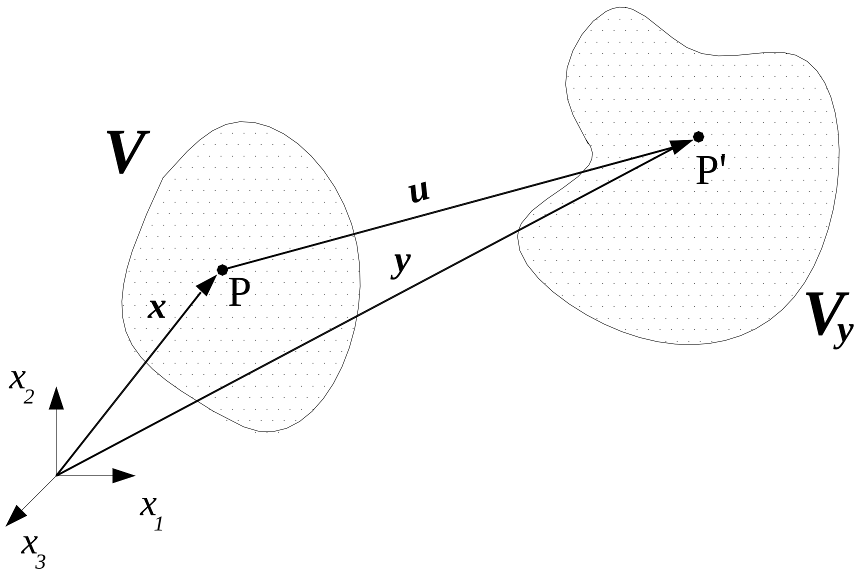
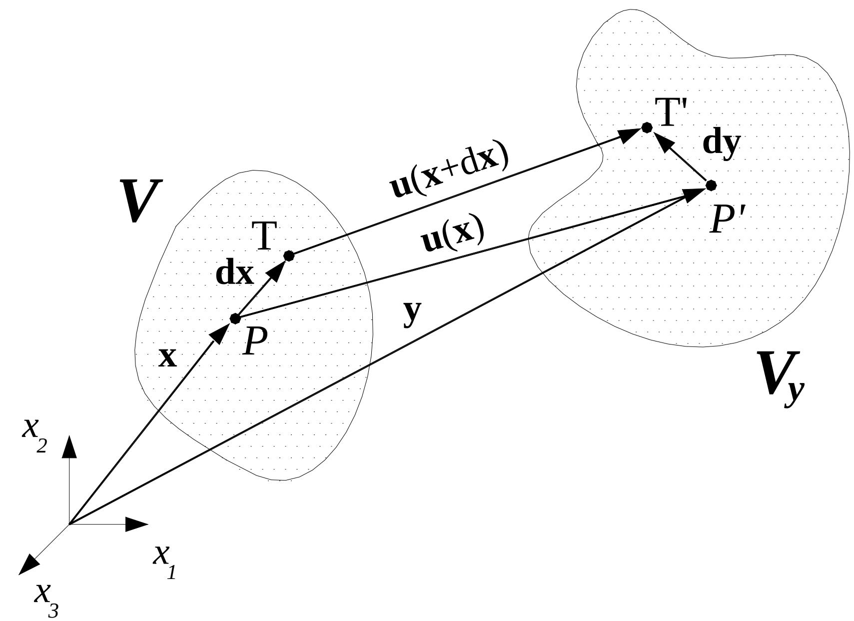
Estudo das deformações





$$\begin{Bmatrix} dy_1 \\ dy_2 \\ dy_3 \end{Bmatrix} = \begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{Bmatrix} + \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1} & \dfrac{\partial u_1}{\partial x_2} & \dfrac{\partial u_1}{\partial x_3} \\ \dfrac{\partial u_2}{\partial x_1} & \dfrac{\partial u_2}{\partial x_2} & \dfrac{\partial u_2}{\partial x_3} \\ \dfrac{\partial u_3}{\partial x_1} & \dfrac{\partial u_3}{\partial x_2} & \dfrac{\partial u_3}{\partial x_3} \end{bmatrix} \begin{Bmatrix} dx_1 \\ dx_2 \\ dx_3 \end{Bmatrix}$$

