Summary of IEEE Standard 1459: Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced, or Unbalanced Conditions

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***Abstract*—This paper describes the new IEEE Standard 1459, Definitions for the Measurement of Electric Power Quantities Under Sinusoidal, Nonsinusoidal, Balanced or Unbalanced Conditions. The information is presented in the context of historical events that explain the reasons for new definitions. The new definitions originate from S_e, the effective apparent power definition attributed to F. Buchholz and W. Goodhue. The resolution of S_e extends from well-established concepts. The need for the separation of 60/50-Hz powers from the non-60/50 Hz-powers is emphasized. The standard serves users who want to evaluate the performance of modern equipment or to design and build the new generation of instrumentation.**

***Index Terms*—Apparent power, power definitions, power factor, power quality.**

I. HISTORICAL BACKGROUND

IT WAS the year 1886 and the American Institute of Electrical Engineers (AIEE) was about 400 strong. Most of the members were listed as Electricians. W. Stanley had build the first ac line at Great Barrington, MA. One year later, T. A. Edison started the infamous campaign against ac applications and a few months later N. Tesla patented the polyphase ac motor. Overnight the alternating current became a primary topic at AIEE meetings. The best and the brightest "Electricians" of the time were struggling to grasp the physical meaning of the phase shift between the current and voltage. The breakthrough came again from W. Stanley, who, in 1887, was also the first to present a paper titled "The Phenomenon of Retardation in Induction Coil" [1]. In 1888, O. B. Shallenberger published a brief paper that explained the flow of instantaneous power and the oscillations of power caused by the energy exchange between inductance and source. The next significant step came in 1893 when A. E. Kennelly, followed by C. Steinmetz, started to apply the complex numbers theory to model impedances and to define current and voltage phasors.

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At least 15 more years had to pass before the rank and file engineers of that time started to understand that ac circuits act differently than dc circuits and to accept the concept of apparent and reactive power. The incentive for this approach stemmed from the engineering economics, the need for a figure of merit, namely, the power factor that helps quantify ac lines utilization. Meanwhile, the proliferation of thee-phase systems was challenging the early years engineers with a new dilemma; as early as 1910 [3] engineers were debating the merits of two types of apparent power definitions: the vector VA and the arithmetic VA. These two definitions yield equal results only when the system is balanced, otherwise, the arithmetic VA is larger than the vector VA and their respective power factors differ. The balanced and symmetrical systems were easily understood, since their behavior was similar to single-phase systems, the unbalanced systems, however, were casting doubt on the correctness of apparent power definition and its consequent effect on the power factor value. In 1920, C. L. Fortescue and other luminaries of that time tried unsuccessfully to solve the puzzle and to decide on the right definition. The records [4] are fascinating to read; a remarkable observation was made then by W. V. Lyon [5], who probably was the first to understand the true meaning of the power factor. He wrote the following.

> The power factor is thus the ratio of the actual power to the greatest possible power that would be absorbed by any load taking the same r.m.s. line currents and the same r.m.s. voltages.

Unfortunately, in 1920 the pillars of our profession were charmed by the elegance of Fortescue's symmetrical components theory, that was fitting hand-in-glove with the vector apparent power model and Lyon's definition was left unexploited and forgotten.

At about the same time mercury rectifiers proliferation to transportation and electrochemical processes starts to steer a special interest for nonsinusoidal conditions. In 1927, C. I. Budeanu [6] described the first model of powers in single-phase systems with distorted waveforms. He proposed a three-dimensional resolution of the apparent power S, with the components P active power, Q reactive power, and D distortion power. The dissemination of Budeanu's theory lead to more soul searching and confusion and to the need for a unified theory that will address all possible conditions in ac networks. In 1933, A. E. Knowlton chaired the famous AIEE Schenectady meeting that turned into one of the most heated debates in the history of AIEE [7]. The discussions were fueled by a set of papers presented by the AIEE elite: C. L. Fortescue, V. G. Smith, W.V. Lyon, and W. H. Pratt. There are 21 recorded discussions. A most noteworthy comment was made by V. Karapetoff.

> Any definition of power factor that cannot be realized with fairly simple practical measuring instruments will remain a dead letter; on the other hand, a definition that may not be quite rigorous theoretically may prove to be of great practical usefulness if the corresponding measurements are simple and can readily be understood by the average operating engineer.

