Testing for Interaction in Binary Logit and Probit Models: Is a Product Term Essential?

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Political scientists presenting binary dependent variable (BDV) models often hypothesize that variables interact to influence the probability of an event, Pr(Y). The current typical approach to testing such hypotheses is (1) estimate a logit or probit model with a product term, (2) test the hypothesis by determining whether the coefficient for this term is statistically significant, and (3) characterize the nature of any interaction detected by describing how the estimated effect of one variable on Pr(Y) varies with the value of another. This approach makes a statistically significant product term necessary to support the interaction hypothesis. We show that a statistically significant product term is neither necessary nor sufficient for variables to interact meaningfully in influencing Pr(Y). Indeed, even when a logit or probit model contains no product term, the effect of one variable on Pr(Y) may be strongly related to the value of another. We present a strategy for testing for interaction in a BDV model, including guidance on when to include a product term.

Many phenomena important to political scientists are binary outcomes: an event occurs ($Y = 1$), or it does not ($Y = 0$).[1] Political scientists studying binary dependent variables (BDVs) frequently hypothesize that two independent variables *interact* in influencing the probability that the event will occur,[2] i.e., that the effect of one independent variable (X_1) on this probability is conditional on the value of the other (X_2).[3] Typically, current practice is to test this hypothesis using logit or probit, being guided by two recommendations from the political methodology literature.

First, scholars have wisely been urged to focus on the presentation and interpretation of substantively relevant quantities—in the BDV context, the probability

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[1]We examined all articles appearing in the 2005 issues of *American Journal of Political Science, American Political Science Review*, and *Journal of Politics*; and the 2007 issues of *British Journal of Political Science*, *Journal of Conflict Resolution*, and *Political Analysis*. Together these journals published 77 quantitative articles analyzing binary dependent variables, representing 41% of all the empirical articles in these journals.

[2]Indeed, of the 77 articles presenting BDV models in the journals referenced in note 1, 26—over one-third—test hypotheses that one or more variables interact.

[3]To be more precise, X_1 is said to *interact* with X_2 in influencing the probability that the event will occur [i.e., $\text{Prob}(Y = 1)$] if given an increment in X_1 [from $X_{1(\text{lo})}$ to $X_{1(\text{hi})}$] and an increment in X_2 [from $X_{2(\text{lo})}$ to $X_{2(\text{hi})}$],

$$\begin{aligned}&[\text{Prob}(Y = 1 \mid X_1 = X_{1(\text{hi})}, X_2 = X_{2(\text{hi})}) - \text{Prob}(Y = 1 \mid X_1 = X_{1(\text{lo})}, X_2 = X_{2(\text{hi})})]\\ &-[\text{Prob}(Y = 1 \mid X_1 = X_{1(\text{hi})}, X_2 = X_{2(\text{lo})}) - \text{Prob}(Y = 1 \mid X_1 = X_{1(\text{lo})}, X_2 = X_{2(\text{lo})})] \neq 0.\end{aligned}$$

When both increments are infinitesimal, this requires that the second derivative, $\partial^2\text{Prob}(Y = 1)/\partial X_1 \partial X_2$, be different from zero. When the difference in X_1 is infinitesimal, but the difference in X_2 is discrete, this requires that the marginal effect of X_1 on $\text{Prob}(Y = 1)$ [i.e., the first derivative, $\partial\text{Prob}(Y = 1)/\partial X_1$] has different values at $X_{2(\text{lo})}$ and $X_{2(\text{hi})}$. When both increments are discrete, this requires that a second difference in probabilities be nonzero.

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that the event will occur [to be denoted Prob($Y = 1$), or more compactly, Pr(Y)]. King, Tomz, and Wittenberg observe that estimated logit or probit coefficients are generally "difficult to interpret and only indirectly related to the substantive issues that motivated the research" (2000, 348; see also Huang and Shields 2000). Similarly, Brambor, Clark, and Golder note that "the [logit or probit] analyst is not concerned with model parameters per se; he or she is primarily interested in the marginal effect of [X_1 on Pr(Y)] for substantively meaningful values of the conditioning variable [X_2]" (2006, 74).

Second, researchers have been warned that—just as with a model having a continuous dependent variable and estimated with ordinary least squares (OLS) regression—a necessary condition for concluding that there is meaningful interaction between X_1 and X_2 is that the logit or probit model analyzed include a product term (X_1X_2) and the coefficient for this term be statistically significant. Nagler (1991, 1994) contends that the functional form of logit and probit forces the marginal effect of each independent variable on Pr(Y) to be conditional on the value of every independent variable in the model. That is, each independent variable must interact with all others. Nagler (1994, 250) argues that meaningful interaction must be "variable specific" and therefore modeled with product terms involving the interacting variables. Any finding of interaction from a model without a product term—or from a model with a product term that proves statistically insignificant—is an "artifact of the methodology," and thus substantively meaningless (Nagler 1991, 1393; see also Brambor, Clark, and Golder 2006, 77; Frant 1991).

In an attempt to be faithful to both of these recommendations, many scholars testing a hypothesis that two variables, X_1 and X_2, interact in influencing a BDV in recent years have used the following procedure:

1. Make the dependent variable the probability that some event will occur: Pr(Y).
2. Develop a hypothesis about how the effect of X_1 on Pr(Y) should vary with the value of X_2, when other independent variables ($X_3, X_4, \ldots, X_k$) are fixed at specified values (typically, their mean).[4]
3. Estimate a model including a product term (X_1X_2),

$$\Pr(Y) = G\left(\beta_0 + \beta_1 X_1 + \beta_2 X_2 + \beta_p X_1 X_2 + \sum_{i=3,\ldots,k} \beta_i X_i\right),$$

   where G () is a logit or probit link function.
4. Test the hypothesis by determining whether the coefficient for the product term (β_p) is statistically significant and in the predicted direction.[5]

   a. **If β_p is significant**, accept the hypothesis of interaction. Then use the model's coefficients to compute estimates of the effect of X_1 on Pr(Y) at different values of X_2 when $X_3, X_4, \ldots, X_k$ are fixed at the specified values, and report how this effect varies with X_2 to clarify the "substantive nature" of the interaction.[6] This can be accomplished in different ways, among them:

      (i) estimate a second difference in Pr(Y) using simulation methods (King, Tomz, and Wittenberg 2000):

$$\begin{aligned}\Delta\Delta[\Pr(Y)] &= [\text{first difference in } \Pr(Y) \\ &\qquad \text{when } X_1 \text{ changes from } lo \text{ to } hi \text{ and } \boldsymbol{X_2 = hi}]- \\ &\quad [\text{first difference in } \Pr(Y) \\ &\qquad \text{when } X_1 \text{ changes from } lo \text{ to } hi \text{ and } \boldsymbol{X_2 = lo}] \\ &= [\Pr(Y \mid X_1 = hi, X_2 = hi) - \Pr(Y \mid X_1 = lo, X_2 = hi)]- \\ &\quad [\Pr(Y \mid X_1 = hi, X_2 = lo) - \Pr(Y \mid X_1 = lo, X_2 = lo)]\end{aligned}$$

      where *lo* and *hi* are specified low and high values of X_1 and X_2.

      (ii) produce a plot of how the estimated marginal effect of X_1 on Pr(Y) varies with X_2 (Brambor, Clark, and Golder 2006).

   b. **If β_p is not significant**, reject the hypothesis of interaction.[7]

[4]The values at which other independent variables are fixed are rarely established in the initial statement of the hypothesis; rather, these values are generally revealed to readers when empirical results are presented. It would be better if analysts made these values explicit from the outset.

[5]In some cases, more than one product term is included to specify interaction between X_1 and X_2 (e.g., X_1X_2 and $X_1X_2^2$). When two or more product terms are used, the joint significance of these terms can be tested using "a log-likelihood ratio test comparing the unrestricted and restricted models, where the restricted model omits [the product terms]" (Nagler 1994, 250).

[6]The direct quote is Senese's (2005, 1773), but the practice is widespread. Other descriptions of the purpose for reporting estimated effects on Pr(Y) include offering a "graphical representation" of the interaction (Shugart, Valdini, and Suominen 2005, 444), "summariz[ing] the joint effect of" variables (Prior 2005, 585), and "illustrat[ing] the range of the effect" of X_1 (Haspel and Knotts 2005, 567). The essential common element in all these purposes is that the results they generate about estimated effects on Pr(Y), while useful for description, are not deemed relevant to the actual test of the hypothesis of interaction—which is based purely on the coefficient for the product term.

[7]Some analysts compute estimates of how the effect of X_1 on Pr(Y) varies with X_2 even when they reject the hypothesis of interaction to "illustrate" that any interaction present is weak (e.g., Valentino

Indeed, of the 26 articles analyzing a BDV model and positing interaction in the issues of the six journals we reviewed (see note 1), all estimate a logit or probit model including a product term, only two claim that interaction is present without finding a statistically significant product term, and 14 report how the estimated effect of one independent variable on $\Pr(Y)$ varies with the value of another.

In this essay, we argue that the two recommendations motivating this widely employed procedure are inconsistent. In a binary logit or probit model, two distinct unobserved dependent variables are of potential interest: (1) an *unbounded* latent variable assumed to be measured by the observed binary outcome, Y, and (2) the probability that the event will occur, $\Pr(Y)$, *a variable constrained to the range between zero and one*. If one's hypotheses are about the effects of X_1 and X_2 on the unbounded latent dependent variable, the situation is fully analogous to the case of a continuous dependent variable model estimated with regression, and a nonzero product term coefficient is necessary for interaction. However, if as is usually the case, one's hypotheses concern the effects of X_1 and X_2 on $\Pr(Y)$, a nonzero product term coefficient is not necessary for substantively meaningful interaction to be present. In some cases, accurate specification of one's theory requires a product term; in other cases it does not. But assuming a logit or probit model is properly specified, even when a product term is not included—or when it is included but proves to be statistically insignificant—interaction detected between variables in influencing $\Pr(Y)$ is substantively meaningful and not an artifact of the methodology.

The upshot of this is that when analyzing a BDV, the typical current approach to testing hypotheses about interaction should change. Fortunately, the prescriptions we offer in our essay can be implemented using existing statistical techniques. These include the simulation methodology invoked by CLARIFY (King, Tomz, and Wittenberg 2000) and the kind of marginal effect plots recommended by Brambor, Clark, and Golder (2006).

The remainder of the article proceeds as follows. First, we review the basic features of binary logit and probit. Then we summarize the rationale for the claim that a statistically significant product term is essential for validating a hypothesis of interaction in a BDV setting. In the next section, we challenge several misconceptions inherent in the typical approach to testing for interaction in this setting, arguing that a statistically significant product term is neither necessary nor sufficient for claiming interaction among independent variables in influencing $\Pr(Y)$. Our final sections offer advice for testing interaction hypotheses involving BDVs, which we illustrate by revisiting several classic studies of the effect of voter registration provisions on the probability of voting.

