GABARITO - JNBo

1. $\vec{v}(t) = A(e^{\frac{t}{T}} - 1)\hat{x} + B\left(\frac{t}{T}\right)^2 \hat{z}$

(a) $\vec{a} = \frac{d\vec{v}}{dt} = \frac{A}{T} e^{\frac{t}{T}} \hat{x} + 2\frac{B}{T}\left(\frac{t}{T}\right) \hat{z}$

(b) $\vec{r} = \vec{r}_0 + \int_0^t \vec{v}(t')\,dt'$

$$\vec{r}(t) = \vec{r}_0 + \left[AT(e^{\frac{t}{T}} - 1) - At\right]\hat{x} + \frac{BT}{3}\left(\frac{t}{T}\right)^3 \hat{z}$$

(c) $A = 2\,m/s \quad B = 0{,}3\,m/s \quad T = 1\,s$

$\vec{r}_0 = 2\hat{x} + \hat{y} \qquad t_1 = 2\,s$

$\vec{r}(2) = 2\hat{x} + \hat{y} + \left[2(e^2 - 1) - 4\right]\hat{x} +$
$0{,}1 \times 8\,\hat{z}$

$\vec{r}(t_1 = 2) = (2e^2 - 2 - 2)\hat{x} + \hat{y} + 0{,}8\,\hat{z}$

$\vec{r}(2) = 2e^2\hat{x} + \hat{y} + 0{,}8\,\hat{z} = 14{,}78\hat{x} + \hat{y} + 0{,}8\hat{z}$

(d) $\langle\vec{v}\rangle = \frac{\Delta\vec{r}}{\Delta t} = \frac{(2e^2 - 2)\hat{x}}{2} + \frac{0{,}8}{2}\hat{z}$

$\langle\vec{v}\rangle = (e^2 - 1)\hat{x} + 0{,}4\,\hat{z}$

$\vec{v}(0) = 0 \qquad \vec{v}(2) = 2(e^2 - 1)\hat{x} + 0{,}3 \times 4\,\hat{z}$

$\vec{v}(2) = 2(e^2 - 1)\hat{x} + 1{,}2\,\hat{z} = 12{,}78\,\hat{x} + 1{,}2\,\hat{z}$

$W_{0\to 2} = \Delta K_{02} = K_2 - K_0 = \frac{1}{2} m\, v^2(2) - 0$

$v^2(2) = 4\,(e^2-1)^2 + 1{,}2^2 = 26{,}752 \ m^2/s^2$

$\frac{1}{2} m\, v^2(2) = 0{,}01 \times 26{,}76 = 0{,}268 \ J$

$W_{0\to} = 0{,}268 \ J$

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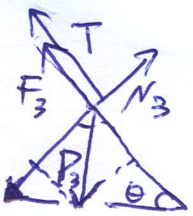
2. (a) $T = (M_1 + M_2)\, a$ (I)



$N_3 = P_3 \cos\theta = M_3 g \cos\theta$

$F_3 = \mu_c N_3 = M_3 g \mu_c \cos\theta$

$M_3 a = \begin{pmatrix} P_3 \,\text{sen}\,\theta - T \\ - M_3 g \mu_c \cos\theta \end{pmatrix} = \begin{pmatrix} M_3 g \,\text{sen}\,\theta - T \\ - M_3 g \mu_c \cos\theta \end{pmatrix}$ (II)

Subst. (I)

$M_3 a = M_3 g \,\text{sen}\,\theta - (M_1 + M_2)\, a - M_3 g \mu_c \cos\theta$

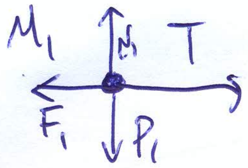
$(M_1 + M_2 + M_3)\, a = M_3 g\, (\text{sen}\,\theta - \mu_c \cos\theta)$

$a = \frac{M_3 g\, (\text{sen}\,\theta - \mu_c \cos\theta)}{(M_1 + M_2 + M_3)} = \frac{2 \times 10 \times \left(0{,}5 - 0{,}231 \frac{\sqrt{3}}{2}\right)}{(0{,}4 + 0{,}6 + 2)}$

$a = \frac{20 \times 0{,}3}{3} = 2 \ m/s^2$

$\cancel{T = 1a = 2 N}$

$T = (M_1 + M_2)\, a = 1a = 2 \ N$

(b) 