The confusion at the meeting is epitomized by the remarks of a young engineer, P. L. Alger, later to become a renowned motor designer, that advocated the negative sign for Q when the load is inductive. In spite of the criticism raised then by Lyon against Budeanu's theory, H. Curtis and F. Silsbee advanced Budeanu's method to three-phase systems [8]. Their resolution of S was ultimately included in the first *American Standard Definitions of Electrical Terms* [9], issued in 1941. Curtis-Silsbee's definitions, found in the 1941 edition, remained practically unchanged over the years and are found in the latest edition of the IEEE Std 100. The definitions pivot around the arithmetic VA (arithmetic apparent power)

$$S_A = |\bar{S}_a| + |\bar{S}_b| + |\bar{S}_c| \tag{1}$$

and the vector VA (vector apparent power)

$$S_V = |\bar{S}_a + \bar{S}_b + \bar{S}_c| = \sqrt{P^2 + Q^2 + D^2} \tag{2}$$

where

$$\bar{S}_a = P_a + jQ_a + kD_a \quad |\bar{S}_a| = V_a I_a \quad V_a^2 = \sum_h V_{ah}^2$$
$$\bar{S}_b = P_b + jQ_b + kD_b \quad |\bar{S}_b| = V_b I_b \quad I_a^2 = \sum_h I_{ah}^2$$
$$\bar{S}_c = P_c + jQ_c + kD_c \quad |\bar{S}_c| = V_c I_c \quad V_b^2 = \sum_h V_{bh}^2$$

and

$$P_i = \sum_h V_{ih} I_{ih} \cos(\theta_{ih}) \quad P = P_a + P_b + P_c, \qquad i = a, b, c$$
$$Q_i = \sum_h V_{ih} I_{ih} \sin(\theta_{ih}) \quad Q = Q_a + Q_b + Q_c$$
$$D_i = \sqrt{S_i^2 - P_i^2 - Q_i^2} \quad D = D_a + D_b + D_c.$$

θ_{ith} is the phase angle between the voltage harmonic phasor V_{ih} and the current harmonic phasor I_{ih} (the subscript i marks the phases a, b, c and h is the harmonic order). The three power axes P, Q, and D have the versors 1, j, and k that are perpendicular to each other.

From (1) and (2) it results that $S_V \leq S_A$ and $PF_V = P/S_V \geq PF_A = P/S_V$. As will be proven in Sections II–VII, both power factors PF_V and PV_A are incorrect when the waves are nonsinusoidal or the load is imbalanced.

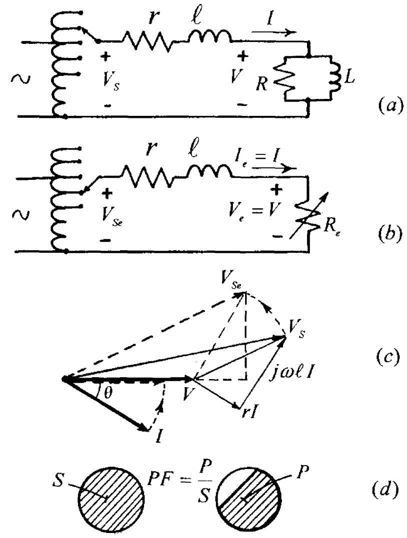

Fig. 1. PF definition. (a) Single-phase system with $PF < 1$. (b) Optimized system with $PF = 1$. (c) Phasors diagram. (d) Conductor utilization.

II. NEED FOR MODIFICATIONS

The proliferation of power solid-state switching devices used in a wide range of applications makes Budeanu's vision reality today; there are indeed many sites where the current waveforms are drastically distorted. Moreover, often the instrumentation design is based on methods developed for sinusoidal waves and their testing or calibration is done with sinusoidal 60/50 Hz. Such W, VA, and varmeters are prone to significant errors when operated in nonsinusoidal situations.

Advancements in circuit analysis and electromagnetic field theory in the last decades led to a better understanding of energy flow mechanism and a multitude of new models for the resolution of S. Progress in signal processing led to new, more effective and more robust algorithms. Developments in computer architecture made available tools for real-time data acquisition and for processing and storing huge amounts of information. Today, we have the means to measure almost any defined quantity.

All the same, we should not forget that electrical industries have a long and successful tradition, and utilities have invested billions of dollars in the metering infrastructure. Any modifications must be an extension of the existing theory and practice; we must follow Karapetoff's advise: the improved definitions must be of practical usefulness and readily understood by the average engineer.

III. APPARENT POWER AND POWER FACTOR

The IEEE Std 1459 presents a set of new definitions that rely on the power factor as defined by W. V. Lyon. In Fig. 1 is sketched a single-phase circuit that helps understand Lyon's concept. The R, L load is supplied from a substation via a line with $r + j\omega\ell$ impedance. The load active power is $P = VI\cos\theta$ and the line power loss is $\Delta P = rI^2$. Now, if the inductance L is compensated by connecting in parallel a capacitance $C = 1/\omega^2 L$ and the load resistance as well as the substation voltage V_S are adjusted such that $R = R_e = V/I$ and $V_{Se} = \sqrt{(V + R_e I)^2 + (\omega\ell I)^2}$, then the transfer of energy is optimized and the greatest possible power absorbed by the load is obtained while both, the load voltage and the line current, $I_e = I$, remain unchanged. This greatest power possible is the apparent power $S = VI$. It is possible to go one step further and give Lyon's definition a better physical meaning based on the fact that the power loss in the actual system equals the power loss in the optimized system. This leads to the following crucial definitions.