The Structure of Binary Dependent Variable Models

Logit and probit models are designed to model a phenomenon of interest that is discrete: i.e., it occurs ($Y = 1$) or it does not ($Y = 0$). In both models, whether the event occurs has a Bernoulli distribution described by the parameter $\Pr(Y)$—the probability that the event occurs (Beck, King, and Zeng 2000, 24). $\Pr(Y)$ is assumed to be determined by a set of independent variables $\{X_1, X_2, \ldots, X_k\}$, and a corresponding set of parameters $\{\beta_0, \beta_1, \beta_2, \ldots, \beta_k\}$, through a nonlinear link function, G, that maps the unbounded index $Y^* = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$ into the bounded probability space $[0, 1]$:

$$\Pr(Y) = G\left(Y^*\right)$$

In the logit model, the link function G is the logistic cumulative density function:

$$G\left(Y^*\right) = \Lambda(Y^*) = \frac{1}{1 + \exp(-Y^*)}$$

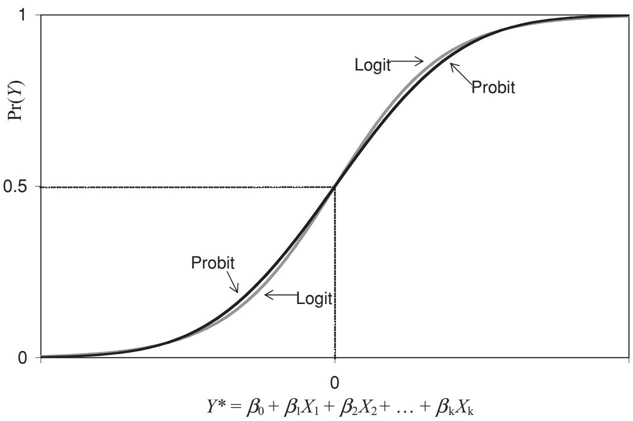
For the probit model, the Gaussian normal cumulative density function is used instead:

$$G(Y^*) = \Phi(Y^*) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{Y^*} \exp\left(-\frac{x^2}{2}\right) dx$$

These link functions are graphed in Figure 1, which shows their great similarity. Both map a Y^* value of 0 into a probability of 0.5. Both are maximally sloped at this point but decline in slope as Y^* departs from 0 and $\Pr(Y)$ departs from 0.5 (in either direction), approaching a slope of 0 as $\Pr(Y)$ approaches its floor of 0 or its ceiling of 1 and Y^* approaches $\pm\infty$. Note that both curves are nearly linear in the range near $Y^* = 0$ and when Y^* is either very large or small, but are strongly nonlinear when Y^* assumes other values. Although $\Pr(Y)$ is the ultimate dependent variable in the logit or probit model, Y^* can also be conceived as a latent (or unobserved) dependent variable influenced by the model's independent variables (since $Y^* = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$). However, the variables $\Pr(Y)$ and Y^* are of fundamentally different character: $\Pr(Y)$ is constrained to the range $[0, 1]$ while Y^* is unbounded. Moreover, although the metric for $\Pr(Y)$ is evident, the scale for Y^* is unknown.

and Sears 2005, 680). Others refrain from reporting any effects on $\Pr(Y)$ in this situation (e.g., Heller and Mershon 2005).

FIGURE 1 The Logit and Probit Link Functions



In both the logit and probit models, for any independent variable (say, X_1), one can determine the marginal (or instantaneous) effect of X_1 on $\Pr(Y)$—the derivative, $\partial \Pr(Y)/\partial X_1$—at any set of values for the independent variables by calculating:

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\partial \Pr(Y)}{\partial Y^*}\right]\left[\frac{\partial Y^*}{\partial X_1}\right] \tag{1}$$

The first term on the right side of this equation is $\partial \Pr(Y)/\partial Y^*$, the marginal effect of the unbounded variable Y^* on $\Pr(Y)$. This marginal effect is the instantaneous slope of the logit or probit curve in Figure 1 at the values for the independent variables. The second term on the right is $\partial Y^*/\partial X_1$, the marginal effect of X_1 on the unbounded index. Thus, in both the logit and probit models, the marginal effect of X_1 on $\Pr(Y)$ is a function of the marginal effect of X_1 on the unbounded variable Y^*, and the marginal effect of Y^* on $\Pr(Y)$.

When there are no product terms among the independent variables, equation (1) takes the form:

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\partial \Pr(Y)}{\partial Y^*}\right]\left[\frac{\partial Y^*}{\partial X_1}\right] = \left[\Lambda(Y^*) \times (1 - \Lambda(Y^*))\right][\beta_1] \quad \text{(logit)}$$

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\partial \Pr(Y)}{\partial Y^*}\right]\left[\frac{\partial Y^*}{\partial X_1}\right] = \left[\Phi'(Y^*)\right][\beta_1] \quad \text{(probit)}$$

Note that in this situation, the marginal effect of X_1 on the unbounded index, $\partial Y^*/\partial X_1$, is constant at β_1, implying that there is no interaction between X_1 and X_2 in influencing Y^*. But the marginal effect of X_1 on $\Pr(Y)$ depends on the values of all independent variables in the model, because $\partial \Pr(Y)/\partial Y^*$ is determined by all independent variables.

Consider, for example, a simple logit model with constant term −4, and two independent variables, X_1 and X_2, both having β_i coefficients of 1:

$$\Pr(Y) = G(-4 + X_1 + X_2) \tag{2}$$

In this model, the marginal effect of X_1 on the unbounded variable Y^* is constant at 1, but the marginal effect of X_1 on $\Pr(Y)$ varies with the values of X_1 and X_2. To illustrate, Figure 2A plots equation (2) for all values of X_1 in the range [0, 8] at three different values of X_2: 0, 1, and 2.

For any of the three curves, start at the point where $\Pr(Y) = 0.5$. As X_1 moves in either direction from this point and pushes $\Pr(Y)$ closer to 1 or 0, X_1's marginal effect on $\Pr(Y)$—the slope of the curve—decreases. The rate of change in the marginal effect is small (i.e., the curve is nearly linear) at first, and becomes small again when X_1 gets either very large or very small (especially outside the range of X_1 values plotted). But at points in between, the slope of the curve changes more quickly. This change in slope is an inherent feature of logit's functional form that keeps $\Pr(Y)$ within the [0, 1] boundaries of probability even as X_1 and X_2 approach $\pm\infty$.

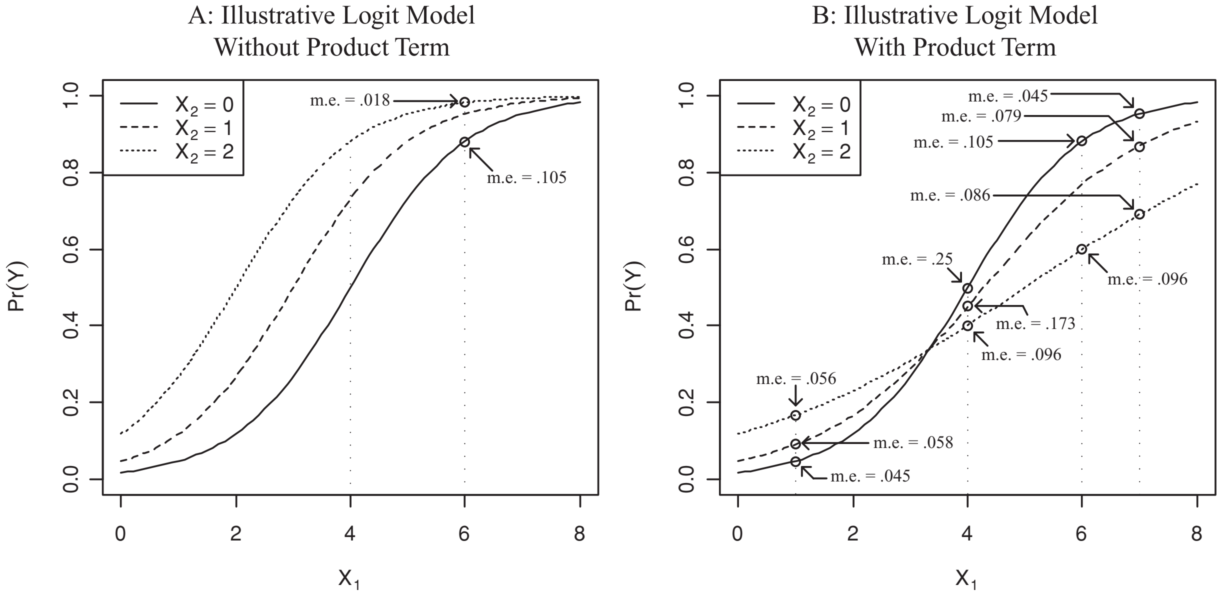
Thus, in any logit or probit model, the marginal effect of X_1 on $\Pr(Y)$ depends on the values of all independent variables; this marginal effect is greatest when $\Pr(Y)$ is 0.5 and declines when a change in either variable pushes $\Pr(Y)$ toward 0 or 1. We refer to the phenomenon in which the marginal effect of a variable on $\Pr(Y)$ is strongest at some value of $\Pr(Y)$ [$P_{(\max)}$] between 0 and 1, and declines in strength as $\Pr(Y)$ gets smaller or larger as *compression*, because deviations of $\Pr(Y)$ away from $P_{(\max)}$ compress further possible change in $\Pr(Y)$ to ever-smaller ranges. But compression is not unique to logit and probit; when independent variables are unbounded, compression will be present in any statistical model in which the link function mapping the index Y^* into $\Pr(Y)$ is monotonic.[8]

Now consider a logit model that includes a product term, X_1X_2, so that $Y^* = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_pX_1X_2 + \beta_3X_3 + \cdots + \beta_kX_k$. For this model,

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\partial \Pr(Y)}{\partial Y^*}\right]\left[\frac{\partial Y^*}{\partial X_1}\right] = \left[\Lambda(Y^*)(1 - \Lambda(Y^*))\right]\left[\beta_1 + \beta_p X_2\right] \quad \text{(logit)}$$

[8] $P_{(\max)}$ does not have to be 0.5—as in the logit and probit models—for this claim to hold. For example, the claim applies to scobit, which allows $P_{(\max)}$ to be any value in the interval (0, 1) (Nagler 1994). If the independent variables have finite ranges, compression is not guaranteed to occur since $\Pr(Y)$ can stay well within (0, 1) even if the marginal effect of each variable remains unchanged as the values of the independent variables shift.

