{\rho}(P, \underline{n}) = \underline{T}\,\underline{n}$$

Equações diferenciais de equilíbrio

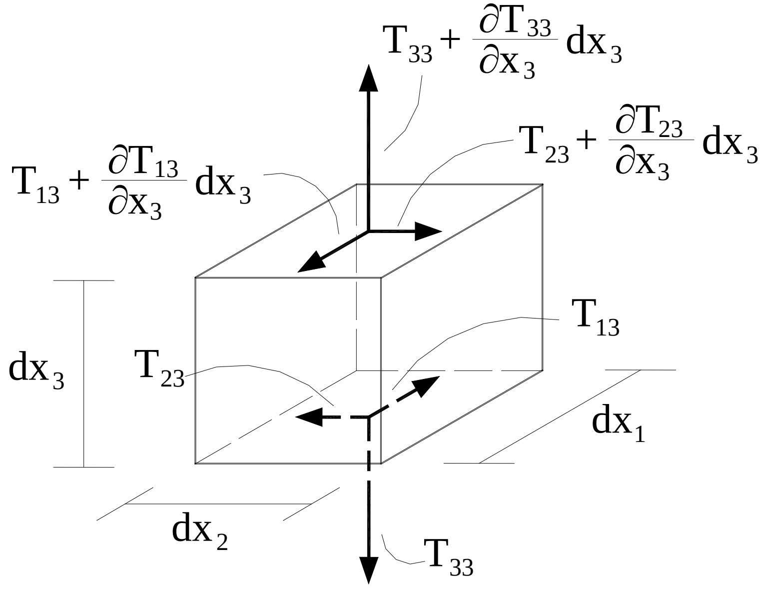


- Equilíbrio do paralelepípedo
- É necessário considerar:
  - variação da tensão entre faces paralelas
  - forças de volume


T_{21}
T_{11}
$T_{31} + \frac{\partial T_{31}}{\partial x_1} dx_1$
T_{31}
dx_3
dx_1
$T_{11} + \frac{\partial T_{11}}{\partial x_1} dx_1$
dx_2
$T_{21} + \frac{\partial T_{21}}{\partial x_1} dx_1$

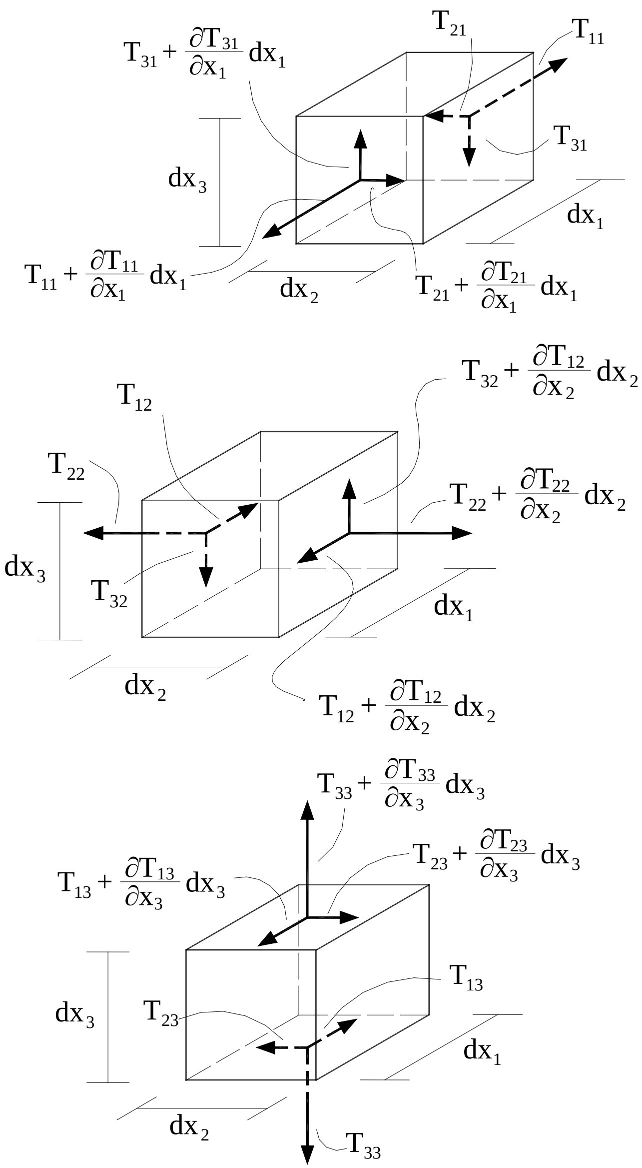


T_{12}
T_{22}
$T_{32} + \frac{\partial T_{12}}{\partial x_2} dx_2$
$T_{22} + \frac{\partial T_{22}}{\partial x_2} dx_2$
dx_3
T_{32}
dx_1
dx_2
$T_{12} + \frac{\partial T_{12}}{\partial x_2} dx_2$



$T_{33} + \frac{\partial T_{33}}{\partial x_3} dx_3$
$T_{23} + \frac{\partial T_{23}}{\partial x_3} dx_3$
$T_{13} + \frac{\partial T_{13}}{\partial x_3} dx_3$
T_{13}
dx_3
T_{23}
dx_1
dx_2
T_{33}

Equilíbrio na direção de $\underline{e}_1$



$$
\begin{aligned}
&-T_{11}dx_2dx_3 + T_{11}dx_2dx_3 + \frac{\partial T_{11}}{\partial x_1}dx_1dx_2dx_3 \\
&-T_{12}dx_1dx_3 + T_{12}dx_1dx_3 + \frac{\partial T_{12}}{\partial x_2}dx_2dx_1dx_3 \\
&-T_{13}dx_1dx_2 + T_{13}dx_1dx_2 + \frac{\partial T_{13}}{\partial x_3}dx_3dx_1dx_2 \\
&+ f_1^{\,B}dx_1dx_2dx_3
\end{aligned}
$$

$$
\frac{\partial T_{11}}{\partial x_1} + \frac{\partial T_{12}}{\partial x_2} + \frac{\partial T_{13}}{\partial x_3} + f_1^{\,B} = 0
$$

Analogamente

$$\frac{\partial T_{21}}{\partial x_1} + \frac{\partial T_{22}}{\partial x_2} + \frac{\partial T_{23}}{\partial x_3} + f_2^{\ B} = 0$$

$$\frac{\partial T_{31}}{\partial x_1} + \frac{\partial T_{32}}{\partial x_2} + \frac{\partial T_{33}}{\partial x_3} + f_3^{\ B} = 0$$