1) *Apparent power is the maximum power transmitted to the load (or delivered by a source) while keeping the same line losses and the same load (or source) voltage and current.*
2) *Power factor is the ratio of the actual power to the maximum power that could be transmitted while keeping the line power loss and the load voltage constant.* (Here, the word *power* can also be replaced with *energy transmitted during a given time interval*).

In Fig. 1(d) is depicted a simple geometrical interpretation of the above definitions: S is assumed proportional with the total cross-sectional area of the conductor, while the active power P covers only a part of the total cross sectional area.

For perfectly sinusoidal and balanced three-phase systems the definitions of S_V or S_A comply well with the above rules. For three-phase four-wire with balanced but nonsinusoidal conditions and for all types of imbalanced systems the available definitions listed in IEEE Std 100 do not comply.

IV. Where the Old Definitions are Lacking

A. Basic Expression $S = 3V_{LN}I = \sqrt{3}V_{LL}I$ is not Always True

Let us assume a perfectly balanced but nonsinusoidal voltage

$$\begin{aligned} v_a &= \sqrt{2}V_1\sin(\omega t) + \sqrt{2}V_3\sin(3\omega t + \alpha_1) \\ &\quad + \sqrt{2}V_5\sin(5\omega t\alpha_5) \\ v_b &= \sqrt{2}V_1\sin(\omega t - 120^\circ) + \sqrt{2}V_3\sin(3\omega t + \alpha_1) \\ &\quad + \sqrt{2}V_5\sin(5\omega t + \alpha_5 + 120^\circ) \\ v_c &= \sqrt{2}V_1\sin(\omega t + 120^\circ) + \sqrt{2}V_3\sin(3\omega t + \alpha_1) \\ &\quad + \sqrt{2}V_5\sin(5\omega t + \alpha_5 - 120^\circ) \end{aligned}$$

where v_a, v_b, and v_c are line-to-neutral voltages. From here, we find the line-to-line voltages

$$\begin{aligned} v_{ab} &= \sqrt{3}\sqrt{2}V_1\sin(\omega t + 30^\circ) + \sqrt{3}\sqrt{2}V_5\sin(5\omega t - 30^\circ) \\ v_{bc} &= \sqrt{3}\sqrt{2}V_1\sin(\omega t - 90^\circ) + \sqrt{3}\sqrt{2}V_5\sin(5\omega t + 90^\circ) \\ v_{ca} &= \sqrt{3}\sqrt{2}V_1\sin(\omega t + 150^\circ) + \sqrt{3}\sqrt{2}V_5\sin(5\omega t - 150^\circ). \end{aligned}$$

If we compare the rms line-to-neutral voltage $V_{LN} = \sqrt{V_1^2 + V_3^2 + V_5^2}$, with the rms line-to-line voltage $V_{LL} = \sqrt{3}\sqrt{V_1^2 + V_5^2}$, we find that $V_{LL} < \sqrt{3}V_{LN}$. This means that for balanced nonsinusoidal systems that contain zero-sequence harmonics

$$S = 3V_{LN}I > \sqrt{3}V_{LL}I.$$

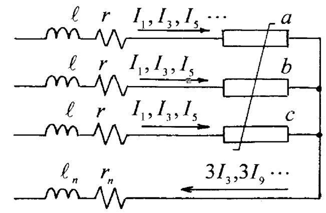

Fig. 2. Balanced three-phase four-wire system with nonlinear load.

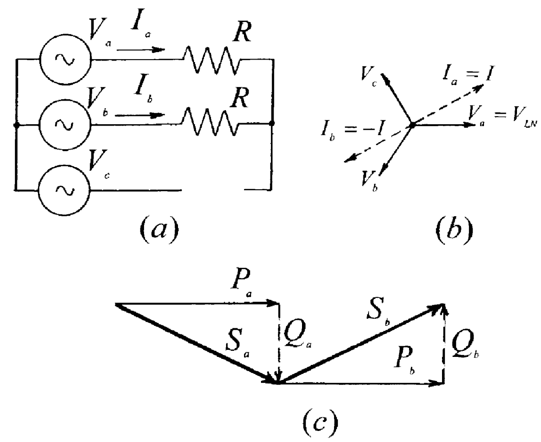

Fig. 3. Unbalanced three-phase load. (a) Circuit. (b) Phasors. (c) Power triangles.