FIGURE 2 Logit Models Illustrating How Marginal Effects on Pr(Y) Vary with the Values of Independent Variables



Panel A depicts the model $\Pr(Y) = G(-4 + X_1 + X_2)$. Panel B depicts the model $\Pr(Y) = G(-4 + X_1 + X_2 - 0.30X_1X_2)$. In both cases, $G()$ is the logit link function. Arrows indicate the marginal effect (m.e.) of the curve, i.e., $\partial\Pr(Y)/\partial X_1$, at the indicated point.

$$\frac{\partial \Pr(Y)}{\partial X_1} = \left[\frac{\partial \Pr(Y)}{\partial Y^*}\right]\left[\frac{\partial Y^*}{\partial X_1}\right] = \left[\Phi'(Y^*)\right]\left[\beta_1 + \beta_p X_2\right] \quad \text{(probit)}$$

With the inclusion of the product term, the marginal effect of X_1 on the unbounded index ($\partial Y^*/\partial X_1$)—while constant across all values of X_1—varies linearly with X_2: $\partial Y^*/\partial X_1 = \beta_1 + \beta_p X_2$. Thus, there is now interaction between X_1 and X_2 in influencing Y^*. The inclusion of the product term influences the marginal effect of X_1 on $\Pr(Y)$ in two ways: by changing the value of Y^* (hence changing $\partial\Pr(Y)/\partial Y^*$), and by adding the $\beta_p X_2$ term to $\partial Y^*/\partial X_1$.

To illustrate the implications of a product term, consider a logit model that adds an X_1X_2 term to equation (2) with coefficient −0.30:

$$\Pr(Y) = G(-4 + X_1 + X_2 - 0.30X_1X_2)$$

Figure 2B plots the relationship between X_1 and $\Pr(Y)$ at various values of X_2 for this new model. We see that a compression effect is also present in this model; the marginal effect of X_1 on $\Pr(Y)$ is still greatest at $\Pr(Y) = 0.5$, and declines when a change in either X_1 or X_2 pushes $\Pr(Y)$ toward either 0 or 1. However, the addition of the product term to the model introduces a second source of variation in the marginal effect of X_1 on $\Pr(Y)$. For example, in both models, the marginal effect of X_1 on $\Pr(Y)$ is 0.105 when $X_1 = 6$ and $X_2 = 0$ (compare Figure 2A with Figure 2B). Hold X_1 at 6, but increase X_2 to 2. In Figure 2A—the model without a product term—$\partial\Pr(Y)/\partial X_1$ declines appreciably to 0.018; in Figure 2B—the model with a product term—this marginal effect declines only slightly to 0.096.

The discussion above makes clear that for any two variables (e.g., X_1 and X_2) in a logit or probit model, the marginal effect of X_1 on $\Pr(Y)$ will vary with X_2—i.e., there will be *some* interaction between X_1 and X_2 in influencing $\Pr(Y)$—even when the model includes no product term, due to what we have called compression. Indeed, in a model without a product term, there is no interaction between X_1 and X_2 in influencing the unbounded variable, Y^*, and all interaction between X_1 and X_2 in influencing $\Pr(Y)$ is due *exclusively* to compression. Adding an X_1X_2 term to the model allows for interaction between X_1 and X_2 in influencing the unbounded variable, Y^*, and creates an additional source of variation in the marginal effect of X_1 on $\Pr(Y)$ owing to the fact that the marginal effect of X_1 on Y^* ($\partial Y^*/\partial X_1$) is no longer constant. Importantly, as we will show below, depending on the range of values for the independent variables in the population being studied, the interaction between X_1 and X_2 in influencing $\Pr(Y)$ can be extensive—with the marginal effect of X_1 on $\Pr(Y)$ varying substantially with the value of X_2—or it can be trivially small—with the marginal effect of X_1 on $\Pr(Y)$ varying only slightly as X_2 changes.

The Prevailing View about Testing for Interaction in Logit and Probit Models

Typical practice is to treat a statistically significant product term as a necessary condition for concluding that X_1 and X_2 interact in influencing $\Pr(Y)$. This practice rests on the contention that interaction between X_1 and X_2 that is due to compression is not substantively relevant. Two reasons have been offered in the literature to support this view.

(a) Compression is inherent in any logit or probit model, making interaction due to compression an artifact of the model and thus substantively meaningless.

There have been several cases in which political scientists finding evidence of what we have termed compression in a logit or probit model have reported their results as indicative of substantively meaningful interaction and been criticized. In one instance, Wolfinger and Rosenstone (1980) analyzed voter turnout in the 1972 presidential election using probit. These researchers were interested in, among other things, the relationship between voter turnout and how easy it was for citizens to register to vote. By comparing the estimated marginal effect of voter registration provisions on the probability that someone will vote at different values of education, Wolfinger and Rosenstone concluded that "[l]iberalizing registration provisions would have by far the greatest impact on the least educated and relatively little effect on well-educated people" (1980, 79). As there were no product terms involving education and any of the voter registration provisions in their model, the decline in the marginal impact of voter registration provisions on the probability of voting as education increased must be due to what we have termed compression, something the authors of the study understood.[9]

In a widely cited *AJPS* article, Nagler argues that Wolfinger and Rosenstone's claim of interaction "does not necessarily follow from [their] data analysis and is instead an artifact of the methodology used" (Nagler 1991, 1393):

> Since persons with the lowest education level are those who on average are closest to having .5 probability of voting, estimates of change based on that group will necessarily be larger than estimates of change for any other group. Thus, the smaller effects of [registration restrictions] on better-educated groups is simply an artifact of the changing slope of the normal density curve, not a result of a unique relationship between individuals' education and state registration requirements. Hence, to infer substantive interaction between registration laws and education based on the differing estimated impacts is incorrect. This interaction is *assumed* by the model specification. (Nagler 1991, 1397)

A similar exchange occurred between Berry and Berry (1990) and Frant (1991). Berry and Berry published conclusions that independent variables interact in influencing the probability of a lottery adoption by an American state based on results from a probit model including no product terms. Frant argued that Berry and Berry's interaction findings were not substantively meaningful:

> [Berry and Berry] present as empirical results what are in reality artifacts of the way the model is specified. . . . They find, for example, that the effect [on the probability of a lottery adoption] associated with an election year is greater (in absolute terms) when the state is in poor fiscal health than when it is in good fiscal health. This sounds like a *conclusion* (and the authors treat it as such); but actually, it is an *assumption* of the model. . . . This is a plausible assumption, but it is only an assumption. When we employ a probit (or logit) model, this assumption is built in. (Frant 1991, 571)

Nagler's and Frant's positions reflect a message common in the political methodology literature that any interaction in influencing $\Pr(Y)$ detected by a logit or probit model without a product term is an artifact of the estimation procedure, and not a substantively meaningful phenomenon that should be interpreted as causal interaction. Recently, Brambor, Clark, and Golder advised readers that compression occurs "whether the analyst's hypothesis is conditional or not—it is just part and parcel of deciding to use a nonlinear model such as probit; it is

[9] Wolfinger and Rosenstone write, "With probit a variable has very little impact on those who are either very unlikely or nearly certain to vote. It has the greatest impact in the middle of the distribution, on those who . . . are most susceptible to the forces pushing them to vote or not to vote" (1980, 11).

always there. If one wants to test a conditional hypothesis in a meaningful way, then the analyst has to include an explicit interaction term . . . " (2006, 77; see also Huang and Shields 2000).

(b) In a logit or probit model, compression is not relevant to an assessment of interaction between two specific variables because compression creates interaction among *all* independent variables. Product terms, by contrast, capture variable-specific interaction.

Compression is present in a logit or probit model because as $\Pr(Y)$ approaches its limits of 1 and 0, even powerful causal variables cannot increase/decrease the probability of an event beyond the upper/lower limit that probabilities can assume. In a model in which three or more variables influence $\Pr(Y)$, these limits may be approached via the influence of varying combinations of variables. Suppose, for example, that Y is influenced by three variables, X_1, X_2, and X_3, as in the following logit model:

$$\Pr(Y) = G\,(X_1 + 0.5X_2 + 0.25X_3)$$

In this model, when $X_1 = X_2 = X_3 = 0$, the unbounded index $Y^* = X_1 + 0.5X_2 + 0.25X_3$ equals 0, a value that is transformed by the logit link function to $\Pr(Y) = 0.5$. At this point, the marginal effect of X_1 on $\Pr(Y)$ is maximized at 0.25. If X_1 and X_3 are left at 0 but X_2 is increased to 4.5, the index Y^* grows to 2.25, at which point $\Pr(Y) = 0.905$, and the marginal effect of X_1 on $\Pr(Y)$ declines to 0.086. Thus, compression in the logit model makes it so that the marginal effect of X_1 on $\Pr(Y)$ declines by 0.164 (= 0.25 – 0.086) as X_2 increases from 0 to 4.5 while X_1 and X_3 are held constant at zero. One might therefore claim that X_1 and X_2 interact in influencing $\Pr(Y)$. But one could also claim that X_1 interacts with X_3 in influencing $\Pr(Y)$ using identical logic: starting at $X_1 = X_2 = X_3 = 0$, if X_1 and X_2 are held constant but X_3 is increased to 9, once again Y^* grows to 2.25, $\Pr(Y) = 0.905$, and the marginal effect of X_1 on $\Pr(Y)$ declines to 0.086.

Nagler (1991, 1994) argues that because compression forces the marginal effect of X_1 on $\Pr(Y)$ to vary with the value of every independent variable in the model, researchers should not interpret compression as relevant to causal interaction between any specific pair of variables. He invokes this criticism to challenge Wolfinger and Rosenstone's (1980) claim that voter registration provisions interact with education in influencing the probability of voting:

> . . . to the extent that those with less education are nearest to the .5 mark in their expected probability of voting (or become nearest to the .5 mark in their expected probability of voting after altering a different independent variable), they will be more affected than those with higher education levels by changes in *any* independent variable. (Nagler 1991, 1397)

To Nagler (1994, 249), the only substantively relevant interaction between two variables is what he calls "variable-specific interaction," and he makes clear his belief that product terms are needed to capture this kind of interaction:

> If adding [a product term to a model] leads to an improvement in the model, then I would argue that we have "variable-specific" interaction between [two variables], as opposed to interaction imposed between all the variables by the functional form of the model. (249–50)

Since the publication of Nagler's 1991 article, political scientists have rarely published claims that variables interact in influencing the probability that an event will occur based on a logit or probit model without a statistically significant product term. This suggests that most political scientists have been convinced that compression does not reflect substantively meaningful interaction, and that as a result, a statistically significant product term is necessary to confirm that X_1 and X_2 interact in influencing $\Pr(Y)$.