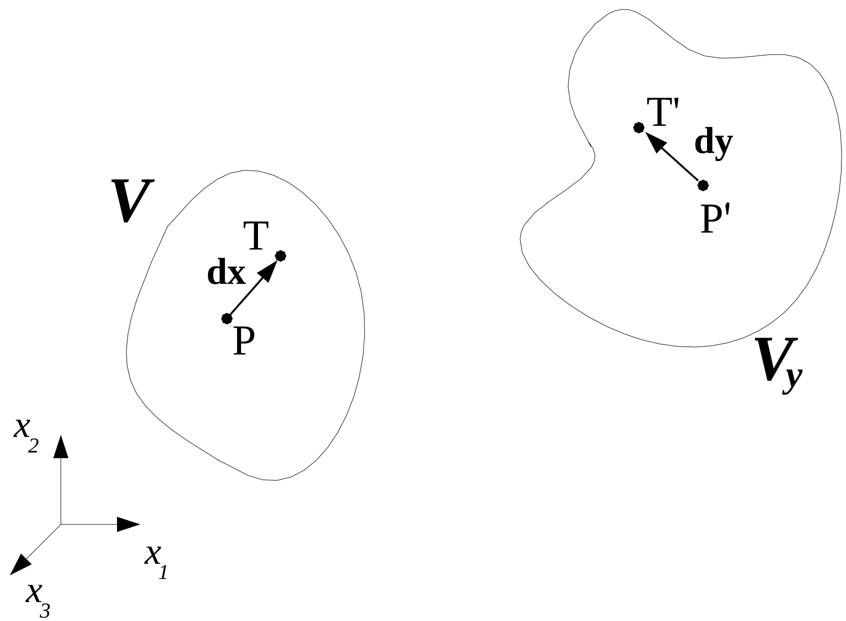
$$\underline{dy} = \underline{dx} + \underline{\nabla u}\,\underline{dx}$$

$$\underline{dy} = (\underline{I} + \underline{\nabla u})\underline{dx} = \underline{F}\,\underline{dx}$$

$$\underline{F} = (\underline{I} + \underline{\nabla u})$$

$\underline{\nabla u}$: Gradiente dos deslocamentos

$\underline{F}$: Gradiente das deformações



$$\underline{\underline{E}} = \frac{1}{2}\left[\underline{\nabla u}^T + \underline{\nabla u}\right]$$

$$[E] = \frac{1}{2}\left([\nabla u]^T + [\nabla u]\right) = \begin{bmatrix} \dfrac{\partial u_1}{\partial x_1} & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3} + \dfrac{\partial u_3}{\partial x_1}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_2} + \dfrac{\partial u_2}{\partial x_1}\right) & \dfrac{\partial u_2}{\partial x_2} & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right) \\ \dfrac{1}{2}\left(\dfrac{\partial u_1}{\partial x_3} + \dfrac{\partial u_3}{\partial x_1}\right) & \dfrac{1}{2}\left(\dfrac{\partial u_2}{\partial x_3} + \dfrac{\partial u_3}{\partial x_2}\right) & \dfrac{\partial u_3}{\partial x_3} \end{bmatrix}$$

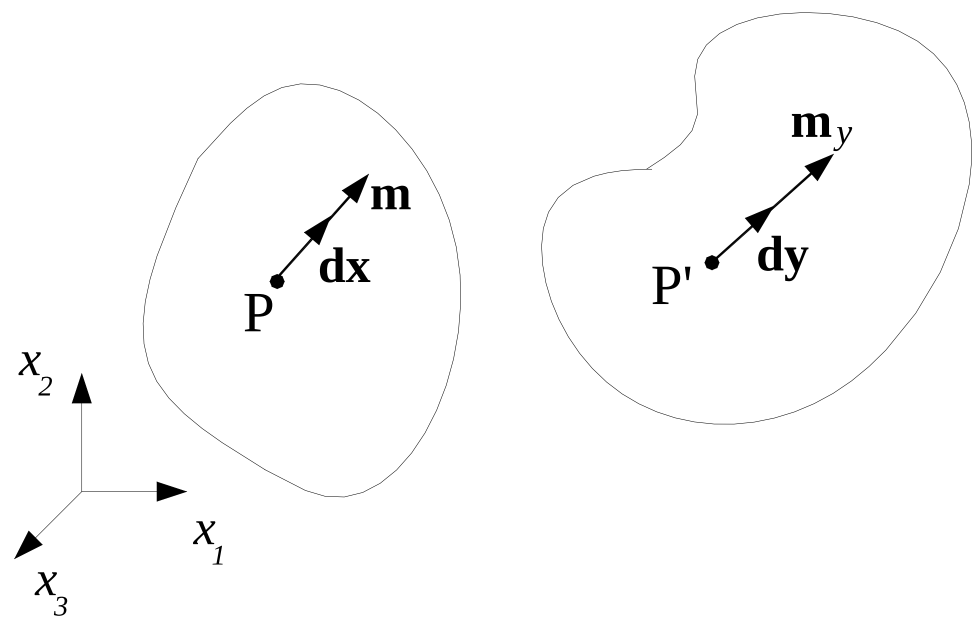
$\underline{\underline{E}}$: Tensor das deformações