B. Neutral Current Path Power Loss is Ignored

In Section III, we learned that the apparent power value corresponds to a certain line power loss. Mathematically, for a single-phase system (Fig. 1) this condition is reflected in the basic expression

$$\Delta P = r\left(\frac{S}{V}\right)^2. \tag{3}$$

In Fig. 2 is shown a three-phase four-wire system with nonlinear but balanced load. The line rms currents are

$$I_a = I_b = I_c = I = \sqrt{I_1^2 + I_3^2 + I_5^2 + \cdots}$$

and assuming pure zero-sequence triplen harmonics it results that the rms neutral current is

$$I_n = 3\sqrt{I_3^2 + I_9^2 + \cdots}.$$

The actual line power loss is

$$\Delta P = 3rI^2 + r_nI_n^2. \tag{4}$$

In this case, $S_A = S_V = 3V_{LN}I$, however, an expression that ties S_V with ΔP similar to (3) is not possible, i.e.,

$$\Delta P > \left(\frac{r}{3}\right)\left(\frac{S_V}{V_{LN}}\right)^2.$$

C. Meaningless Results When the Load Is Unbalanced

1) First example, Fig. 3: The active power is

$$P = P_a + P_b = 2V_{LN}I\cos(30^\circ) = \frac{3V_{LN}^2}{2R}$$

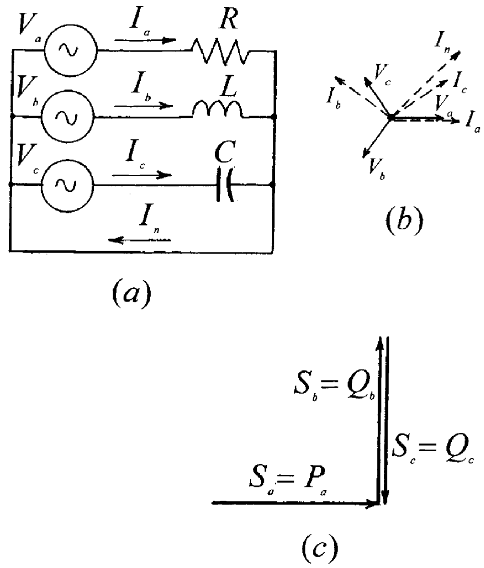

Fig. 4. Unbalanced three-phase load. (a) Circuit. (b) Phasors. (c) Power triangles.

and the reactive power is

$$Q = Q_a + Q_b = V_{LN} I[\sin(30°) - \sin(30°)] = 0.$$

Power triangles [Fig. 3(c)] lead readily to the arithmetic apparent power

$$S_A = S_a + S_b = 2V_{LN} I = \frac{\sqrt{3}V_{LN}^2}{R}$$

the vector apparent power

$$S_V = P_a + P_b = P$$

and to puzzling results for the power factors

$$PF_A = \frac{P}{S_A} = \frac{3}{2\sqrt{3}} = 0.866 \quad PF_V = \frac{P}{S_V} = 1.0.$$

2) Second example, Fig. 4: Here, the three impedances have equal absolute values, $R = \omega L = 1/\omega C$. The power triangles have the powers

$$P_a = \frac{V_{LN}^2}{R} \quad P_b = P_c = 0 \quad P = P_a$$
$$Q_a = 0 \quad Q_b = -Q_c = \frac{V_{LN}^2}{R}$$
$$\bar{S}_a = P_a \quad \bar{S}_b = -\bar{S}_c = jP_a$$

with the total apparent powers

$$S_A = 3P_a \text{ and } S_V = P_a$$

and the respective power factors

$$PF_A = 0.333 \quad PF_V = 1.0.$$

Since both circuits are far from the optimum energy transfer condition the unity power factor cannot be the right answer. Also, the PF_A values are incorrect (to be explained in the Section V).

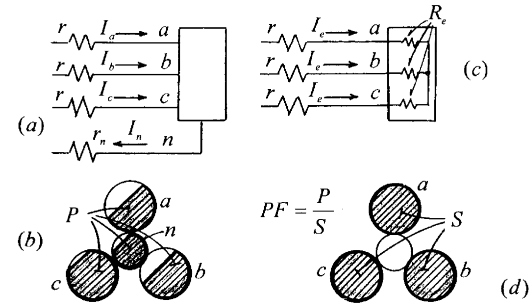

Fig. 5. PF definition for three-phase circuits. (a) System with imbalanced load. (b) Conductor utilization. (c) Optimized system with $PF = 1$. (d) Conductor utilization in the optimized system.

V. STD 1459 Approach: The Effective Apparent Power

The general case of a three-phase system is sketched in Fig. 5(a), where an imbalanced load is fed by a four-wire system. The incomplete conductors utilization is symbolically depicted in Fig. 5(b). The line power loss is

$$\Delta P = 3r\left(I_a^2 + I_b^2 + I_c^2\right) + r_n I_n^2. \tag{5}$$

The optimized system with unity power factor [Fig. 5(c)] must dissipate exactly the same power in the line, however, the load consists of three equal resistances, hence, the three line currents are identical, $I_a = I_b = I_c = I_e$, $I_n = 0$, and

$$\Delta P = 3rI_e^2. \tag{6}$$

From (5) and (6) results the key expression for the *effective, or equivalent, current*

$$I_e = \sqrt{\frac{1}{3}\left(I_a^2 + I_b^2 + I_c^2 + \rho I_n^2\right)} \quad \rho = \frac{r_n}{r}. \tag{7}$$

In typical four-wire low- and medium-voltage installations, $\rho = 0.2$–4. Std 1459 assumes $\rho = 1$. Modern instruments have the capability to adjust ρ to an agreed upon value.