Common Misconceptions about Testing for Interaction in the BDV Setting

Contrary to the prevailing view, we argue that in the BDV setting when one is interested in the effects of independent variables on $\Pr(Y)$, compression creates substantively relevant interaction. When substantial compression is present, it is a fundamental feature of the data generating process (DGP); as such, taking it into account is essential to accurately describe how the marginal effect of one variable, X_1, on $\Pr(Y)$ varies with the value of another variable, X_2, and thus essential to validly test a hypothesis that X_1 and X_2 interact in influencing $\Pr(Y)$. Sometimes, but not always, a product term is needed to correctly specify the nature of interaction between two variables. But whether there is a product term in the model or not, compression effects cannot be ignored. Our argument involves seven points, most of which represent a challenge to widely held views about interaction in BDV models.

In our discussion we will frequently refer to the notion of *substantively meaningful interaction* between X_1 and X_2 in influencing $\Pr(Y)$. By this we mean a situation in which (1) the interaction is statistically significant, i.e., the effect of one variable, say X_1, on $\Pr(Y)$ varies with the value of the other (X_2) to a statistically significant degree, and (2) the estimated magnitude of the interaction is large enough to be deemed consequential, i.e., the estimated response of the effect of X_1 on $\Pr(Y)$ to a change in X_2 is of nontrivial magnitude.[10] If substantively meaningful interaction in influencing $\Pr(Y)$ is detected by a logit or probit model without a product term, the interaction is due solely to compression. In a model including an X_1X_2 term (with a nonzero coefficient), the interaction is due to some combination of variable-specific interaction in influencing the unbounded latent variable Y^* (as reflected by the product term coefficient) and compression.

1. Compression in a logit or probit model should not be viewed as theoretically irrelevant. It is an appropriate theoretical rationale for expecting interaction between independent variables in their joint influence on $\Pr(Y)$.

Compression in a logit or probit model is often held to be theoretically irrelevant due to the fact that it occurs simply because logit and probit always constrain the dependent variable, $\Pr(Y)$, to be between 0 and 1 regardless of the event being studied. However, the restricted range of $\Pr(Y)$ in a logit or probit model should not be viewed as an assumption imposed on $\Pr(Y)$ by the specific statistical model chosen by the researcher. Rather, no matter what event is being studied, any researcher who wants to correctly specify the DGP must choose a model that constrains $\Pr(Y)$ to the $[0, 1]$ range because this range is a fundamental characteristic of the DGP. Probability values of 1.5 and −0.3 are impossible, as an event cannot occur 150 times out of 100 or –30 times out of 100, and any statistical model inconsistent with a dependent variable constrained to the [0, 1] range must *ipso facto* misspecify the DGP. Indeed, the primary reason logit and probit are viewed as an improvement over a linear probability model [LPM] (i.e., OLS regression with a BDV) is that with an LPM, predicted $\Pr(Y)$ values for some observations may fall outside [0, 1]. Logit and probit force the marginal effects of independent variables to decline as $\Pr(Y)$ nears its limits, thereby avoiding these meaningless predictions.

Compression should not be viewed as theoretically irrelevant. Indeed, it can be a strong theoretical rationale for expecting interaction between variables in their influence on $\Pr(Y)$. Consider, for example, the factors determining whether countries will go to war. Two variables found to negatively affect the probability that a pair of countries (or dyad) will fight a war are the *geographic distance between the countries* (e.g., Bremer 1992; Diehl 1985) and the *joint level of democracy in the dyad* (e.g., Oneal and Russett 1997; Ray 1998).[11] The "democratic peace" effect has been found to be so strong that the probability of war is nearly zero when the level of democracy is very high (i.e., both countries are fully democratic) and no other variable takes an extreme value (e.g., Levy 1988; Maoz 1998). Imagine first that some dyad is *not* highly democratic. With all other variables held at their mean, increasing the distance between the countries should lead to a substantial decrease in the likelihood of war: the states may conflict over an issue, but the farther they are from each other the more expensive war becomes, thereby decreasing the utility of war to the countries. Now imagine that the same dyad is highly democratic. This lowers the probability of war to nearly zero. It is still reasonable to expect that a change in the distance between the states would prompt a sizeable change in the (unbounded) utility to the countries of a war, but because the countries are highly democratic, the utility of war to them is already so low that a marginal change in utility should not appreciably change the near zero probability of war. This rationale justifies the hypothesis that the distance between countries and their level of joint democracy interact in influencing their probability of going to war: as the level of democracy rises, the negative effect of distance on the probability of war decreases in magnitude.

Similarly, Wolfinger and Rosenstone (1980) rely on the [0, 1] restriction of a probability to defend their proposition that influences on an individual's decision to vote are conditional on the individual's level of education. For example, the authors write, "... a high-status occupation or a high income has less impact on a college graduate, who is 90 percent likely to vote, than it has on a high school dropout, who is only 55 percent likely to vote" (1980, 11).

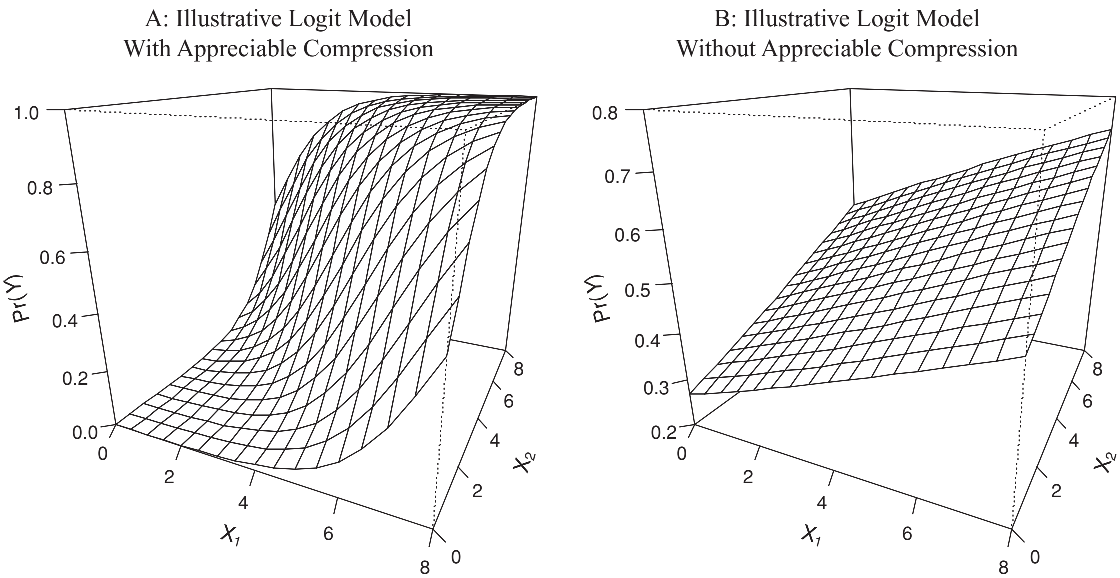
[10] Mathematically, by condition (2) we mean that the quantity defined in note 3,

$$\begin{aligned}&[\mathrm{Prob}(Y=1 \mid X_1 = X_{1(\mathrm{hi})}, X_2 = X_{2(\mathrm{hi})})\\ &\qquad - \mathrm{Prob}(Y=1 \mid X_1 = X_{1(\mathrm{lo})}, X_2 = X_{2(\mathrm{hi})})]\\ -&[\mathrm{Prob}(Y=1 \mid X_1 = X_{1(\mathrm{hi})}, X_2 = X_{2(\mathrm{lo})})\\ &\qquad - \mathrm{Prob}(Y=1 \mid X_1 = X_{1(\mathrm{lo})}, X_2 = X_{2(\mathrm{lo})})],\end{aligned}$$

has magnitude greater than some threshold, c (the value of which may be subject to debate within a substantive field).

[11] The level of democracy in a dyad is typically defined as the level in the less democratic of the two countries.

FIGURE 3 Logit Models with and without Appreciable Compression



Panel A depicts the model $\Pr(Y) = G\,(-8 + X_1 + X_2)$, and panel B illustrates the model $\Pr(Y) = G\,(-1 + 0.125X_1 + 0.125X_2)$, both for DGPs in which X_1 and X_2 are confined to the [0, 8] range.

These variables have "very little impact on those who are either very unlikely or nearly certain to vote. [They have] the greatest impact in the middle of the distribution, on those who are between 40 and 60 percent likely to vote and are most susceptible to the forces pushing them to vote or not to vote" (1980, 11).[12]

2. Logit and probit will find compression of nontrivial magnitude *only* when it is actually there; if just trivial compression is present in the data generating process, logit and probit will not mistakenly make it appear more substantial.

It is true that with any logit or probit model, if independent variables are unbounded, a sufficiently extreme value for *any* independent variable having a nonzero coefficient will move $\Pr(Y)$ close enough to one of its boundaries that the possible marginal impact of any independent variable will be severely restricted. But we must not confuse the logit and probit models as *mathematical functions*—which are defined for all positive and negative values for each independent variable, regardless of how large or small—with these models when they are used by political scientists to characterize data generating processes in real-world populations. In the latter situation, the values for independent variables will always be confined to finite ranges. When independent variables are constrained to finite ranges, a DGP for an event that is accurately specified by a logit or probit model need not be characterized by substantial (i.e., more than trivial) compression. Consider, for example, the two-independent variable DGP reflected in the logit equation plotted in Figure 3A, in which all cases in the population have values for X_1 and X_2 that are between 0 and 8.[13] In this DGP, there is substantial compression, as evidenced in the greater rate of ascent of the surface near its center than when both X_1 and X_2 assume their lowest values, or when both variables are at their highest. The DGP in Figure 3B, in which population values for X_1 and X_2 are also constrained to the [0, 8] range, is a logit equation as well. But despite this, the surface is very nearly flat, as the marginal effect of X_1 on $\Pr(Y)$ [i.e., $\partial \Pr(Y)/\partial X_1$] is always between 0.025 and 0.031 (a very narrow range). Thus, in the case of this DGP that takes the form of a logit model, it would be reasonable to claim that the effects of X_1 and X_2 on $\Pr(Y)$ are essentially linear and additive, and that nearly no compression is present.

[12]Huang and Shields call what we term compression "built-in interaction" (2000, 81). They share our view that when logit or probit models accurately specify the DGP, built-in interaction/compression should be viewed as meaningful, as it constitutes a component of the total interaction among the independent variables. Huang and Shields differ from us in that they agree with Nagler that claims of interaction in a logit or probit model always "must be modeled by a statistically significant multiplicative term in the equation" (81).