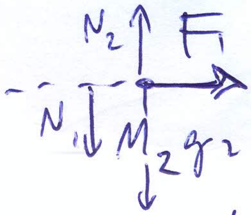
$$M_1 a = T - F_1 = T - \underset{\uparrow \text{máx}}{\mu_e} M_1 g$$

$$\mu_e M_1 g = T - M_1 a = 2 - 0,4 \times 2$$

$$\mu_e M_1 g = 2 - 0,8 = 1,2\,N$$

$$\mu_e = \frac{1,2}{4} = 0,3$$

Conferindo:



$$\mu_e M_1 g = M_2 a$$

$$\mu_e = \frac{M_2 a}{M_1 g} = \frac{0,6}{0,4} \times \frac{2}{10} = 0,3 \checkmark$$

(c) $W_{at} = -\mu_e M_3 g \cos\theta\, x_2 = -0,231 \times 2 \times \frac{\sqrt{3}}{2}$

$W_{at} = -0,4\,J$

$$E_0 = \frac{1}{2}(M_1 + M_2 + M_3) v_0^2 = \frac{3}{2} \times (2,45)^2 = 9,0\,J$$

$$E_2 = \frac{1}{2} k x_2^2 - M_3 g x_2 \sin\theta - W_{at}$$

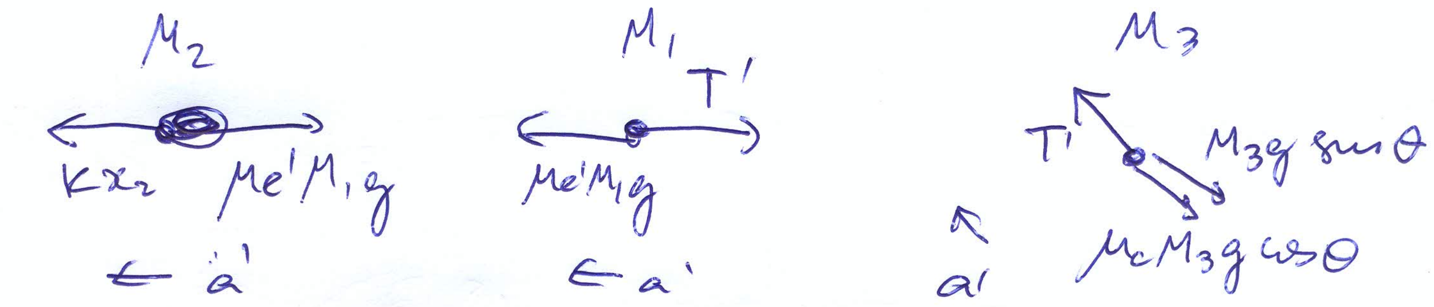
$$E_2 = E_0 = \frac{1}{2} k x_2^2 - \frac{20 \times 1,5}{2} + 0,4 = 9,0\,J$$

$$\frac{1}{2} k x_2^2 = 9,0 - 0,4 + 15 = 23,6 = 23,6\,J$$

$$k = \frac{47,2}{(1,5)^2} \qquad k = 20,98\,N/m$$

$$k \simeq 21\,N/m$$

(d) Em x_2 (quando $v=0$)



$$M_2 a' = K x_2 - \mu_e' M_1 g$$

$$M_1 a' = \mu_e' M_1 g - T'$$

pode ser + dependendo da hipótese

$$M_3 a' = T' - M_3 g(\sin\theta - \mu_c \cos\theta)$$

$$(M_1 + M_3) a' = M_1 g \mu_e' - M_3 g (\sin\theta - \mu_c \cos\theta)$$

$$a' = \frac{M_1 g}{M_1 + M_3} \mu_e' - \frac{M_3 g}{(M_1 + M_3)} (\sin\theta - \mu_c \cos\theta)$$

$M_2 a' =$

$$K x_2 - \mu_e' M_1 g = \frac{M_1 M_2 g}{M_1 + M_3} \mu_e' - \frac{M_2 M_3 g}{(M_1 + M_3)} (\sin\theta - \mu_c \cos\theta)$$

0,3

$$\left(\frac{M_1 M_2 g}{M_1 + M_3} + M_1 g \right) \mu_e' = \frac{M_2 M_3 g (\sin\theta - \mu_c \cos\theta)}{M_1 + M_3} + K x_2$$

$$5 \mu_e' = 1,5 + 31,5 \qquad \mu_e' = 6,6$$