A similar approach is used to find the *effective voltage* V_e. The load is represented by three equivalent equal resistances R_Y, connected in Y and three resistances R_Δ connected in Δ. The respective powers are

$$P_Y = \frac{3V_e^2}{R_Y}$$
$$P_\Delta = \frac{9V_e^2}{R_\Delta}.$$

Using the ratio

$$\xi = \frac{P_\Delta}{P_Y} = \frac{3R_Y}{R_\Delta}$$

the power equality between the actual load and the optimized system is

$$\frac{V_a^2 + V_b^2 + V_c^2}{R_Y} + \frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{3R_Y}\xi = 3\frac{V_e^2}{R_Y} + 3\frac{(\sqrt{3}V_e)^2}{3R_Y\xi}$$

yielding

$$V_e = \sqrt{\frac{3\left(V_a^2 + V_b^2 + V_c^2\right) + \xi\left(V_{ab}^2 + V_{bc}^2 + V_{ca}^2\right)}{9(1+\xi)}}. \tag{8}$$

The Standard 1459 uses $\xi = 1$. For three-wire lines the standard recommends the simplified expressions

$$V_e = \sqrt{\frac{V_{ab}^2 + V_{bc}^2 + V_{ca}^2}{9}}$$
$$I_e = \sqrt{\frac{I_a^2 + I_b^2 + I_c^2}{3}}. \tag{9}$$

The *effective apparent power*

$$S_e = 3V_e I_e \tag{10}$$

correctly reflects the power loss in the neutral current path as well as the effect of imbalance [Fig. 5(c) versus Fig. 5(b)]. This is the maximum active power that can be transmitted through the given three-phase line to a perfectly balanced three-phase load, supplied with the effective voltage V_e, while keeping the line losses unchanged.

Expressions (9) and (10) were suggested in 1922 by F. Buchholz [11] and explained in 1933 by W. M. Goodhue [12], one of the participants at the AIEE Schenectady meeting.

Now, we can apply the concept of S_e to the previous two examples. From Fig. 3 (first example), we compute the effective current

$$I_e = \sqrt{\frac{1}{3}(I^2 + I^2)} = \sqrt{\frac{2}{3}}I = \frac{V_{LN}}{\sqrt{2}R}$$

and assuming $V_e \approx V_{LN}$ results

$$S_e = \frac{3V_{LN}^2}{\sqrt{2}R} \text{ and } PF = \frac{P}{S_e} = \frac{\sqrt{2}}{2} = 0.707.$$

From Fig. 4 (second example), we find the rms neutral current $I_n = \sqrt{2}I = \sqrt{2}V/R$ and the effective current

$$I_e = \sqrt{\frac{1}{3}(I^2 + I^2 + I^2 + 2I^2)} = \sqrt{\frac{5}{3}}\frac{V_{LN}}{R}$$

and assuming again $V_e \approx V_{\mathrm{IN}}$ it results that

$$S_e = 3\sqrt{\frac{5}{3}}\frac{V_{LN}^2}{R} \text{ and } PF = \frac{P}{S_e} = \frac{1}{\sqrt{15}} = 0.258.$$

These are results that conform to Lyon's concept of apparent power and power factor. We observe that for both examples $PF < PF_V < PF_A$, meaning that the values S_V and S_A are not correct.

VI. Effective Apparent Power Resolution

The electric energy generated by modern power plants is almost harmonics free and void of negative- and zero-sequence voltage and currents. At the user's site, induction and synchronous motors develop useful, dominant torques that are the result of interaction between rotor currents and the positive-sequence rotating magnetic flux. Voltage distortion and imbalance cause only parasitic torques and additional losses. Customers expect to be supplied with 60/50-Hz sinusoidal voltages, void of negative and zero sequence. It seems perfectly logical to separate the 60/50-Hz powers from the rest. This separation is conveniently obtained [10], if we split the effective current and voltage into a fundamental effective component and a harmonic effective component, i.e.,

$$I_e^2 = I_{e1}^2 + I_{eH}^2 \text{ and } V_e^2 = V_{e1}^2 + V_{eH}^2 \tag{11}$$

where the subscript 1 means fundamental (60 or 50 Hz) and the subscript H the inclusion of all the harmonics, thus,

$$\begin{aligned}
I_{e1}^2 &= \frac{1}{3}\left(I_{a1}^2 + I_{b1}^2 + I_{c1}^2 + \rho_1 I_{n1}^2\right)\\
I_{eH}^2 &= I_e^2 - I_{e1}^2\\
V_{e1}^2 &= \frac{1}{18}\left[3\left(V_{a1}^2 + V_{b1}^2 + V_{c1}^2\right) + V_{ab1}^2 + V_{bc1}^2 + V_{ca1}^2\right]\\
V_{eH}^2 &= V_e^2 - V_1^2 \quad \xi = 1.
\end{aligned}$$