[13]We chose the values 0 and 8 arbitrarily; we could just as easily have constructed an example using any other pair of values.

Is there any reason to fear that if we estimated a logit model with independent variables X_1 and X_2 using a data set generated from the DGP represented in Figure 3B we will be led to conclude there is substantively meaningful interaction due to compression even though very little compression is actually present? The answer must be no. The true model is a logit equation and it is well known that an accurately specified logit model yields consistent parameter estimates (Aldrich and Nelson 1984). Consistent parameter estimates will generate consistent estimates of the marginal effects of independent variables on $\Pr(Y)$. Thus, with a sufficiently large random sample, estimated marginal effects on $\Pr(Y)$ will, on average, be on target. Since the true marginal effects of X_1 and X_2 are nearly constant over the range of values for X_1 and X_2 in the population, estimated marginal effects on $\Pr(Y)$ can also be expected to be nearly constant.[14] Note, however, that this claim holds only if we confine estimates of marginal effects to the actual range of X_1 and X_2 in the population being studied; if we predict marginal effects outside this range, we may mistakenly conclude that there is substantial compression. Of course, predictions outside the bounds of available data (i.e., inferences about counterfactuals) are dangerous regardless of the econometric technique employed (King and Zeng 2006).

In sum, a logit or probit model will not mistakenly detect substantial compression where little is present in the DGP as long as the model accurately specifies the DGP. Except to the normal extent we would expect due to estimation uncertainty in finite samples, logit and probit will find nontrivial compression in the data only when there is nontrivial compression in the true model.

3. The large number of researchers who use logit or probit to test a hypothesis of interaction and, as part of their procedure, report how the estimated effect of one variable on $\Pr(Y)$ varies with the value of another variable are treating compression effects as substantively meaningful, even if they do not explicitly recognize that they are doing so.

In the introduction, we describe a widely used procedure for testing a hypothesis that X_1 and X_2 interact in influencing $\Pr(Y)$ that involves calculating how the estimated effect of X_1 on $\Pr(Y)$ varies with X_2. We applaud the practice of reporting estimated effects on $\Pr(Y)$, but simply note that when one does this, one is tacitly acknowledging that compression *is* substantively relevant. As long as a probit or logit model accurately specifies the true DGP, any estimate of the effect of a variable on $\Pr(Y)$ derived from the model's coefficients will reflect whatever compression is present in the DGP. Indeed, the only way for a user of probit or logit to avoid reporting effects of independent variables that take into account compression is to restrict attention to effects on the unbounded latent variable Y^*—i.e., to limit attention to the model's coefficients—and say nothing about impacts on $\Pr(Y)$.

4. In a logit or probit model, a statistically significant product term is not a *necessary* condition for substantively meaningful interaction among independent variables in their influence on $\Pr(Y)$.

If a large product term coefficient were necessary for substantial interaction among independent variables in influencing $\Pr(Y)$, then a logit or probit model with no product term (i.e., one in which the product term's coefficient is constrained to be zero) would never exhibit substantial interaction among variables in their effects on $\Pr(Y)$. But our earlier examination of the model depicted in Figure 2A made clear that even in a model without a product term, compression can lead to substantial variation in the marginal effect of one variable on $\Pr(Y)$ across different values of another variable. For example, when $X_1 = 6$, a shift in X_2 from 0 to 2 reduces the marginal effect of X_1 on $\Pr(Y)$ from 0.105 to 0.018; this constitutes a nontrivial amount of interaction between the independent variables.[15]

Of course, we do not know, in practice, how often political scientists who rejected a hypothesis of

[14]To illustrate our conclusion, we conducted a Monte Carlo analysis. We constructed 100 samples of 1,000 observations, where values of X_1 and X_2 were drawn randomly from uniform distributions between 0 and 8, and the value of the BDV, Y, was generated by the DGP in Figure 3B, which is characterized by the nearly complete absence of compression. Using each sample, we (1) estimated a logit model including X_1 and X_2; (2) calculated $\partial^2\Pr(Y)/\partial X_1\partial X_2$—a second derivative measuring the marginal effect of X_2 on the marginal effect of X_1, and therefore the extent of interaction between X_1 and X_2 in influencing $\Pr(Y)$—at each of 81 points spread evenly over the X_1-X_2 space (both variables ranging over $0, 1, 2, \ldots, 8$); and (3) determined whether each derivative was statistically significant at the .05 level (two-tail test). Only 6.5% of the 8,100 second derivatives proved statistically significant, and the largest of the 8,100 point estimates was 0.003. Thus, even the derivatives that were statistically significant indicated interaction that is trivially small.

[15]To better illustrate our conclusion, we conducted a Monte Carlo analysis. We examined the case in which the DGP in Figure 2A (which contains no product term) characterizes a population in which X_1 and X_2 are confined to the intervals [4, 6] and [0, 2], respectively. We constructed 100 samples of 1,000 observations, where values of X_1 and X_2 were drawn randomly from uniform distributions—between 4 and 6 for X_1, and between 0 and 2 for X_2—and the value of the BDV, Y, was generated by the given DGP. Using each sample, we (1) estimated a logit model including X_1 and X_2; (2) calculated the second derivative, $\partial^2\Pr(Y)/\partial X_1\partial X_2$, at the center of the range for X_1 and X_2 (i.e., $X_1 = 5$, $X_2 = 1$); and (3) determined whether the derivative is statistically significant at the .05 level (two-tail test). For each sample, we also computed a second difference in $\Pr(Y)$ across the range for X_1 and X_2,

interaction based on a statistically insignificant product term and did not calculate estimated effects on $\Pr(Y)$ would have found interaction had they done so. Yet replicating one study drawn from our search of recent issues of journals (see note 1) reveals a case in which a test based on the product term coefficient and a test based on effects on $\Pr(Y)$ yield different conclusions about the statistical significance of interaction. Heller and Mershon's (2005) study of party switching in the Italian Chamber of Deputies examines whether *electoral rules* (first-past-the-post [FPTP] vs. proportional representation [PR]) and the nature of a legislator's *party label* (blurred vs. clear) interact in influencing the probability that the legislator will switch across political blocs [Pr(*switched*)]. The authors estimate a logit model including *electoral rules*, *party label*, *electoral rules* ∗ *party label*, and other variables. In the first column of their Table 4 (2005, 550), they report an estimated product term coefficient of −1.44 that is not statistically significant at the .05 level (Z = −1.09), having a 95% confidence interval of [−4.02, 1.14].[16] This provides statistical grounds for rejecting a hypothesis of interaction between *electoral rules* and *party label* in influencing the unbounded latent dependent variable, Y^*. But Heller and Mershon's hypotheses relate to effects on Pr(*switched*). To see whether there is statistically significant interaction between *electoral rules* and *party label* in influencing this probability, we use CLARIFY to estimate a second difference in Pr(*switched*) showing the change in the impact of *electoral rules* on Pr(*switched*) when *party label* shifts from clear to blurred, but all other independent variables are fixed at their mean:

$$\begin{aligned}\Delta\Delta[\Pr(\textit{switched})] &= [\Pr(\textit{switched} \mid \textit{electoral rules} = \text{FPTP}, \textit{party label} = \text{blurred}) \\ &\quad - \Pr(\textit{switched} \mid \textit{electoral rules} = \text{PR}, \textit{party label} = \text{blurred})] \\ &\quad - [\Pr(\textit{switched} \mid \textit{electoral rules} = \text{FPTP}, \textit{party label} = \text{clear}) \\ &\quad - \Pr(\textit{switched} \mid \textit{electoral rules} = \text{PR}, \textit{party label} = \text{clear})].\end{aligned}$$

We obtain a second difference of [(0.039)−(0.084)]−[(0.008)− (0.006)] = −0.047, which is statistically significant at the .05 level with a 95% confidence interval of [−0.107, −0.001].[17]

5. In a logit or probit model, a statistically significant product term is not a *sufficient* condition for substantively meaningful interaction among independent variables in their influence on $\Pr(Y)$.

Consider the logit model of Figure 2B, which includes a product term of nontrivial magnitude. There are certainly many values for X_1 and X_2 at which there is strong interaction between X_1 and X_2. For instance, when $X_1 = 4$, the slopes of the three curves are quite different ($\partial\Pr(Y)/\partial X_1 = 0.25$, 0.173, and 0.096, respectively), indicating that the marginal effect of X_1 on $\Pr(Y)$ changes considerably as X_2 rises from 0 to 2. But there are other values for the independent variables at which there is little interaction. When $X_1 = 1$, for example, the slopes of the three curves are very similar ($\partial\Pr(Y)/\partial X_1 = 0.045$, 0.058, and 0.056, respectively). Indeed, if in the population of interest both X_1 and X_2 were constrained to the [0, 2] range, the three curves are close enough to being parallel that we would be comfortable rejecting a hypothesis of interaction and concluding that the effects of X_1 and X_2 are approximately additive despite the large negative coefficient for the product term.[18]

$$\begin{aligned}\Delta\Delta[\Pr(Y)] &= [\Pr(Y \mid X_1 = 6, X_2 = 2) - \Pr(Y \mid X_1 = 4, X_2 = 2)] \\ &\quad - [\Pr(Y \mid X_1 = 6, X_2 = 0) - \Pr(Y \mid X_1 = 4, X_2 = 0)],\end{aligned}$$

and determined whether it was statistically significant. All of the second derivatives and second differences were indeed significant. This illustrates that even when the coefficient for the product term in a logit model is zero, there are situations in which one can expect statistically significant interaction in influencing $\Pr(Y)$ with a sample size typical in political science research.

[16]We are grateful to Carol Mershon and William Heller for providing us with a replication data set. Heller and Mershon do not report this confidence interval; we computed it after replicating their results exactly.

[17](1) A second difference of −0.047 in the probability of switching blocs seems to us to indicate interaction of nontrivial magnitude. But readers may draw their own conclusions. (2) Note that *electoral rules* and *party label* are dichotomous variables, and hence only one change in each variable is possible (that from one value to the other). Thus, the second difference we calculate represents a complete description of the extent of interaction between these variables when all other independent variables are fixed at their mean.