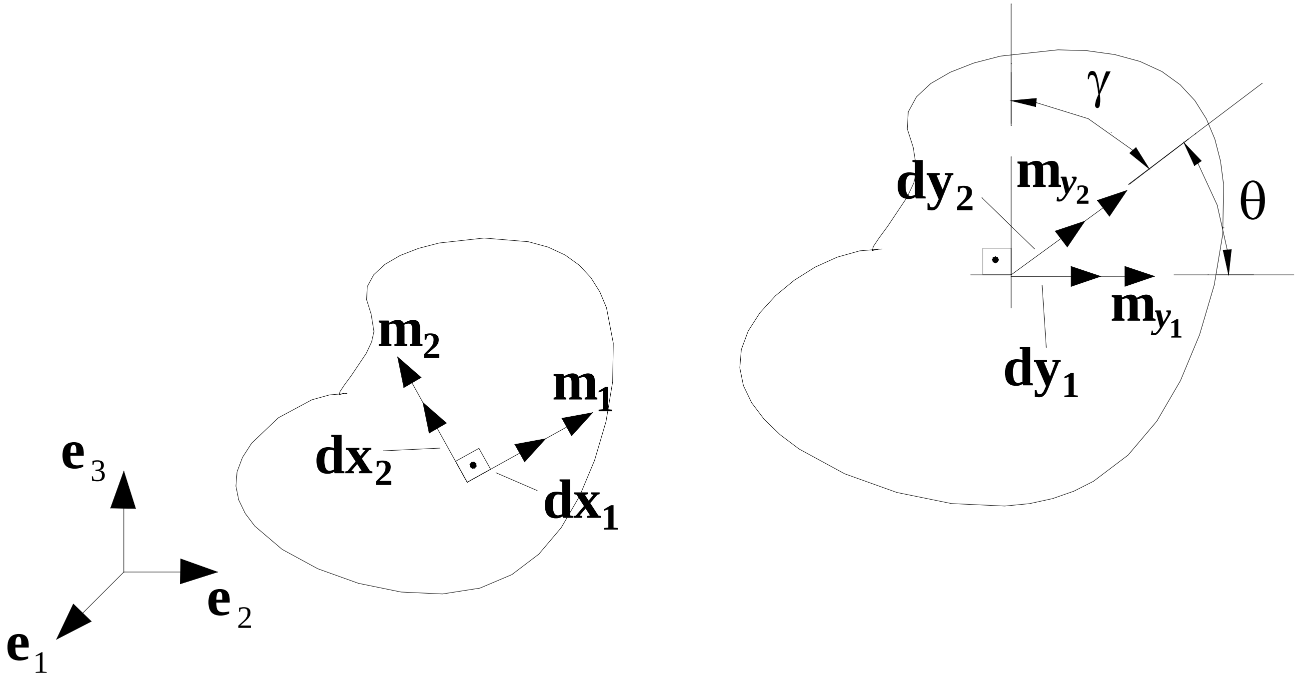
Para deslocamentos infinitesimais

$$ds = \left\|\underline{dx}\right\|$$

$$ds^* = \left\|\underline{dy}\right\|$$

$$\varepsilon_\ell = \frac{ds^* - ds}{ds} = \underline{m} \cdot \underline{\underline{E}}\,\underline{m}$$





$$\gamma = 2\underline{m}_1 \cdot \underline{\underline{E}}\,\underline{m}_2 = 2\underline{m}_2 \cdot \underline{\underline{E}}\,\underline{m}_1$$

$\gamma :$ distorção

Interpretação das componentes do tensor das deformações

$$[E]=\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}$$

O alongamento linear de uma fibra infinitesimal na direção do versor $\underline{e}_1$

$$\varepsilon_\ell(\underline{e}_1) = \underline{e}_1 \cdot \underline{\underline{E}}\,\underline{e}_2$$

$$= \{1 \quad 0 \quad 0\}\begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix}\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$$

$$\varepsilon_\ell(\underline{e}_1) = E_{11}$$

Analogamente

$$\varepsilon_\ell(\underline{e}_2) = E_{22}$$

$$\varepsilon_\ell(\underline{e}_3) = E_{33}$$

Pode-se calcular a distorção de fibras ortogonais de direções $\underline{e}_1$ e $\underline{e}_2$

$$\gamma = 2\underline{e}_1 \cdot \underline{E}\underline{e}_2$$

$$= \begin{Bmatrix} 1 & 0 & 0 \end{Bmatrix} \begin{bmatrix} E_{11} & E_{12} & E_{13} \\ E_{21} & E_{22} & E_{23} \\ E_{31} & E_{32} & E_{33} \end{bmatrix} \begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$$

$$\gamma(\underline{e}_1, \underline{e}_2) = 2E_{12}$$

Analogamente

$$\gamma(\underline{e}_2, \underline{e}_3) = 2E_{23}$$

$$\gamma(\underline{e}_1, \underline{e}_3) = 2E_{13}$$