From (11), it results that S_e has four terms

$$S_e^2 = (3V_{e1}I_{e1})^2 + (3V_{e1}I_{eH})^2 + (3V_{eH}I_{e1})^2 + (3V_{eH}I_{eH})^2.$$

The first term is the *fundamental effective apparent power*

$$S_{e1} = 3V_{e1}I_{e1}.$$

In turn, this power can be separated into *positive-sequence fundamental apparent power* S_1^+ and the remaining component S_{U1} attributed to system imbalance. S_1^+ is conveniently resolved into *active and reactive fundamental positive-sequence powers* $(S_1^+)^2 = \left(P_1^+\right) + (Q_1^+)^2$,

$$P_1^+ = 3V_1^+ I_1^+ \cos\theta_1^+ \quad Q_1^+ = 3V_1^+ I_1^+ \sin\theta_1^+.$$

The next three terms constitute the *nonfundamental (non-60 Hz) effective apparent power*

$$S_{eN} = \sqrt{S_e^2 - S_{e1}^2} = \sqrt{D_{eI}^2 + D_{eV}^2 + S_{eH}^2}$$

where

$$D_{eI} = 3V_{e1}I_{eH} = 3S_{e1}(THD_{eI})$$

is the *effective current distortion power* and $THD_{eI} = I_{eH}/I_{e1}$ is the equivalent total harmonic distortion of the current.

$$D_{eV} = 3V_{eH}I_{e1} = 3S_{e1}(THD_{eV})$$

is the *effective voltage distortion power* and $THD_{eV} = V_{eH}/V_{e1}$ is the equivalent total harmonic distortion of the voltage.

$$S_{eH} = 3V_{eH}I_{eH} = 3S_{e1}(THD_{eI})(THD_{eV})$$

is the *effective harmonic apparent power* that contains the *harmonic active power*

$$P_H = \sum_{\substack{h\neq 1\\ i=a,b,c}} V_{ih}I_{ih}\cos\theta_{ih} = P - P_1$$

and the remaining nonactive term

$$D_{eH} = \sqrt{S_{eH}^2 - P_H^2}.$$

The resolution of S_e is depicted in Fig. 6. This approach has the following advantages.

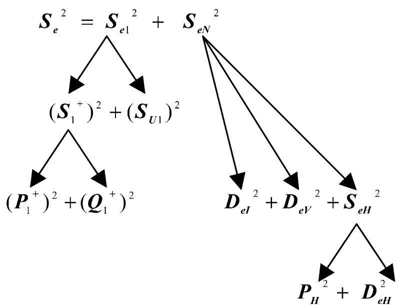

Fig. 6. Resolution of S_e according to STD 1459.

1) The "main product," P_1^+, is separated from the remaining active power components. Nonlinear loads convert a small part of P_1^+ in P_H, P_1^- and P_1^0, which are injected back into the power network and dissipated. Usually, P_H, P_1^-, $P_1^0 < 0$, and compared to P_1^+ have minute values, this making their measurement quite challenging. Typically, $P_H/P_1 < 0.02$.
2) The positive-sequence reactive power Q_1^+ is separated, too, thus helping estimate and judge the need for linear capacitor banks needed to correct the fundamental power factor, $\cos\theta_1^+$.
3) The nonfundamental apparent power S_{eN} is a most useful quantity that permits to evaluate at a glance the severity of distortion. It indicates if a particular load, or group of loads, operate under low, moderate, or excessive harmonic pollution conditions. S_{eN} helps determine the size of static, active or hybrid filters, or other types of power compensation equipment. This quantity may affect customer penalty and may serve to detect and protect equipment that is sinking harmonics and acts like a harmonic filter.
4) The effective apparent power S_e and its components stem directly from the classical sinusoidal definitions that were the norm for one century and are well understood.

VII. Comprehensive Example

The studied case is a three-phase load containing 90-kVA single-phase nonlinear loads and 45-kVA linear loads consisting of small motors and electric heaters. The load is supplied by a slightly asymmetrical three-phase, four-wire system, 480 V, 60 Hz [Fig. 7(a)]. Voltage and current oscillogrames are presented in Fig. 7(b) and (c). Line current waveforms are characterized by 60-Hz sinusoids (due to the linear loads) that carry the pulses caused by nonlinear loads. The line-to-neutral voltage wave shows a distinct distortion that can be traced to the third harmonic. The line-to-line voltages, not being affected by the third harmonic, seem undistorted. The neutral current is dominated by the third harmonic. Table I lists the rms voltages and currents at the mains a, b, c, and n [Fig. 7(a)].