[18]To better illustrate point 5, we conducted a Monte Carlo analysis. We examined the case in which the DGP in Figure 2B (which contains a sizeable product term) characterizes a population in which X_1 and X_2 are both confined to the interval [0, 2]. We constructed 100 samples of 1,000 observations, where values of X_1 and X_2 were drawn randomly from uniform distributions between 0 and 2, and the value of the BDV, Y, was generated by the given DGP. Using each sample, we (1) estimated a logit model including X_1, X_2, and X_1X_2; (2) computed a second difference in $\Pr(Y)$ across the range for X_1 and X_2,

$$\begin{aligned}\Delta\Delta[\Pr(Y)] &= [\Pr(Y \mid X_1 = 2, X_2 = 2) - \Pr(Y \mid X_1 = 0, X_2 = 2)] \\ &\quad - [\Pr(Y \mid X_1 = 2, X_2 = 0) - \Pr(Y \mid X_1 = 0, X_2 = 0)];\end{aligned}$$

and (3) determined whether this second difference was statistically significant at the .05 level (two-tail test). The product term was significant in all 100 samples, but the second difference in $\Pr(Y)$ was significant only 8% of the time. This illustrates that even when the coefficient for the product term in a logit model is statistically significant, there may not be significant interaction between the two variables in influencing $\Pr(Y)$.

We can offer a substantive illustration that interaction between two variables in their effect on $\Pr(Y)$ can fail to be statistically significant even when their corresponding product term is significant by replicating a study identified by our content analysis of recent issues of journals (see note 1). Marinov's (2005) research on the effectiveness of international economic sanctions in destabilizing the governments they target seeks to determine whether the presence of *sanctions* (a dichotomous variable indicating whether a country is subject to sanctions in a year) and the nature of political institutions (whether the country is a *democracy*) interact in influencing the probability that the country's leaders will survive in office [Pr(*survive*)]. The author estimates a logit model including *sanctions*, *democracy*, *sanctions* ∗ *democracy,* and other covariates. We estimate a slightly revised model and obtain a product term coefficient of 1.22, which is statistically significant even at the .01 level (Z = 3.27) and has a 95% confidence interval of [0.49, 1.95].[19] We use CLARIFY to estimate a second difference in Pr(*survive*) showing the change in the impact of *sanctions* on Pr(*survive*) when one moves from a nondemocratic target country to one that is democratic, when all other independent variables are fixed at their mean:[20]

$$\begin{aligned}\Delta\Delta[\Pr(\textit{survive})] &= [\Pr(\textit{survive} \mid \text{sanctions are } \textit{present}\text{, country is democratic})\\ &\quad - \Pr(\textit{survive} \mid \text{sanctions are } \textit{absent}\text{, country is democratic})]\\ &\quad - [\Pr(\textit{survive} \mid \text{sanctions are } \textit{present}\text{, country is } \textit{not} \text{ democratic})\\ &\quad - \Pr(\textit{survive} \mid \text{sanctions are } \textit{absent}\text{, country is } \textit{not} \text{ democratic})].\end{aligned}$$

We obtain a second difference of $[(0.780) - (0.084)] - [(0.630) - (0.080)] = 0.146$, which fails to be statistically significant at the .01 level, and barely misses significance at the .05 level as well (having a 95% confidence interval of $[-0.0003, 0.356]$).

6. In a logit or probit model, the sign of the coefficient for a product term may give a misleading signal about the "direction" of the interaction between independent variables in influencing $\Pr(Y)$. Put differently, the direction of interaction among independent variables in influencing the unbounded latent variable Y^* can be different from the direction of interaction in influencing $\Pr(Y)$.

By the *direction* of interaction between X_1 and X_2 in influencing some dependent variable, V, we refer to the sign (positive or negative) of the "difference between differences" in note 3. Stated verbally, the direction of interaction refers to whether the effect of X_1 on V (1) increases (i.e., becomes either more positive or less negative) as X_2 increases (making the direction of interaction positive), or (2) decreases (i.e., becomes less positive or more negative) as X_2 increases (making the direction of interaction negative). Given a logit or probit model including a product term, X_1X_2, so that $Y^* = \beta_0 + \beta_1X_1 + \beta_2X_2 + \beta_pX_1X_2 + \beta_3X_3 + \cdots + \beta_kX_k$, the magnitude of the interaction between X_1 and X_2 in influencing the latent dependent variable Y^* (as measured by the second derivative, $\partial^2Y^*/\partial X_1\partial X_2$) is β_p at all values for X_1 and X_2: at all values, a unit increase in X_2 prompts a change of β_p in the marginal effect of X_1 on Y^*.[21] For the same model, as Norton, Wang, and Ai (2004, 158) have shown, the magnitude of interaction between X_1 and X_2 in influencing $\Pr(Y)$ is given by

$$\begin{aligned}\partial^2\Pr(Y)/\partial X_1\partial X_2 &= \Pr(Y)(1-\Pr(Y))\beta_p\\ &\quad + [\Pr(Y)(1-\Pr(Y))(1-2\Pr(Y))\\ &\qquad (\beta_1+\beta_pX_2)(\beta_2+\beta_pX_1)]\end{aligned}$$

Thus, the direction (and magnitude) of interaction between X_1 and X_2 in influencing $\Pr(Y)$ is determined in part by the product term coefficient (β_p), but also by the values of X_1 and X_2, and the coefficients for these variables. As an example, consider the logit model of Figure 2B. Since the coefficient for X_1X_2 is negative (-0.30), the marginal effect of X_1 on the unbounded index Y^* declines as X_2 increases regardless of the values for X_1 and X_2, making the direction of interaction in influencing Y^* negative throughout. However, the figure shows clearly that there are values of X_1 and X_2 at which the marginal effect of X_1 on $\Pr(Y)$ *increases* as X_2

[19] We thank Marinov for providing replication data. Our results correspond to those reported by Marinov in the right-most column of his Table 2. (1) He uses conditional logit to obtain the coefficients in this table, but switches to ordinary logit to generate estimated effects on Pr(*survive*), a choice he justifies in his note 26. We also use ordinary logit to compute effects on Pr(*survive*), but to achieve comparability in results, we report the product term coefficient for the same ordinary logit model rather than the conditional logit estimation. (2) One of the variables in Marinov's model is *wealth*, which is entered alone and in a product term with *sanctions*. After we discovered some discrepancies in the replication data set (available at http://pantheon.yale.edu/%7Envm4/replicate/replicate.htm), Marinov reported to us that his published results rely on a different indicator of *wealth* for the product term containing this variable (GNP) than for the stand-alone variable (GNP per capita). In our estimation, we use GNP for both variables. These two differences combine to produce only slight disparities between our coefficients and those in Marinov's Table 2; for example, Marinov's coefficient for *sanctions* ∗ *democracy* is 1.20 (Z = 3.33).

[20] When Marinov analyzes effects on Pr(*survive*), he computes different effects for each country, setting variables besides *sanctions* and *democracy* at their *country-specific* means (over the period of analysis). We do not challenge his choice to do this, but for the purpose of our illustration, we fix other variables at their global means (over country-years), since this is the typical approach in the literature for estimating effects on $\Pr(Y)$.

[21] This is because $\partial Y^*/\partial X_1 = \beta_1 + \beta_pX_2$, and $\partial(\beta_1 + \beta_pX_2)/\partial X_2 = \beta_p$.

increases, making the interaction in influencing $\Pr(Y)$ positive. This is true, for example, when $X_1 = 7$, at which $\partial \Pr(Y)/\partial X_1 = 0.045, 0.079$, and 0.086, respectively, when $X_2 = 0$, 1, and 2.

Demonstrating that the direction of the interaction between X_1 and X_2 in influencing $\Pr(Y)$ *can* be different than the sign of the coefficient for the product term for some values of X_1 and X_2 does not provide evidence that this occurs with any frequency in the research setting. Indeed, we suspect that it is more common in political science applications of logit and probit for the sign of the product term coefficient estimate to be the same as the direction of interaction in influencing $\Pr(Y)$ over the relevant values for the independent variables. But we can point to an example in which this is not the case. Scholz and Wang (2006) posit that the strength of local institutions [*institutions*] interacts with the liberalism of state elites [*liberalism*] in influencing the probability of a violation of the Clean Water Act by a plant regulated by a state environmental agency [Pr(*violation*)], predicting that the negative effect of *institutions* on Pr(*violation*) becomes weaker (i.e., less negative) as *liberalism* increases (i.e., positive interaction). They estimate a probit model including *institutions*, *liberalism*, *institutions* ∗ *liberalism*, and other variables (with a sample of nearly 38,000 cases); and use the coefficients to compute a second difference in Pr(*violation*) involving changes in *liberalism* and *institutions* from one standard deviation below their mean to one standard deviation above, when the other independent variables are held at central values:[22]

$$\begin{aligned}
&\Delta\Delta[\Pr(\textit{violation})] = \\
&[\Pr(\textit{violation} \mid \textit{institutions} = \text{mean} + 1\text{ SD}, \textit{liberalism} = \text{mean} + 1\text{ SD}) - \\
&\Pr(\textit{violation} \mid \textit{institutions} = \text{mean} - 1\text{ SD}, \textit{liberalism} = \text{mean} + 1\text{ SD})] - \\
&[\Pr(\textit{violation} \mid \textit{institutions} = \text{mean} + 1\text{ SD}, \textit{liberalism} = \text{mean} - 1\text{ SD}) - \\
&\Pr(\textit{violation} \mid \textit{institutions} = \text{mean} - 1\text{ SD}, \textit{liberalism} = \text{mean} - 1\text{ SD})].
\end{aligned}$$

Scholz and Wang obtain a point estimate for this second difference with large magnitude, 0.15, and a positive sign indicating support for their interaction hypothesis.[23] But they report that the coefficient for their product term is *negative* and statistically significant at the .10 level (Scholz and Wang 2006, Table 1). Thus, the sign of the product term coefficient is different than the direction of the interaction between *institutions* and *liberalism* in influencing Pr(*violation*).[24]

7. The fact that in a logit or probit model the marginal effect of each independent variable on $\Pr(Y)$ varies with the values of all independent variables does not invalidate the substantive importance of interaction between a pair of these variables of theoretical interest.

One argument for not regarding interaction due to compression as meaningful [point (b) discussed above] is that the variation in the marginal effect of a variable associated with compression is not "variable specific" (Nagler 1994): in a logit or probit model, the marginal effect of X_1 on $\Pr(Y)$ will change when any variable in the model changes, and therefore, one ought not interpret the change in this marginal effect as some other specific variable changes as substantively meaningful.

We contend that this position falsely equates (for a model with k independent variables) the hypothesis that X_1 and X_2 interact with the hypothesis that X_1 does *not* interact with any of $X_3, X_4, \ldots, X_k$. In fact, advancing the former hypothesis does not imply a belief that the latter is true. To continue with the voting turnout example of Wolfinger and Rosenstone, a claim that high levels of education desensitize a person to changes in voter registration requirements (because highly educated people are already very likely to vote) does *not* imply that extreme values of other variables (e.g., a high age) do not cause a similar desensitization (because senior citizens are also highly likely to vote).

Even when all variation in the marginal effects of variables on $\Pr(Y)$ is due to compression—as in a model with no product term—an analyst interested in whether X_1 and X_2 interact in influencing $\Pr(Y)$ is perfectly justified in measuring and reporting how the marginal effect of X_1 on $\Pr(Y)$ varies with the value of X_2 when $X_3, \ldots, X_k$ are held constant at specified values (e.g., their mean), and describing substantial variation in the effect of X_1 as indicating interaction between X_1 and X_2. Indeed, to do otherwise would involve mischaracterizing the nature of these variables' effects. Given an accurately specified logit or probit model, the only way for an analyst to assess the amount of interaction present in a way that excludes from

[22]To be precise, Scholz and Wang use a detection-controlled estimation (DCE) procedure with a binary probit link function to estimate a three-equation model of which the equation for Pr(*violation*) is a part.

[23]The authors do not report a test of statistical significance for the second difference, but given the magnitude of the point estimate and a sample size in excess of 37,000, it seems virtually certain to be significant. (The authors also report in a private communication that the 95% confidence intervals for the two first differences that are subtracted to yield the second difference have no overlap.)

[24]Scholz and Wang (2006, 93) recognized the very point we are making about the possibility of different directions for interaction depending on whether one is assessing effects on Pr(*violation*) or on the latent unbounded variable. The authors justify their choice to ignore the negative product term coefficient when testing their hypothesis about effects on Pr(*violation*) by citing an earlier empirical study making a similar choice (Berry, Berkman, and Schneiderman 2000).

consideration interaction due to compression and takes into account just variable-specific interaction would be to abandon all interest in effects on $\Pr(Y)$ and restrict attention to impacts on the unbounded latent variable, Y^*.

Deciding Whether to Include a Product Term

We have argued that in a logit or probit model, a statistically significant product term is neither necessary nor sufficient for concluding that there is substantively meaningful interaction among independent variables in their influence on $\Pr(Y)$. How, then, should a researcher positing interaction among independent variables in influencing $\Pr(Y)$ decide whether to include a product term? We believe that this decision must be based on an explicit theory about the effects of variables on the unbounded latent dependent variable, Y^*, assumed by the model.[25] This is because even though the researcher's hypothesis is about the effects of variables on $\Pr(Y)$, in both logit and probit it is Y^*—rather than $\Pr(Y)$—that is specified as a linear function of the independent variables (i.e., $Y^* = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$).

If one's theory predicts that the independent variables interact in influencing Y^*, a logit or probit model specifying the interaction with one or more product terms should be estimated, and the statistical significance of these terms should be determined to test for interaction; here the statistical tests for interaction advocated by Nagler (1991) are appropriate.[26] If the expected interaction in influencing Y^* is confirmed, the researcher should use the coefficients for the model to compute estimated effects of variables on $\Pr(Y)$ suitable for testing her hypothesis about interaction in influencing $\Pr(Y)$. For this purpose she can calculate second differences measuring how the response of $\Pr(Y)$ to a discrete change in one variable varies with the value of another variable, perhaps using CLARIFY (King, Tomz, and Wittenberg 2000). Alternatively, she can construct plots showing how the marginal effect of one variable on $\Pr(Y)$ varies with the value of another variable, as suggested by Brambor, Clark, and Golder (2006). If the expected interaction in influencing Y^* is not found, the researcher may still derive estimates of the impacts of variables on $\Pr(Y)$ to test her hypothesis, but she should recognize that one element of the theory underlying the hypothesis has been called into question.[27] Nevertheless, compression may produce the expected interaction among variables in influencing $\Pr(Y)$ even if these variables do not interact in their influence on Y^*.

On the other hand, a researcher may have no reason to believe that the effects of independent variables on the latent variable Y^* are interactive and may base his hypothesis that independent variables interact in influencing $\Pr(Y)$ strictly on an expectation of compression. In this case, there is no need to include a product term in the model. Put differently, no product term is required if the analyst's only reason for positing interaction between X_1 and X_2 in influencing $\Pr(Y)$ is an expectation that when the value of X_1 is extreme, $\Pr(Y)$ is near its limit of zero or one and thus there is little room for $\Pr(Y)$ to change as X_2 changes.[28] In any event, the analyst should use the parameter estimates for the model (with no product term)

[25]An alternative justification for choosing whether to include a product term that would not require introducing a theory involving an unbounded latent dependent variable would be evidence that one choice can be expected to yield estimated effects on $\Pr(Y)$ that more closely approximate the true effects. One might guess that because a model with a product term allows logit and probit to maximize fit over a larger family of functions, it would routinely lead to a better approximation of true effects on $\Pr(Y)$ even if the DGP did not take the exact form of a logit or probit model. However, this reasoning ignores the reduction in the efficiency of parameter estimates as additional terms are added to a model. Indeed, Monte Carlo analysis shows that a model excluding a product term outperforms a model with a product term in approximating the true relationship among X_1, X_2, and $\Pr(Y)$ for many types of DGPs that do not take the exact form of logit or probit even when they involve substantial interaction between X_1 and X_2 in influencing $\Pr(Y)$ (Berry, DeMerritt, and Esarey 2009).

[26]Although in most applications by political scientists, interaction between X_1 and X_2 is specified by a single product term—X_1X_2—properly specifying some forms of interaction requires multiple terms involving X_1 and X_2 (see note 5). Since the latent variable Y^* is unbounded, one should include the same terms one would include if one were modeling the effects of X_1 and X_2 on an observed unbounded variable in an OLS regression model. For good discussions of regression models with product terms, see Friedrich (1982), Braumoeller (2004), Brambor, Clark, and Golder (2006), and Kam and Franzese (2007).

[27]In this situation, since the statistical test failed to support the expectation of interaction in influencing Y^*, the researcher must decide whether to compute estimated effects on $\Pr(Y)$ using the coefficients for (1) the original model including the product term(s), or (2) a revised model deleting product terms and therefore specifying additive effects among variables in their influence on Y^*. We offer no advice on this choice except to note that social scientists regularly face similar trade-offs when conducting empirical tests of their theories. For example, the situation is comparable to regressing a dependent variable on a set of independent variables, finding that some (call them Xs) have statistically significant effects while others (call them Zs) do not, and having to decide whether to report as the best estimates of the effects of the Xs the original coefficients or coefficients derived from revised models from which the Zs have been excluded.

[28]Of course, the researcher can test his belief that the independent variables do not interact in influencing Y^* by including one or more product terms that are expected to have coefficients of zero, and determining whether their estimated values are in fact near zero.

to generate estimates of the effects of variables on $\Pr(Y)$ to test his hypothesis; the same tools used for models with a product term—second differences or marginal effect plots—can be employed.[29]

Consider, for example, the case of voting. A utility maximization theory of election turnout assumes that a citizen votes when the expected utility of voting is greater than the expected utility of not voting, i.e., $E[U_{vote}] - E[U_{no\ vote}] > 0$. This net utility of voting, $E[U_{vote}] - E[U_{no\ vote}]$, is an unbounded variable, and a theory about the determinants of this net utility can provide the basis for a theoretically informed decision about whether to include a product term in a logit model designed to test hypotheses about the determinants of the probability that someone will vote. If the net utility of voting is subject to an error component, ε, then a citizen votes whenever

$$E[U_{vote}] - E[U_{no\ vote}] + \varepsilon > 0,$$

or alternatively,

$$E[U_{no\ vote}] - E[U_{vote}] < \varepsilon.$$

$\Pr(E[U_{no\ vote}] - E[U_{vote}] < \varepsilon)$ is the probability that a citizen turns out to vote. The distribution of ε determines the form of the appropriate statistical model; a Gaussian normal ε implies probit, while a logistic ε implies logit.[30] The quantity $E[U_{no\ vote}] - E[U_{vote}]$ corresponds to the index $Y^* = \beta_0 + \beta_1 X_1 + \beta_2 X_2 + \cdots + \beta_k X_k$, where observable X variables (like education or voter registration laws) are influences on the size of this net utility.

If one's theory predicts interaction between X_1 and X_2 in influencing the net utility of voting, a logit or probit model should include an X_1X_2 term. If one does not expect interaction between X_1 and X_2 in influencing the net utility of voting, and all interaction between X_1 and X_2 is expected to be due to compression, a product term is not needed. In either case, the logit coefficients do not directly provide information about interaction between X_1 and X_2 in influencing the *probability* of voting; one must use the model's coefficients to compute estimated effects on this probability. These estimated effects would incorporate not only any interaction between X_1 and X_2 in influencing the net utility of voting, as reflected in a nonzero coefficient for X_1X_2, but whatever compression is present in the DGP as well.

An Illustration of an Appropriate Test for Interaction

To illustrate our recommendations, we examine the relationship among education, voter registration provisions, and the probability of voting in a presidential election [to be denoted Pr(*vote*)] initially investigated by Wolfinger and Rosenstone (1980), and later analyzed by Nagler (1991, 1994) and Huang and Shields (2000); we consider the hypothesis that education and voter registration provisions interact in influencing the probability of voting, such that "[l]iberalizing registration provisions would have by far the greatest impact on the least educated and relatively little effect on well-educated people" (Wolfinger and Rosenstone 1980, 79). We employ 1984 Current Population Survey data ($n = 99{,}676$) from a replication data set for Nagler's (1994) study provided by Altman and McDonald (2003). The operational definition of voter registration provisions is *closing date*, the number of days before an election at which registration is closed.[31] The variable *education* measures number of years of schooling collapsed into eight categories, coded 1 (0–4 years), 2 (5–7 years), 3 (8 years), 4 (9–11 years), 5 (12 years), 6 (1–3 years of college), 7 (4 years of college), and 8 (5 or more years of college). The control variables are a respondent's *age*, whether the respondent lives in the *south*, and whether a *gubernatorial election* occurred in the respondent's state during the presidential election year.

Wolfinger and Rosenstone (1980) test the proposition that *education* and *closing date* interact in influencing Pr(*vote*) using a probit model without product terms; Nagler (1991) tests the hypothesis with a model including two product terms: $closing\ date * education$ and $closing\ date * education^2$. Which specification is correct hinges on the theory underlying the empirical analysis: only a theory about the effects of these independent variables on a latent unbounded variable, Y^*, for which the BDV is assumed to be an indicator (such as the utility of voting) can reasonably guide the choice about whether to include product terms.

Although they do not explicitly offer these theories, the specifications chosen by Nagler and Wolfinger and Rosenstone are consistent with different utility maximization theories about the influence of education and voter

[29]A Stata program designed by Brambor, Clark, and Golder to construct marginal effect plots for logit and probit models including a product term (available at http: //homepages.nyu.edu/%7emrg217/interaction.html) can easily be modified for use with models without a product term.