The power loss dissipated by the three lines is

$$\Delta P_L = 3r\left(I_a^2 + I_b^2 + I_c\right) = 310.0\text{ W}.$$

The rms neutral current is larger than the line current; this fact combined with $\rho = r_n/r = 6/4.4 = 1.43$, leads to power loss

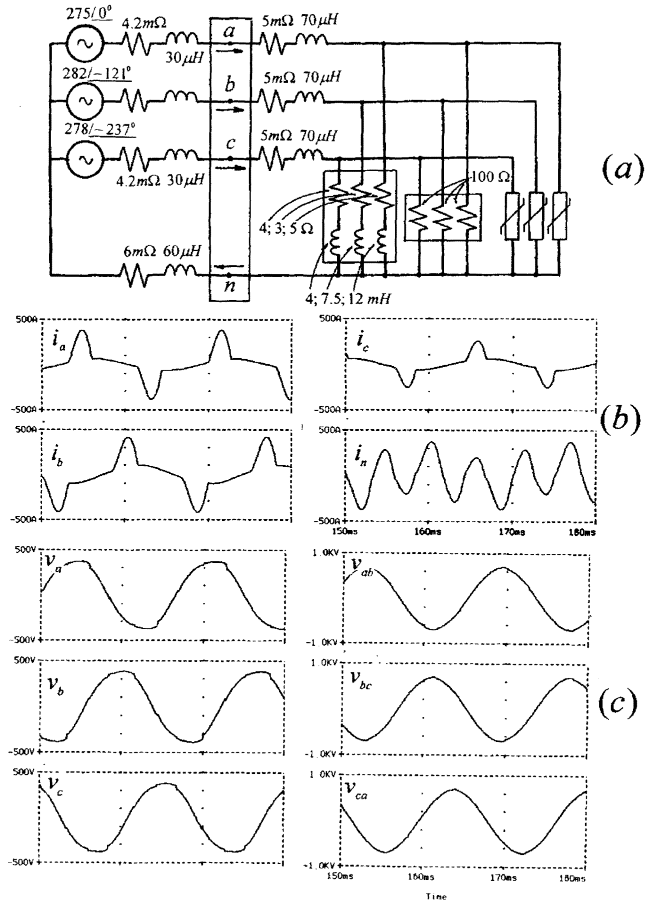

Fig. 7. Example. (a) Circuit diagram. (b) Lines and neutral current oscillograms. (c) Line-to-neutral and line-to-line voltages.

TABLE I
RMS Voltages (V) and Currents (A)

V_a	I_a	V_b	I_b	V_c	I_c	I_n
273.21	163.46	282.94	188.69	279.85	107.11	207.56

in the neutral current path, ΔP_n, comparable with the power loss in the three lines,

$$\Delta P_n = r_n I_n^2 = 258.5\text{ W}.$$

This result proves the claim that the neutral path losses must be reflected in the PF value via the definition of S.

Voltage and current phasors measured at the input terminals a, b, c, and n are summarized in Tables II and III.

The effective currents and voltages values are

$$\begin{aligned} I_e &= 203.25\text{ A} \quad I_{e1} = 137.17\text{ A} \quad I_{eH} = 149.99\text{ A} \\ V_e &= 278.24\text{ V} \quad V_{e1} = 277.82\text{ V} \quad V_{eH} = 15.22\text{ V} \end{aligned}$$

with the equivalent total harmonic distortions

$$THD_{Ie} = 1.093 \quad THD_{Ve} = 0.055.$$

The values of the normalized powers to the fundamental effective apparent power, $S_{e1} = 114.30$ kVA, are presented in Table IV. The results reveal the following characteristics of the load.

TABLE II
PERCENTAGE VOLTAGE HARMONIC PHASORS. BASE CURRENT: $V_{al} = 272.45$ V

h	V_{ah}	V_{bh}	V_{ch}
1	100/0°	103.65/−126°	102.28/123°
3	5.77/49°	5.79/50°	5.37/50°
5	1.59/−144°	1.14/−70°	0.49/170°
7	0.49/−37°	0.77/−129°	0.54/168°
9	1.02/96°	1.04/91°	0.99/109°
11	0.75/179°	0.52/−132°	0.49/174°
13	0.31/−124°	0.47/148°	0.41/154°
15	0.56/40°	0.57/33°	0.47/65°
17	0.57/133°	0.41/173°	0.47/140°
%THD	6.20	5.92	5.48

TABLE III
PERCENT CURRENT HARMONIC PHASORS. BASE CURRENT: $I_{al} = 133.92$ A

h	I_{ah}	I_{bh}	I_{ch}
1	100/−23.67°	123.12/−121.6°	68.48/98.0°
3	57.60/142°	57.50/145°	31.52/151°
5	35.80/−65°	33.89/60°	22.75/−169°
7	14.52/82°	12.25/−38°	12.46/−133°
9	3.17/161°	4.33/151°	4.33/−111°
11	5.12/−144°	4.91/21°	2.16/−165°
13	3.43/21°	2.37/−101°	2.85/−159°
15	1.21/97°	1.90/90°	1.85/−137°
17	2.00/−173°	1.79/−30°	0.74/−160°
%THD	69.74	55.45	60.03