[30]In McFadden's (1974) original random utility maximization multinomial choice model, ε is a random component of the utility for each choice and is assumed to take a Type I extreme value distribution. While this assumption is necessary in the multinomial context, in the binary context it is also acceptable to assume that ε is a random component over the comparison of the two choices and takes a simpler logistic form (Maddala 1983, 22).

[31]Wolfinger and Rosenstone originally included additional operationalizations of voter registration provisions that are excluded in the Nagler data, and therefore excluded from our analysis.

TABLE 1 Probit Models of 1984 Presidential Voting Turnout

	Dependent Variable: Pr(*vote*)	
Independent Variable	**(1) Model with Product Terms (i.e., Nagler Specification)**	**(2) Model without Product Terms (i.e., Wolfinger & Rosenstone Specification)**
closing date	0.0006	−0.0078*
	(0.0037)	(0.0004)
education	0.2645*	0.1819*
	(0.0416)	(0.0144)
education^2	0.0051	0.0123*
	(0.0042)	(0.0014)
age	0.0697*	0.0697*
	(0.0013)	(0.0013)
age^2	−0.0005*	−0.0005*
	(0.0000)	(0.0000)
south	−0.1155*	−0.1159*
	(0.0110)	(0.0110)
gubernatorial election	0.0034	0.0034
	(0.0116)	(0.0116)
closing date ∗ *education*	−0.0032*	
	(0.0015)	
closing date ∗ *education*^{2}	0.00028	
	(0.00015)	
constant	−2.7431*	−2.5230*
	(0.1074)	(0.0486)
N	99,676	99,676
Log-likelihood	−55815.28	−55818.03

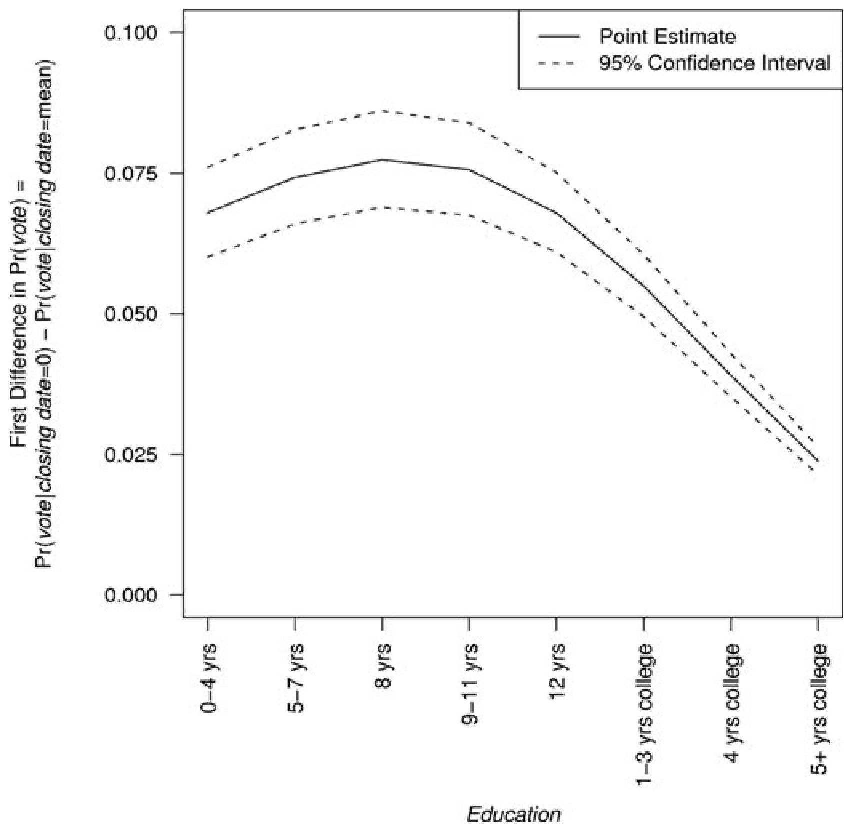
Standard errors in parentheses.
* $p \leq .05$.

registration provisions on the utility of voting. Wolfinger and Rosenstone's specification is consistent with a theory that the independent variables in the model have additive effects on the (unbounded) utility of voting, and thus that any interaction between education and voter registration provisions in influencing the (bounded) probability of voting results from compression. In this explanation, people at all levels of education derive the same utility from a relaxation of voter registration restrictions, but the *behavior* of less-educated people—whether they vote—is more likely to be affected by such a relaxation because these are the individuals who are most likely to be nearly indifferent between voting and not voting. The utility highly educated people derive from voting is already so likely to be above the threshold necessary to justify voting that additional benefit from easier voter registration will probably not make these people appreciably more likely to vote. This theory predicts that the coefficients for the product terms involving *education* and *closing date* should be zero. In contrast, Nagler's specification is consistent with a theory that the independent variables interact in their influence on the latent utility of voting: the magnitude of the effect of voter registration provisions on an individual's utility of voting declines as her level of education rises.[32] This theory predicts nonzero coefficients for the product terms.

Table 1 presents parameter estimates from the probit model containing *closing date* ∗ *education* and *closing date* ∗ *education*^{2} (reflecting Nagler's specification) in column 1, and from the model excluding these two product terms (reflecting Wolfinger and Rosenstone's specification) in column 2. A likelihood-ratio test comparing the unrestricted model (containing product terms) to the restricted model (omitting them) shows that the two product terms fail to be jointly significant at the .05 level ($p = .06$). When we drop *closing date* ∗ *education*^{2}

[32] Nagler's inclusion of both *closing date* ∗ *education* and *closing date* ∗ *education*^{2} allows the relationship between *education* and the strength of the marginal effect of *closing date* on the utility of voting to be nonlinear.

FIGURE 4 First Difference Measuring the Effect of *Closing Date* on Pr(*Vote*) at All Levels of *Education*



Results are generated using CLARIFY based on the probit model in column 2 of Table 1. All first differences assume that all independent variables except *education* and *closing date* are fixed at central values.

from the model—thereby constraining the relationship between *education* and the magnitude of the effect of *closing date* on the utility of voting to be linear—we get a similar result: the coefficient for *closing date* ∗ *education* is statistically insignificant ($p = .15$).[33] Thus, there is no empirical evidence that education and voter registration provisions interact in influencing the latent utility of voting. On this basis, we reject the theory predicting such interaction. However, the data are consistent with Wolfinger and Rosenstone's specification assuming that *closing date* and *education* have additive effects on the utility of voting, and consequently that any interaction between these variables in influencing the probability of voting is due to compression. This specification yields the coefficients in column 2. Therefore, to test Wolfinger and Rosenstone's hypothesis that voter registration provisions have their strongest impact on the probability of voting among the least educated Americans, we use the coefficients for the model in column 2 to derive the predicted probabilities of voting reported below.

We test the Wolfinger and Rosenstone hypothesis by analyzing second differences in Pr(*vote*) using CLARIFY. We begin by calculating *at each level of education* a first difference: the estimated effect on the probability of voting of relaxing the stringency of voter registration provisions (i.e., *closing date*) from its mean (registration closes 24.8 days before the election) to 0 (registration is allowed right up to the day of the election) when all other variables are fixed at central values (dichotomous variables at their mode, all others at their mean).[34] For someone with 0–4 years of schooling (*education* = 1), this relaxation in voter registration provisions increases the probability of voting by 0.068, with a 95% confidence interval of [0.060, 0.076]. Figure 4 presents a graph

[33] Contrary to expectation, the point estimates for the product terms in both models imply that the magnitude of the effect of *closing date* on the utility of voting increases as *education* rises.

[34] We focus on this increment in *closing date* because it represents the change that a state with typical voter registration provisions would experience if it were to adopt the policy innovation of same-day registration.

showing this first difference in Pr(*vote*) at all eight levels of education.

The figure shows that the effect of voter registration provisions on the probability of voting is maximized at eight years of schooling (i.e., *education* = 3). As education increases from this level, the effect of *closing date* on Pr(*vote*) declines substantially. Specifically, an increase in education from eight years to its maximum yields a decline in the first difference measuring the effect of *closing date* on the probability of voting of 0.054 (= 0.077 – 0.024). This second difference has a 95% confidence interval of [0.047, 0.060]; since this interval excludes zero, the difference is statistically significant at the .05 level. As education decreases from eight years, the impact of *closing date* on the probability of voting also declines, but at a much lower rate. For example, a decrease in education from eight years to 0–4 years prompts a decline in the first difference measuring the effect of *closing date* on Pr(*vote*) of only 0.009 (= 0.077 – 0.068). This second difference is also statistically significant at the .05 level (with a 95% confidence interval of [0.008, 0.011]), but its magnitude is small. Indeed, 9–11 years of schooling (*education* = 4) seems to be a critical threshold: above this point, as education rises there is a steady and substantial decline in the impact of *closing date* on the probability of voting; below this point, the consequences of lowering *closing date* from its mean to zero is nearly constant, always increasing the probability of voting by an amount between 0.068 and 0.077.

In summation, although there is no evidence of interaction between education and voter registration provisions in influencing the latent utility of voting, there is evidence of substantial interaction between these variables in influencing the *probability* of voting. When all other independent variables are fixed at central values, reducing the number of days before an election at which registration is closed from its mean to zero results in an increase in the probability of voting at any level of education. But the magnitude of the effect of voting registration provisions on this probability increases substantially as education declines from its highest level to eight years of schooling. In contrast, the effect of voting registration provisions on the probability of voting is nearly the same at eight years of education as it is at all lower levels. Since there are no product terms in the probit model used to derive these results, this interaction between education and voting registration provisions in influencing the probability of voting is due strictly to compression: when other independent variables are at central values, highly educated persons derive sufficient utility from voting to give them a high probability of turning out to vote even when voter registration provisions are fairly restrictive.

Conclusion

We have argued that one can glean no definitive information about the nature of interaction among independent variables in influencing $\Pr(Y)$ from the sign and magnitude of a product term coefficient in a binary logit or probit model. This is because (1) there can be substantial interaction among independent variables in influencing $\Pr(Y)$ even when the coefficient for all product terms is zero, (2) there can be little interaction among independent variables in influencing $\Pr(Y)$ even when product term coefficients are large, and (3) when there is both strong interaction between two independent variables in influencing $\Pr(Y)$ and a statistically significant product term, the direction of their interaction in influencing $\Pr(Y)$ can be opposite from the direction of their interaction in influencing the unbounded latent variable Y^* (as indicated by the sign of the product term coefficient). Testing the statistical significance of the product term is necessary to confirm a hypothesis that independent variables interact in influencing the unbounded latent dependent variable. But this test does not shed light on the nature of the interaction between the variables in influencing $\Pr(Y)$. Whether the variables interact in influencing $\Pr(Y)$ should be tested by direct examination of estimated effects on $\Pr(Y)$.

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