TABLE IV
PERCENT POWERS. BASE VALUE:
$S_{e1} = 3V_{e1}U_{e1} = 3 \times 277.17 = 114.30$ kVA

$S_e = 148.40$	$S_{e1} = 100.00$ $S_1^+ = 94.89$	$S_{eN} = 109.71$
$P = 86.87$	$P_1 = 87.13$ $P_1^+ = 86.71$ $P_1^- = 0.117$ $P_1^0 = 0.309$	$P_H = -0.266$
$N = 109.64$	$Q_1^+ = 38.54$ $Q_1^- = -0.180$ $Q_1^0 = -0.420$	$D_{eI} = 109.34$ $D_{eV} = 5.47$ $D_{eH} = 5.98$
$PF_e = \frac{P}{S_e} = 0.585$ $PF_V = \frac{P}{S_V} = 0.776$	$PF_{e1} = \frac{P_1}{S_{e1}} = 0.871$ $PF_1^+ = \frac{P_1^+}{S_1^+} = 0.914$	
$S_V = 111.92$ $S_A = 112.01$	$Q_B = 35.04$ (Budeanu)	$D_B = 61.27$ (Budeanu)

Apparent powers: The highly distorted currents, as well as the presence of zero-sequence harmonics, cause $S_e > S_{e1} > S_1^+$. Moreover, the overall contribution of voltage and current harmonics yields a significant level of non-60-Hz apparent power, S_{eN}, $(S_{eN}/S_{e1} = 1.097)$.

Active powers: The fundamental active power P_1 is the dominant term. However, due to the fact that the harmonic power is injected from load ino the network $P_1 > P$, i.e., $P_H < 0$, $P_H = -258.32$ W. Since the supplying voltages are not symmetrical we find that negative and zero-sequence 60-Hz active powers are positive (dissipated in the linear loads).

Nonactive powers: All the nonactive powers are lumped in $N = \sqrt{S_e - P^2}$, however, the ability to differentiate between the 60-Hz reactive power and higher frequencies nonactive powers is lost in N.

The normalized fundamental positive-sequence reactive power $Q_1^+/S_{e1} = 0.385$ gives the size of the shunt capacitance $(0.385 \times 114.3 = 44.00$ kvar) that will help make the 60-Hz positive-sequence power factor equal to 1.0. If the reactive power is measured using Budeanu's definition, Q_B, given in IEEE Std 100, we find $Q_B < Q_1^+$. This result is caused by the fact that for $h \neq 1$ all the terms $V_h I_h \sin\theta_h < 0$. Moreover, Budeanu's distortion power D_B is significantly smaller than S_{eN} and D_{eI}, therefore, it is not a useful distortion indicator. The huge contribution of current harmonics to nonactive power is due to the $h\omega L$ effect; unless resonance conditions exist, the Thevenin impedance is highly inductive and the phase angles between the current and voltage harmonic phasors are near 90°. For comparison purposes, apparent power values calculated from the old definitions S_V, S_A are also included in Table IV.

Engineers familiar with IEEE Std 1459 point to the fact that S_{eN} cannot make the distinction between a case when harmonics flow into the load and the case when the flow is from the load into the power network. There is only one way to detect direction of the overall harmonic power: the measurement of P_H [10]. However, even this may not be sufficient since at the same bus some power harmonics may enter the load and others (of different orders) may exit the load. For detailed studies, the ultimate solution is the measurement of each voltage and current phasor harmonic. Due to $\theta_h \approx \pm 90°$, the accurate measurement of the phase angles is not a trivial task.

If a load, or a cluster of loads, is linear, then the total harmonic distortions of voltage and current at the mains must be comparable, $THD_{Ie} \approx THD_{Ve}$. If resonance conditions do occur, due to interaction of capacitors, transformers, and motor inductances [13], the monitoring of S_{eN} may warn the customer, or the utility, of the fact that harmonic currents are sunk into the customer equipment and significant useful equipment lifespan can be spared.

The definitions found today in IEEE Std 100 do not offer a better solution for this case. None of the components of S_V or S_A, except P_H, can help determine if a load pollutes or is polluted.

VIII. CONCLUSION

The new standard restores and emphasizes Buchholz–Goodhue effective apparent power S_e, the only definitions true to the physical meaning of apparent power in polyphase circuits. Another major feature of the standard is the recognition of the 60-Hz positive-sequence powers as essential components. The remaining non-60 –Hz powers are lumped in one component, S_{eN}, that helps measure the amount of kVA harmonics flowing through the monitored bus, as well as the relative degree of distortion, S_{eN}/S_{e1}. Measurements of S_{eN} can be used for warning customers or utilities, or for penalty purposes. Modern instrumentation computer systems can readily help measure the powers recommended in IEEE Std 1459.

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